Letter

Piezoresistive effect in two-dimensional Dirac materials

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Applying the Bir-Picus ansatz for strain-induced corrections to the electron momentum scattering time on impurities in a transition metal dichalcogenide monolayer, and taking the parameters of MoS_2 for our estimations, we derive general analytical expressions describing the piezoresistive effect, the strain-induced corrections to (longitudinal) Drude conductivity, linear magnetoresistance, and the Hall conductivity of the monolayer for an arbitrary dependence of electron momentum scattering time on its energy. We show that a two-band model, even with the account of the trigonal warping of electron valleys, should be revisited for the description of the piezoresistive effect in the case of strongly degenerate electrons. Therefore, we extend the two-band model by accounting for the deformation of higher-energy bands and derive general expressions describing strain-induced corrections to the kinematic coefficients of the monolayer. Thus, the developed approach allows to estimate the deformation constants of higher-energy bands.

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Introduction. Mechanical strain affects the electrical and optical properties of materials [1]. An external applied mechanical force can result in modifications of the crystal lattice of a solid-state object by influencing the bounding between atoms. From an application-oriented point of view, the alteration of the properties of a strained material allows for a tool detecting external perturbations. For instance, an increase (or a decrease) of conductivity under external stress embodies the piezoresistive (also called tensoresistive) effect (PRE) underlying the operation of various pressure sensors. The majority of existing micromechanical sensors [2] and piezoresistors are silicon and germanium based. These are the materials where the PRE was initially discovered [3,4].

However, strain-induced effects also remain in the focus of recent research involving two-dimensional (2D) Dirac materials such as graphene and transition metal dichalcogenide (TMD) monolayers [5–7]. TMD materials may be used for various known and undiscovered fundamental effects and potential technological applications in nano- and optoelectronics. Due to specific characteristics of the band structure, TMDs exposed to external electromagnetic (EM) fields allow for nontrivial transverse electron transport [8–11].

A typical example of a 2D TMD material is molybdenum disulfide (MoS_2), a noncentrosymmetric direct-band-gap superconductor of the D_{3h} point symmetry group, having two nonequivalent valleys in its Brillouin zone. It is the absence of inversion symmetry which allows for the manifestation of the PRE in MoS_2, which accounts for the induction of electrical polarization under external strain [12,13] allowing for its potential applicability in strain optronics [14] and flexible electronics [15,16].

The goal of this Letter is a theoretical study of the PRE in 2D TMD materials using the parameters of MoS_2 as a testing ground. The key entities describing the effect are the longitudinal Drude conductivity, the transverse Hall conductivity, and the longitudinal Drude conductivity in the external magnetic field (or magnetoresistance) of the monolayer exposed to an external in-plane uniform strain.

It is rather well known that in 2D TMD monolayers, the interaction of carriers of charge with deformation can be described as their interaction with an effective gauge pseudo-EM field expressed by pseudoscalar and pseudovector potentials [17]. From this perspective, PRE can be viewed as the consequence of the joint action of (i) "real" and (ii) "pseudo" -EM fields. Let us assume that both fields are weak, thus (i) induces the linear response of the system and both the longitudinal (Drude) and transverse (Hall) conductivities, and the field (ii) results in the emergence of corrections to both the Drude and Hall conductivities.

Then, phenomenologically, PRE lies within the formula $\sigma_{\alpha\beta} = \sigma^0_{\alpha\beta} + \lambda_{\alpha\beta ij} u_{ij}$, where the fourth-rank tensor $\lambda_{\alpha\beta ij}$ describes the corrections to the bare conductivity tensor $\sigma^0_{\alpha\beta}$ due to the deformation described by the strain tensor u_{ij}. Certainly, $\lambda_{\alpha\beta ij}$ depends on the properties of the electron gas (nondegenerate or degenerate), the inclusion of scattering on short-range or Coulomb impurities, and electron densities driven by the external gate voltage. The bare conductivity tensor according to classical Drude theory is diagonal $\sigma^0_{\alpha\beta} = \sigma^0 \delta_{\alpha\beta}$, whereas a weak classical magnetic field produces the corrections $\sigma^0_{xy} = -\sigma^0_{yx} \propto \omega_c$, where ω_c is a cyclotron frequency, whereas $\sigma^0_{xx} \propto \omega_c^2$.

In cubic-lattice semiconductors (such as germanium and silicon), PRE strongly depends on the degree of degeneracy of the electron gas [1]. In the case of Boltzmann statistics, the main effect comes from the modulation of the material band gap due to strain [18,19]. In contrast, if the electron gas is strongly degenerate and the Fermi level is located within the conduction band, the effect predominantly results in a strong modification of the kinematic characteristics of charge carriers, such as the electron velocity, distribution function, and the relaxation time. In the case of 2D Dirac materials, the band gap is affected by the pseudoscalar potential of the pseudo-EM field, whereas the kinematic characteristics are mainly determined by the pseudovector potential. Here, we consider the key nontrivial effects coming from the pseudovector potential, which considerably affects degenerate electrons.

The eigenproblem in the presence of strain. The bare Hamiltonian in the framework of the two-band model of the MoS_2 monolayer band structure including the terms describing the trigonal warping of the valleys reads

$$H_0(\mathbf{p}) = \frac{\Delta}{2}\sigma_z + v(\tau_v\sigma_x p_x + \sigma_y p_y) + \begin{pmatrix} 0 & \mu p_+^2 \\ \mu p_-^2 & 0 \end{pmatrix}, \quad (1)$$

where Δ is the band gap in the absence of strain, v is the interband velocity parameter, $\tau_v = \pm 1$ is the valley index, μ reflects the warping strength, and the Pauli matrices σ_i, $i = x, y, z$ describe the triangle sublattices of the hexagonal lattice of the monolayer.

The presence of strain with the components of displacement vector of the media u_i can be described by the pseudovector potentials $\mathbf{A}_i = \eta_i \mathbf{A} = \eta_i \tau_v A_0(u_{yy} - u_{xx}, 2u_{xy})$ [20], where η_i are characteristic parameters of the TMD material, and the components of the strain tensor read

$$u_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \quad (2)$$

The total Hamiltonian including the carrier-strain interaction reads $H = H_0(\mathbf{p} - \mathbf{A}_1) + (\alpha|\mathbf{p} - \mathbf{A}_2|^2 + \beta|\mathbf{p} - \mathbf{A}_3|^2\sigma_z)/4m_0$, where α and β are band parameters of a TMD monolayer [20]. In matrix form,

$$H = \begin{pmatrix} \frac{\Delta}{2} + \Xi_{\mathbf{p}}^c & h_{\mathbf{p}} \\ h_{\mathbf{p}}^* & -\frac{\Delta}{2} + \Xi_{\mathbf{p}}^v \end{pmatrix}, \quad (3)$$

where $h_{\mathbf{p}} = v(p - A_1)_- + \mu(p - A_1)_+^2$ is the off-diagonal part of the system Hamiltonian, responsible for the interband processes and the trigonal warping of the valleys, and $(p - A_1)_\pm = \tau_v(p_x - (A_1)_x) \pm i(p_y - (A_1)_y)$; $\Xi_{\mathbf{p}}^{c,v} = (\alpha|\mathbf{p} - \mathbf{A}_2|^2 \pm \beta|\mathbf{p} - \mathbf{A}_3|^2)/4m_0$ with m_0 the free-electron mass. The energy eigenvalues of the Hamiltonian (3) are

$$\begin{aligned} E_{c,v} &= \frac{\Xi_c + \Xi_v}{2} \pm E_{\mathbf{p}}, \\ E_{\mathbf{p}} &= \sqrt{\left(\frac{\Delta + \Xi_c - \Xi_v}{2}\right)^2 + |h_{\mathbf{p}}|^2}, \\ |h_{\mathbf{p}}|^2 &= v^2|\mathbf{p} - \mathbf{A}_1|^2 + \mu^2|\mathbf{p} - \mathbf{A}_1|^4 \\ &\quad + v\mu[(\mathbf{p} - \mathbf{A}_1)_-^3 + (\mathbf{p} - \mathbf{A}_1)_+^3], \end{aligned} \quad (4)$$

and the wave functions read

$$\begin{aligned} \psi_c(\mathbf{r}) &= \begin{pmatrix} \cos\left(\frac{\theta_{\mathbf{p}}}{2}\right) \\ \sin\left(\frac{\theta_{\mathbf{p}}}{2}\right)\frac{h_{\mathbf{p}}^*}{|h_{\mathbf{p}}|} \end{pmatrix} \frac{e^{i\mathbf{pr}}}{\sqrt{S}}, \\ \psi_v(\mathbf{r}) &= \begin{pmatrix} \sin\left(\frac{\theta_{\mathbf{p}}}{2}\right) \\ -\cos\left(\frac{\theta_{\mathbf{p}}}{2}\right)\frac{h_{\mathbf{p}}^*}{|h_{\mathbf{p}}|} \end{pmatrix} \frac{e^{i\mathbf{pr}}}{\sqrt{S}}, \end{aligned} \quad (5)$$

where $\cos\theta_{\mathbf{p}} = (\Delta + \Xi_c - \Xi_v)/2E_{\mathbf{p}}$, S is a sample area, and the subscripts c and v denote conduction and valence bands, correspondingly.

From Eq. (4) it becomes clear why it is important to take into account the deformation of higher-energy bands beyond the two-band model. Indeed, in the absence of higher-energy band terms $\alpha = \beta = 0$, and hence $\Xi_c = \Xi_v = 0$, the electron energy only contains the deformation in the form of a shift $\mathbf{p} \to \mathbf{p} - \mathbf{A}_1$. Thus, the valleys shift as a whole without a change of their form. Such a shift does not influence the intravalley scattering processes and kinematic electron characteristics, such as its dispersion, velocity, etc., resulting in the absence of PRE. Therefore, the account of the higher-energy band deformation ($\alpha, \beta \neq 0$) is of crucial importance for the phenomena in question.

The energy dispersion (4) can be simplified assuming the inequality $\Delta \gg |h_{\mathbf{p}}|$, which is legitimate due to the large value of the gap in MoS_2, $\Delta \sim 2$ eV, and that the other characteristic energies in the system are usually much smaller than the gap [21,22].

Furthermore, counting the electron energy in the conduction band from its bottom and disregarding the terms proportional to p^4, we find for the conduction band $E_c = \varepsilon_{\mathbf{p}}$, where

$$\begin{aligned} \varepsilon_{\mathbf{p}} &= \frac{1}{4m_0}(\alpha|\mathbf{p} - \mathbf{A}_2|^2 + \beta|\mathbf{p} - \mathbf{A}_3|^2) \\ &\quad + \frac{1}{\Delta}[v^2|\mathbf{p} - \mathbf{A}_1|^2 + v\mu((\mathbf{p} - \mathbf{A}_1)_-^3 + (\mathbf{p} - \mathbf{A}_1)_+^3)]. \end{aligned} \quad (6)$$

In what follows we are only interested in the linear-in-strain tensor u_{ij} contribution to the conductivity of the system since we assume a relatively small deformation, as it is commonly supposed in works on strain-induced effects (see, e.g., Ref. [1]). In this case, it is instructive to make a shift in the reciprocal space, $\mathbf{p} - e\mathbf{A} \to \mathbf{p}$ in expressions (5) and (6).

Keeping only the linear-in-$\mathbf{A}$ terms gives the electron dispersion in a given valley,

$$\varepsilon_{\mathbf{p}} \approx \frac{p^2}{2m} + \gamma(\mathbf{A} \cdot \boldsymbol{\nabla}_{\mathbf{p}})W(\mathbf{p}), \quad (7)$$

where $W(\mathbf{p}) = \tau_v W(p_x^3 - 3p_x p_y^2)$ [21], $\gamma = (\alpha\eta_2 m + \beta\eta_3 m)/(2m_0)$, $(2m)^{-1} = (\alpha\eta_2 + \beta\eta_3)/(2m_0) + v^2/\Delta \approx v^2/\Delta$, and $W = 2v\mu/\Delta$. The wave function of an electron in the conduction band (in the limit $\tilde{\Delta} \gg |h_{\mathbf{p}}|$) reads

$$\psi_c(\mathbf{r}) = \frac{1}{\Delta}\begin{pmatrix} \Delta \\ h_{\mathbf{p}+\gamma\mathbf{A}}^* \end{pmatrix} \frac{e^{i(\mathbf{p}+\gamma_0\mathbf{A})\mathbf{r}}}{\sqrt{S}}, \quad (8)$$

where $\gamma_0 = (\alpha\eta_2 m + \beta\eta_3 m)/(2m_0) + \eta_1$.

Drude piezoconductivity. We consider a degenerate electron gas statistics characterized by the Fermi energy E_F. It is

placed in a 2D monolayer sample illuminated by an external EM field with the frequency ω, which is much smaller than both the band gap of the material, $\omega \ll \Delta$, and the Fermi energy, $\omega \ll E_F$ (here and below we count the conduction-band energy from its bottom). These approximations allow us to use the Boltzmann transport equation [23,24] to analyze PRE,

$$\frac{\partial f}{\partial t} + [\mathbf{F}(t)\cdot\nabla_{\mathbf{p}}]f + \hat{Q}\{f\} = \mathbf{0}, \tag{9}$$

where $\mathbf{F}(t) = e\mathbf{E}(t)$ is a force acting on the electrons in the uniform alternating electric field $\mathbf{E}(t) = \mathbf{E}e^{-i\omega t}$, and $\hat{Q}$ is the electron-impurity collision integral.

Electron scattering on short-range impurities. First, let us consider the case of electron scattering on short-range defects (later, we will consider the generalization of these processes: electron scattering on long-range Coulomb centers). The first-order solution of Eq. (9) with respect to the external EM field, which determines the ac current density, reads

$$j_\alpha = -e\int \frac{d\mathbf{p}}{(2\pi\hbar)^2} v_\alpha(-i\omega + \hat{Q})^{-1}(\mathbf{F}\cdot\nabla_{\mathbf{p}})f_{\mathbf{p}}, \tag{10}$$

where v_α is the electron velocity, and $f_{\mathbf{p}}$ is the equilibrium (in the presence of strain) electron distribution function. For an arbitrary function $\chi_{\mathbf{p}}$, the collision integral can be found from the Fermi golden rule,

$$\hat{Q}\chi_{\mathbf{p}} = \frac{2\pi n_i}{\hbar}\sum_{\mathbf{p}'} |M_{\mathbf{p}\mathbf{p}'}|^2\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})(\chi_{\mathbf{p}} - \chi_{\mathbf{p}'}), \tag{11}$$

where n_i is the density of impurities, and $M_{\mathbf{p}\mathbf{p}'}$ is the intraband scattering matrix element of the impurity potential over the wave functions Eq. (5). The functions v_α, $f_{\mathbf{p}}$, and $\hat{Q}$ include strain-induced corrections. Thus, all these quantities should be expanded up to linear order with respect to $\mathbf{A}$.

The strain-induced correction to the collision operator consists of two contributions. The first one originates from the strain-induced modification of the electron dispersion (7), and the second one is due to the correction to the scattering matrix elements coming from the strain-induced additions to the electron wave functions (8). Denoting the bare collision operator as $\hat{Q}_0$ and the strain-induced corrections as $\hat{Q}_1$, we can expand the collision operator up to the first order with respect to $\hat{Q}_1$ as

$$(-i\omega + \hat{Q}_0 + \hat{Q}_1)^{-1} \approx (-i\omega + \hat{Q}_0)^{-1} - (-i\omega + \hat{Q}_0)^{-1}\hat{Q}_1(-i\omega + \hat{Q}_0)^{-1}. \tag{12}$$

The action of bare collision operator (in the case of short-range impurities) on the nth harmonic of the distribution function gives

$$(-i\omega + \hat{Q}_0)^{-1}e^{in\varphi} = e^{in\varphi}(-i\omega + 1/\tau_n)^{-1},$$
$$\frac{1}{\tau_n} = \frac{2\pi n_i u_0^2}{\hbar}\sum_{\mathbf{p}'}\delta\left(\varepsilon^0_{\mathbf{p}} - \varepsilon^0_{\mathbf{p}'}\right)(1-\cos n\theta), \tag{13}$$

where u_0 is a strength of the short-range impurity potential.

Furthermore, the action of the first-order correction $\hat{Q}_1$ (for the short-range impurities) can be found from a straightforward derivation [25],

$$\hat{Q}_1\{\chi_{\mathbf{p}}\} = \frac{2\pi n_i u_0^2}{\hbar}\sum_{\mathbf{p}'}(\delta\varepsilon_{\mathbf{p}} - \delta\varepsilon_{\mathbf{p}'})\delta'\left(\varepsilon^0_{\mathbf{p}} - \varepsilon^0_{\mathbf{p}'}\right)(\chi_{\mathbf{p}} - \chi_{\mathbf{p}'}) + \frac{2\pi n_i}{\hbar}\sum_{\mathbf{p}'} D_{\mathbf{p}\mathbf{p}'}\delta\left(\varepsilon^0_{\mathbf{p}} - \varepsilon^0_{\mathbf{p}'}\right)(\chi_{\mathbf{p}} - \chi_{\mathbf{p}'}). \tag{14}$$

Here, $\varepsilon^0_{\mathbf{p}} = \mathbf{p}^2/2m$ is a bare electron energy, and $\delta\varepsilon_{\mathbf{p}} = \gamma(\mathbf{A}\cdot\nabla_{\mathbf{p}})W(\mathbf{p})$ is a strain-induced correction [see also Eq. (7)]. In Eq. (14), $D_{\mathbf{p}\mathbf{p}'}$ is a strain-induced correction to the scattering matrix element induced by the wave-function renormalization in the presence of strain (8).

Now, the strain-induced correction to the current density can be written as a sum of two terms. The first term describes the correction to the current density due to the electron kinematic quantities, such as velocity δv_α ($\delta\mathbf{v} = \nabla_{\mathbf{p}}\delta\varepsilon_{\mathbf{p}}$) and energy $\delta\varepsilon_{\mathbf{p}}$,

$$j^{\mathrm{I}}_\alpha = -e\int \frac{d\mathbf{p}}{(2\pi\hbar)^2}[\delta v_\alpha(-i\omega + \hat{Q}_0)^{-1}(\mathbf{F}\cdot\nabla_{\mathbf{p}})f_0 + v^0_\alpha(-i\omega + \hat{Q}_0)^{-1}(\mathbf{F}\cdot\nabla_{\mathbf{p}})(\delta\varepsilon_{\mathbf{p}}f_0')]. \tag{15}$$

The second term originates from the corrections to the scattering integral (14),

$$j^{\mathrm{II}}_\alpha = e\int \frac{d\mathbf{p}}{(2\pi\hbar)^2} v^0_\alpha(-i\omega + \hat{Q}_0)^{-1} \times\hat{Q}_1(-i\omega + \hat{Q}_0)^{-1}(\mathbf{F}\cdot\nabla_{\mathbf{p}})f_0. \tag{16}$$

From Eq. (15) we can derive the general expression for the strain-induced correction to the conductivity,

$$\frac{\delta\sigma_{\alpha\beta}}{\sigma_0} = \lambda\begin{pmatrix} u_{xx} - u_{yy} & u_{xy} + u_{yx} \\ u_{xy} + u_{yx} & -(u_{xx} - u_{yy}) \end{pmatrix}, \tag{17}$$

where the coefficient λ determines PRE. From the same Eq. (15) we find

$$\lambda^{\mathrm{I}} = 12\gamma W m A_0\left(1 - \frac{1}{4}\frac{\langle(\varepsilon^2\tau_{1\omega})''\rangle}{\langle(\varepsilon\tau_{1\omega})'\rangle}\right), \tag{18}$$

where the primes stand for the derivative with respect to electron bare energy $\varepsilon \equiv \varepsilon^0_{\mathbf{p}}$, and $\tau_{n\omega} = \tau_n(1 - i\omega\tau_n)^{-1}$. The energy distribution averaging in Eq. (18) is defined as

$$\langle X\rangle = \int_0^\infty d\varepsilon g(\varepsilon)X(\varepsilon)f_0(\varepsilon)\Big/\int_0^\infty d\varepsilon g(\varepsilon)f_0(\varepsilon), \tag{19}$$

where $g(\varepsilon) = \nu m/2\pi\hbar^2$ is a 2D density of states and $\nu = 4$ is the electron degeneracy factor accounting for the valley and spin degrees of freedom.

The coefficient λ^{II} is determined by the integrals in Eq. (16) with $\hat{Q}_1$ taken from Eq. (14). For short-range impurities, the first term in Eq. (14) vanishes, and the second term is determined by the lower component of the wave function (8). This component has an additional smallness $vp_F/\Delta \ll 1$ as compared with Eq. (18). Therefore, we can disregard λ^{II} as it is much smaller than λ^{I}.

It should be noted that when deriving Eq. (18), we assumed an arbitrary dependence $\tau_{n\omega}(\varepsilon)$ for generality. In the case of

short-range impurities, the scattering time τ_1 does not depend on energy in 2D systems with a parabolic dispersion. In this case, Eq. (18) can be simplified,

$$\lambda^{\mathrm{I}} \approx 6\gamma W m A_0. \tag{20}$$

Let us make an estimation of this parameter. Using $\eta_2 = -5.655$, $\eta_3 = 1.633$, $\alpha = -0.01$, $\beta = -1.54$ [20], and $m = 0.5\,m_0$, we find $\gamma \approx -0.6$. Taking the warping $W \approx -3.5$ eV $\mathrm{A}^3/\hbar^3$ [21] and $A_0 = \hbar/a_0$, where $a_0 = a/\sqrt{3}$ with the MoS_2 lattice constant $a = 0.315$ nm [26], we find $\lambda^{\mathrm{I}} \sim 1$. Then, from Eq. (17) we conclude that the relative correction to conductivity is of the order of relative deformation. For a single-layer MoS_2, the breaking deformation is about 10% [27,28]. Thus, the upper bound is $\delta\sigma_{\alpha\beta} \propto 0.1\sigma^0$.

Electron scattering on Coulomb impurities. In the case of Coulomb impurities, the general expressions derived above still hold, but the matrix element of the impurity potential is different, $u_0 \to u(\mathbf{p} - \mathbf{p}')$ [29,30]. It results in an energy dependence of the first-harmonic relaxation time $\tau_1(\varepsilon)$. Another difference is in the integrals in Eq. (14). The second integral in Eq. (14) is still small as it is proportional to Δ^{-1}, and can be neglected. The first term in Eq. (14) describes the relaxation time correction due to the strain-induced renormalization of electron dispersion.

The direct calculation of the first term for the case of Coulomb impurities is cumbersome. To simplify the derivations, we can use the ansatz suggested in Ref. [1]. The key idea here is to use an energy-dependent expression for the relaxation time $\tau(\varepsilon)$ derived for the isotropic electron energy for the case of arbitrary electron dispersion $\varepsilon_{\mathbf{p}}$. Following this approach, we use $\tau_{1\omega}(\varepsilon_{\mathbf{p}})$ with the electron energy including the deformation, Eq. (7). Expanding $\tau_{1\omega}(\varepsilon_{\mathbf{p}}) \approx \tau_{1\omega}(\varepsilon^0_{\mathbf{p}}) + \tau'_{1\omega}(\varepsilon^0_{\mathbf{p}})\delta\varepsilon_{\mathbf{p}}$ yields Eq. (18) for $\tau_{1\omega}(\varepsilon^0_{\mathbf{p}})$ with $\tau_{1\omega} \to \tau_{1\omega}(\varepsilon^0_{\mathbf{p}})$, whereas the scattering time correction $\tau'_{1\omega}(\varepsilon^0_{\mathbf{p}})\delta\varepsilon_{\mathbf{p}}$ gives an additional contribution to the current density,

$$j^{\mathrm{III}}_{\alpha} = -e\int \frac{d\mathbf{p}}{(2\pi\hbar)^2} v_\alpha \tau'_{1\omega}\delta\varepsilon_{\mathbf{p}}(\mathbf{F}\cdot\nabla_{\mathbf{p}})f_0. \tag{21}$$

Taking this integral and using Eq. (18) yields

$$\lambda = 12\gamma W m A_0\left(1 - \frac{\langle(\varepsilon^2\tau_{1\omega})''\rangle - \langle(\varepsilon^2\tau'_{1\omega})'\rangle}{4\langle(\varepsilon\tau_{1\omega})'\rangle}\right) \tag{22}$$

for an arbitrary dependence of the first-harmonic relaxation time on bare electron energy. Since $\langle(\varepsilon^2\tau_{1\omega})''\rangle - \langle(\varepsilon^2\tau'_{1\omega})'\rangle = 2\langle(\varepsilon\tau_{1\omega})'\rangle$, Eq. (22) resembles Eq. (20). Hence, formula (20) is universal, and it represents a key analytical result of this Letter.

Strain-induced magnetoresistance and the Hall effect. In this section, we consider the Hall effect and strain-induced magnetoresistance in the case of weak classical magnetic fields, when $\omega_c\tau \ll 1$ and in the static limit of the external electric field, $\omega \to 0$. In the presence of an external magnetic field $\mathbf{B}$, the expression for the electric current density acquires a linear-in-$\mathbf{B}$ contribution,

$$j_\alpha = e^2\int \frac{d\mathbf{p}}{(2\pi\hbar)^2} v_\alpha Q^{-1}([\mathbf{v}\times\mathbf{B}]\cdot\nabla_{\mathbf{p}})Q^{-1}(\mathbf{F}\cdot\nabla_{\mathbf{p}})f_{\mathbf{p}}. \tag{23}$$

Assuming $\mathbf{B} = (0, 0, B)$ and $\mathbf{E} = (E_x, 0, 0)$, and following the same steps of the derivation as it was done above, we find that the correction to the relaxation time $\tau'_1(\varepsilon^0_{\mathbf{p}})\delta\varepsilon_{\mathbf{p}}$ gives a vanishing contribution to the current density in the presence of strain. The remaining part stemming from $\tau_1(\varepsilon^0_{\mathbf{p}})$ gives

$$\frac{\delta\sigma_{xx}}{\sigma^0} = 3\gamma\tau_v W A_y m\omega_c \times \frac{\langle[\varepsilon^2(\tau^2)']'\rangle + 4\langle(\varepsilon\tau^2)'\rangle - 2\langle(\varepsilon^2\tau^2)''\rangle}{\langle(\varepsilon\tau)'\rangle} \tag{24}$$

for the longitudinal part of the conductivity (magnetoresistance), whereas the strain-induced correction to the Hall effect reads

$$\frac{\delta\sigma_{yx}}{\sigma^0} = 3\gamma\tau_v W A_x m\omega_c \frac{\langle[\varepsilon^2(\tau^2)']'\rangle + 4\langle(\varepsilon\tau^2)'\rangle}{\langle(\varepsilon\tau)'\rangle}, \tag{25}$$

where $\sigma_0 = e^2 N\langle(\varepsilon\tau)'\rangle/m$ is a Drude conductivity, $A_x = \tau_v A_0(u_{yy} - u_{xx})$, and $A_y = 2\tau_v A_0 u_{xy}$. Obviously, strain results in the emergence of a linear-in-$\mathbf{B}$ magnetoresistance effect, whereas in the absence of strain, magnetoresistance behaves as $\propto \omega_c^2$ in the case of a classically weak magnetic field. Since the fraction in Eq. (25) is proportional to τ with some coefficient of the order of unity, and $\omega_c\tau \ll 1$, then as long as $\lambda \sim 1$, the contribution of the $\delta\sigma_{yx}$ term under uniaxial strain $u_{xx} \neq 0$ can be roughly estimated as $\delta\sigma_{yx} \sim \sigma^0(\omega_c\tau)u_{xx}$.

Another specific feature of this effect is that it exists only if the electron-impurity scattering time depends on the electron energy. Indeed, Eq. (24) gives zero if τ is constant. It holds for the case of short-range impurity scattering in 2D systems with parabolic band dispersion. Hence, the magnetoresistance effect described by Eq. (24) may occur for the dominating short-range impurity electron scattering mechanism only if we take into account the nonparabolicity of the electron bands in the electron-impurity scattering probability and the energy dependence of the electron density of states.

In the case of electron scattering on Coulomb impurities (as a dominant mechanism), the momentum relaxation time in 2D systems does depend on energy even for a simple parabolic band. Thus, the linear magnetoresistance [Eq. (24)] effect occurs.

If both mechanisms are present and we denote as $\tau_c(\varepsilon)$ the momentum relaxation time describing the electron scattering on Coulomb centers, then the total scattering time reads $\tau^{-1}(\varepsilon) = \tau_i^{-1} + \tau_c^{-1}(\varepsilon)$. It depends on energy even if short-range impurity time τ_i is a constant. Thus, in the case of a combined action of two impurity scattering mechanisms, linear strain-induced magnetoresistance occurs.

Conclusions. We developed an analytical theoretical description of the piezoresistive effect for degenerate electron gas in two-dimensional Dirac materials under the action of a uniform strain both in the absence and presence of a weak classical magnetic field. We derived analytical expressions for the conductivity correction coefficients, Eqs. (22), (24), and (25), for an arbitrary dependence of the electron momentum scattering time on energy. We showed that for a degenerate electron gas, strain-induced corrections to the conductivity stem from the combined action of warping of electron

dispersion and the emerging corrections to the higher-energy electron bands.

We want to note that the effects in question are present even at room temperatures. Moreover, the developed theory is general: It can describe not only scattering on impurities but also other scattering mechanisms, characterized by the electron momentum relaxation time $\tau(\varepsilon)$.

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