

Spin-dependent edge states in two-dimensional Dirac materials with a flat bandLi-Li Ye¹,^{*} Chen-Di Han,¹ and Ying-Cheng Lai^{1,2,*}¹*School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, Arizona 85287, USA*²*Department of Physics, Arizona State University, Tempe, Arizona 85287, USA* (Received 7 August 2023; revised 8 November 2023; accepted 20 November 2023; published 4 December 2023)

The phenomenon of spin-dependent quantum scattering in two-dimensional (2D) pseudospin-1/2 Dirac materials leading to a relativistic quantum chimera was recently uncovered. We investigate spin-dependent Dirac electron optics in 2D pseudospin-1 Dirac materials, where the energy-band structure consists of a pair of Dirac cones and a flat band. In particular, with a suitable combination of external electric fields and a magnetic exchange field, electrons with a specific spin orientation (e.g., spin-down) can be trapped in a class of long-lived edge modes, generating resonant scattering. The spin-dependent edge states are a unique feature of flat-band Dirac materials and have no classical correspondence. However, electrons with the opposite spin (i.e., spin up) undergo conventional quantum scattering with a classical correspondence, which can be understood in the framework of Dirac electron optics. A consequence is that the spin-down electrons produce a large scattering probability with broad scattering angle distribution in both near- and far-field regions, while the spin-up electrons display the opposite behavior. Such characteristically different behaviors of the electrons with opposite spins lead to spin polarization that can be as high as nearly 100%.

DOI: [10.1103/PhysRevB.108.235404](https://doi.org/10.1103/PhysRevB.108.235404)**I. INTRODUCTION**

Dirac electron optics concerns the behaviors of ballistic electrons in two-dimensional (2D) Dirac materials [1–7], in which a paradigm is graphene p - n junction systems [8]. In particular, due to the relativistic quantum phenomenon of Klein tunneling and the gapless Dirac-cone dispersion relation, the transmission of Dirac electrons through the p - n junction interface resembles a highly transparent focusing lens with a negative refractive index [4]. It corresponds to a Veselago lens [9] for chiral Dirac fermions in graphene and provides an experimental way to tune the refractive index by varying the gate potential, making it possible to realize graphene-based electron lens [10] and transistors [11]. In previous studies of Dirac electron optics, the electronic counterparts of various optical phenomena were observed, such as Fabry-Pérot resonances [12,13], cloaking [14], Dirac fermion microscopes [15], and electron Mie scattering [16–20]. In addition, in the framework of Dirac electron optics, diverse unconventional relativistic quantum phenomena such as anti-super-Klein tunneling in phosphorene p - n junctions [8] and tilted energy dispersion effect [21] were studied. A rigorous semiclassical theory beyond the standard WKB approximation for the 2D Dirac equation was developed [22] as the foundation of Dirac electron optics. Experimentally, Dirac fermion flows were imaged through a circular Veselago lens using the polarized tip of a scanning gate microscope [23] and nanoscale quantum electron optics was tested in graphene with atomically sharp p - n junctions [24]. Spin-dependent Dirac electron optics in

graphene was also studied, leading to the phenomenon of relativistic quantum chimera [4].

In this paper, we investigate spin-dependent Dirac electron optics in 2D pseudospin-1 Dirac materials whose energy-band structure consists of a pair of Dirac cones and a flat band. Compared with graphene, pseudospin-1 Dirac materials exhibit unconventional physical phenomena such as super-Klein tunneling [25], conical diffraction [26–28], and chaos Q -spoiling defiance with edge states [29]. An example of pseudospin-1 materials is the dice lattice, as illustrated in Fig. 1(a), where the quasiparticles can be described by the generalized 2D Dirac-Weyl Hamiltonian [30]. Following the previous work on spin-dependent electron optics in graphene [4], we consider an eccentric circular cavity of dice lattice consisting of a large circle and a small circular domain inside the large one, where the centers of the two circles do not coincide, as shown in Fig. 1(b). The real spin degree of freedom of the electron becomes relevant when the whole device is placed on a ferromagnetic substrate [31,32], as described by a magnetic exchange potential in the Hamiltonian. Now apply two distinct gate voltages to the cavity: one to the large circular domain excluding the small circle and another to the small circular domain. With appropriate combinations of the magnetic exchange field (MEF) strength and the gate voltages, the quantum scattering behaviors of the spin-up and spin-down electrons can be characteristically distinct. For example, spin-up electrons can exhibit lensing modes while spin-down electrons would focus on the edge of the large cavity. As a result, the spin-down electrons produce a large scattering probability with broad scattering angle distribution in both the near-field and far-field regions, while the spin-up electrons display the opposite behavior. Such characteristically different behaviors of the electrons with opposite spins lead to spin

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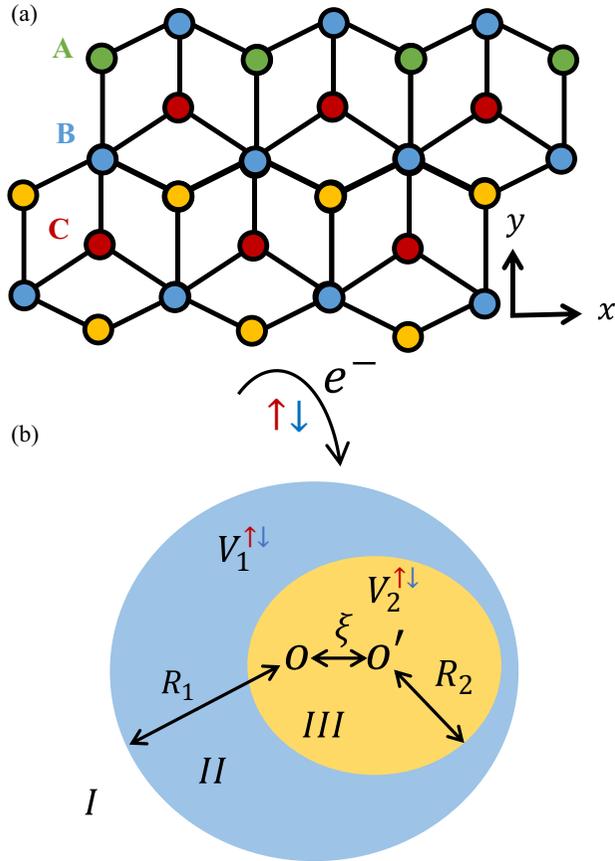


FIG. 1. Configuration of the eccentric circular cavity of 2D pseudospin-1 lattice. (a) Dice lattice (a potential material to realize the pseudospin-1 Dirac Weyl Hamiltonian), whose energy band structure consists of a pair of Dirac cones and a flat band and (b) a concrete device configuration, and spin-dependent potentials $V_i^{\uparrow\downarrow}$ ($i = 1, 2$) generated by the gate voltages v_1 and v_2 and the magnetic exchange potential $\mu_1 = \mu_2 = \mu$, which are applied to the blue and yellow domains, respectively. The radii of the two circular regions are $R_1 = r_0$ and $R_2 = 0.6r_0$, with $r_0 \sim 100$ nm being the characteristic length.

polarization that can be as high as nearly 100%. We note that the edge modes for spin-down electrons break the ray-wave correspondence and confine the electrons for a relatively long time [29]. In contrast, the lensing modes for spin-up electrons have a classical correspondence in the small wavelength limit and tend to leak from the cavity in a short time. The phenomenon of spin-dependent quantum scattering, particularly the scattering from the edge mode, is also studied using a classically chaotic system: the stadium cavity.

The reasons to exploit MEF instead of an external magnetic field are as follows. While an external magnetic field can induce spin polarization and other intriguing physical phenomena, the required magnitude of the magnetic field can be on the order of tesla, which may be challenging in experiments. Magnetic insulators or magnetic impurities provide a viable solution, as the induced MEF is equivalent to a strong magnetic field of the required magnitude [33]. In particular, in the absence of an external magnetic field, the MEF can be induced by an adjacent magnetic insulator, such as EuS

[31] or a ferromagnetic insulator [32] for spin generation and modulation [34,35]. Remarkably, the MEF in magnetic multilayers can be exploited to realize a magnetic field of tens or even hundreds of tesla [31,36]. In addition, electronic spin lenses [37], i.e., the counterpart of the photonic chiral metamaterials generated by spin-resolved negative refraction Klein tunneling, were studied with the MEF in graphene in a ferromagnetic configuration. The MEF thus makes it possible to study electron optics [22–24,38]. For these reasons, we chose to study the behaviors of 2D flat-band Dirac materials in combined electrical and magnetic fields, where the latter is realized through the MEF interaction.

In a finite graphene sheet, edge states are common. In previous works on pseudospin-1 materials [29,39–41], a class of robust but spin-independent edge modes was found, which can resist even fully developed classical chaos and Klein tunneling [29,41]. The edge modes were deemed a unique feature of pseudospin-1 Dirac materials in the absence of a magnetic exchange potential where the real spin degrees of freedom are degenerate. The focus of the present paper is on the effect of the MEF on the spin-dependent quantum scattering dynamics in pseudospin-1 materials. It is also noteworthy that, in a previous work [42], 100% spin polarization was realized in monolayer transition metal dichalcogenides assisted by total external reflection with spin-orbit coupling and an electrostatic potential barrier. The system configuration is less complicated and experimentally more feasible than our design. The main feature of our work is spin-dependent edge states. In particular, in our system, two types of quantum states arise and coexist: Electronic quasibound states with a well-defined classical correspondence and spin-dependent edge states without such a classical correspondence, where the latter can produce a much broader scattering angle distribution and stronger scattering compared with the former. It is the drastically different scattering behaviors that make a near 100% spin polarization possible. The physical mechanism leading to such a high spin polarization is thus different than that reported in Ref. [42]. We report on this phenomenon of coexistence of two different types of relativistic quantum states, one with and another without a classical correspondence.

II. PSEUDOSPIN-1 ECCENTRIC CIRCULAR SCATTERING SYSTEM

We consider (real) spin-1/2 Dirac electron scattering from the 2D pseudospin-1 Dirac system in Fig. 1(b). The eccentric circular scattering cavity is created by an electric gate potential $\mathcal{V}_{\text{gate}}(\mathbf{r})$ [19,20] and the magnetic exchange potential $\mathcal{M}(\mathbf{r})$ induced by an adjacent magnetic insulator within the gate region [4]. The total Hamiltonian is

$$\hat{H} = v_F \sigma_0 \otimes \mathbf{S} \cdot \hat{\mathbf{p}} + \hbar v_F [\sigma_0 \otimes S_0 \mathcal{V}_{\text{gate}}(\mathbf{r}) - \sigma_z \otimes S_0 \mathcal{M}(\mathbf{r})], \quad (1)$$

with pseudospin-1 matrix vector \mathbf{S} , spin-1/2 Pauli matrix σ_z , and identity matrices $\sigma_0^{2 \times 2}$ and $S_0^{3 \times 3}$. Using the relation $[\sigma_z \otimes S_0, \hat{H}] = 0$, we block diagonalize the Hamiltonian as

$$\hat{H} = \text{diag}[\hat{H}_1, \hat{H}_{-1}], \quad (2)$$

where

$$\hat{H}_s = v_F \mathbf{S} \cdot \hat{\mathbf{p}} + \hbar v_F [\mathcal{V}_{\text{gate}}(\mathbf{r}) - s\mathcal{M}(\mathbf{r})] \quad (3)$$

for spin index s ($s = 1$ or \uparrow for spin-up and $s = -1$ or \downarrow for spin-down). The total potential is thus dependent upon the real spin:

$$V^s(\mathbf{r}) \equiv \mathcal{V}_{\text{gate}}(\mathbf{r}) - s\mathcal{M}(\mathbf{r}). \quad (4)$$

The radii of the two eccentric circles are R_1 and $R_2 < R_1$ whose origins are located at O and O' , respectively, with the eccentric distance ξ , as shown in Fig. 1(b). For $\xi \neq 0$, classical chaos can arise [29]. The whole physical space can be divided into three parts: region I ($r > R_1$), region II [$r < R_1$ (origin O) and $r' > R_2$ (origin O')], and region III ($0 < r' < R_2$). The gate potentials $\mathcal{V}_{\text{gate}}(\mathbf{r})$ are v_1 and v_2 applied to regions II and III, respectively, and the magnetic exchange potential $\mathcal{M}(\mathbf{r})$ is $\mu_1 = \mu_2 \equiv \mu$. The total magnetic exchange and electric potential with spin index s is

$$V_i^s = v_i - s\mu \quad (5)$$

for $i = 1, 2$ in regions II and III, respectively. In region I, we have $V_0 = 0$. The energy is $\epsilon = \hbar v_F E$, where E is the normalized energy with the same dimension of the wavelength in units of $1/r_0$ with the characteristic length r_0 . The wave vectors in the three regions are

$$\begin{aligned} k_I &= |E|, \\ k_{\text{II}}^s &= |E - V_1^s|, \\ k_{\text{III}}^s &= |E - V_2^s|. \end{aligned}$$

Using the principle of Dirac electron optics [4,6] and spin-resolved Snell's law, we have that the effective refractive indices are $n_0 = (E - V_0)/E = 1$ (vacuum) and $n_i^s = (E - V_i^s)/E$ with $i = 1, 2$.

The characteristic units of energy, including the electronic energy, electrostatic energy, and the energy of the MEF, are in terms of $\hbar v_F/r_0 \sim 0.01$ eV with $r_0 \sim 100$ nm (the radius of the large circular cavity) and the Fermi velocity $v_F \sim 10^6$ m/s. The typical wavelength of a Dirac electron inside the cavity is $\lambda = \hbar v_F/E_d \sim 10$ nm, where E_d is the energy difference between the electronic energy and the total potential of magnitude on the order ~ 0.1 eV. The Dirac electron inside the cavity then exhibits a particlelike behavior in some reasonable Fermi energy range [43,44] ~ 0.1 eV, where the width of the p - n junction edge is as sharp as [3,18,45] $d \sim 1$ nm. In this energy regime, the concepts of Dirac electron optics are applicable. For convenience, we use dimensionless quantities in our analysis and computation.

We use the S -matrix approach to calculate various scattering cross sections, as detailed in Appendices A–D.

III. RESULTS

A. Spin-dependent edge states in the eccentric circular cavity

With the configuration in Fig. 1(b), the edge modes are relativistic quantum resonant states that confine the electrons to a quasi-1D region, where the resonant energy is about half the potential. Figure 2(a) demonstrates an edge mode associated with spin-down electrons (the left inset) confined

around $r' \approx R_2$ with $E \approx 15 = (V_1 + V_2)/2$. For comparison, the right inset shows a conventional pseudospin-1 scattering mode [29]. Spin-up electrons, however, exhibit characteristically different scattering behaviors, as illustrated in Fig. 2(b) for two energy values. The corresponding scattering probability distributions for the spin-down and spin-up electrons are shown in Figs. 2(c) and 2(d), respectively. The edge mode produces a large scattering probability with wide directional distribution in both the near- and far-field regions. (Section III D provides a detailed analysis of the edge-mode enhanced scattering for spin-down electrons.) In contrast, the scattering patterns for the spin-up electrons are reminiscent of lensing modes in geometric optics that arise in the small wavelength limit: $k_{\text{II}} = |E - V_1| \approx 73$, $k_{\text{III}} = |E - V_2| \approx 23$, and $k_I = |E| \approx 15$. The distinct scattering behaviors for spin-down and spin-up electrons can also be characterized by the momentum-transport cross section, defined as $\sigma_{\text{tr}} \equiv \oint d\theta f(\theta) \sigma_{\text{diff}}$ with incident direction $\theta' = 0$, where $f(\theta) \equiv 1 - \cos \theta$, the differential cross section σ_{diff} is determined by the scattering matrix, and σ_{tr} is proportional to the resistance $\sigma_{\text{tr}}^{\uparrow\downarrow} \propto R^{\uparrow\downarrow}$ (see Appendices A–D for details). The edge modes generate a much larger resistance than the lensing states, as shown by the differential momentum-transport cross section in Figs. 2(e) and 2(f), respectively.

The physical reason underlying the emergence of the edge modes lies in the boundary condition for the three-component spinor stipulated by the generalized Dirac-Weyl equation for pseudospin-1 quasiparticles [46]. In particular, the radial or normal current density across the boundary of the scatterer must be continuous, but it is not necessary for the angular or tangent component of the current density to be continuous. In addition, the probability density needs not be continuous across the boundary. In fact, a larger difference in the probability density can arise if there is a significant imbalance in the first and third components of the spinor across the boundary. If the scattering potential redistributes the spinor wave-function components properly, there will be a significant increase in the probability density from the exterior to the interior of the scattering boundary, leading to strong boundary trapping of the quasiparticles inside the potential and thereby to robust edge modes. This phenomenon of boundary confinement is most pronounced when the Fermi energy of the particle is about half of the potential height—the Klein tunneling regime [46].

We now demonstrate that spin-dependent edge modes can lead to unusually nearly complete spin polarization. Figure 3(a) shows, in the 2D parameter plane (ξ, μ), color-coded values of the spin polarization averaged over a relevant range of the Fermi energy, which is defined as (Appendix C)

$$\langle P_z \rangle = \langle (\sigma_{\text{tr}}^{\downarrow} - \sigma_{\text{tr}}^{\uparrow}) / (\sigma_{\text{tr}}^{\downarrow} + \sigma_{\text{tr}}^{\uparrow}) \rangle.$$

There exists a relatively large area in the parameter plane in which the spin polarization exceeds 85%. Figure 3(b) shows the maximum spin polarization versus μ , which can reach a value as high as 97% (for $\mu \approx 24$), due to the drastically different scattering behaviors associated with the spin-down and spin-up electrons. Figure 3(c) shows, for $\mu = 24$, the energy-averaged momentum-transport cross sections $\langle \sigma_{\text{tr}} \rangle$ versus ξ for spin-down and spin-up electrons,

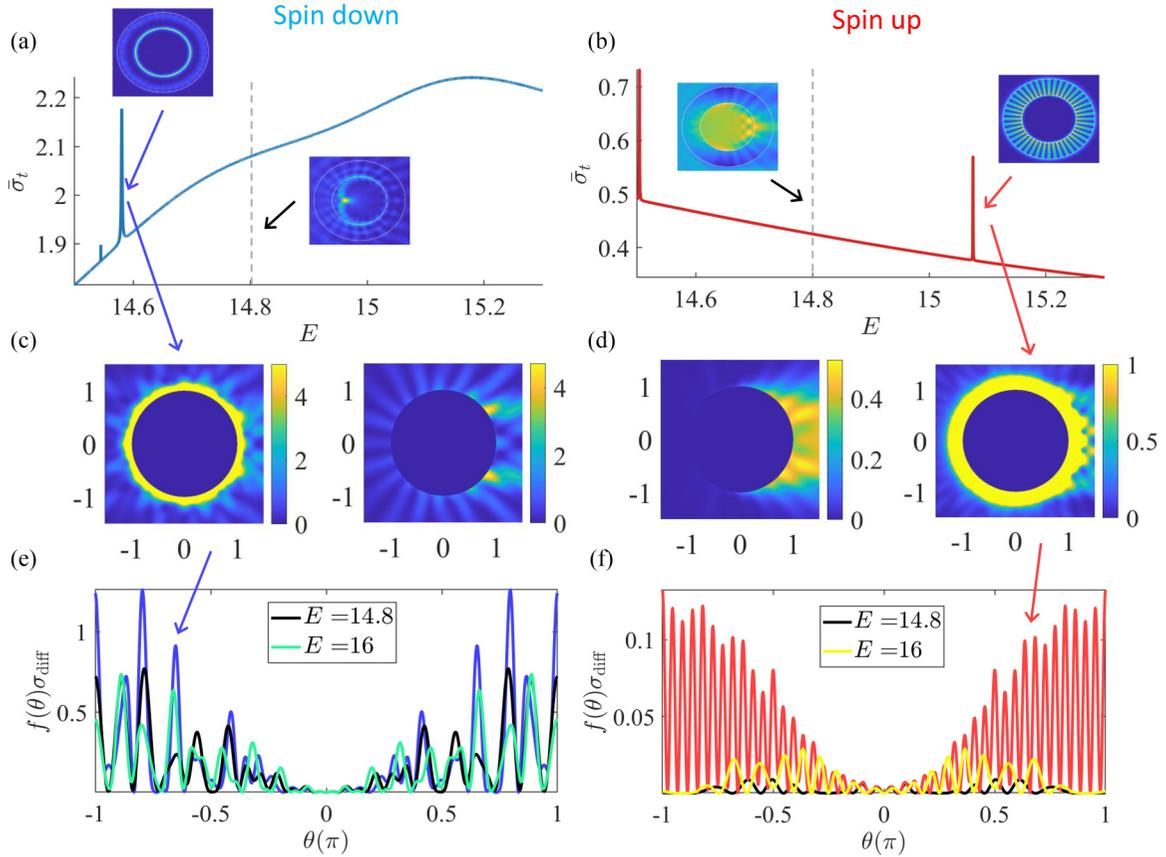


FIG. 2. Emergence of spin-specific edge states in an annular cavity ($\xi = 0$). (a), (b) The average total scattering cross section σ_t in the Fermi energy range [43,44]: ~ 0.1 eV with $E \sim 0.01$ eV, for spin-down and spin-up electrons, respectively, where σ_t is obtained by averaging the total scattering cross section $\sigma_t(\theta') = \oint d\theta |f(\theta, \theta')|^2$, over all possible incident directions θ' . The insets display the probability distribution patterns. In (a), the left inset corresponds to an edge state with no classical correspondence while the right inset is a conventional state. In (b), the scattering states are lensing-like with a classical correspondence. (c), (d) The scattering probability distribution (defined in Appendix C in the near-field region I for spin-down and spin-up electrons, respectively). (e), (f) Differential momentum-transport cross section [differential cross section σ_{diff} times $f(\theta) \equiv (1 - \cos \theta)$] versus the scattering angle θ for spin-down and spin-up electrons, respectively. For the former, the total potentials are $V_1^\downarrow = -10$ and $V_2^\downarrow = 40$. For the latter, the corresponding parameter values are $V_1^\uparrow = -10 - 2\mu$ and $V_2^\uparrow = 40 - 2\mu$ ($\mu = 24$). The incident plane wave is along the x axis with $\theta' = 0$ and the scattering angle is between $-\pi$ and π (at the resolution of 1000 points).

where the cross section values for spin-down electrons are markedly larger than those for spin-up electrons. The difference is the largest for $\xi \gtrsim 0$, leading to the highest spin polarization there. For a fixed value of μ , as ξ increases from zero (integrable classical dynamics) to, e.g., 0.3 (chaotic classical dynamics), the spin polarization can be maximized by some value of ξ . Figure 3(d) shows the average spin polarization versus ξ for $\mu = 24$. Since ξ is a geometric parameter controlling the degree of classical chaos (as ξ increases from zero, the classical dynamics become more chaotic), the result shows that classical chaos deteriorates spin polarization.

The top left inset in Fig. 3(b) shows a mode corresponding to a classical geometric optic lensing pattern. This type of lensinglike mode can shrink an incident parallel beam into a narrow parallel emission flow, as illustrated in Fig. 4.

The characteristic difference between the edge modes for spin-down electrons and the lensing modes for spin-up electrons can also be revealed by the maximum Wigner-Smith

time delay defined as

$$\tau(E) \equiv -i\hbar \text{Tr}[S^\dagger \partial S / \partial E],$$

with S being the scattering matrix in Appendix C. Figure 5 shows, for $\mu = 24$, the maximum delay τ_{max} (over Fermi energy) versus the geometric parameter ξ for spin-down (blue) and spin-up (red) electrons, where the former is significantly larger than that for the latter. A remarkable feature is that, as ξ increases from zero so the classical dynamics changes from being integrable to mixed and then to chaotic, τ_{max} for spin-down electrons hardly vary, indicating that the edge modes have no classical correspondence. In contrast, τ_{max} for spin-up electrons continue to decrease with ξ , which agrees with the classical intuition that, as the dynamics become more chaotic, the average time that an electron can stay inside the cavity should decrease. Because of the classical-quantum correspondence for the lensing modes, their properties can be understood using ray tracing from geometric optics in Sec. III C.

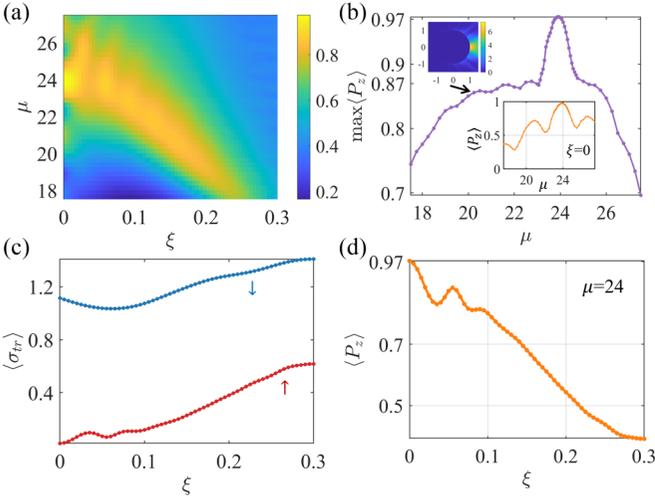


FIG. 3. Realization of nearly complete spin polarization in the eccentric circular cavity. (a) Color-coded values of the average spin polarization $\langle P_z \rangle$ in the parameter plane (ξ, μ) averaged over the Fermi energy. High spin polarization can be achieved in a substantial area in the plane. (b) Maximum average spin polarization $\max\langle P_z \rangle$ about ξ versus μ . The upper inset displays the scattering probability density of the lensing modes for $\mu = 20$, $E = 14.8$, and $\xi = 0.165$, and the lower inset shows $\langle P_z \rangle$ versus μ for $\xi = 0$. Near-perfect spin polarization characterized by $\max\langle P_z \rangle \lesssim 1$ is achieved. (c) Momentum-transport cross section $\langle \sigma_{tr} \rangle$ averaged over the Fermi energy versus ξ for the spin-down and spin-up electrons, for $\mu = 24$. (d) Average spin polarization $\langle P_z \rangle$ versus ξ for $\mu = 24$.

B. Scattering-direction dependent spin polarization

The average momentum-transport cross section is defined as

$$\langle \sigma_{tr} \rangle = \frac{1}{E_1 - E_0} \int_{E_0}^{E_1} dE \int_0^{2\pi} d\theta (1 - \cos \theta) |f(\theta, \theta')|^2, \quad (6)$$

where $|f(\theta, \theta')|^2$ is the probability for scattering associated with incident angle θ' and scattering angle θ as in Eq. (C8). The weighting factor $1 - \cos \theta$ is used to quantify the scattering angle deviation from the incident angle $\theta' = 0$. The quantity $\langle \sigma_{tr} \rangle$ contains two implicit parts: the total scattering probability (or scattering background σ_t) and the scattering angle θ distribution with respect to the incident direction. To separate the effect of scattering direction on spin polarization, we remove the background by normalizing the scattering probability over θ with the total scattering cross section, $|f(\theta, \theta')|^2 / \sigma_t$, and define an alternative spin polarization that depends on the scattering direction as

$$q_z = \frac{\sigma_{tr}^\downarrow / \sigma_t^\downarrow - \sigma_{tr}^\uparrow / \sigma_t^\uparrow}{\sigma_{tr}^\downarrow / \sigma_t^\downarrow + \sigma_{tr}^\uparrow / \sigma_t^\uparrow}, \quad (7)$$

where the ratio σ_{tr} / σ_t is proportional to the average momentum transfer cross section [47] over the scattering angle with

$$\langle \Delta \mathbf{p} \rangle_\Omega = q \hat{\mathbf{x}} \sigma_{tr} / \sigma_t, \quad (8)$$

where the incident direction is along $\hat{\mathbf{x}}$, $\Delta \mathbf{p} \equiv \mathbf{p}_{in} - \mathbf{p}_{out}$, q is the incident momentum magnitude, and Ω is the scattering

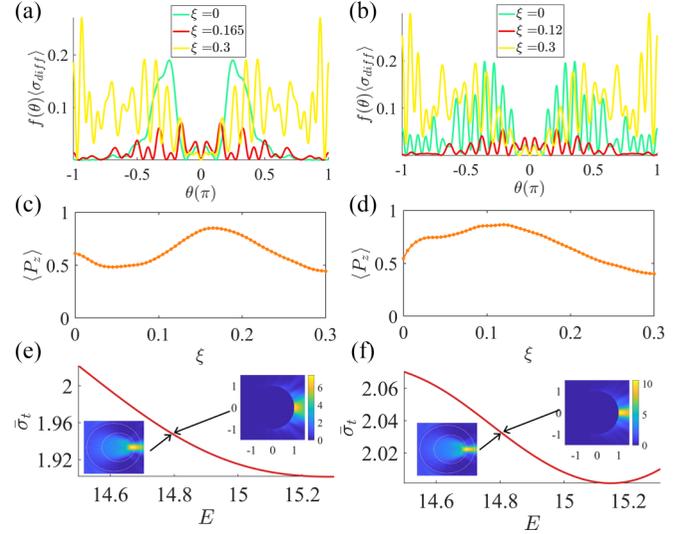


FIG. 4. Lensing-like pattern for a spin-up Dirac fermion. (a), (b) The quantity $f(\theta) \langle \sigma_{diff} \rangle$ versus θ for $\mu = 20$ and $\mu = 22$, respectively, where $f(\theta) \equiv 1 - \cos \theta$ and the differential cross section σ_{diff} is averaged over the Fermi energy. In each case, results from three values of ξ are displayed. (c), (d) Average spin polarization $\langle P_z \rangle$ over Fermi energy versus the eccentric parameter ξ for $\mu = 20$ and $\mu = 22$, where the maximum value of $\langle P_z \rangle$ occurs at $\xi = 0.165$ and 0.12 , respectively. (e), (f) Average total cross section $\bar{\sigma}_t$ versus Fermi energy for $\mu = 20$ and $\mu = 22$, respectively. In each panel, the lower-left inset shows the probability distribution while the upper-right inset displays the scattering probability distribution for the specific energy value as indicated by the arrows. Other parameters are the same as those in Fig. 2. These modes have a well-defined classical correspondence: the second kind of classical lensing ray pattern satisfying the conditions \mathcal{C}_2 (see Sec. III C for details).

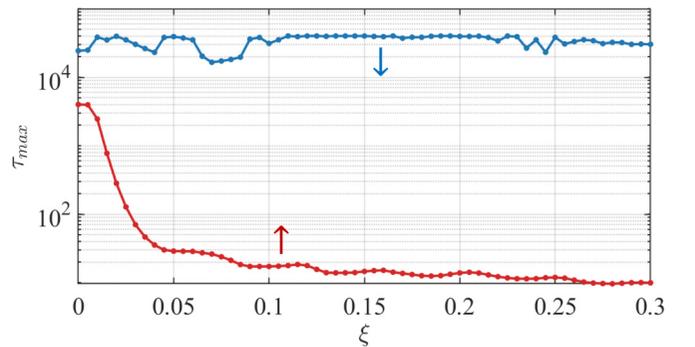


FIG. 5. Contrast between edge modes and lensing modes in terms of the Wigner-Smith time delay. Shown is a typical case of the delay maximized over the Fermi energy versus the geometric parameter ξ associated with the edge modes for spin-down (blue) and the lensing modes for spin-up (red) electrons. The magnetic exchange potential is $\mu = 24$. The delay time for the edge modes is independent of the classical dynamics and is significantly longer than that for the lensing modes, where for the latter, the delay time decreases continuously as the classical dynamics become more chaotic.

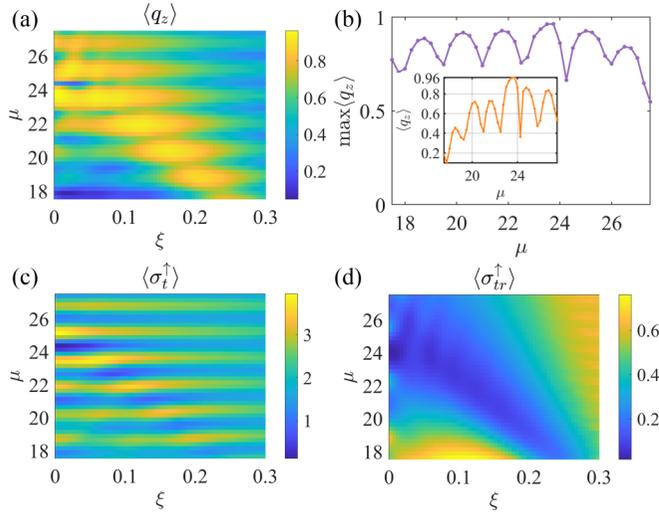


FIG. 6. Scattering-direction dependent spin polarization. (a) Average direction-dependent spin polarization $\langle q_z \rangle$ in the (ξ, μ) parameter plane. (b) The maximum value $\max(q_z)$ (over ξ) versus μ . The inset corresponds to the case of $\xi = 0$. (c) The removed average scattering background cross section $\langle \sigma_t^\uparrow \rangle$ in the (ξ, μ) plane. (d) Average momentum-transport cross section $\langle \sigma_{tr}^\uparrow \rangle$ in the (ξ, μ) plane.

solid angle. Figure 6(a) shows the numerically calculated $\langle q_z \rangle$ over Fermi energy in the parameter plane (ξ, μ) . It can be seen that high spin polarization can be achieved. Figure 6(b) shows, the maximum value $\max(q_z)$ versus μ , which can be as large as 96%! Figure 6(c) shows the removed average scattering background cross section $\langle \sigma_t^\uparrow \rangle$ in the (ξ, μ) plane, which exhibits a periodic structure in μ . For reference, Fig. 6(d) shows the average momentum-transport cross section $\langle \sigma_{tr}^\uparrow \rangle$ in the (ξ, μ) plane.

C. Understanding spin-up fermion lensing modes based on Dirac electron optics

We provide a geometric-optics-based interpretation to understand the lensinglike scattering states associated with spin-up Dirac fermions through two kinds of classical lensing ray patterns. Figures 7(a) and 7(b) show the total potential and effective refractive index versus the exchange potential μ , respectively, for spin-down and spin-up electrons.

The set of conditions under which the first type of classical lensing ray pattern arises (denoted as \mathcal{C}_1), as shown in Fig. 7(c), is (1) an infinitesimal refractive angle δ_1 , (2) approximately equal lengths of the solid and dashed blue ray paths, (3) $\Delta_2 \approx \Delta_0 + \delta\Delta$ with infinitesimal term $\delta\Delta$, and (4) $\Delta_0 < \Delta_0^c < \pi/2$. The Snell's law, $\sin \Delta_2 \approx 2\delta_1 n_1/n_2$ and $\sin \Delta_0 \approx \delta_1 n_1/n_0$, gives

$$\sin(\Delta_0 + \delta\Delta) \approx \sin(\Delta_0)2n_0/n_2 \gtrsim \sin(\Delta_0).$$

Condition \mathcal{C}_1 requires $2n_0/n_2 \gtrsim 1$, so the refractive index n_2 should be at least $n_2 \approx 2 - \delta n \lesssim 2$. The first kind of classical lensing pattern displayed in Fig. 7(c) corresponds to the $\mu = 24$ case with the effective refractive index in the small wavelength limit: $n_2 \lesssim 2$, $n_1 \approx 5$, and $n_0 = 1$, as shown in Fig. 7(b). In this case, the average spin polarization $\langle P_z \rangle$

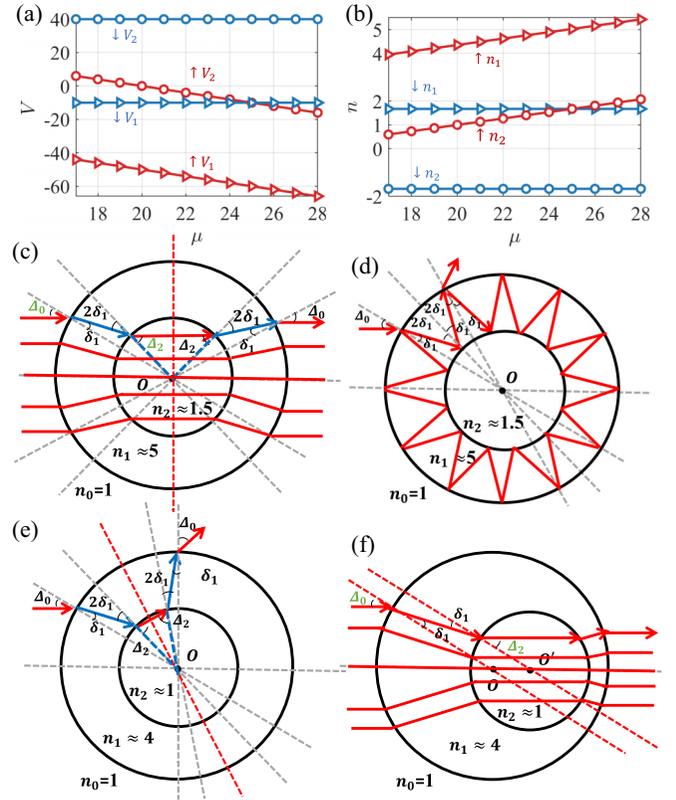


FIG. 7. Geometric optics interpretation for the quantum lensinglike scattering states associated with spin-up Dirac fermions. (a) Total potential versus the exchange potential μ for spin-down and spin-up electrons. (b) Effective refractive index in the small wavelength limit versus μ for spin-down and spin-up electrons. (c) For $\mu = 24$, the annular circular cavity displays one classic lensing pattern analogous to the one produced by two convex lenses. An incident ray with $\Delta_0 < \Delta_0^c$ exhibits no significant scattering. (d) For $\Delta_0 \geq \Delta_0^c$, total internal reflection occurs at the inner interface between regions II and III. However, at the outer boundary, total internal reflection does not occur under the condition \mathcal{C}_1 (see text) and spin-resolved Snell's law, generating a broad scattering angle distribution. (e) For $\mu = 20$, an annular cavity produces large scattering angles even for $\Delta_0 < \Delta_0^c$ due to the distinct refractive index configuration, away from both conditions \mathcal{C}_1 and \mathcal{C}_2 . (f) For $\mu = 20$, an eccentric circular cavity generates a different kind of lensing pattern analogous to the one created by one convex and another concave lens.

reaches maximum for the annular cavity with $\xi = 0$ and the ray pattern in Fig. 7(c) resembles the scattering probability of the corresponding lensinglike mode in the left panel in Fig. 2(d) in the main text for $E = 14.8$. For $\mu = 24$, the critical incident angle is determined by

$$\sin \Delta_0^c \approx \delta_1^c n_1/n_0 \approx 0.75,$$

with $\sin \Delta_2 = 2\delta_1^c n_1/n_2 = 1$, so $\Delta_0^c \approx 48.6^\circ$. For $\Delta_0 \geq \Delta_0^c$, total internal reflections occur at the inner interface between regions II and III but will not at the outer boundary, resulting in a vast scattering angle distribution, as shown in Fig. 7(d), which resembles the pattern with the resonant quantum state in the Fermi energy range of lensinglike modes in Fig. 2(b) in the main text. In principle, the directional distribution of the

leaking of the quantum resonant states can be quantitatively understood by semiclassical simulation [6,48].

The set of conditions C_2 under which the second type of lensing pattern arises, as shown in Fig. 7(f), is (1) infinitesimal refractive angle δ_1 , (2) the two red dashed ray segments in Fig. 7(f) being approximately parallel, (3) $\Delta_2 \approx \Delta_0$, and (4) $\Delta_0 < \Delta_0^c < \pi/2$. Starting from the conditions C_1 , if n_2 is away from two, such as for $\mu = 20$ with $n_2 \approx 1$, the annular cavity shape produces large scattering angles because of the large deviation of Δ_2 from Δ_0 , as shown in Fig. 7(e), breaking both conditions C_1 and C_2 . For the potential configuration with $\mu = 20$, condition C_2 is satisfied for an eccentric circular cavity, producing the lensing pattern in Fig. 7(f), which resembles the corresponding lensinglike mode in the insets of Figs. 4(e) and 4(f) and the upper inset of Fig. 3(b) in the main text. The corresponding critical incident angle is $\Delta_0^c \approx 30^\circ$, which is smaller than that in the $\mu = 24$ case.

In general, total internal reflections disrupt parallel rays, where a small critical incident angle will generate a large spread of the emitted rays. While all rays in the effective refractive index configuration associated with the classical-quantum correspondence for $\mu \in [20, 24]$ can produce the classical lensing ray pattern with the proper incident angle and eccentric parameter ξ , an enlarged critical angle is indicative of the contribution to scattering from the lensing patterns. As a result, in the corresponding quantum regime, the spin polarization increases from $\mu = 20$ to $\mu = 24$. In principle, if μ is increased further, the corresponding classical lensing ray pattern will occur for $\xi < 0$ and generate patterns similar to those for $\xi > 0$.

We note that the edge states of spin-down electrons break the ray-wave correspondence [29], their scattering behaviors cannot be explained by geometric optics.

D. Spin-dependent edge mode in a chaotic stadium cavity

In a previous work [29], it was demonstrated that a spin-independent edge mode can confine a particle for a long time, defying any Q -spoiling effect induced by classical chaos. To further demonstrate the peculiar behavior of the edge modes, we set up and study a spin-resolved scattering cavity of the stadium shape, whose geometric boundary is shown as the blue curve in Fig. 8(a), where a is a so-called chaotic parameter in the sense that the classical dynamics are chaotic for $a > 0$. To calculate the scattering cross sections, we use a previously developed method, the multiple-multipole method originated from optics [49–53] and adopted to photonic crystal waveguides [54], to surface plasmons in metallic nanostructures [52], and to Dirac-Weyl spinor systems under different geometrical and mass settings [29,39,55,56]. By this method, two sets of dipoles, one inside and another outside the cavity, as shown in Fig. 8(a), are used as the sources to produce the far-field scattering wave function. For spin-down electrons, the total potential in the cavity is $V_0 = 50$. There are quasibound edge modes with Fermi energy about half of the total potential, as shown by the peaks in the total cross section in Fig. 8(b). For spin-up electrons, the total potential in the cavity is $V_0 = 70$ and the classical dynamics are chaotic, which smooths out the sharp resonances, as shown in Fig. 8(c). For the edge mode associated with spin-down

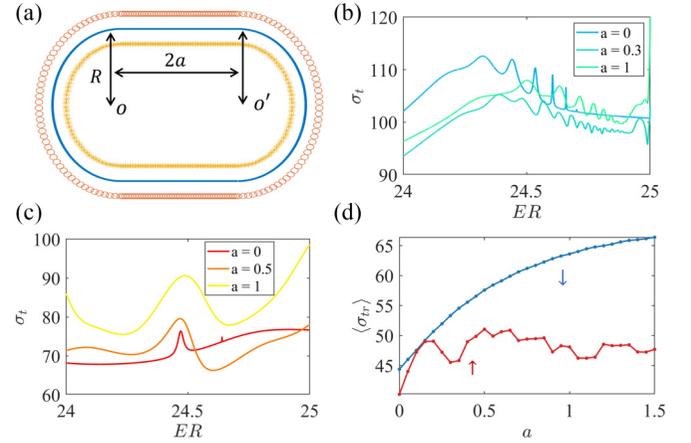


FIG. 8. Spin-resolved Dirac electron scattering from a stadium cavity of 2D pseudospin-1 Dirac material. (a) The stadium geometry (blue) defined by two parameters: the chaotic parameter a (the classical dynamics are chaotic for $a > 0$) and R , the radius of the two semicircles. Two sets of dipoles are displayed, one inside and another outside the stadium, which are used to calculate the scattering cross sections according to the multiple-multipole method developed for pseudospin-1 relativistic quantum scattering [29]. (b) Total cross section versus the Fermi energy parameter ER for a spin-down Dirac fermion for three values of a , where the total potential within the stadium is $V_0 = v + \mu = 50$. (c) Total cross section versus ER for a spin-up Dirac fermion for three values of a , where the total potential inside the stadium is $V_0 = v - \mu = 70$. (d) Momentum-transport cross section $\langle \sigma_{tr} \rangle$ versus the chaotic parameter for a spin-down (blue) and a spin-up (red) electron.

electrons, the resonant peaks have also been smoothed out. Intuitively, a larger potential in the cavity produces stronger scattering. However, the edge mode leads to strong scattering even with a small potential, as shown in Fig. 8(d), the momentum-transport cross section versus the stadium parameter a . It can be seen that a spin-down electron, due to its large momentum-transport cross section $\langle \sigma_{tr} \rangle$ as the result of the edge mode, produces larger and larger equivalent scattering resistance than that from a spin-up electron as the chaotic parameter increases.

IV. DISCUSSION

We investigated spin-dependent quantum scattering in 2D pseudospin-1 Dirac materials. The scatterer is placed on a magnetic insulator substrate so the real spin degree of the electrons becomes physically relevant, where the spin-up and spin-down electrons experience a different magnetic exchange potential and exhibit distinct scattering behaviors. We studied an eccentric circular scatterer that was previously used to uncover spin-dependent scattering behaviors in graphene. We found that a class of edge modes can arise for electrons with a specific spin orientation, where the electrons can be confined around the edge modes for a long time, generating resonant scattering with a large momentum-transport cross section. The quantum scattering behaviors of these electrons do not have a classical correspondence. On the contrary, electrons with the opposite spin do not possess such edge modes: they

tend to stay in the scattering region for a much shorter time with a small cross section. For these electrons, the quantum scattering dynamics have a classical correspondence, so ray tracing with Dirac electron optics can be used to understand their behaviors. The remarkable difference in the spin-specific scattering cross sections leads to tunable spin polarization and can even generate near-perfect spin polarization. The physical principles laid out in this paper can be useful for applications in spintronics.

The basic principle of spintronics is to manipulate the spin degree of freedom to bring unique capabilities to microelectronics and information technology with applications such as magnetic memories and sensors, radio-frequency and microwave devices, and logic and non-Boolean devices [57]. In spintronics, a key requirement is to achieve high spin polarization in functional materials [58], which has remained a challenge. For example, the early proposition of spin field-effect transistors for large-scale integrated circuits [59] requires high spin polarization [58,60–64]. Graphene spintronics [65] based on relativistic quantum mechanics of pseudospin-1/2 fermions possess certain advantages such as room-temperature spin transport with long spin diffusion lengths of several micrometers [66,67], gate-tunable carrier concentration, high electronic mobility, and efficient spin injection [68,69]. An early work [70] realized a nearly $\pm 100\%$ spin-polarized current in two-terminal bipolar spin diodes of zigzag graphene nanoribbons. A recent work reported a breakthrough [42]: 100% spin and valley polarization was achieved in monolayer transition metal dichalcogenides assisted by total external reflection with spin-orbit coupling and an electrostatic potential barrier. While an external magnetic field can also produce spin polarization and other intriguing physical effects, the required field strength can be on the order of tesla. Placing the 2D material on a substrate with magnetic insulators [31,32] or magnetic impurities [33] to generate an MEF provides a viable solution [31,36]. More generally, MEF facilitates studies of Dirac electron optics [1–7,38].

Experimentally, it has become feasible to implement electron scattering in 2D Dirac materials. For example, the width of p - n junction edge in Dirac materials can already be made sufficiently sharp [3,18,45] (e.g., $d \sim 1$ nm compared with the typical Fermi wavelength $\lambda_F \sim 10$ nm). In addition, the materials can be fabricated on the scale of micrometers to reach the small wavelength limit at which Dirac electron optics is applicable [71]. The required magnetic exchange potential has been realized in experiments [31,32]. For electrostatic potential in the eccentric circular shape, a previous experimental work [19] demonstrated that a circular p - n junction (a local embedded gate) in a graphene/hBN heterostructure can be created by local defect charge and STM tip with a square voltage pulse. Also, the Dirac electron scattering in multicircular quantum dots was studied [20]. Moreover, experimental material platforms already existed to create pseudospin-1 Dirac systems with a flat band, such as transition-metal oxide SrTiO₃/SrIrO₃/SrTiO₃ trilayer heterostructures [72], SrCu₂(BO₃)₂ [73], and graphene-In₂Te₂ bilayer [74]. These works suggest the feasibility of fabricating the eccentric circular cavity in our work by using the STM technology.

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APPENDIX A: S-MATRIX APPROACH TO ELASTIC DIRAC ELECTRON SCATTERING

Consider electronic scattering from a cavity made of 2D Dirac materials with a flat band. At low energies, the effective Hamiltonian describes the dynamics of a pseudospin-1 Dirac-Weyl quasiparticle. The cavity is subject to external electrical and MEFs: Its properties are controlled by an electric gate potential $\mathcal{V}_{\text{gate}}(r)$ and a magnetic exchange potential $\mathcal{M}(r)$ induced by the magnetic insulator substrate within the gate region [4]. The total Hamiltonian is

$$\hat{H} = v_F \sigma_0 \otimes \mathbf{S} \cdot \hat{\mathbf{p}} + \hbar v_F [\sigma_0 \otimes S_0 \mathcal{V}_{\text{gate}}(\mathbf{r}) - \sigma_z \otimes S_0 \mathcal{M}(\mathbf{r})], \quad (\text{A1})$$

where \mathbf{S} denotes the vector of spin-1 matrices, σ_0 and S_0 are the 2×2 and 3×3 identity matrices, respectively, σ_z is the Pauli z matrix, and v_F is the Fermi velocity. Tensor product of the three-component pseudospin-1 quasiparticles and two-component real spin 1/2 electron, so the Hamiltonian matrix is 6×6 , which can be block-diagonalized as $\hat{H} = \text{diag}[\hat{H}_1, \hat{H}_{-1}]$ with the following two 3×3 sub-Hamiltonian matrices \hat{H}_s for real spin index $s = \pm 1$:

$$\hat{H}_s = v_F \mathbf{S} \cdot \hat{\mathbf{p}} + \hbar v_F [\mathcal{V}_{\text{gate}}(\mathbf{r}) - s \mathcal{M}(\mathbf{r})], \quad (\text{A2})$$

where the identity $[\sigma_z \otimes S_0, \hat{H}] = 0$ has been used. The total potential is spin dependent:

$$V^s(r) \equiv \mathcal{V}_{\text{gate}}(r) - s \mathcal{M}(r).$$

The prototypical system we use to demonstrate achieving high spin polarization is an eccentric circular cavity defined by two distinct radii: R_1 and $R_2 < R_1$, where the centers of the two circles are located at O (the larger disk) and O' (the smaller disk) with the eccentric distance ξ between OO' , as shown in Fig. 1(b) in the main text. For convenience, we define three regions in the position space: region I with $V_0 = 0$ for $r > R_1$, region II with V_1^s for $r < R_1$ and $r' > R_2$, and region III with V_2^s for $r' < R_2$. The wave vectors in the three regions are given by

$$\begin{aligned} k_{\text{I}} &= |E|, \\ k_{\text{II}}^s &= |E - V_1^s|, \\ k_{\text{III}}^s &= |E - V_2^s|. \end{aligned}$$

The wave functions in the three regions can be written down according to the standard form of the spinor wave eigenvector of \hat{H}_s in the cylindrical coordinates, which are given by

$${}^k g_m = \frac{1}{\sqrt{2}} \begin{pmatrix} f_{m-1}(kr) e^{-i\theta} \\ i\alpha \sqrt{2} f_m(kr) \\ -f_{m+1}(kr) e^{i\theta} \end{pmatrix} e^{im\theta}, \quad (\text{A3})$$

where $\alpha \equiv \text{sign}(E - V)$, $k = |E - V|$. There are two cases for the function $f_m(kr)$: (i) $f_m = H_m^{(1,2)}$, the Hankel functions of the first and the second kind, and (ii) $f_m = J_m$, the Bessel function. For cases (i) and (ii), ${}^k g_m$ is given by ${}^k g_m = {}^k h_m^{(1,2)}$

and ${}^k g_m = {}^k j_m$, respectively. In particular, in region I, the wave function can be expanded in the spinor cylindrical wave basis as

$$\Psi^{(I)}(\vec{r}) = \sum_{m=-\infty}^{+\infty} a_m^I \left[{}^{k_1} h_m^{(2)} + \sum_{j=-\infty}^{+\infty} S_{mj} {}^{k_1} h_j^{(1)} \right]. \quad (\text{A4})$$

In region II, the wave function can be written as

$$\Psi^{(II)}(\vec{r}) = \sum_{m=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} m a_{lj}^{II} \left[{}^{k_{II}} h_l^{(2)} + \sum_{j=-\infty}^{+\infty} S_{lj}^{\text{od}} {}^{k_{II}} h_j^{(1)} \right], \quad (\text{A5})$$

where S^{od} is the off-diagonal scattering matrix for the eccentric circular cavity and S^{cd} is the diagonal matrix to characterize the scattering from a circular domain [4], which are related by $S^{\text{od}} = U^{-1} S^{\text{cd}} U$, or

$$\begin{aligned} S_{lj}^{\text{od}} &= \sum_{l',j'} (U^{-1})_{ll'} S_{l'j'}^{\text{cd}} U_{j'j} \\ &= \sum_{l',j'} J_{l-l'}(k_{II}\xi) S_{l'l'}^{\text{cd}} \delta_{l'j'} J_{j-j'}(k_{II}\xi) \\ &= \sum_{l'} J_{l-l'} S_{l'l'}^{\text{cd}} J_{j-l'}. \end{aligned}$$

The boundary conditions for a pseudospin-1 quasiparticle [46] stipulate continuity of the second component of the spinor wave function and conservation of the radial current density:

$$\Psi_2^I(R_1) = \Psi_2^{II}(R_1), \quad (\text{A6})$$

$$\Psi_1^I(R_1)e^{i\theta} + \Psi_3^I(R_1)e^{-i\theta} = \Psi_1^{II}(R_1)e^{i\theta} + \Psi_3^{II}(R_1)e^{-i\theta}. \quad (\text{A7})$$

In matrix form, the boundary conditions can be expressed as

$$A^I[X^{(2)} + SX^{(1)}] = \alpha_I \alpha_{II} A^{II}[x^{(2)} + S^{\text{od}}x^{(1)}], \quad (\text{A8})$$

$$\begin{aligned} A^I[[Z^{(2)} - Y^{(2)}] + S[Z^{(1)} - Y^{(1)}]] \\ = A^{II}[[z^{(2)} - y^{(2)}] + S^{\text{od}}[z^{(1)} - y^{(1)}]], \end{aligned} \quad (\text{A9})$$

where $\alpha_I \equiv \text{sign}(E)$, $\alpha_{II} \equiv \text{sign}(E - V_1)$, $A^I \equiv [a_m^I \delta_{mj}]$, $A^{II} \equiv [{}^m a_j^{II}]$, and

$$\begin{aligned} X^{(1,2)} &\equiv [H_m^{(1,2)}(k_I R_1) \delta_{mj}], \quad x^{(1,2)} \equiv [H_m^{(1,2)}(k_{II} R_1) \delta_{mj}], \\ Y^{(1,2)} &\equiv [H_{m+1}^{(1,2)}(k_I R_1) \delta_{mj}], \quad y^{(1,2)} \equiv [H_{m+1}^{(1,2)}(k_{II} R_1) \delta_{mj}], \\ Z^{(1,2)} &\equiv [H_{m-1}^{(1,2)}(k_I R_1) \delta_{mj}], \quad z^{(1,2)} \equiv [H_{m-1}^{(1,2)}(k_{II} R_1) \delta_{mj}]. \end{aligned} \quad (\text{A10})$$

The spinor wave function can be written as

$${}^k h_m^{(1,2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} H_{m-1}^{(1,2)}(kr) e^{-i\theta} \\ i\alpha \sqrt{2} H_m^{(1,2)}(kr) \\ -H_{m+1}^{(1,2)}(kr) e^{i\theta} \end{pmatrix} e^{im\theta}, \quad (\text{A11})$$

where the general form of the basis is described by Eq. (A3). The scattering matrix can be written as

$$S = -\frac{Z^{(2)} - Y^{(2)} - \alpha_I \alpha_{II} X^{(2)} \mathcal{T}}{Z^{(1)} - Y^{(1)} - \alpha_I \alpha_{II} X^{(1)} \mathcal{T}}, \quad (\text{A12})$$

where $\mathcal{T} \equiv F^{-1}[H - G]$, and

$$\begin{aligned} F &\equiv x^{(2)} + S^{\text{od}}x^{(1)}, \\ G &\equiv y^{(2)} + S^{\text{od}}y^{(1)}, \\ H &\equiv z^{(2)} + S^{\text{od}}z^{(1)}. \end{aligned} \quad (\text{A13})$$

The coefficient A^I is determined by the incident wave function (see Appendix C) and the coefficient A^{II} is given by

$$A^{II} = \alpha_I \alpha_{II} A^I [X^{(2)} + SX^{(1)}] F^{-1}.$$

Using the Graf's addition theorem [20], we have, for $r' > \xi$,

$$H_m^{(1,2)}(kr) e^{im\theta} = \sum_{n=-\infty}^{+\infty} J_{m-n}(k\xi) e^{in\theta'} H_n^{(1,2)}(kr'), \quad (\text{A14})$$

which gives

$${}^k h_m^{(1,2)} = \sum_{n=-\infty}^{+\infty} J_{m-n}(k\xi) {}^k \tilde{h}_n^{(1,2)}. \quad (\text{A15})$$

For convenience, in the following, we use the tilde symbol to denote the quantities in the circular region of origin at O' . We have

$$\begin{aligned} {}^{k_{II}} h_l^{(2)} + \sum_{j=-\infty}^{+\infty} S_{lj}^{\text{od}} {}^{k_{II}} h_j^{(1)} &= \sum_{n=-\infty}^{+\infty} J_{l-n}(k_{II}\xi) {}^{k_{II}} \tilde{h}_n^{(2)} + \sum_{j=-\infty}^{+\infty} S_{lj}^{\text{od}} \left[\sum_{n=-\infty}^{+\infty} J_{j-n}(k_{II}\xi) {}^{k_{II}} \tilde{h}_n^{(1)} \right] \\ &= \sum_{l'=-\infty}^{+\infty} J_{l-l'} {}^{k_{II}} \tilde{h}_{l'}^{(2)} + \sum_{n,l'=-\infty}^{+\infty} J_{l-l'} S_{l'l'}^{\text{cd}} \sum_{j=-\infty}^{+\infty} (J_{j-n} J_{j-l'}) {}^{k_{II}} \tilde{h}_n^{(1)} \\ &= \sum_{l'=-\infty}^{+\infty} J_{l-l'}(k_{II}\xi) [{}^{k_{II}} \tilde{h}_{l'}^{(2)} + S_{l'l'}^{\text{cd}} {}^{k_{II}} \tilde{h}_{l'}^{(1)}], \end{aligned} \quad (\text{A16})$$

where

$$\delta_{l'n} = \sum_{j=-\infty}^{+\infty} J_{j-l'}(k_{II}\xi) J_{j-n}(k_{II}\xi).$$

The wave function in region II with origin O can be rewritten as a wave function with origin O' as $\Psi^{\text{II}}(r, \theta) = \tilde{\Psi}^{\text{II}}(r', \theta')$, where

$$\tilde{\Psi}^{\text{II}}(r', \theta') = \sum_{m=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} m \tilde{a}_l^{\text{II}} [k_{\text{II}} \tilde{h}_l^{(2)} + S_{ll}^{\text{cd}} k_{\text{II}} \tilde{h}_l^{(1)}],$$

with $m \tilde{a}_l^{\text{II}} \equiv \sum_{l'} m a_{l'}^{\text{II}} J_{l'-l}(k_{\text{II}} \xi)$. In region III with origin O' , the wave function is given by

$$\tilde{\Psi}^{\text{III}}(r', \theta') = \sum_{m=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} m \tilde{b}_l^{k_{\text{III}}} \tilde{j}_l. \quad (\text{A17})$$

Using the boundary condition Eq. (A6), we obtain

$$m \tilde{b}_l = \alpha_{\text{II}} \alpha_{\text{III}} m \tilde{a}_l^{\text{II}} \frac{H_l^{(2)}(k_{\text{II}} R_2) + S_{ll}^{\text{cd}} H_l^{(1)}(k_{\text{II}} R_2)}{J_l(k_{\text{III}} R_2)}. \quad (\text{A18})$$

$$S_{mm}^{\text{cd}} = -\frac{J_m(k_{\text{III}} R_2) k_{\text{III}} h_m^{(2)}(R_2) - \alpha_{\text{II}} \alpha_{\text{III}} H_m^{(2)}(k_{\text{II}} R_2) k_{\text{III}} j_m(R_2)}{J_m(k_{\text{III}} R_2) k_{\text{III}} h_m^{(1)}(R_2) - \alpha_{\text{II}} \alpha_{\text{III}} H_m^{(1)}(k_{\text{II}} R_2) k_{\text{III}} j_m(R_2)}, \quad (\text{B5})$$

with $\alpha_{\text{III}} \equiv \text{sign}(E - V_2)$.

APPENDIX C: SCATTERING CROSS SECTIONS

The wave function in region I from Eq. (A4) can be rewritten as the sum of the contributions from the incident and scattering waves:

$$\begin{aligned} \Psi^{(1)}(\vec{r}) &= \sum_{m=-\infty}^{\infty} a_m^{\text{I}} \left[2 k_1 j_m + \sum_{m'=-\infty}^{\infty} (S_{mm'} - \delta_{mm'}) k_1 h_{m'}^{(1)} \right] \\ &= \chi_{\text{in}} + \sum_{m=-\infty}^{\infty} a_m^{\text{I}} \sum_{m'=-\infty}^{\infty} T_{mm'} k_1 h_{m'}^{(1)}. \end{aligned} \quad (\text{C1})$$

The incident wave function corresponds to

$$\chi_{\text{in}} \equiv \sum_{m=-\infty}^{\infty} 2 a_m^{\text{I}} k_1 j_m. \quad (\text{C2})$$

The norm square of the second term in Eq. (C1), which is the scattering wave function, is defined as the scattering probability in the near field measured from the cavity in region I, with the transmission matrix defined as $T_{mm'} \equiv S_{mm'} - \delta_{mm'}$. The coefficient a_m^{I} for each angular momentum channel is determined by the incident plane wave function:

$$\chi_{\text{in}}(r, \theta) = \frac{1}{2} \begin{pmatrix} e^{-i\theta'} \\ \sqrt{2s} \\ e^{i\theta'} \end{pmatrix} e^{i\mathbf{k}_{\text{in}} \cdot \mathbf{r}}, \quad (\text{C3})$$

with the incident wave vector $\mathbf{k}_{\text{in}} = k_1(\cos \theta', \sin \theta')$. Expanding the incident wave function for each angular momentum

APPENDIX B: SCATTERING MATRIX FOR A CIRCULAR CAVITY

To obtain the scattering matrix S^{cd} , we consider a circular cavity of radius R_2 centered at O where $r > R_2$ and $0 < r < R_2$ define regions II and III, respectively. Due to the circular symmetry, the wave function for each angular momentum channel can be written as

$$\begin{aligned} \Psi_m^{\text{II}} &= k_{\text{II}} h_m^{(2)} + S_{mm}^{\text{cd}} k_{\text{II}} h_m^{(1)}, \\ \Psi_m^{\text{III}} &= B_m^{k_{\text{III}}} j_m. \end{aligned} \quad (\text{B1})$$

Applying the boundary conditions gives

$$\begin{aligned} \alpha_{\text{II}} [H_m^{(2)}(k_{\text{II}} R_2) + S_{mm}^{\text{cd}} H_m^{(1)}(k_{\text{II}} R_2)] &= \alpha_{\text{III}} B_m J_m(k_{\text{III}} R_2), \\ k_{\text{II}} h_m^{(2)}(R_2) + S_{mm}^{\text{cd}} k_{\text{II}} h_m^{(1)}(R_2) &= B_m k_{\text{III}} j_m(R_2), \end{aligned} \quad (\text{B2})$$

where

$$k_{\text{II}} h_{1+3}^{(1,2)}(R_2) \equiv H_{m-1}^{(1,2)}(k_{\text{II}} R_2) - H_{m+1}^{(1,2)}(k_{\text{II}} R_2), \quad (\text{B3})$$

$$k_{\text{III}} j_m(R_2) \equiv J_{m-1}(k_{\text{III}} R_2) - J_{m+1}(k_{\text{III}} R_2). \quad (\text{B4})$$

We thus have

channel by the Jacobi-Anger formula

$$e^{ik_1 r \cos(\theta - \theta')} = \sum_{m=-\infty}^{+\infty} i^m J_m(k_1 r) e^{im(\theta - \theta')}, \quad (\text{C4})$$

we obtain

$$\chi_{\text{in}}(r, \theta) = \sum_{m=-\infty}^{+\infty} \frac{1}{\sqrt{2}} i^{m-1} e^{-im\theta'} k_1 j_m. \quad (\text{C5})$$

Note that j_m is the three-component vector defined by Eq. (A3), while J_m is the scalar Bessel function. The coefficient $a_m^{\text{I}}(\theta')$ in Eqs. (C1) and (C2) is then given by

$$a_m^{\text{I}}(\theta') = i^{m-1} e^{-im\theta'} / (2\sqrt{2}). \quad (\text{C6})$$

Scattering cross section characterizes the behavior of particles in the far-field region $kr \gg 1$ (from the cavity). In the far field, the wave function from Eq. (C1) tends to

$$\lim_{kr \gg 1} \psi(r, \theta) = \chi_{\text{in}} + \frac{f(\theta, \theta')}{2\sqrt{-ir}} \begin{pmatrix} e^{-i\theta} \\ \sqrt{2s} \\ e^{i\theta} \end{pmatrix} e^{ik_1 r}, \quad (\text{C7})$$

with the scattering angle distribution in the far field as

$$f(\theta, \theta') = \frac{2}{\sqrt{\pi} k_1} \sum_{m=-\infty}^{\infty} a_m^{\text{I}}(\theta') \sum_{m'=-\infty}^{\infty} T_{mm'} (-i)^{m'} e^{im'\theta}, \quad (\text{C8})$$

a result of the asymptotic behavior of the Hankel function

$$\lim_{x \rightarrow \infty} H_m^{(1,2)}(x) \rightarrow \sqrt{\frac{2}{\pi x}} e^{\pm i(x - m\pi/2 - \pi/4)},$$

and the standard plane-wave normalization requirement. The differential cross section σ_{diff} is given in terms of $f(\theta, \theta')$ as

$$\sigma_{\text{diff}} \equiv \frac{d\sigma}{d\theta} = |f(\theta, \theta')|^2, \quad (\text{C9})$$

and the total scattering cross section, which records the probability of scattering events under all possible directions, is given by

$$\sigma_t(\theta') = \oint d\theta |f(\theta, \theta')|^2. \quad (\text{C10})$$

The momentum-transport cross section is defined as

$$\sigma_{tr}(\theta') = \oint d\theta (1 - \cos\theta) |f(\theta, \theta')|^2. \quad (\text{C11})$$

Averaging over the incident angle θ' leads to

$$\bar{\sigma}_t = \frac{1}{2\pi} \oint d\theta' \sigma_t(\theta'), \quad (\text{C12})$$

$$\bar{\sigma}_{tr} = \frac{1}{2\pi} \oint d\theta' \sigma_{tr}(\theta'). \quad (\text{C13})$$

Performing an average over some Fermi energy interval, we get

$$\langle \sigma_{tr} \rangle = \frac{1}{E_1 - E_0} \int_{E_0}^{E_1} dE \sigma_{tr}(E). \quad (\text{C14})$$

The momentum transport cross section determines the transport relaxation time τ_{tr} through

$$\frac{1}{\tau_{tr}} = n_c v_F \sigma_{tr}, \quad (\text{C15})$$

where n_c is the concentration of identical scatters. Our scattering system is sufficiently dilute so multiple scattering events

can be neglected. For ballistic transport and elastic scattering with system size comparable with the mean-free path: $\mathcal{L}_s \approx \mathcal{L}_{\text{mean-free}} = v_F \tau_{tr}$, the semiclassical Boltzmann transport theory gives that the conductivity is inverse of the σ_{tr} :

$$G \propto \frac{1}{\sigma_{tr}}. \quad (\text{C16})$$

The spin polarization is defined by the spin-resolved transmission coefficient as [75]

$$P_z = (T^\downarrow - T^\uparrow)/(T^\downarrow + T^\uparrow).$$

We thus have

$$P_z = \frac{\sigma_{tr}^\downarrow - \sigma_{tr}^\uparrow}{\sigma_{tr}^\downarrow + \sigma_{tr}^\uparrow}, \quad (\text{C17})$$

with $\sigma_{tr}^{\downarrow\uparrow} \propto R^{\downarrow\uparrow}$, where the resistance R is the inverse of the conductivity G .

APPENDIX D: VALIDATION OF S-MATRIX APPROACH

1. Reduction from eccentric circular to annular cavity

For an annular scattering cavity ($\xi = 0$), the scattering matrix can be analytically calculated, providing a way to validate the scattering-matrix approach to the general case of $\xi \neq 0$. For this purpose, we consider the annular scattering cavity $\xi = 0$ but with two boundaries: one at R_1 and another at R_2 . In the three regions, the wave functions associated with an angular momentum channel are

$$\begin{aligned} \Psi_m^I &= k_I h_m^{(2)} + S_{mm} k_I h_m^{(1)}, \\ \Psi_m^{II} &= A_m [k_{II} h_m^{(2)} + S_{mmm}^{\text{cd}} k_{II} h_m^{(1)}], \\ \Psi_m^{III} &= B_m k_{III} j_m. \end{aligned} \quad (\text{D1})$$

Imposing the boundary conditions at $r = R_1$ and $r = R_2$ gives

$$\begin{bmatrix} k_{II} h_m^{(2)}(R_1) & 0 & k_{II} h_m^{(1)}(R_1) & -k_I h_m^{(1)}(R_1) \\ k_{II} h_m^{(2)}(R_1) & 0 & k_{II} h_m^{(1)}(R_1) & -k_I h_m^{(1)}(R_1) \\ k_{II} h_m^{(2)}(R_2) & -k_{II} j_m(R_2) & k_{II} h_m^{(1)}(R_2) & 0 \\ k_{II} h_m^{(2)}(R_2) & -k_{II} j_m(R_2) & k_{II} h_m^{(1)}(R_2) & 0 \end{bmatrix} \begin{bmatrix} A_m \\ B_m \\ C_m \\ S_m \end{bmatrix} = \begin{bmatrix} k_I h_m^{(2)}(R_1) \\ k_I h_m^{(2)}(R_1) \\ 0 \\ 0 \end{bmatrix}, \quad (\text{D2})$$

where $C_m \equiv A_m S_{mmm}^{\text{cd}}$ and

$$\begin{aligned} k_I h_m^{(1,2)}(R_j) &= s_i H_m^{(1,2)}(k_i R_j), \\ k_{II} h_m^{(1,2)}(R_j) &= H_{m-1}^{(1,2)}(k_{II} R_j) - H_{m+1}^{(1,2)}(k_{II} R_j), \\ k_I j_m(R_j) &= s_i J_m(k_i R_j), \\ k_{II} j_m(R_j) &= J_{m-1}(k_{II} R_j) - J_{m+1}(k_{II} R_j), \end{aligned}$$

with $i = \text{I, II, III}$ and $j = 1, 2$. Note that $k_{II} h_m^{(2)}(R_1)$ and $k_{II} h_m^{(2)}(R_1)$ are scalars, roughly corresponding to the second component and the sum of the first and third components of the radial part of $k_{II} h_m^{(2)}(R_1)$, respectively. We have

$$A_m = \frac{s_I H_m^{(2)}(k_I R_1) + s_I H_m^{(1)}(k_I R_1) S_{mm}}{s_{II} H_m^{(2)}(k_{II} R_1) + s_{II} H_m^{(1)}(k_{II} R_1) S_{mmm}^{\text{cd}}}, \quad (\text{D3})$$

$$B_m = A_m \frac{s_{II} H_m^{(2)}(k_{II} R_2) + s_{II} H_m^{(1)}(k_{II} R_2) S_{mmm}^{\text{cd}}}{s_{III} J_m(k_{III} R_2)}. \quad (\text{D4})$$

The scattering matrix is given by

$$S_{mm} = -\frac{s_I y_m H_m^{(2)}(k_I R_1) - s_{II} x_m [H_{m-1}^{(2)}(k_I R_1) - H_{m+1}^{(2)}(k_I R_1)]}{s_I y_m H_m^{(1)}(k_I R_1) - s_{II} x_m [H_{m-1}^{(1)}(k_I R_1) - H_{m+1}^{(1)}(k_I R_1)]}, \quad (\text{D5})$$

where

$$\begin{aligned} x_m &= H_m^{(2)}(k_{II} R_1) + H_m^{(1)}(k_{II} R_1) S_{mm}^{\text{cd}}, \\ y_m &= [H_{m-1}^{(2)}(k_{II} R_1) - H_{m+1}^{(2)}(k_{II} R_1)] \\ &\quad + [H_{m-1}^{(1)}(k_{II} R_1) - H_{m+1}^{(1)}(k_{II} R_1)] S_{mm}^{\text{cd}}. \end{aligned}$$

For the eccentric circular cavity, the scattering matrix can be determined by Eq. (A12). We can reduce the eccentric cavity to an annular cavity by taking the limit $\xi \rightarrow 0$. In that case, the off-diagonal scattering matrix will reduce to the diagonal matrix: $S_{lj}^{\text{cd}} \rightarrow S_{lj}^{\text{cd}} \delta_{jl}$ and $S_{mm'} \rightarrow S_{mm} \delta_{mm'}$. We have that Eq. (A12) reduces to the same form of Eq. (D5) as

$$S_{mm} = -\frac{Z_m^{(2)} - Y_m^{(2)} - s_I s_{II} X_m^{(2)} \mathcal{T}_m}{Z_m^{(1)} - Y_m^{(1)} - s_I s_{II} X_m^{(1)} \mathcal{T}_m}, \quad (\text{D6})$$

with $\mathcal{T}_m = y_m/x_m$. We find that, numerically, the difference between the scattering matrix in Eq. (D6) and that in Eq. (D5) is on the order of computer round-off error (about 10^{-15}). The excellent agreement between the analytic S matrix for $\xi = 0$ and the numerically calculated matrix in the limit $\xi \rightarrow 0$ validates the S -matrix approach manifested through Eq. (A12).

Figure 9(a) shows the convergence of the S matrix in a large angular momentum range. For large angular momenta, the S -matrix elements are negligibly small, suggesting that these angular-momentum channels contribute little to the scattering process. More specifically, Fig. 9(a) is the color map of the S -matrix elements in the angular momentum representation. The near-zero components in the high angular momentum basis mean convergence. Note that the diagonal term in Fig. 9(a) will be removed in the transmission matrix $\mathcal{T} = S - I$, which determines the scattering cross sections.

2. Mirror symmetry

An eccentric circular cavity possesses the mirror (parity) symmetry. The parity operator for pseudospin-1/2 quasiparticles is given by [4] $\mathcal{P}_x = i\sigma_x R_y$, where R_y denotes the mirror transform in the position space [e.g., $x \rightarrow x$ ($k_x \rightarrow k_x$), $y \rightarrow -y$ ($k_y \rightarrow -k_y$), and $\theta \rightarrow -\theta$], and $i\sigma_x$ arises from the $-\pi$ rotation in the counterclockwise direction about the x axis in the spin space, i.e., $e^{i\pi\sigma_x/2}$, which is equivalent to the mirror-transform operation in the three-dimensional space. For pseudospin-1 Dirac-Weyl quasiparticles, using Rodrigues's rotation formula [76], we obtain the rotation operator as

$$e^{i\theta(\hat{n}\cdot\mathbf{J})} = I_3 + i(\hat{n}\cdot\mathbf{J}) \sin \theta + (\hat{n}\cdot\mathbf{J})^2 (\cos \theta - 1), \quad (\text{D7})$$

where \mathbf{J} denotes the total angular momentum, \hat{n} specifies the rotation axis and θ is the rotation angle in the clockwise direction around \hat{n} . Consider the rotation operation with $\theta = \pi$

around x axis, we have $\tilde{S}_x \equiv e^{i\pi S_x} = I_3 - 2S_x^2$, so

$$\tilde{S}_x = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

where $\tilde{S}_x^2 = I_3$ and $\tilde{S}_x^{-1} = \tilde{S}_x$. The parity operator is given by $\mathcal{P}_x = \tilde{S}_x R_y$ with $\mathcal{P}_x \mathcal{P}_x^{-1} = I_3$. The Hamiltonian Eq. (A2) is invariant under this parity operation:

$$\begin{aligned} \mathcal{P}_x \hat{H}_y \mathcal{P}_x^{-1} &= v_F \mathcal{P}_x \mathbf{S} \cdot \hat{\mathbf{p}} \mathcal{P}_x^{-1} + \mathcal{V}_{\text{gate}}(x, -y) - s\mathcal{M}(x, -y) \\ &= v_F (S_x \hat{p}_x + S_y \hat{p}_y) + \mathcal{V}_{\text{gate}}(x, -y) - s\mathcal{M}(x, -y). \end{aligned}$$

where

$$\begin{aligned} \mathcal{P}_x S_x \mathcal{P}_x^{-1} \mathcal{P}_x \hat{p}_x \mathcal{P}_x^{-1} &= S_x \hat{p}_x, \\ \mathcal{P}_x S_y \mathcal{P}_x^{-1} \mathcal{P}_x \hat{p}_y \mathcal{P}_x^{-1} &= (-S_y)(-\hat{p}_y) = S_y \hat{p}_y. \end{aligned} \quad (\text{D8})$$

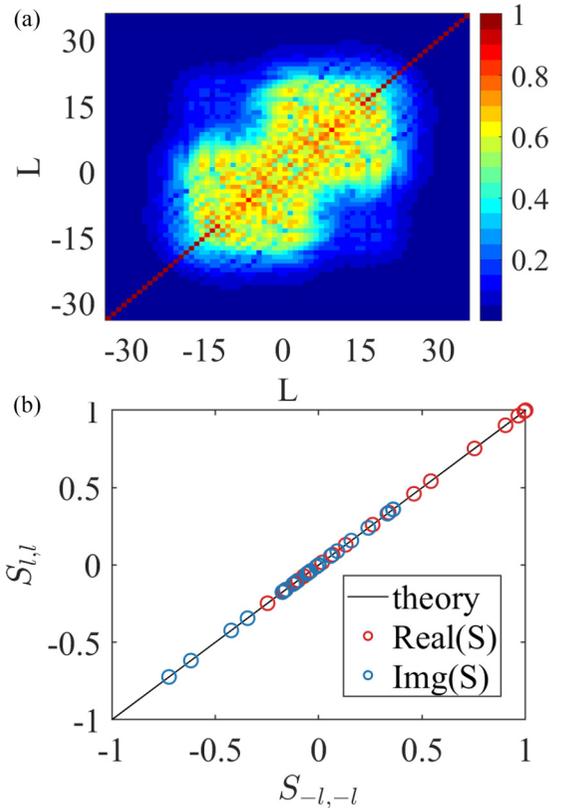


FIG. 9. Validation of the S -matrix approach. (a) Distribution of the elements of $|S|^{1/4}$ in a large angular momentum interval. Since the matrix elements in the S matrix are between zero and one, the elements of $|S|^{1/4}$ are used for better visualization. (b) Mirror symmetry constraint of the real (red) and imaginary (blue) parts of the S -matrix elements for $\xi = 0.165$. Other parameters are potentials $V_1 = -10 - 2\mu$, $V_2 = 40 - 2\mu$, with $\mu = 20$ for the spin-up electrons. The angular momentum range is $L = -35 : 1 : 35$ and the Fermi energy range is $E = 14.5 : (10^{-3}/2) : 15.3$.

Finally, using

$$\begin{aligned}\mathcal{V}_{\text{gate}}(x, -y) &= \mathcal{V}_{\text{gate}}(x, y), \\ \mathcal{M}(x, -y) &= \mathcal{M}(x, y),\end{aligned}$$

we obtain $\mathcal{P}_x \hat{H}_s \mathcal{P}_x^{-1} = \hat{H}_s$. As a result, the parity operation on the wave function is also a solution of the system. The cylindrical spinor basis under the parity transform has the form

$$\mathcal{P}_x^k h_m^{(1,2)} = (-1)^{m+1} \frac{1}{\sqrt{2}} \begin{pmatrix} H_{-m-1}^{(1,2)}(kr) e^{-i\theta} \\ is\sqrt{2} H_{-m}^{(1,2)}(kr) \\ -H_{-m+1}^{(1,2)}(kr) e^{i\theta} \end{pmatrix} e^{-im\theta} = (-1)^{m+1} k h_{-m}^{(1,2)}. \quad (\text{D9})$$

The wave function in region I in the eccentric circular cavity is

$$\begin{aligned}\mathcal{P}_x \Psi^I &= \sum_{m=-\infty}^{+\infty} \mathcal{P}_x a_m^I \mathcal{P}_x^{-1} \left[\mathcal{P}_x^{k_I} h_m^{(2)} + \sum_{m'=-\infty}^{+\infty} \mathcal{P}_x S_{mm'} \mathcal{P}_x^{-1} \mathcal{P}_x^{k_I} h_{m'}^{(1)} \right] \\ &= \sum_{m=-\infty}^{+\infty} \mathcal{P}_x a_m^I \mathcal{P}_x^{-1} (-1)^{m+1} \left[k_I h_{-m}^{(2)} + \sum_{m'=-\infty}^{+\infty} \mathcal{P}_x S_{mm'} \mathcal{P}_x^{-1} (-1)^{m'-m} k_I h_{-m'}^{(1)} \right] \\ &= \sum_{n=-\infty}^{+\infty} \tilde{A}_n^I \left[k_I h_n^{(2)} + \sum_{n'=-\infty}^{+\infty} S_{nn'}^{k_I} h_{n'}^{(1)} \right],\end{aligned} \quad (\text{D10})$$

with $n \equiv -m$, $n' \equiv -m'$ and $\tilde{A}_n^I \equiv \mathcal{P}_x a_{-n}^I \mathcal{P}_x^{-1} (-1)^{-n+1}$. We have

$$S_{nn'} \equiv S_{-m, -m'} = \mathcal{P}_x S_{mm'} \mathcal{P}_x^{-1} (-1)^{m'-m} = (-1)^{m'-m} S_{mm'}.$$

For $m = m'$, we get $S_{m,m} = S_{-m,-m}$. Thus, the real and imaginary parts of the S matrix obey this relation, as shown in Fig. 9(b).

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