Letter

Noise-induced universal diffusive transport in fermionic chains

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We develop a microscopic transport theory in a randomly driven fermionic model with and without a linear potential. The operator dynamics arise from the competition between noisy and static couplings, leading to diffusion regardless of ballistic transport or Stark localization in the clean limit. The universal diffusive behavior is attributed to a noise-induced bound state arising in the operator equations of motion at small momentum. By mapping the noise-averaged operator equation of motion to a one-dimensional non-Hermitian hopping model, we analytically solve for the diffusion constant, which scales nonmonotonically with noise strength, revealing regions of enhanced and suppressed diffusion from the interplay between on-site and bond dephasing noise, and a linear potential. For large on-site dephasing, the diffusion constant vanishes, indicating an emergent localization. On the other hand, the operator equation becomes the diffusion equation for strong bond dephasing and is unaffected by additional arbitrarily strong static terms that commute with the local charge, including density-density interactions. The bound state enters a continuum of scattering states at finite noise and vanishes. However, the bound state reemerges at an exceptional-like point in the spectrum after the bound-to-scattering state transition. We then characterize the fate of Stark localization in the presence of noise.

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Introduction. An outstanding challenge of many-body physics is a complete explanation of how phenomenological laws governing irreversible macroscopic transport behavior emerge from reversible microscopic dynamics, a process encapsulated by the eigenstate thermalization hypothesis [1–3]. This challenge only magnifies in interacting quantum many-body systems in both equilibrium and nonequilibrium processes [4,5]. Along these lines, one-dimensional systems [6,7] are attractive because quantum fluctuations have a pronounced effect, leading to a wide array of quantum phenomena ranging from ballistic transport to localization, in particular, the observation of superdiffusive transport [8–14] beyond the expected ballistic behavior in integrable systems. However, a complete characterization of quantum transport in solvable models remains challenging despite having access to the eigenenergies and excitations [15].

Randomly driven models, in which couplings are random variables uncorrelated in time, help understand the spreading of a local operator under Heisenberg evolution, known as the operator dynamics. Systems with added stochasticity ought to lose their microscopic properties, such as conservation laws, permitting the emergence of universal behavior. These systems have recently been revitalized with discrete time evolution involving dual unitary circuits [16,17] and replica disorder-averaged random unitary circuits [18–20]. On the other hand, stochastic dynamics of continuous-time models in random Hamiltonians [21–26], noisy spin chains [27–32], and (a)symmetric simple exclusion processes [33–36] have provided deep insights. Random unitary dynamics have also attracted experimental interest in cold atoms [37–39], trapped ions [40–42], and paraxial optics [43].

Despite tremendous progress, a complete characterization of the ingredients necessary for unorthodox transport to arise in interacting many-body systems remains open. One approach is introducing a static term as a perturbation [29,44] to access more generic information about late-time transport. A recent study [45] of a spin-1/2 chain with exchange couplings that fluctuate in space-time around a nonzero mean revealed, through perturbation theory, late-time spin diffusion, albeit with a superdiffusive enhancement suggesting normal diffusion [46].

In this Letter, we extend these results to nonperturbative static terms. We develop a microscopic transport theory in a fermionic chain without and in the presence of a linear potential. In both cases, the operator dynamics arise from the competition between randomly driven and arbitrarily strong static couplings. We analytically solve for the diffusion constant by exactly mapping the noise-averaged operator equation of motion to a one-dimensional non-Hermitian hopping model—the diffusion constant scales nonmonotonically with noise strength, revealing enhanced and suppressed diffusion regions.

We uncover for all noise models that a diffusive mode governs the late-time hydrodynamics at small k, attributed to an emergent bound state in the operator equations of motion. As k increases, the bound state enters a scattering state continuum and vanishes. From the non-Hermitian structure of the operator equations, the bound state reemerges at an exceptional-like point where a pair of complex energies form. However, for strong bond dephasing noise, the operator equation becomes the diffusion equation and is *unaffected* by additional arbitrarily strong static terms that commute with

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the local charge, including density-density interactions. Moreover, we then characterize the fate of Stark localization in the presence of noise. Ultimately, noise destabilizes the Stark ladder, allowing transport to occur albeit nonmonotonically.

Model. We explore the dynamics of one-dimensional noninteracting fermions with time-dependent noise [47,48], through the Hamiltonian

$$H_t = \sum_{x,y}[J_{x,y} + \Gamma_{x,y}(t)]c_x^\dagger c_y, \tag{1}$$

where $c_x^\dagger$ (c_x) creates (annihilates) an electron at site index x. The off-diagonal elements of $J_{x,y}$ and $\Gamma_{x,y}(t)$ represent either static or driven hopping, while the diagonal elements represent a static or driven potential. The amplitudes $\{\Gamma_{x,y}\}$ are drawn independently for each pair of sites (x, y) from a Gaussian distribution with zero mean and variance,

$$\mathbb{E}[\Gamma_{x,y}(t)\Gamma_{l,m}(t')] = \Gamma_{xy}\delta_{x,l}\delta_{y,m}\delta(t-t'), \tag{2}$$

where $\mathbb{E}[\cdot]$ denotes the average over disorder, $\Gamma_{x,y}$ sets the energy scale of the noise, and $\delta(t-t')$ implies the couplings are correlated at a single instance in time.

We study analytically and numerically time-dependent correlation functions to reveal the long-distance late-time hydrodynamic transport in the presence of noise. In the Heisenberg picture, the infinitesimal operator evolves stochastically, $\mathcal{O}_{t+dt} = e^{iH_t dt}\mathcal{O}_t e^{-iH_t dt}$. The evolution equation for a generic noise-averaged operator follows from expanding the flow of $\mathcal{O}_t$ up to second order in dt and averaging the noise [49–51],

$$d\bar{\mathcal{O}}_t = \sum_{x,y}[iJ_{x,y}[c_x^\dagger c_y, \bar{\mathcal{O}}_t] + \Gamma_{x,y}\mathcal{L}_{x,y}[\bar{\mathcal{O}}_t]]dt. \tag{3}$$

Here, the average dynamics are governed by an effective Lindblad description [34,52–54] where $\mathcal{L}_{x,y}[*] = L_{x,y}^\dagger * L_{x,y} - \frac{1}{2}\{L_{x,y}^\dagger L_{x,y}, *\}$ with $L_{x,y} = c_x^\dagger c_y + \text{H.c.}$, and $\{,\}$ standing for the anticommutator [55]. The jump operators are derived explicitly in the Supplemental Material (SM) [56]. Competition between coherent and incoherent dynamics drive the time-evolved noise-averaged operator in the late-time limit to the steady state $\lim_{t\to\infty}\bar{\mathcal{O}}_t = \sum_x n_x$ from charge conservation.

Characterizing transport. Universal behavior of the random unitary dynamics is ascertained through the infinite-temperature fermion density-density correlation function,

$$C_{x,y}(t) = \frac{1}{2^N}\text{tr}\left[\left(n_x(t) - \frac{1}{2}\right)\left(n_y - \frac{1}{2}\right)\right], \tag{4}$$

where $n_x(t)$ denotes the time-evolved density operator at site index x in the Heisenberg picture. The density-density correlation function Eq. (4) decays with an algebraic tail at late times,

$$\lim_{t\to\infty}\lim_{N\to\infty} C_{N/2,N/2}(t) \sim t^{-1/z}. \tag{5}$$

The dynamical exponent z classifies the universal hydrodynamic transport behavior, for example, $z = 1$ for ballistic, $1 < z < 2$ for superdiffusive, $z = 2$ for diffusive, $z > 2$ is subdiffusive, and $z = \infty$ for localized.

Operator dynamics. The Heisenberg operator $n_x(t)$ remains a two-body operator under evolution due to the absence of interactions, permitting the expansion,

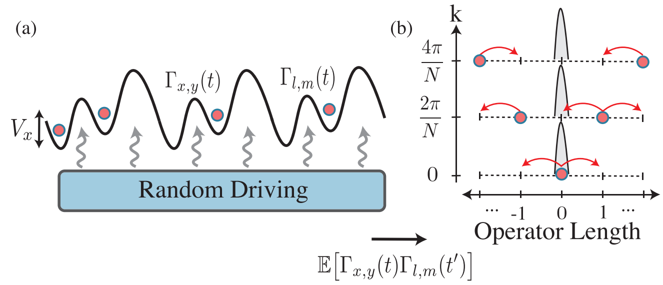

FIG. 1. Noise-induced non-Hermitian hopping model. (a) Randomly driven noninteracting fermions in a spatially dependent potential V_x. Classical noise $\Gamma_{x,y}(t)$ models the random drive by coupling locally to the hopping or density. (b) Noise-averaged operator equations of motion map onto a set of one-dimensional non-Hermitian hopping models with a repulsive delta function. The x axis is the operator length, and k is the center-of-mass momentum.

$$n_x(t) = \sum_{m,n=1}^{N} A_{m,n}(t)c_m^\dagger c_n. \tag{6}$$

With the initial condition, $A_{m,n}(0) = \delta_{m,x}\delta_{n,x}$. We transform into the coordinates $\ell = n - m$ [57] and $\mathcal{R} = n + m$ representing the operator length and center of mass. Because the noise-averaged operator equation is translation invariant in $\mathcal{R}$ in our models, a Fourier transformation maps Eq. (3) to equations for $A_{\ell,k}$ describing a one-dimensional hopping model on a fictitious lattice of operator length ℓ with the center-of-mass momentum k (see Fig. 1). The correlation function, in terms of the coefficients, is given by $\frac{1}{8\pi}\int dk A_{0,k}(t)e^{ik(x-y)}$, where $A_{\ell,k}(t)$ is the time-evolved wave function of the effective hopping model and $A_{\ell,k}(0) = \delta_{\ell,0}$. At finite noise, the effective model is non-Hermitian, where the nonpositive real parts of the eigenvalues drive the system to the steady state in the late-time limit, corresponding to the eigenvalue with the maximal real part, namely, the eigenstate decays slowest during time evolution.

Bond and on-site dephasing noise. We now focus our model in Eq. (1) on nearest-neighbor hopping with dephasing noise on both bonds and sites. Specifically, we define the parameters,

$$J_{x,x+1} = J, \quad \Gamma_{x,x} = \mathcal{V}, \quad \Gamma_{x,x+1} = \Gamma. \tag{7}$$

Here, J is the nearest-neighbor coherent hopping, and $\mathcal{V}$ and Γ are the on-site and bond dephasing strengths, which are respectively real scalar values. The eigenvalue equations of Eq. (3) take the form

$$\begin{aligned}
\mathcal{E}_q A_0 &= t_k[A_1 - A_{-1}] - 4\Gamma\sin^2(k)A_0,\\
\mathcal{E}_q A_{\pm 1} &= \pm t_k[A_{\pm 2} - A_0] + \Gamma A_{\mp 1} - [\mathcal{V} + 2\Gamma]A_{\pm 1},\\
\mathcal{E}_q A_\ell &= t_k[A_{\ell+1} - A_{\ell-1}] - [\mathcal{V} + 2\Gamma]A_\ell.
\end{aligned} \tag{8}$$

We dropped the index k in $A_{\ell,k}$ for simplicity, and q labels different levels of the eigenvalue equation. The first two equations are the boundary conditions near the origin of the fictitious operator length lattice, and the third describes the bulk for $|\ell| > 1$ with the effective hopping, $t_k = 2J\sin(k)$. There are two well-known limits of Eq. (8): no noise,

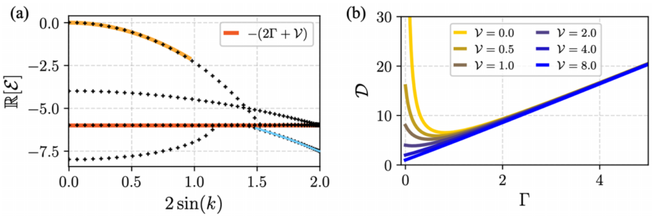

FIG. 2. Bond and on-site dephasing noise. (a) Real part of eigenvalue spectrum with both on-site and bond dephasing noise. The yellow curve is the diffusive mode corresponding to Eq. (13). The red line indicates the continuum of scattering states, and the blue curve is a degenerate set of complex energies. (b) Diffusion constant from Eq. (13). When $\Gamma = 0$ the diffusion constant decreases from a ballistic ($\mathcal{V} \to 0$) to an emergent localization regime when $\mathcal{V} \to \infty$. As Γ reaches the minimum, $\sqrt{6}J - \mathcal{V}$ then *increases* monotonically into a noise-assisted transport regime. Parameters: (a) $N = 400$, $\gamma = 0$, $\Gamma/J = 2$, $\mathcal{V}/J = 2$.

$\Gamma = \mathcal{V} = 0$, and pure dephasing, $J = 0$. In the former case, the model is purely coherent, leading to the correlation function,

$$C_{x,y}(t) = \tfrac{1}{4}\mathcal{J}_{x-y}^2(2Jt). \tag{9}$$

Here, $\mathcal{J}_{x-y}(2Jt)$ is the Bessel function of the first kind of order $x - y$. The asymptotic behavior of the correlation function, $\lim_{t\to\infty} C_{N/2,N/2}(t) = 1/\pi t$, indicates ballistic transport with an exponent $z = 1.0$. In the latter case ($J = 0$ or equivalently $t_k = 0$), the operator length $\ell = 0$ decouples from all other operator lengths, mapping to the diffusion equation, with the solution

$$C_{x,y}(t) = \tfrac{1}{4}e^{-2\Gamma t}\mathcal{I}_{x-y}(2\Gamma t). \tag{10}$$

Here, $\mathcal{I}_{x-y}(2\Gamma t)$ is the modified Bessel function of the first kind of order $x - y$. The asymptotic scaling of Eq. (10) is $\lim_{t\to\infty} C_{N/2,N/2}(t) = 1/2\sqrt{t\pi}$ corresponding to the exponent $z = 2$. Including a static potential that couples to the density does not affect the diffusive mode because it commutes with the local charge n_x and bond dephasing leaves n_x unchanged. Generically, including any static term that commutes with the local charge, even the density-density interaction, $n_x n_y$, will not affect the diffusive hydrodynamic mode.

Now we solve Eq. (8) for general J, $\mathcal{V}$, and Γ. It is similar to the standard Schrödinger equation with a δ potential; both scattering and bound states exist in the spectrum, whereby the bulk equation fixes the real part of the scattering states energy to be $-[\mathcal{V} + 2\Gamma]$ [see the red line in Fig. 2(a)]. Translation invariance of Eq. (8) permits the ansatz,

$$A_\ell = \begin{cases} A_{-1}e^{q(1+\ell)} & \text{if } \ell \leqslant -1, \\ -A_1 e^{q(1-\ell)+i\pi\ell} & \text{if } \ell \geqslant 1. \end{cases} \tag{11}$$

Inserting the above solution into the bulk equation gives the energy $\mathcal{E}_q = 4\sin(k)\sinh(q) - \mathcal{V} - 2\Gamma$. The boundary conditions for $|\ell| \leqslant 1$ constraint the values of q through (see SM [56])

$$[\mathcal{E}_q + 4\Gamma\sin^2(k)][t_k e^q + \Gamma] = -2t_k^2. \tag{12}$$

The above equation is an exactly solvable cubic equation, which at small k admits two physical solutions, one that begins at $\mathcal{E}_q = 0$ [see the yellow curve in Fig. 2(a)] and the other at $\mathcal{E}_q = -[3\Gamma + \mathcal{V}]$ [the lowest branch in Fig. 2(a)]. The branch in Fig. 2(a) beginning at $\mathcal{E}_q = -[\Gamma + \mathcal{V}]$ is determined by solving Eq. (8) assuming $A_0 = 0$. Moreover, the gapless bound-state energy is given by

$$\mathcal{E}_q = -4\left[\Gamma + \frac{2J^2}{\mathcal{V} + 3\Gamma}\right]k^2. \tag{13}$$

A diffusive mode always exists at small momentum regardless of whether the sites or the hopping have finite dephasing [see the yellow curve in Fig. 2(a)]. When both $\mathcal{V}, \Gamma \to 0$, the diffusion constant diverges, which is reminiscent of ballistic transport in the coherent limit. Previously obtained was the result with either only on-site or bond dephasing noise [36,58]. In general, the diffusion constant decreases monotonically with increasing on-site dephasing $\mathcal{V}$ because an energy barrier from site to site impedes coherent hopping. In particular, in the absence of bond dephasing, the diffusion constant is zero in the large $\mathcal{V}$ limit, indicating an emergent localization. As illustrated in Fig. 2(b), the diffusion constant displays nonmonotonic behavior as a function of bond dephasing Γ. Specifically, as Γ increases, the diffusion constant reaches a minimum at $\Gamma = (\sqrt{6}J - \mathcal{V})/3$ (assuming $\mathcal{V} < \sqrt{6}J$), and then *increases* monotonically, entering a regime of noise-assisted transport [42,59,60].

As momentum increases, two interesting characteristics become apparent. First, the diffusive mode undergoes a bound-to-scattering state phase transition upon entering a scattering state continuum at $\mathcal{E}_q = -2\Gamma - \mathcal{V}$. Then, from the non-Hermitian characteristic of Eq. (8), there is an exceptional-like point [61,62] where the two physical solutions of Eq. (12) collide and coalesce, becoming a complex conjugate pair of energies visualized by the doubly degenerate points in Fig. 2(a) indicated with a blue curve.

Linear potential with bond and on-site dephasing. In the clean limit of the previous examples, the system exhibited ballistic transport [see Eq. (9)]. However, no matter how weak or the location, finite noise causes diffusive transport. We now turn our attention to the opposite limit, where in the clean limit ($\Gamma = \mathcal{V} = 0$), the system is localized, and the diffusion constant vanishes. We will study Wannier-Stark localization in the presence of noise [63–66]. Specifically, we consider the linear potential $J_{x,x} = -\gamma x$ where γ is the slope with the noise coupled to the hopping and density. We now study the competition between these two noise models through the equation

$$\mathcal{E}_q A_{\ell,k} = t_k[A_{\ell+1,k} - A_{\ell-1,k}] + [i\gamma\ell - 2\Gamma - \mathcal{V}]A_{\ell,k}. \tag{14}$$

The bulk operator equation is no longer translation invariant in ℓ, which permitted the plane wave ansatz Eq. (11). Solving the recursion relation, A_ℓ instead takes the form

$$A_{\ell,k} = \begin{cases} A\mathcal{I}_{\nu_-}(-2it_k/\gamma) & \text{if } \ell < -1, \\ B\mathcal{I}_{\nu_+}(-2it_k/\gamma) & \text{if } \ell > 1, \end{cases} \tag{15}$$

where $\nu_\pm = i(\mathcal{E}_q + 2\Gamma + \mathcal{V})/\gamma \pm \ell$. For $\mathcal{V} = \Gamma = 0$ the operator equations are anti-Hermitian leading to an equally spaced tower of purely imaginary eigenvalues, $\mathcal{E}_q = i\gamma q$ for $q \in \{-\ell_{\max}, \ell_{\max}\}$ independent of momentum k. The corresponding unnormalized eigenstates are $A_{\ell,k} = \mathcal{I}_{\ell-q}[-4iJ\sin(k)/\gamma]$ which are Wannier-Stark localized [67–70]. Finite noise

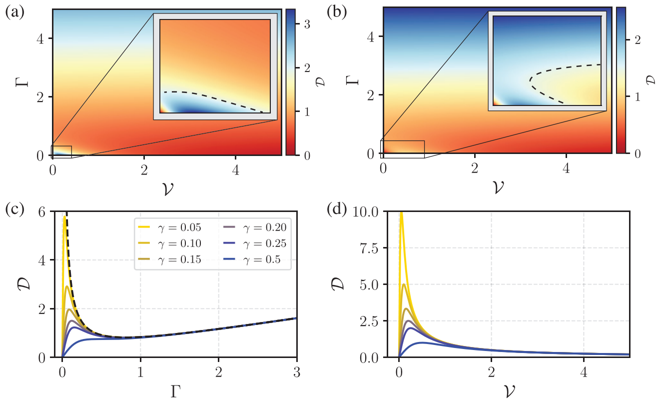

FIG. 3. Diffusion constant phase diagram. (a) Diffusion constant from Eq. (16) with the linear potential strength $\gamma = 0.15$. Inset: Illustration of the nonmonotonicity along both axes. (b) Same as in (a) with $\gamma = 0.50$ where the nonmonotonic behavior arises only along $\Gamma = 0$. Inset: Illustration of nonmonotonicity along the on-site dephasing axis only. (c) Diffusion constant with $\mathcal{V} = 0$. Provided $\gamma < 0.5$, there is an initial noise-assisted regime to a maximum value, where then bond dephasing introduces an energy barrier, suppressing diffusion. Once $\Gamma > \gamma$, diffusion enhances as if the linear potential was absent [see the black curve for $\gamma = 0$ or Fig. 2(b)]. As $\gamma \to 0.5$, the nonmonotonic behavior is lost, and diffusion immediately enters a noise-assisted transport regime, (d) while when $\Gamma = 0$, noise compensates for the energy barrier from the linear potential, enhancing transport to a maximum. As $\mathcal{V}$ increases further, the on-site dephasing dominates the linear potential, introducing an energy barrier and decreasing the diffusion constant. Parameters: The dotted black curves in (a) and (b) indicate a maximum or minimum.

renders the operator equations non-Hermitian, causing an eigenvalue to become purely real, which is the long-wavelength mode. In the SM [56], we determine the scaling of the hydrodynamic mode,

$$\mathcal{E}_q = -8\left[\frac{\Gamma}{2} + \frac{J^2(\mathcal{V}+\Gamma)}{\gamma^2 + (\mathcal{V}+\Gamma)(\mathcal{V}+3\Gamma)}\right]k^2, \qquad (16)$$

which is diffusive for finite noise, similar to Anderson localized models with global noise [29,47,71], but different from local noise models [72,73]. In the limit $\gamma = 0$, we recover the bound-state energy Eq. (13), while in the limit either $\mathcal{V}$ or Γ is large, the bound-state energy is finite, specifically 4Γ, indicating Stark localization instability to noise.

In Figs. 2(a) and 2(b), we plot the heat map of the diffusion constant with $\gamma < 0.5$ and $\gamma = 0.5$. In both cases, the model is Stark localized when $\mathcal{V} = \Gamma = 0$. When $\gamma < 0.5$, initially, there is a regime where increasing Γ or $\mathcal{V}$ leads to noise-assisted transport to a maximum value [see Fig. 2(c) or 2(d)]. Increasing noise further in either direction introduces an energy barrier that overcomes the linear potential, suppressing diffusion; however, when $\Gamma > \gamma$, diffusion enhances once more as if the linear potential was nonexistent [see the black curve for $\gamma = 0$ in Fig. 3(c) or Fig. 2(b)]. As

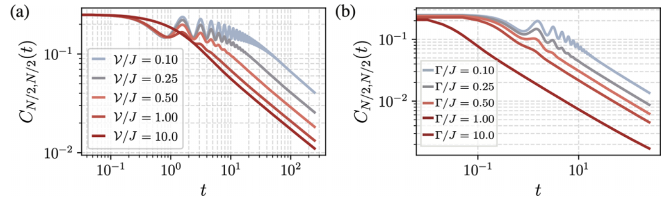

FIG. 4. Noisy linear potential operator dynamics. (a) The autocorrelation function $C_{N/2,N/2}(t)$ with on-site dephasing. The oscillating behavior is a signature of the underlying Stark localization, which pushes the onset of diffusion to late times. (b) Same as (a) but bond dephasing noise. Parameters: (a) and (b) $N = 400$, $dt = 0.05$, $\gamma = 4$, $J = 1$.

$\gamma \to 0.5$, the nonmonotonic behavior decreases and is lost when $\gamma > 0.5$, whereby diffusion immediately enters a noise-assisted transport regime. On the other hand, the on-site dephasing dominates the linear potential as $\mathcal{V}$ increases [see Fig. 3(d)], introducing an energy barrier and decreasing the diffusion constant.

We first study the operator dynamics of Eq. (14) with only on-site dephasing present, i.e., $\Gamma = 0$. When $\mathcal{V} \ll \gamma$ the diffusion constant is small, and Bloch oscillations push diffusion to later times [see Fig. 4(a)], rather than when $\mathcal{V}$ is the dominant energy scale. In contrast, diffusion almost immediately occurs when the noise is on the bonds [see Fig. 4(b)], i.e., $\mathcal{V} = 0$, a consequence of the diffusion constant always being finite regardless of the linear potential strength.

Conclusion. Through a combination of analytics and large-scale numerics, this work developed a transport model where the operator dynamics arise from the competition between randomly driven and static couplings. We exactly solve for the diffusion constant by determining the emergent bound state of an effective one-dimensional non-Hermitian hopping model. In contrast to standard hydrodynamic theories [74,75], the diffusion constant scales nonmonotonically with noise strength. For pure dephasing, the noise-averaged equation satisfies the diffusion equation, which is robust to arbitrarily strong static terms that commute with the local charge, including interactions. As momentum increases, the bound state enters a continuum of scattering states and vanishes. Surprisingly, beyond the bound-to-scattering state phase transition, the bound state reemerges at an exceptional-like point. We further find Stark localization is unstable to on-site and bond dephasing noise, but illustrates a rich phase diagram where diffusion enters regimes of enhancement and suppression. Future work could be understanding transport when the model has long-range hopping or correlated noise [76].

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[56] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.108.L180303 for explicit derivations of the equations of motion, diffusion constants, and additional numerics that illustrate the validity of our approximations and accuracy of our solutions.
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