Letter

Particle current statistics in driven mesoscale conductors

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We propose a highly scalable method to compute the statistics of charge transfer in driven conductors. The framework can be applied in situations of nonzero temperature, strong coupling to terminals, and in the presence of nonperiodic light-matter interactions, away from equilibrium. The approach combines the so-called mesoscopic leads formalism with full counting statistics. It results in a generalized quantum master equation that dictates the dynamics of current fluctuations and higher order moments of the probability distribution function of charge exchange. For generic time-dependent quadratic Hamiltonians, we provide closed-form expressions for computing noise in the nonperturbative regime of the parameters of the system, reservoir, or system-reservoir interactions. Having access to the full dynamics of the current and its noise, the method allows us to compute the variance of charge transfer over time in nonequilibrium configurations. The dynamics reveal that in driven systems, the average noise should be defined operationally with care over which period of time is covered.

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Current fluctuations are inherent to out-of-equilibrium mesoscopic devices operating in the quantum regime [1–5]. Their categorization and quantification is relevant to the understanding of fundamental thermodynamics as well as the operation of quantum thermal machines [6–9]. Recent experimental advances include nonperiodic modulation in light-induced currents [10,11] and the control of the system-reservoir interactions in superconducting circuits [12] as well as single-molecule junctions [13,14]. These advances call for methodologies that allow one to cope with the effects of these physical properties at finite-temperature to understand fluctuations in their regimes of operation.

Most of the existing methods for computing current fluctuations, however, are only applicable in restricted regimes of operation. When solely coherent quantum effects are important and there is no time-dependence in the Hamiltonian, the *Levitov-Lesovik approach* [15,16], which extends the *Landauer scattering theory* [17], provides nonperturbative exact results. *Green's function techniques* can be formulated to treat strong system-reservoir coupling [18]. However, to include in this approach either a time-periodic drive or incoherent effects arising from many-body interactions typically requires treatments via nonequilibrium Green's functions [19–22]. These methods are perturbative either in the Hamiltonian parameters or the drive parameters and naturally cannot be applied to cases lacking a perturbative parameter, such as when applying a strong nonperiodic drive on the nanoscale conductor.

If the system-reservoir coupling energy is weak, *quantum master equations* (QMEs) offer an alternative, flexible route to evaluate both average currents and their fluctuations [1,23–26].

While for small systems, QME methods can handle many-body interactions in a nonperturbative manner, these methods are fundamentally limited in their ability to accurately and consistently describe the system's quantum state [27]. Furthermore, it has been recently argued that an appropriate thermodynamic description at the *fluctuating* level may only be obtained after applying the secular approximation on the *Redfield QME* [28].

We introduce a novel method that allows for the nonperturbative characterization of current fluctuations in out-of-equilibrium configurations for arbitrarily driven systems, overcoming the aforementioned limitations. Our scalable method combines a *full-counting statistics* (FCS) treatment [1] with the so-called *mesoscopic leads description* [29–32] and brings together advantages of both approaches. Mesoscopic leads build the reservoirs by a finite collection of fermionic modes, each of which is subject to damping, intended to bring the discrete modes of the bath to their equilibrium state with respect to a fixed temperature and chemical potential. In its basic form, the mesoscopic leads approach has been shown to build the correct thermodynamic state [33–40] and it has been adopted to study noninteracting [41–44],

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periodically driven [40,45,46], and impurity [47–49] models, as well as thermal machines [39] in the strongly interacting, finite-temperature and strong system-reservoir coupling regimes, away from equilibrium. The method presented in this work bridges and combines in a nontrivial way two established but separate frameworks, namely the mesoscopic leads approach and the FCS, yielding the charge current and its fluctuations for arbitrary system-reservoir coupling strength, temperature, bias-voltage and time-dependent driving fields. Further, in the case of Gaussian time-dependent quantum systems, our framework leads to elegant expressions for the instantaneous dynamics of the currents and its noise. Studying as an example a periodically driven system, we compute the instantaneous charge current noise. We reveal the subtle nature of fluctuations under driving far away from equilibrium with the noise showing crucial dependency on the time interval under investigation.

Mesoscopic reservoirs. We consider a fermionic system S described by a set of L annihilation operators $\{\hat{c}_j\}$ and a Hamiltonian $\hat{H}_{\mathrm{S}}(t)$, possibly interacting and driven. The system is coupled to Q fermionic reservoirs, each modelled by a set of operators $\{\hat{b}_{n,\alpha}\}$, Hamiltonians $\hat{H}_{\mathrm{B},\alpha} = \sum_{n=1}^{\infty} \omega_{n,\alpha} \hat{b}_{n,\alpha}^{\dagger} \hat{b}_{n,\alpha}$ (we set $\hbar = 1, k_{\mathrm{B}} = 1$) and prepared in grand-canonical states at temperatures T_α and chemical potentials μ_α. Each reservoir α is assumed to couple to a specific system operator $\hat{c}_{p_\alpha}$ via $\hat{H}_{\mathrm{SB}_\alpha} = \sum_{n=1}^{\infty} \lambda_{n,\alpha} (\hat{c}_{p_\alpha}^{\dagger} \hat{b}_{n,\alpha} + \hat{b}_{n,\alpha}^{\dagger} \hat{c}_{p_\alpha})$, which is not necessarily weak. The corresponding bath spectral densities are $\mathcal{J}_\alpha(\omega) = 2\pi \sum_{n=1}^{\infty} |\lambda_{n,\alpha}|^2 \delta(\omega - \omega_{n,\alpha})$. The combination of time-dependent drives and/or interactions in $t \mapsto \hat{H}_{\mathrm{S}}(t)$, together with strong couplings between the system and the fermionic baths, makes the above problem notoriously difficult to handle.

The mesoscopic leads approach has been successful in this regard [29–31,39,40,45,50,51]. Here, each reservoir α is mapped into a *finite* set of N_α *lead modes* $\{\hat{a}_{k,\alpha}\}$, $k = 1, \dots, N_\alpha$, each of which is coupled to a residual reservoir, as depicted in Fig. 1. The method is designed so as it converges to the true dynamics when $N_\alpha \to \infty$.

The Hamiltonian of the leads reads $\hat{H}_{\mathrm{L}} = \sum_\alpha \sum_{k=1}^{N_\alpha} \varepsilon_{k,\alpha} \hat{a}_{k,\alpha}^{\dagger} \hat{a}_{k,\alpha}$, with each lead mode $\hat{a}_{k,\alpha}$ assigned an energy $\varepsilon_{k,\alpha}$, designed to homogeneously sample the spectral bandwidth of $\hat{H}_{\mathrm{B}_\alpha}$. Moreover, S only interacts with the lead modes, and not their residual reservoirs. It follows that $\hat{H}_{\mathrm{SB}_\alpha} \mapsto \hat{H}_{\mathrm{SL}_\alpha} = \sum_{k=1}^{N_\alpha} \kappa_{k,\alpha} (\hat{c}_{p_\alpha}^{\dagger} \hat{a}_{k,\alpha} + \hat{a}_{k,\alpha}^{\dagger} \hat{c}_{p_\alpha})$, with new coupling strengths $\kappa_{k,\alpha} = \sqrt{\mathcal{J}_\alpha(\varepsilon_{k,\alpha}) \gamma_{k,\alpha}/(2\pi)}$, where $\gamma_{k,\alpha} = \varepsilon_{k+1,\alpha} - \varepsilon_{k,\alpha}$ will be small whenever N_α is large. Crucially, via this mapping the residual environment of each lead mode has a flat spectral density, governed by $\gamma_{k,\alpha}$ [30,31]. Thus, even if the original SB coupling is not weak, the coupling of the lead modes to their residual baths becomes small, provided N_α is sufficiently large.[1] This condition allows one to trace out the residual environments and obtain a master equation for the joint system-state $\hat{\rho}_{\mathrm{SL}}$.

Full counting statistics. The mesoscopic leads approach only gives access to average currents through continuity equations. Our goal is to take this method a step further and construct the full probability distribution of charge fluctuations. Letting $I_\nu(t)$ denote the stochastic charge current to reservoir ν and $N_\nu(t, t_0) = \int_{t_0}^{t} dt'\, I_\nu(t')$ the corresponding integrated (net) charge in the interval $[t_0, t]$, our interest will be on the probability $P(n, t, t_0) = P(N_\nu(t, t_0) = n)$. We have that [1]

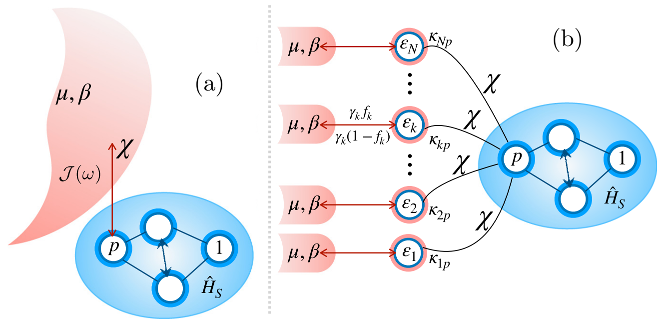

FIG. 1. Mesoscopic lead description of an open quantum system. (a) A depiction of an infinite bath at temperature T and chemical potential μ with spectral density function $\mathcal{J}(\omega)$ coupled locally to the pth fermionic site of a system. (b) The bath is discretized by a finite collection of N fermionic modes with self-energies ε_k, which are coupled locally to the pth site of the system with strength $\kappa_{k,p}$. Each of the modes is subject to dissipation intended to drive the mode to thermal and chemical equilibrium state. In (a), FCS is performed with a counting field χ *embedded in the reservoir*. In contrast, in (b) the counting fields turn up in the *internal* system-modes couplings.

$$P(n, t, t_0) = \int_{-\pi}^{\pi} \frac{d\chi}{2\pi} e^{-in\chi} G(\chi, t, t_0). \tag{1}$$

As one of our main results, we show in Ref. [52] that $G(\chi, t, t_0) := \mathrm{Tr}[\hat{\rho}_{\mathrm{SL}}(\chi, t, t_0)]$ and $\hat{\rho}_{\mathrm{SL}}(\chi, t, t_0)$ satisfies the generalized master equation $\frac{d}{dt} \hat{\rho}_{\mathrm{SL}}(\chi, t, t_0) = \mathcal{L}_\chi(t) \hat{\rho}_{\mathrm{SL}}(\chi, t, t_0)$, with the tilted Liouvillian

$$\mathcal{L}_\chi(t)\hat{\rho} = -\mathrm{i}\big[\hat{H}_{\mathrm{S}}(t) + \hat{H}_{\mathrm{L}} + \hat{H}_{\mathrm{SL}}^{\chi}, \hat{\rho}\big]_\chi + \sum_\alpha \mathcal{D}_\alpha \hat{\rho}. \tag{2}$$

Here, χ is the counting field, $[\hat{A}_\chi, \hat{B}]_\chi := \hat{A}_\chi \hat{B} - \hat{B}\hat{A}_{-\chi}$,

$$\hat{H}_{\mathrm{SL}}^{\chi} = \sum_{\alpha=1}^{Q} \sum_{k=1}^{N_\alpha} \kappa_{k,\alpha} \big(\hat{c}_{p_\alpha}^{\dagger} \hat{a}_{k,\alpha}\, e^{-i\chi\delta_{\alpha,\nu}/2} + \hat{a}_{k,\alpha}^{\dagger} \hat{c}_{p_\alpha}\, e^{i\chi\delta_{\alpha,\nu}/2} \big), \tag{3}$$

and

$$\mathcal{D}_\alpha \hat{\rho} = \sum_{k=1}^{N_\alpha} \gamma_{k,\alpha}(1 - f_{k,\alpha}) \left[\hat{a}_{k,\alpha} \hat{\rho} \hat{a}_{k,\alpha}^{\dagger} - \frac{1}{2} \{ \hat{a}_{k,\alpha}^{\dagger} \hat{a}_{k,\alpha}, \hat{\rho} \} \right] + \sum_{k=1}^{N_\alpha} \gamma_{k,\alpha} f_{k,\alpha} \left[\hat{a}_{k,\alpha}^{\dagger} \hat{\rho} \hat{a}_{k,\alpha} - \frac{1}{2} \{ \hat{a}_{k,\alpha} \hat{a}_{k,\alpha}^{\dagger}, \hat{\rho} \} \right]. \tag{4}$$

The Lindblad dissipators $\mathcal{D}_\alpha$ are generators of quantum dynamical semi-groups: It is important to note that in this picture, they are made time-independent. They act only locally on the individual lead modes $\hat{a}_{k,\alpha}$, with strength $\gamma_{k,\alpha}$ and Fermi-Dirac occupation $f_{k,\alpha} := (e^{(\varepsilon_{k,\alpha} - \mu_\alpha)/T_\alpha} + 1)^{-1}$. Setting $\chi = 0$, one recovers the traditional mesoscopic leads master equation [39]. With $\mathcal{L}_\chi$, however, we now have access to the full $P(n, t, t_0)$. Note that we have included an explicit

[1]More quantitatively, the condition is that $\gamma_{k,\alpha}$ needs to remain the smallest energy scale in the problem [50,51].

dependence on the initial condition at t_0. As we shall see, it is important to keep track of this argument to evaluate charge statistics in systems with an explicit time-dependent Hamiltonian. The counting field χ specifies which physical process we are monitoring. Charge transport is usually associated with quantum jumps in the master equation, with χ placed in the terms $\hat{a}_{k,\alpha}\hat{\rho}\hat{a}^\dagger_{k,\alpha}$ and $\hat{a}^\dagger_{k,\alpha}\hat{\rho}\hat{a}_{k,\alpha}$ of Eq. (4). Instead, a crucial aspect of our result (2) is that χ is placed in the unitary system-leads interactions, $\hat{c}^\dagger_p\hat{a}_{k,\alpha}$ and $\hat{a}^\dagger_{k,\alpha}\hat{c}_p$. This is a consequence of the mapping, which implies that the exchange of particles between S and B is mapped to an exchange between S and the lead modes $\hat{a}_{k,\alpha}$. In nondriven systems at steady state, such a distinction is immaterial. However, for driven systems, and during transients, it is crucial.

We note that the proposed scheme is based on the two-point measurement protocol FCS [1] which can be justified with the assumption that initial total density matrix is a product state of system and environment states (see Ref. [52] for further details).

Noise. The average current, $J_\nu(t) := \langle I_\nu(t)\rangle = \frac{d}{dt}\langle N_\nu(t,t_0)\rangle$ is given by [39,40]

$$J_\nu(t) = \mathrm{i}\sum_{k=1}^{N_\nu}\kappa_{k,\nu}\mathrm{Tr}\{(\hat{c}^\dagger_{p_\nu}\hat{a}_{k,\nu} - \hat{a}^\dagger_{k,\nu}\hat{c}_{p_\nu})\hat{\rho}_{\mathrm{SL}}(\chi = 0, t, t_0)\}, \tag{5}$$

and therefore does not require the tilted dynamics. For all higher order moments, however, $\mathcal{L}_\chi$ is required. Here, we focus on the charge variance $\mathrm{var}[N(t,t_0)] := \langle N^2_\nu(t,t_0)\rangle - \langle N_\nu(t,t_0)\rangle^2$ or, more conveniently, the *noise*

$$D_\nu(t,t_0) := \frac{d}{dt}\mathrm{var}[N_\nu(t,t_0)] = 2\int_{t_0}^{t} dt'\,\langle \delta I_\nu(t)\delta I_\nu(t')\rangle, \tag{6}$$

where $\delta I_\nu(t) = I_\nu(t) - J_\nu(t)$ and the last equality follows from $N_\nu(t,t_0) = \int_{t_0}^t dt'\,I_\nu(t')$.

A major advantage of our approach is the ability to describe arbitrary drives and transient dynamics. In such cases, it is crucial to note that while $J_\nu(t)$ is an instantaneous quantity, $D_\nu(t,t_0)$ depends on the time interval $[t_0,t]$ in question. At the stochastic level, the charge is additive as $N_\nu(t_2,t_0) = N_\nu(t_2,t_1) + N_\nu(t_1,t_0), \forall t_2 > t_1 > t_0$. In contrast, the variance *is not additive* since $\mathrm{var}(A+B) = \mathrm{var}(A) + \mathrm{var}(B) + 2\mathrm{cov}(A,B)$. Equation (6) thus yields

$$D_\nu(t_2,t_0) = D_\nu(t_2,t_1) + 2\frac{d}{dt_2}\mathrm{cov}[N_\nu(t_2,t_1), N_\nu(t_1,t_0)], \tag{7}$$

which shows a dependence on the correlation between the transferred charge at different intervals. For systems with autonomous steady states, it suffices to work with $\lim_{t\to\infty} D_\nu(t,t_0)$, and no such subtlety arises. However, this is not the case in driven systems. For example, in the case of periodic drives (with characteristic driving period τ), $D_\nu(t_0+\tau,t_0)$ reflects fluctuations over a single period while $\lim_{t\to\infty} D_\nu(t,t_0)$ portrays the fluctuations over many periods. To our knowledge, there is currently no method capable to account for this distinction, and demonstrate its ramifications.

Gaussian states and dynamics. Our description thus far has made no assumption about the structure of $\hat{H}_{\mathrm{S}}(t)$. Arbitrary interacting systems are accessible and can be simulated using, e.g., *tensor networks*, as put forth in Ref. [39]. However, if $\hat{H}_{\mathrm{S}}(t)$ is quadratic in fermionic operators, the tilted Liouvillian (2) is Gaussian-preserving. Let $\{\hat{b}_i\} = \{\hat{c}_j, \hat{a}_{k,\alpha}\}$ denote a combined set of fermionic operators of the system plus the Q leads. A quadratic $\hat{H}_{\mathrm{S}}(t)$ implies that we may write $\hat{H}(t) = \hat{H}_{\mathrm{S}}(t) + \sum_{\alpha=1}^{Q}(\hat{H}_{\mathrm{L}_\alpha} + \hat{H}_{\mathrm{SL}_\alpha}) := \sum_{i,j} h_{i,j}(t)\hat{b}^\dagger_i\hat{b}_j$, for a matrix $\mathbf{H}(t)$ with matrix elements $h_{i,j}(t)$ of dimension $L + \sum_{\alpha=1}^{Q} N_\alpha$. In the untilted case ($\chi = 0$), it is well-known that the particle-number preserving *covariance matrix* $\mathbf{C} \geqslant 0$ with entries $[\mathbf{C}(t)]_{i,j} := \mathrm{Tr}[\hat{b}^\dagger_j\hat{b}_i\hat{\rho}(t)]$ evolves according to the Lyapunov equation [40,53,54]

$$\frac{d\mathbf{C}(t)}{dt} = -[\mathbf{W}(t)\mathbf{C}(t) + \mathbf{C}(t)\mathbf{W}^\dagger(t)] + \mathbf{F}, \tag{8}$$

where $[\mathbf{W}(t)]_{i,j} = \mathrm{i}h_{i,j}(t) + \gamma_{i,j}/2$ and $\boldsymbol{\gamma}$ is a diagonal matrix with entries $\gamma_{k,\alpha}$ [Eq. (4)] in the sector of the leads. Similarly, $\mathbf{F}$ is a diagonal matrix with entries $\gamma_{k,\alpha}f_{k,\alpha}$. The average current (5) can then be written as $J_\nu(t) = \mathrm{i}\mathrm{Tr}[\mathbf{G}_\nu\mathbf{C}(t)]$, where $\mathbf{G}_\nu$ is an antisymmetric matrix with entries $\pm\kappa_{k,\nu}$ in the sectors connecting $\hat{c}_{p_\nu}$ and $\hat{a}_{k,\nu}$ [52].

The noise can be obtained using the method shown in the Ref. [52]. It consists of writing the noise over any interval $[t_1,t_2]$ as $D_\nu(t_2,t_1) = 2\mathrm{Tr}[\mathbf{G}_\nu\tilde{\mathbf{C}}(t_2,t_1)]$, where $\tilde{\mathbf{C}}(t_2,t_1)$ is an auxiliary matrix, obtained by integrating the modified Lyapunov equation

$$\begin{aligned}\frac{d\tilde{\mathbf{C}}(t,t_1)}{dt} = &-[\mathbf{W}(t)\tilde{\mathbf{C}}(t,t_1) + \tilde{\mathbf{C}}(t,t_1)\mathbf{W}^\dagger(t)]\\ &-\frac{1}{2}[\mathbf{C}(t)\mathbf{G}_\nu[\mathbf{1}-\mathbf{C}(t)] + [\mathbf{1}-\mathbf{C}(t)]\mathbf{G}_\nu\mathbf{C}(t)],\end{aligned} \tag{9}$$

with initial condition $\tilde{\mathbf{C}}(t_1,t_1) = \mathbf{0}$. The second line contains $\mathbf{C}(t)$, which is the solution of Eq. (8), with initial condition at time $t = 0$ (and not t_1). Physically, we can interpret the solution $\tilde{\mathbf{C}}(t_2,t_1)$ as turning a detector on at t_1 and then off at t_2. The real dynamics $\mathbf{C}(t,0)$ evolves from $t = 0$ onward, indefinitely. Given a window $[t_1,t_2]$, we obtain the corresponding fluctuations by integrating Eq. (9). With these expressions, we can therefore analyze fluctuations over arbitrary intervals, for Hamiltonians with arbitrary time dependence. Equations (8) and (9) can be integrated using standard Runge-Kutta methods.

Time-dependent current and noise in two-terminal junctions. We consider two metal electrodes kept at different equilibrium states and bridged by a two-site fermionic system, which is modulated via a time-periodic electric field,

$$\hat{H}_{\mathrm{S}}(t) = \left(\frac{eaE(t)}{2}\right)(\hat{c}^\dagger_1\hat{c}_1 - \hat{c}^\dagger_2\hat{c}_2) - \Delta(\hat{c}^\dagger_1\hat{c}_2 + \hat{c}^\dagger_2\hat{c}_1). \tag{10}$$

Here e is the electric charge, a is the spacing between the two sites and $E(t) = A\cos(\omega t)$ is the electric field. We fix the internal coupling Δ as the energy scale of the problem. The two reservoirs have the same temperatures, $T_{\mathrm{L}} = T_{\mathrm{R}} = 0.1\Delta$, but a chemical potential bias $\mu_{\mathrm{L}} = 24\Delta$ and $\mu_{\mathrm{R}} = -24\Delta$ [45]. The spectral function of the baths are taken as $\mathcal{J}_{\mathrm{L}}(\omega) = \mathcal{J}_{\mathrm{R}}(\omega) = \Gamma,\ \forall\,\omega \in [-W, W]$, and zero otherwise, where W is a cutoff energy and Γ the effective coupling. We discretize each reservoir into N lead-modes with energies ε_k between $-W$ and W, such that $\gamma_{k,\alpha} = 2W/N$ and $\kappa_{k,p} = \sqrt{\Gamma\gamma_{k,\alpha}/2\pi}$. Throughout,

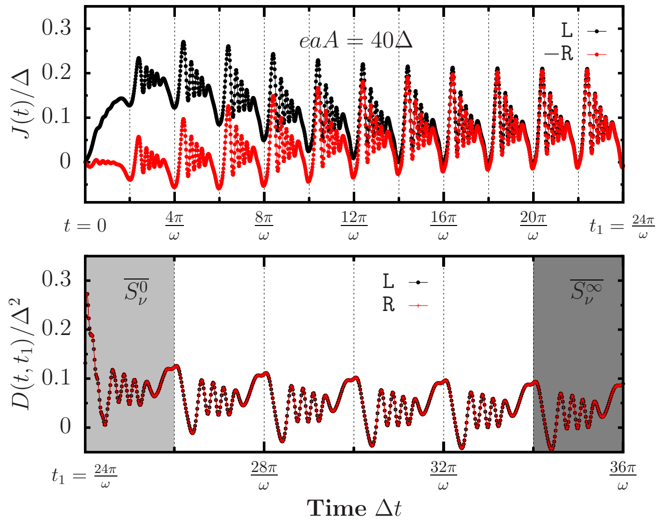

FIG. 2. (Top) average currents $J_{\mathrm{L}}(t)$ and $-J_{\mathrm{R}}(t)$ [Eq. (5)] during multiple periods $\tau = 2\pi/\omega$ of the external drive, up until the LC is reached. (Bottom) noise $D_{\mathrm{L/R}}(t, t_1)$, starting at the LC $t_1 = 24\pi/\omega$. Integrating over the first period yields $\overline{S^0_\nu}$ in Eq. (12). Waiting for multiple periods and then integrating yields instead $\overline{S^\infty_\nu}$ in Eq. (13). Parameters are described in the main text.

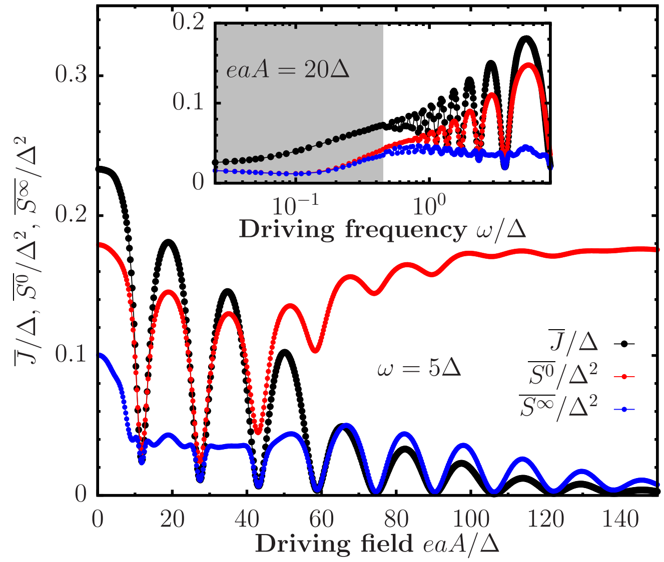

FIG. 3. $\overline{J}$, $\overline{S^0}$, and $\overline{S^\infty}$ [Eqs. (11)–(13)] as a function of the driving field amplitude eaA/Δ, in the limit cycle, with fixed frequency $\omega = 5\Delta$. (Inset) same, but as a function of ω/Δ, with fixed $eaA = 20\Delta$. Other parameters are as in Fig. 2.

we fix $\Gamma = 0.5\Delta$, $W = 100\Delta$, and $N = 400$ (which sufficed to guarantee convergence of all simulations). The integration of Eqs. (8) and (9) was carried out through fourth-order Runge-Kutta integration with a time-step $\delta t = 0.01/\Delta$.

Figure 2 (top) displays the instantaneous currents Eq. (5) of the left and right reservoirs during several drive periods $\tau = 2\pi/\omega$, starting at $\tilde{\mathbf{C}}(0) = \mathbf{0}$, with a fixed $eaA = 40\Delta$ and $\omega = 5\Delta$. As can be seen, $J_{\mathrm{L/R}}$ gradually tend to the *limit cycle* (LC), where $J_\nu(t+\tau) = J_\nu(t)$. This suggests we define the LC-averaged current as

$$\overline{J}_\nu = \frac{1}{\tau}\int_{t_1}^{t_1+\tau} dt' J_\nu(t'), \tag{11}$$

where t_1 is a large enough time such that $J_\nu(t_1+\tau) = J_\nu(t_1)$.

To analyze the noise, we wait until the LC has been reached at time t_1, so that we eliminate any dependence on the arbitrary initial condition.[2] In Fig. 2 (bottom), we plot $D_{\mathrm{L/R}}(t, t_1)$, starting at $t_1 = 24\pi/\omega$. The choice of t_1 is arbitrary, as long as it is large enough such that $J_\nu(t_1+\tau) = J_\nu(t_1)$. Note that $D_{\mathrm{L}}(t, t_1) = D_{\mathrm{R}}(t, t_1)\ \forall t$; this is a consequence of the symmetric model parameters and it is nongeneric behavior (see Ref. [52] for details). At $t = t_1$, we start counting particles to analyze the instantaneous noise. We find that $D_\nu(t+\tau, t_1) \neq D_\nu(t, t_1)$ over the first period [left-most grey region in Fig. 2 (bottom)]. In fact, integrating $D_\nu(t, t_1)$ from t_1 to $t_1+\tau$ yields

$$\overline{S^0_\nu} := \frac{1}{\tau}\int_0^\tau dt' D_\nu(t_1+t', t_1), \tag{12}$$

which is the average variance of the charge transferred over a single period after the LC. Similarly, integrating from t_1 to $t_1 + 2\tau$ yields the average fluctuation over two periods, and so forth, as it can also be measured. For $t \gg t_1, \tau$, we see in Fig. 2 (bottom) that eventually the noise itself becomes periodic, and $|D_\nu(t+\tau, t_1) - D_\nu(t, t_1)| \to 0$ for $t \to \infty$.[3] This suggests we define

$$\overline{S^\infty_\nu} := \lim_{t\to\infty}\frac{1}{\tau}\int_0^\tau dt' D_\nu(t+t', t_1), \tag{13}$$

depicted in the right-most grey region in Fig. 2 (bottom). $\overline{S^\infty_\nu}$ is in fact the so-called *LC-averaged zero-frequency component* of the noise [19]. Despite being a more standard quantity in the context of systems with autonomous steady states, it lacks the clear physical interpretation as $\overline{S^0_\nu}$ when time-dependent drives are present.

Figure 3 displays $\overline{J}$, $\overline{S^0}$ and $\overline{S^\infty}$ as a function of the driving field strength A. We have suppressed the ν index, as L and R quantities are equivalent in the present case with symmetric driving and bias. This model is known to display current-suppressed minima for certain values of the driving field [45]. A key feature of this is that one can also systematically suppress $\overline{S^\infty}$ [19]. In contrast, the single-period variance $\overline{S^0}$ displays a fundamentally different behavior, remaining nonzero even for arbitrarily large drive amplitudes. This means that even though the average current is suppressed, the fluctuations of the charge exchanged within each period are not. This fundamental difference between $\overline{S^\infty}$ and $\overline{S^0}$ is a feature of driven systems, and depends on the frequency in question. In the inset of Fig. 3, we plot $\overline{J}$, $\overline{S^0}$ and $\overline{S^\infty}$

[2]Given that the Lindblad master equation is expected to be gapped, there will be a finite time until this occurs.

[3]This is a consequence of Eq. (7), which in this case can be written as $D(t+\tau, t_1) = D(t, t_1) + 2\frac{d}{dt}\mathrm{cov}[N(t, t_1), N(t_1, t_1-\tau)]$. The last term is the rate of change of the covariance, which becomes vanishingly small when $t \gg t_1, \tau$.

as a function of ω. We see that $\overline{S^0}$ and $\overline{S^\infty}$ coincide asymptotically in the low frequency regime, becoming identical in the nondriven case, $\omega = 0$ (see Ref. [52]). Conversely, for large frequencies, they deviate substantially.

Conclusions. Accurate, nonperturbative computation of fluctuations of observables in driven, nonequilibrium quantum settings is a long-standing problem. Here, we have put forward a powerful, flexible method that paves the pathway into investigations of fluctuations of quantum systems out of equilibrium. Having access to the full dynamics of the noise we revealed that the average fluctuations contain correlations between different time periods in periodically driven systems, arising from the nonadditivity of the variance. Specifically our method allowed us to uncover the fundamental distinction between two measures for noise, $\overline{S^0}$ and $\overline{S^\infty}$. While the former is a measure for the pure variance of a physical quantity within a certain time interval, the latter contains covariance terms over time intervals [Eq. (7)]. In driven systems, $\overline{S^0}$ thus has a clearer physical interpretation of charge fluctuations, contrasting $\overline{S^\infty}$.

Future prospects include studies of light-driven materials under nonperiodic modulations, relevant to proposals for petahertz signal processing [10,55]. In these scenarios, the full transient dynamics of currents and charge fluctuations are of the essence.

Current fluctuations can further reveal fundamental aspects of electron-electron interactions, as demonstrated in nondriven systems [56]. By combining our FCS-mesoscopic lead framework with tensor-network techniques [39], one could uncover correlated-electron phenomena in nanoscale devices from the behavior of both transient currents and their noise signals.

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