Letter

Counting edge modes via dynamics of boundary spin impurities

Umar Javed,[1] Jamir Marino,[2] Vadim Oganesyan,[3,4,5] and Michael Kolodrubetz[1]

[1]*Department of Physics, The University of Texas at Dallas, Richardson, Texas 75080, USA*
[2]*Institut für Physik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany*
[3]*Physics Program and Initiative for the Theoretical Sciences, The Graduate Center, CUNY, New York, New York 10016, USA*
[4]*Department of Physics and Astronomy, College of Staten Island, CUNY, Staten Island, New York 10314, USA*
[5]*Center for Computational Quantum Physics, Flatiron Institute, 162 5th Avenue, New York, New York 10010, USA*

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We study the dynamics of the one-dimensional Ising model in the presence of a static symmetry-breaking boundary field via the two-time autocorrelation function of the boundary spin. We find oscillatory correlations that decay as power laws. We uncover a phase diagram of dynamical responses where, upon tuning the strength of the boundary field, we observe distinct power laws that directly correspond to changes in the number of edge modes as the boundary and bulk magnetic field are varied. We suggest how the universal physics can be demonstrated in current experimental setups, such as Rydberg chains.

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The interplay of many-body interactions and correlations [1,2] lies at the foundation of emergent phenomena from condensed matter and atomic and molecular systems to high-energy physics. The challenge posed by treating interparticle interactions nonperturbatively in extended systems suggests that one should search for simpler setups to serve as stepping stones towards increasingly complex problems. An archetypal class of such systems is quantum impurity models in strongly correlated systems. By embedding one or a few degrees of freedom in a many-body medium, one can often treat strong impurity-environment couplings exactly [3–7], with the goal of building understanding and applying it towards even more complex scenarios. Examples include the Anderson orthogonality catastrophe in local quenches of gapless systems [8–12], interaction-dependent transport in one-dimensional junctions [13,14], the buildup of entanglement among magnetic impurities and their surrounding fermionic or bosonic environments [15–17], and the formation of polarons in solid state systems or cold atomic clouds [18–23].

This work aims to examine fresh aspects of a quintessential impurity model hosting edge modes, namely the one-dimensional interacting Ising chain with a strong symmetry-breaking boundary field. The impact of boundary fields on the critical point of an extended system is a subject of active interest in both classical and quantum statistical mechanics [24–30]. Although, at leading order, impurities appear to be a subdominant correction to bulk properties of a large system, renormalization-group-relevant (RG-relevant) boundary perturbations can actually induce the formation of new phases with associated critical exponents [31]. In addition to its fundamental importance, the response of bulk systems to relevant boundary perturbations can yield novel edge modes, which in turn have the potential to be utilized as a resource in quantum computing. Here, we uncover a dynamical signature of these important edge phenomena.

Model. We consider a one-dimensional (1D) transverse field Ising model (TFIM) with a local boundary field,

$$H = -J\sum_{n=1}^{L-1}\sigma_n^z\sigma_{n+1}^z - h\sum_{n=1}^{L}\sigma_n^x - h_b\sigma_1^z, \tag{1}$$

where $\sigma_j^{(z,x)}$are Pauli matrices, $J = 1$ is the exchange interaction, h is the transverse field, and h_b is a static boundary field along the z direction. In the absence of a boundary field ($h_b = 0$), the TFIM has $\mathbb{Z}_2$ symmetry and undergoes a continuous phase transition at $h = J$, separating the ferromagnetic phase ($h < J$) and paramagnetic phase ($h > J$).

Numerical results. Motivated by the search for dynamical probes of edge modes [25], we now focus on the dynamics of the boundary spin. Specifically, we calculate the connected autocorrelation function of the boundary spin's magnetization,

$$C(t) = \langle\sigma_1^z(0)\sigma_1^z(t)\rangle - \langle\sigma_1^z(0)\rangle\langle\sigma_1^z(t)\rangle, \tag{2}$$

within the ground state. We find that in the presence of h_b, $C(t)$ decay as a power law, $C(t) \propto t^{-\alpha}$, similar to results in other integrable models [5,6]. However (as shown later in Fig. 2), critical lines emerge in which the power α changes sharply. On the critical lines between these boundary "phases of matter," other power laws emerge. The goal of the remainder of this Research Letter will be to understand these emergent power laws in the dynamical response.

Edge states. Previous works on the TFIM with either longitudinal or transverse boundary field have demonstrated the existence of edge states via the Jordan-Wigner mapping to free Majorana fermions [26–30], as well as in other related spin models [4]. To study the connection between these edge states and the boundary spin dynamics, we perform a Jordan-Wigner transformation, given by [32]

$$\sigma_n^x = i\eta_{A,n}\eta_{B,n}, \quad \sigma_n^z = i\gamma\left(\prod_{j=1}^{n-1} i\eta_{A,j}\eta_{B,j}\right)\eta_{A,n}, \tag{3}$$

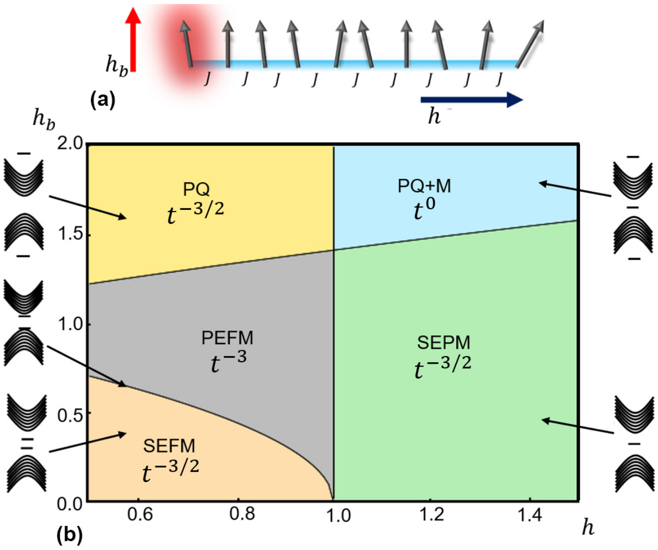

FIG. 1. (a) We study magnetization dynamics of the edge spin in the 1D transverse field Ising model in the presence of boundary field h_b, resulting in power-law relaxation of the boundary autocorrelation function [Eq. (2)] as a function of h_b and transverse field h. (b) Phase transitions between different power laws correspond to a change in the number of edge modes. Phases are labeled according to the behavior of their edge mode: soft-edge paramagnetic and ferromagnetic (SEPM and SEFM, respectively) for cases where the edge mode relaxes slowly as $t^{-3/2}$, pinned-edge ferromagnetic (PEFM) where the edge is held fixed by the field, and protected qubit (PQ) plus Majorana (+M) where a large boundary field creates protected edge modes.

where an ancilla Majorana $\gamma = \gamma^\dagger$ is added to the usual Jordan-Wigner string such that the boundary field also maps to a Majorana hopping term:

$$H = -iJ\sum_{n=1}^{L-1}\eta_{B,n}\eta_{A,n+1} - ih\sum_{n=1}^{L}\eta_{A,n}\eta_{B,n} - ih_b\gamma\eta_{A,1}. \quad (4)$$

In the absence of h_b, Eq. (4) represents the Kitaev chain [33], which has a topological Majorana zero mode on the ferromagnetic (FM) side ($h < J$). Furthermore, the ancilla Majorana gives a separate (artificial) zero mode, which nevertheless couples into the Kitaev chain for $h_b \neq 0$. In the ferromagnetic phase, the ancillary zero mode gaps out the topological zero mode, yielding a gapped fermion. By contrast, there is no topological zero mode on the paramagnetic (PM) side; so the ancillary zero mode remains fixed at $E = 0$ despite hybridizing with the Kitaev chain [33,34].

At higher h_b, a richer edge state structure emerges, as illustrated in Fig. 1. For instance, at $h_b = \sqrt{1-h}$ and $h < J$, the edge state merges into the bulk. At $h_b = \sqrt{1+h}$, a second (gapped) edge state emerges out of the top of the band. Analytical expressions for the edge mode wave functions and energies can be found in the Supplemental Material [35]; they can be exactly solved either for the lattice model or within the low-energy field theory. The field theory calculation further supports the idea that the phase transitions at small h_b and $|h-J| \ll 1$ are universal. By contrast, the edge mode which emerges at $h_b = \sqrt{1+h}$ does not show up in the Ising field theory, indicating that it is a nonuniversal lattice effect. We also note that identical edge states have been found in previous studies of transverse boundary fields ($h\sigma_1^z \to h_{bx}\sigma_1^x$) upon interchanging h and J [26]. This comes from the mathematical fact that the transverse boundary field maps to an identical Majorana chain but shifted by one site due to lack of the ancilla Majorana. However, the role of the transverse boundary field is nonuniversal in the Ising field theory; $h_b\sigma_1^z$ is a relevant boundary perturbation with scaling dimension 1/2, while $h_{bx}\sigma_1^x$ is marginal [32,36]. Therefore we expect our predictions of symmetry-breaking boundary dynamics to be more robust when taken beyond the clean, integrable TFIM.

Crucially, we see that the transition lines where edge modes are gained or lost are precisely the lines where the exponent α dictating edge spin decay changes. This connection between emergent fermionic edge modes and boundary spin dynamics is highly nontrivial and has not been explored in detail. We now seek to model the behavior of the boundary spin and explain the origin of this connection.

Boundary spectral function. To understand the connection between edge modes and boundary dynamics, consider the spectral function

$$C(\omega) = 2\pi\sum_{n\neq 0}\left|\langle\psi_n|\sigma_1^z|\psi_0\rangle\right|^2\delta[\omega-(E_n-E_0)], \quad (5)$$

where $|\psi_0\rangle$ is the ground state and $|\psi_n\rangle$ are the excited states. Diagonalizing the Hamiltonian in the fermionic basis,

$$H = \sum_{\ell=1}^{N_{\rm edge}}\epsilon_\ell(2c_\ell^\dagger c_\ell - 1) + \sum_k\epsilon_k(2c_k^\dagger c_k - 1), \quad (6)$$

we can separate the edge (ℓ) and bulk (k) modes in a semi-infinite strip. Note that this solution involves combining the original Majoranas into Dirac fermions c_k. This form of H allows for both gapped edge modes and Majorana zero modes, for which $c_\ell = \gamma_{\rm edge}$ with $\epsilon_\ell = 0$. One can choose $\epsilon \geqslant 0$ for all modes, such that $|\psi_0\rangle$ is the vacuum state of the c fermions. Then we immediately see that, since $\sigma_1^z = i\gamma\eta_{A1}$ is a two-fermion operator, $|\psi_n\rangle$ is restricted to states with two fermion excitations above the vacuum, $|\psi_n\rangle = c_\alpha^\dagger c_\beta^\dagger|\psi_0\rangle$, in order for the matrix element not to vanish. Going back to the expression of $C(t)$, we have

$$C(t) = \frac{1}{2}\sum_{\alpha\neq\beta}e^{-2i(\epsilon_\alpha+\epsilon_\beta)t}\underbrace{\left|\langle\psi_0|c_\alpha c_\beta\sigma_1^z|\psi_0\rangle\right|^2}_{f_{\alpha\beta}}, \quad (7)$$

where α, β iterate over edge and bulk modes.

At late times, we can solve Eq. (7) via a saddle-point approximation. There are three separate situations to consider.

(1) If α and β are both edge states, which is possible for $N_{\rm edge} \geqslant 2$, then one has infinitely long-lived oscillations proportional to $\cos[2(\epsilon_\alpha+\epsilon_\beta)t]$ as long as the matrix element $f_{\alpha\beta}$ is of order 1, as expected for edge states.

(2) If α is an edge state and $\beta = k$ is a bulk state, then in the thermodynamic limit we can replace $\sum_k \to (L/2\pi)\int dk$. For $t \gg 1/J$, this integral is dominated by the saddle points of the fast-oscillatory term which, for the bulk TFIM, are at $k = 0$ and $k = \pi$. As shown [35], this matches the numerically found exponent $t^{-3/2}$ if the matrix element scales as $f_{\alpha k} \sim k^2$. Such a scaling emerges naturally in the field theory limit

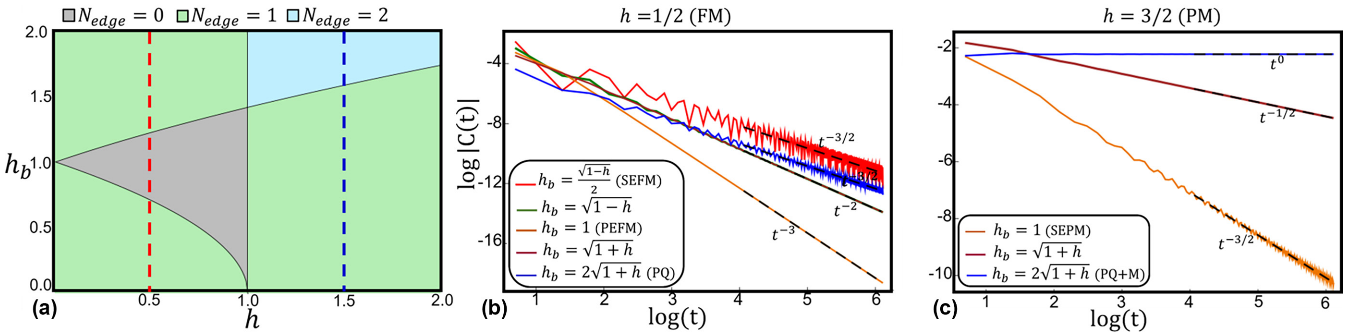

FIG. 2. (a) Phase diagram representing the edge states in the presence of boundary field h_b. (b) and (c) Plots of the autocorrelation function for $h = 0.5$ and $h = 1.5$ with different values of h_b taken across the different phases of edge states represented by red and blue lines in the phase diagram, respectively. Power laws are shown as guides to the eye and match those shown in Fig. 1.

from the bulk modes with open boundary conditions, whose (Majorana) wave functions are proportional to $\sin(k) \sim k$ at low momentum. This power-law decay is an envelope for $e^{-2i(\epsilon_\alpha+\epsilon_{\mathbf{k}=\mathbf{0}})t}$ oscillations due to the edge mode.

(3) If $\alpha = k$ and $\beta = k'$ are both bulk states, then the sum becomes an integral over k and k'. Assuming separability of $f_{kk'} \sim k^2k'^2$, we find $C(t) \sim (t^{-3/2})^2 = t^{-3}$, as seen numerically.

For $N_{\rm edge} = 0$, only case 3 is possible, while cases 2 and 3 are possible for $N_{\rm edge} = 1$. However, the late-time dynamics will be dominated by the slowest decaying exponent, leading to the prediction $|C(t)| \sim t^{-(3/2)(2-N_{\rm edge})}$ as seen in Fig. 2.

Boundary phases of matter. Having established the existence of edge states and their connection to the edge spin dynamics, $C(t)$, we now discuss the physical meaning of these power-law decays and provide labels for the boundary "phases of matter." Let us start with the low-field limit, $h_b \ll 1$, for which the physics near the critical point are universal. In this regime, there are three phases of matter, which we now discuss in detail.

(a) *Soft-edge paramagnet.* The soft-edge paramagnetic (SEPM) phase ($h > J$, $h_b < \sqrt{1+h}$) extends from the $h_b = 0$ paramagnet in which a Majorana zero mode persists, causing slow $t^{-3/2}$ relaxation of the edge magnetization. Perturbing away from the critical point at $h = J$ and $h_b = 0$, one can think of this phase as where the bulk mass gap $\sim h - J$ is more relevant than the boundary perturbation, which corresponds to an energy scale $E_b \sim h_b^2$ [32]. Since the symmetry-breaking field is not important in defining the paramagnetic phase, the boundary dynamics of the SEPM is smoothly connected to the conventional paramagnet at $h_b = 0$.

(b) *Soft-edge ferromagnet.* The soft-edge ferromagnetic (SEFM) phase ($h < J$, $h_b < \sqrt{1-h}$) extends up from the $h_b = 0$ ferromagnet in which the ancilla Majorana couples to the topological edge Majorana and opens a gap, again causing slow $t^{-3/2}$ relaxation of the edge magnetization. In the spin language, this corresponds to a finite gap between the symmetry-breaking ground states which is proportional to the symmetry-breaking field h_b. This destruction of spontaneous symmetry breaking results in an increase in the edge spin relaxation from the ferromagnet, for which it must decay to a constant: $|C(t)| \sim t^0$ as $t \to \infty$ for $h_b = 0$. From a field theory perspective, this is the phase where the symmetry-breaking mass gap $\sim J - h$ is more relevant than the boundary perturbation. However, unlike the SEPM, the soft-edge ferromagnet is not smoothly connected to the $h_b = 0$ ferromagnet because the symmetry-breaking boundary field fundamentally changes the symmetry-breaking ferromagnetic phase.

(c) *Pinned-edge ferromagnet.* In the pinned-edge ferromagnetic (PEFM) phase ($h < J$, $\sqrt{1-h} < h_b < \sqrt{1+h}$), all edge modes have merged into the bulk, resulting in fast t^{-3} relaxation of the edge magnetization. In the spin language, this corresponds to a case where one of the original symmetry-breaking ground states, namely $|\Downarrow\rangle$, has merged into the bulk continuum, meaning that single itinerant domain wall excitations become less costly than a global flip of the Ising spins. In this case, the bulk (and edge) is pinned to a single ground state, removing any meaningful notion of symmetry breaking at the boundary [37]. Field theoretically, this is the phase where the boundary perturbation becomes the dominant scale, being more relevant than the mass gap $\sim J - h$. This phase of matter bears resemblance to the fixed-boundary-condition case of boundary conformal field theory (CFT) [32], but with the important caveat that the bulk is weakly gapped in a symmetry-breaking fashion.

Note that bulk two-time connected correlation functions are unaffected by small h_b throughout, as they only involve excitation to the bulk continuum and not between the degenerate ground states. Therefore the gap is similar to the one induced by other symmetry-breaking perturbations, which are well known to have no effect on connected correlations in the bulk. The bulk-boundary autocorrelations will show signatures of the boundary transition, since they involve finite overlap with the emergent edge mode. However, the bulk-boundary autocorrelation will involve a nontrivial string operator and so will be harder to calculate. This will be left for future work.

A useful analogy for thinking of these low-field phases of matter is that the pinned-edge ferromagnet is the boundary dynamical critical fan emerging from the bulk critical point. The shape of the fan is dictated by critical exponents from the boundary CFT [38]. Unlike a conventional thermal critical fan, however, there are phase transitions between the boundary dynamics in the different phases, rather than crossovers [39].

The high-field phases of matter for $h_b > \sqrt{1+h}$ are not universal, in the sense that they come from high-momentum lattice physics that is not present in the Ising field theory; they are nevertheless robust within the lattice model. The key point in both phases is that a fermionic edge state emerges out of the top of the single particle band. For $h_b \to \infty$, this can be thought of as the edge qubit, which is in a large magnetic field, $h_b\sigma_1^z$. The question is then how this edge qubit is dressed by excitations of the bulk continuum. For $h_b > \sqrt{1+h}$, the edge spin hybridizes with the bulk but remains stable. For $h_b < \sqrt{1+h}$, bulk domain walls hybridize with the edge qubit and destabilize it. Therefore we refer to these phases of matter as the protected qubit (PQ, $h < J$) and protected qubit plus Majorana (PQ+M, $h > J$) phases to reflect the fact that the edge Majorana remains stable for $h > J$ as well. It is particularly notable that the edge correlation function behaves asymptotically to oscillate with finite amplitude ($|C(t)| \sim t^0$) within the PQ+M phase, reflecting the fact that both a fermion and Majorana edge mode coexist, both of which are excited by the σ_1^z operator.

Experimental realizations. Recent experimental advances have made it possible to simulate spin systems in a well-controlled manner. A particularly well-developed platform to explore the physics studied here is with kinetically constrained spin models as realized in tilted Mott insulators of bosons [40] or, more recently, one- and two-dimensional arrays of Rydberg atoms [40–44]. In Rydberg atoms, the ground state $|g\rangle$ and Rydberg state $|r\rangle$ of the atom can be mapped to a spin 1/2 by considering $|g\rangle = |\uparrow\rangle$ and $|r\rangle = |\downarrow\rangle$. Adding strong dipole-dipole interactions between the Rydberg atoms gives the Hamiltonian

$$H = \hbar\Omega \sum_i \sigma_i^x - \sum_i \Delta\sigma_i^z + \sum_{i\neq j} V_{ij}\big(1+\sigma_i^z\big)\big(1+\sigma_j^z\big), \quad (8)$$

where Ω is the Rabi frequency of an external drive and Δ is its detuning frequency, both of which in principle can be controlled locally. The interactions V_{ij} can be made sufficiently strong that no nearest neighbors can simultaneously be in the Rydberg state, a limit known as the PXP model, for which one finds an antiferromagnetic ground state that breaks $\mathbb{Z}_2$ symmetry at large Ω. This model is in the 1D Ising universality class, can be prepared in its ground state deep within the phases, can be locally controlled, and has the nice property that a boundary σ^z field acts precisely as the symmetry-breaking field required above. This suggests that some of our universal predictions, such as the different power laws [3–7] deep within the boundary phases at low h_b and near the criticality $|h - J| \ll 1$, might be realizable experimentally. A notable concern that may cut off this power law is the finite lifetime of the Rydberg excitations, but recent experiments showing coherent dynamics out to relatively long time scales [45] give hope that this regime may be accessible in the nonintegrable Rydberg model. It is worth noting that sharp changes in power laws across transition lines in Fig. 1 are accompanied by kink discontinuities in the frequency of the damped oscillations [cf. Eq. (7)], which provides another route to locating the transitions experimentally.

Finally, we propose two routes to measure the relevant dynamics. First, $C(t)$ can be measured directly using the Hadamard test by directly coupling the boundary spin to an ancilla qubit such that the boundary autocorrelation function maps to coherence of the ancilla [46]. Second, one could instead measure σ_1^z, time evolve, and then measure again, which is within the operational capabilities of Rydberg tweezer arrays.

Conclusion. In conclusion, we have uncovered an unexpected dynamical signature of emergent edge states in the transverse field Ising model with a symmetry-breaking boundary field. Despite sharing a common origin with well-studied effects such as dynamics in boundary conformal field theories (bCFTs) [32,47–51] or boundary phase transitions (e.g., wetting transitions) [28,30], these edge dynamics have a distinct signature. We show that the dynamics are universal at low boundary field and near criticality. Explicitly, for a system with quantum phase transition in the (1+1)D Ising universality class, adding a symmetry-breaking boundary perturbation will give a phase diagram equivalent to Fig. 1 in the regime near criticality. Power-law relaxation of the order parameter field at the boundary will match the Ising model, as will the shape of the phase transition line ($h_b \sim |h - h_c|^{1/2}$). While the high-boundary-field regime is not universal, similar phases to those in Fig. 1 are nevertheless likely to emerge in similar lattice models due to their simple physical origin. An experimentally relevant example where we propose to test this universality further is in the PXP model of Rydberg atoms, whose ground state realizes an effective antiferromagnetic-to-paramagnetic transition.

Based on universality arguments for this transition, we expect similar boundary phase transitions to occur in a much wider class of systems, including critical points in other universality classes. Furthermore, interesting nonunitary dynamics may appear when the boundary spin is driven by white noise exhibiting an intriguing interplay between the many-body Zeno effect and the physics of edge modes [47,52,53]. Our current research efforts are focused in this direction.

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[37] Note that there is still a meaningful notion of symmetry breaking in the bulk, as the dynamics of a large domain wall in the bulk of the system that is oriented opposite to the boundary field is, nevertheless, infinitely long-lived in the infinite-domain limit.
[38] Near the critical point, a new time scale t_b emerges with $t_b \sim h_b^{-2}$; cf. Ref. [32]. Comparing this with the bulk time scale $\xi_t \sim |h - h_c|^{-\nu z}$, where $\nu = z = 1$, we see that $h_b \sim |h - h_c|^{1/2}$ as seen in our phase diagram.
[39] Note that classical 2D Ising analogs of these boundary correlators do not oscillate, but rather decay exponentially. In some parts of the phase diagram, the boundaries will presumably have softer exponents than the bulk correlators and therefore may be dominant as well. While in principle it may be possible to study power-law prefactors of interest here in the classical (Euclidean) setting, we have not explored this possibility.
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