

## Interaction-induced Liouvillian skin effect in a fermionic chain with a two-body loss

Shu Hamanaka,<sup>1,\*</sup> Kazuki Yamamoto<sup>1,2</sup> and Tsuneya Yoshida<sup>1</sup><sup>1</sup>*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*<sup>2</sup>*Department of Physics, Tokyo Institute of Technology, Meguro, Tokyo 152-8551, Japan*

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Despite recent intensive research on topological aspects of open quantum systems, effects of strong interactions have not been sufficiently explored. In this paper, we demonstrate that complex-valued interactions induce the Liouvillian skin effect by analyzing a one-dimensional correlated model with two-body loss. We show that, in the presence of complex-valued interactions, eigenmodes and eigenvalues of the Liouvillian strongly depend on boundary conditions. Specifically, we find that complex-valued interactions induce localization of eigenmodes of the Liouvillian around the right edge under open boundary conditions. To characterize the Liouvillian skin effect, we define the topological invariant by using the Liouvillian superoperator. Then, we numerically confirm that the topological invariant captures the Liouvillian skin effect. Furthermore, the presence of the localization of eigenmodes results in the unique dynamics observed only under open boundary conditions: particle accumulation at the right edge in transient dynamics. Our result paves the way to realize topological phenomena in open quantum systems induced by strong interactions.

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## I. INTRODUCTION

In the past decade, a lot of theoretical and experimental studies have uncovered topological aspects of condensed-matter systems [1–10]. In particular, it has been elucidated that strong correlations alter topological phases and lead to novel phenomena. For example, it has turned out that interactions change the  $\mathbb{Z}$ -classification to the  $\mathbb{Z}_8$ -classification for one-dimensional topological superconductors [11,12], and another study has shown that strong correlations generate topological Mott phases [13]. Moreover, in Ref. [14], the interaction-enabled topological insulator has been proposed, which has no counterpart in noninteracting systems.

On the other hand, non-Hermitian physics has attracted broad interest in classical and open quantum systems [15–33]. One of the most remarkable phenomena induced by non-Hermiticity is the non-Hermitian skin effect, which is characterized by the extreme sensitivity of eigenvalues and eigenstates to boundary conditions [34–44]. The non-Hermitian skin effect has been experimentally observed in ultracold <sup>87</sup>Rb atoms [45] as well as electric circuits [46], quantum walks [47], and mechanical metamaterials [48]. In noninteracting systems, theoretical studies have shown that the non-Hermitian skin effect is caused by the nontrivial point-gap topology, which is intrinsic to non-Hermitian systems [35–37]. Furthermore, the non-Hermitian skin effect has been extended to open quantum systems following the Lindblad master equation [39,49–53]. In particular, the Liouvillian skin effect manifests as the extreme dependence of eigenvalues and eigenmodes of the Liouvillian on boundary conditions. In particular, the eigenmode localized near the

edge is referred to as the skin mode. It has been pointed out that the Liouvillian skin effect has a striking influence on the relaxation processes. Specifically, it has been reported that the maximal relaxation time to the steady state can diverge while maintaining the Liouvillian gap finite [50,53].

In addition to the above progress of the non-Hermitian topological band theory, it has become possible to implement dissipative correlated systems in ultracold atoms [54–62]. This development has opened up a new direction in studies of novel phases and phenomena, such as nonequilibrium steady states [63,64] and dynamical phase transitions [65,66]. Previous studies have revealed that particle losses induce unique phenomena. In particular, two-body loss brings about unusual behavior, e.g., the sign reversal of magnetic correlations [54,67]. Moreover, a lot of theoretical studies have been conducted on a variety of quantum many-body phenomena with atom losses [68–76], such as unconventional superfluid phase transitions in a dissipative BCS model [77,78] and anomalous dissipation-induced renormalization-group flows in a non-Hermitian Kondo model [79].

In view of the pivotal role of interactions in enriching topological phases in Hermitian systems and inducing unique phenomena in open quantum systems, one may naturally expect the presence of novel phenomena induced by the interplay between strong interactions and non-Hermitian topology. So far, the effects of interactions of non-Hermitian topological phases have been studied in several works [42,80–97]. However, previous studies have mainly focused on the effective Hamiltonian, which captures the time evolution of a single trajectory between successive quantum jumps [98]. Thus, it seems that the effects of interaction on the topological property of the Liouvillian remain unclear [99–103]. More specifically, whether many-body interactions can induce the Liouvillian skin effect has not been addressed.

\*hamanaka.shu.45p@st.kyoto-u.ac.jp

In this work, we demonstrate that complex-valued interaction can induce the Liouvillian skin effect in one-dimensional open quantum systems. Specifically, we analyze the correlated fermionic systems with two-body loss. We show that due to strong interactions, eigenmodes and eigenvalues of the Liouvillian become extremely sensitive to boundary conditions. In particular, eigenmodes of the Liouvillian exhibit localization near the edge. To characterize the Liouvillian skin effect, we introduce the topological invariant defined by the Liouvillian superoperator. Then, we numerically reveal that the above topological invariant characterizes the Liouvillian skin effect. Moreover, the Liouvillian skin effect significantly affects the dynamics. In particular, in transient dynamics, particles accumulate near the right edge under open boundary conditions (OBC).

The rest of this paper is organized as follows. In Sec. II, we first introduce the dissipative one-dimensional correlated model. We then briefly explain the methods to analyze the Lindblad equation via the vectorization of the density matrix. Section III provides the definition of the topological invariant and the right-state particle density, which measures the degree of localization of eigenmodes in many-body systems. Then, in Sec. IV, a numerical demonstration of the interaction-induced Liouvillian skin effect is conducted. We give the conclusions in Sec. V. In Appendix A, we discuss the relation between the symmetry of the Liouvillian and the topological number. In Appendix B, we compute the topological number analytically and give the characterization of the Liouvillian skin effect reported in Ref. [50]. We numerically show the absence of the Liouvillian skin effect in noninteracting systems in Appendix C. Appendix D is devoted to the sensitivity of eigenvalues of the Liouvillian to boundary conditions. We provide the derivation of an alternative method for calculating the topological number in Appendix E. Appendix F gives the results about the slowing down process corresponding to the Liouvillian skin effect. In Appendix G, we demonstrate that the Liouvillian skin effect survives for other configurations of down-spins. In Appendix H, we give the reason why particles are localized near the right edge.

## II. MODEL AND METHOD

### A. Falicov-Kimball model with two-body loss

We consider the two-orbital Falicov-Kimball model [104]

$$H = \sum_{\langle ij \rangle \alpha \beta} h_{i\alpha j\beta} c_{i\alpha\uparrow}^\dagger c_{j\beta\uparrow} + U \sum_j n_{jb\uparrow} n_{jb\downarrow}, \quad (1)$$

where  $c_{j\alpha\sigma}^\dagger$  ( $c_{j\alpha\sigma}$ ) is a fermionic creation (annihilation) operator at site  $j = 1, \dots, L$  in orbital  $\alpha = a, b$  with the spin  $\sigma = \uparrow, \downarrow$  state [105].  $h_{i\alpha j\beta}$  is the hopping Hamiltonian between site  $i$  in orbital  $\alpha$  with the spin-up state and site  $j$  in orbital  $\beta$  with the spin-up state.  $U$  denotes the strength of interactions. The summation of the first term  $\langle ij \rangle$  runs over all pairs of nearest-neighbor sites  $i$  and  $j$ . By applying the Fourier transformation to the first term in Eq. (1), the Bloch Hamiltonian  $h_{\alpha\beta}(k)$  in the orbital space reads

$$h(k) = b_2(k)\sigma_2 + b_3(k)\sigma_3, \quad (2)$$

with

$$b_2 = 2t_h - 0.5t_h \sin k, \quad (3a)$$

$$b_3 = 2t_h \cos k. \quad (3b)$$

Here,  $\sigma_j$  ( $j = 1, 2, 3$ ) express the Pauli matrices in the orbital space. The Hamiltonian given in Eq. (1) is obtained from the two-orbital Hubbard model by turning off the hopping of fermions in the down-spin states. It is worth noting that the above model breaks the inversion symmetry. Breaking the inversion symmetry is essential for the emergence of the interaction-induced Liouvillian skin effect (see Appendix A), rather than the specific values of the specific value  $2t_h$ ,  $-0.5t_h$ , and  $2t_h$  in Eqs. (3a) and (3b). We also note that the multibandness is essential to inducing the skin effect without asymmetric hopping.

When dissipation is introduced into this model, under the Markov approximation, the dynamics is described by the Lindblad equation [106,107]

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) = -i[H, \rho] + \sum_j \left[ L_j \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho\} \right]. \quad (4)$$

Here,  $\mathcal{L}$  denotes the Liouvillian, which is the superoperator acting on the density matrix  $\rho$ , the operator  $H$  is the Hamiltonian, and the Lindblad operator  $L_j$  characterizes the effect of dissipation. The Lindblad operator is given by the on-site two-body loss

$$L_j = \sqrt{2\gamma} c_{jb\uparrow} c_{jb\downarrow}. \quad (5)$$

We decompose the Liouvillian  $\mathcal{L}(\rho)$  as

$$\mathcal{L}(\rho) = \mathcal{L}_0(\rho) + \mathcal{L}_J(\rho), \quad (6)$$

where we have introduced

$$\mathcal{L}_0(\rho) = -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) \quad (7)$$

and

$$\mathcal{L}_J(\rho) = \sum_j L_j \rho L_j^\dagger. \quad (8)$$

Here, the non-Hermitian Hamiltonian given by

$$\begin{aligned} H_{\text{eff}} &= H - \frac{i}{2} \sum_j L_j^\dagger L_j \\ &= \sum_{\langle ij \rangle \alpha \beta} h_{i\alpha j\beta} c_{i\alpha\uparrow}^\dagger c_{j\beta\uparrow} + (U - i\gamma) \sum_j n_{jb\uparrow} n_{jb\downarrow} \end{aligned} \quad (9)$$

describes the dynamics of the single quantum trajectory between the quantum jumps [98]. In the following, we demonstrate that the complex-valued interaction  $U - i\gamma$  induces the Liouvillian skin effect [108].

### B. Vectorization of the density matrix

In this subsection, we rewrite the Liouvillian superoperator  $\mathcal{L}$  as an operator  $\mathcal{L}$  acting on the doubled Hilbert space by vectorizing the density matrix. Following the procedure of Refs. [64–66,109], we identify the density matrix  $\rho$  as a vector

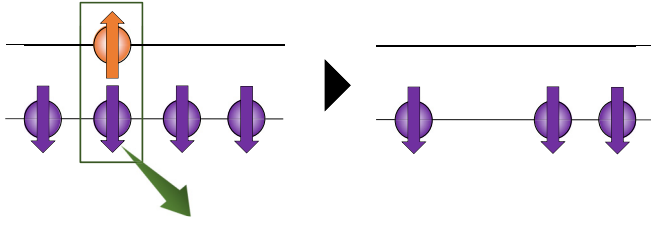


FIG. 1. Schematic illustration of the two-body loss process for an initial state with  $N_{\uparrow} = 1$ . Due to the jump operator of the two-body loss  $\mathcal{L}_J$  given in Eq. (13), a fermion with an up-spin and that with a down-spin form pairs and are scattered into environments. Down-spin configurations are changed from  $\{n_{\downarrow}\} = \{1, 1, 1, 1\}$  to  $\{n_{\downarrow}\}' = \{1, 0, 1, 1\}$  after the two-body loss process.

$|\rho\rangle\rangle$  in the doubled Hilbert space  $\mathcal{H} \otimes \mathcal{H}$  through the mapping

$$\rho = \sum_{ij} \rho_{ij} |i\rangle\langle j| \mapsto |\rho\rangle\rangle = \sum_{ij} \rho_{ij} |i\rangle \otimes |j\rangle. \quad (10)$$

We note that the first (second) space of the doubled Hilbert space  $\mathcal{H} \otimes \mathcal{H}$  is referred to as the ket (bra) space. When the density matrix  $\rho$  is given by the vectorized form  $|\rho\rangle\rangle$ , the Liouvillian superoperator  $\mathcal{L}$  is written as the operator  $\mathcal{L}$  that acts on the doubled Hilbert space

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_J. \quad (11)$$

Here, we define

$$\mathcal{L}_0 = -i(H_{\text{eff}} \otimes I - I \otimes H_{\text{eff}}^*) \quad (12)$$

and

$$\mathcal{L}_J = \sum_j L_j \otimes L_j^*, \quad (13)$$

where  $I$  is the identity operator acting on the ket or bra space [110]. Thus, the Liouvillian superoperator  $\mathcal{L}$  is mapped to the non-Hermitian operator  $\mathcal{L}$  acting on the doubled Hilbert space. After the vectorization of the density matrix, the  $n$ th eigenmode  $|\rho_R^{(n)}\rangle\rangle$  and the  $n$ th eigenvalue  $\Lambda_n$  are obtained by solving the eigenvalue equation

$$\mathcal{L}|\rho_R^{(n)}\rangle\rangle = \Lambda_n |\rho_R^{(n)}\rangle\rangle \quad (14)$$

for  $n = 1, \dots, \dim \mathcal{L}$ . As demonstrated in Sec. IV, eigenmodes and eigenvalues of the Liouvillian exhibit a strong dependence on boundary conditions.

### C. Two-body loss process

Let us consider the two-body loss process as illustrated in Fig. 1. First, we consider an initial state where only one fermion is in an up-spin state with down-spin configurations  $\{n_{\downarrow}\}$ . As the commutation relation

$$[\mathcal{L}_0, N_{\uparrow} \otimes N_{\uparrow}] = [\mathcal{L}_0, n_{j\downarrow} \otimes n_{j\downarrow}] = 0 \quad (15)$$

indicates that the density matrix  $|\rho\rangle\rangle$  is labeled by the total number of fermions in up-spin states  $N_{\uparrow}$  and down-spin configurations  $\{n_{\downarrow}\}$ , the density matrix for  $N_{\uparrow} = 1$  with down-spin configurations  $\{n_{\downarrow}\}$  is spanned by the basis

$$|(N_{\uparrow} = 1)\rangle\rangle = c_{j_1\alpha\uparrow}^\dagger \otimes c_{j_2\beta\uparrow}^\dagger |\{n_{\downarrow}\}\rangle \otimes |\{n_{\downarrow}\}\rangle \quad (16)$$

for  $j_1, j_2 = 1, \dots, L$ ,  $\alpha, \beta = a, b$  in the absence of the jump operator  $\mathcal{L}_J$ . Then, due to the jump operator  $\mathcal{L}_J$  that describes the two-body loss process, a fermion in an up-spin state and that in a down-spin state form pairs and are scattered out into environments. For example, when the total number of down spins in the initial state is  $N_{\downarrow} = 4$ , the two-body loss process changes down-spin configurations into one of the following four (see Fig. 1):

$$\begin{aligned} \{n_{\downarrow}\} &= \{1, 1, 1, 1\} \\ \rightarrow \{n_{\downarrow}\}' &= \{0, 1, 1, 1\}, \{1, 0, 1, 1\}, \{1, 1, 0, 1\}, \{1, 1, 1, 0\}. \end{aligned} \quad (17)$$

Here,  $\{n_{\downarrow}\}'$  denotes the down-spin configurations after the two-body loss process. Then, the density matrix for the  $N_{\uparrow} = 0$  state is spanned by the basis

$$|(N_{\uparrow} = 0)\rangle\rangle = |\{n_{\downarrow}\}'\rangle \otimes |\{n_{\downarrow}\}'\rangle. \quad (18)$$

We construct the basis set  $\{|i\rangle \otimes |j\rangle\}$ , which spans the doubled Hilbert space  $\mathcal{H} \otimes \mathcal{H}$  given in Eq. (10), by identifying the basis set  $\{|i\rangle \otimes |j\rangle\}$  with the basis sets combining  $\{|(N_{\uparrow} = 1)\rangle\rangle\}$  and  $\{|(N_{\uparrow} = 0)\rangle\rangle\}$ :

$$|i\rangle \otimes |j\rangle = \{|(N_{\uparrow} = 1)\rangle\rangle, |(N_{\uparrow} = 0)\rangle\rangle\}. \quad (19)$$

The matrix representation of the Liouvillian with respect to these bases  $\{|i\rangle \otimes |j\rangle\}$  takes the form

$$\mathcal{L} = \left( \begin{array}{c|c} \mathcal{L}_0^{(N_{\uparrow}=1)} & \\ \hline \mathcal{L}_J & \mathcal{L}_0^{(N_{\uparrow}=0)} \end{array} \right). \quad (20)$$

Here,  $\mathcal{L}_0^{(N_{\uparrow}=1)}$  [ $\mathcal{L}_0^{(N_{\uparrow}=0)}$ ] denotes the matrix representation of  $\mathcal{L}_0$  for the  $N_{\uparrow} = 1$  [ $N_{\uparrow} = 0$ ] sector. It is known that the block triangular structure is a general property of the Liouvillian for particle losses [111,112]. This structure simplifies the calculation of the winding number of the Liouvillian as shown in Sec. IV.

## III. TOPOLOGICAL INVARIANT AND SKIN MODE OF THE LIOUVILLIAN

### A. Topological invariant

In this subsection, we first present the topological number defined by the Liouvillian superoperator, and then we discuss the relation between the topological number and the Liouvillian skin effect.

First, we introduce the following topological invariant by using the Liouvillian superoperator:

$$v(\Lambda_{\text{ref}}) = \oint_0^{2\pi} \frac{d\theta}{2\pi i} \frac{d}{d\theta} \log \det[\mathcal{L}(\theta) - \Lambda_{\text{ref}}], \quad (21)$$

where we have imposed the twisted boundary condition only on the ket space. Here,  $\Lambda_{\text{ref}} \in \mathbb{C}$  denotes the reference point, specifying the point-gap on which we focus. The reference point  $\Lambda_{\text{ref}}$  is chosen from case by case in the same manner as the topology of the non-Hermitian Hamiltonian [17,36].  $\mathcal{L}(\theta)$  is defined as

$$i\mathcal{L}(\theta) = H_{\text{eff}}(\theta) \otimes I - I \otimes H_{\text{eff}}^* + i \sum_j L_j(\theta) \otimes L_j^*, \quad (22)$$

where operators  $H_{\text{eff}}(\theta)$  and  $L_j(\theta)$  are defined by multiplying  $e^{\pm i\theta}$  to the hopping term at the boundary, e.g.,  $c_{1\alpha\sigma}^\dagger c_{L\alpha'\sigma'}$  is replaced by  $c_{1\alpha\sigma}^\dagger c_{L\alpha'\sigma'} e^{i\theta}$ . More precisely, in the case of the Falicov-Kimball model with the two-body loss given in Eq. (9),  $H_{\text{eff}}(\theta)$  is written by

$$H_{\text{eff}}(\theta) = H_{\text{eff}}^{\text{bulk}} + H_{\text{eff}}^{\text{edge}}(\theta), \quad (23)$$

where  $H_{\text{eff}}^{\text{bulk}}$  is the Hamiltonian in the bulk, which is independent of  $\theta$  and is written down as

$$H_{\text{eff}}^{\text{bulk}} = \sum_{\langle ij \rangle' \alpha \beta} h_{i\alpha j\beta} c_{i\alpha\uparrow}^\dagger c_{j\beta\uparrow} + (U - i\gamma) \sum_{j=1}^L n_{j\beta\uparrow} n_{j\beta\downarrow}. \quad (24)$$

The summation  $\langle ij \rangle'$  runs over all pairs of nearest neighbor sites  $i$  and  $j$ , excluding the hopping at the boundary between site 1 and site  $L$ . The boundary term of the Hamiltonian  $H_{\text{eff}}^{\text{edge}}(\theta)$  is given by

$$H_{\text{eff}}^{\text{edge}}(\theta) = \sum_{\alpha\beta} (h_{1\alpha L\beta} c_{1\alpha\uparrow}^\dagger c_{L\beta\uparrow} e^{i\theta} + \text{H.c.}). \quad (25)$$

Here,  $h_{1\alpha L\beta}$  is the hopping Hamiltonian between site 1 and site  $L$ . Since we consider the on-site dissipator given in Eq. (5), the Lindblad operator is independent of  $\theta$ , i.e.,  $L_j(\theta) = L_j$ . Because of the relation  $\mathcal{L}(\theta) = \mathcal{L}(\theta + 2\pi)$ , the winding number  $\nu(\Lambda_{\text{ref}})$  given in Eq. (21) is quantized. Hereafter, when the winding number given in Eq. (21) takes a nonzero value, we denote that the point-gap topology of the Liouvillian is nontrivial.

Second, we discuss the relation between the topological number defined in Eq. (21) and the Liouvillian skin effect. Even in the single-particle system, the topological characterization of the Liouvillian skin effect has not been accomplished so far. Importantly, in the single-particle system, the topological invariant  $\nu(\Lambda_{\text{ref}})$  defined in Eq. (21) gives the characterization of the Liouvillian skin effect, provided that the Lindblad operator is given by the asymmetric hopping (see Appendix B for details). In Appendix B, we compute the topological invariant  $\nu(\Lambda_{\text{ref}})$  defined in Eq. (21) analytically, and we discuss the validity of the characterization of the Liouvillian skin effect in the single-particle system. Significantly, the topological invariant  $\nu(\Lambda_{\text{ref}})$  can be computed even in many-body systems. Such a definition of a topological invariant, which is independent of momentum, has already been introduced for the non-Hermitian skin effect in many-body systems [84–86]. In the following section, we numerically calculate the topological invariant  $\nu(\Lambda_{\text{ref}})$  and observe the nontrivial value of the topological invariant  $\nu(\Lambda_{\text{ref}})$  corresponding to the Liouvillian skin effect in many-body systems [113].

## B. Skin mode of the Liouvillian

In this subsection, we first introduce the right-state particle density of the  $n$ th eigenmode of the Liouvillian superoperator  $\mathcal{L}$  as  $\Delta_{j\alpha\sigma}^{(n)}$ , which measures the degree of localization of eigenmodes of the Liouvillian superoperator in many-body systems. Then, we show that in the single-particle system, the right-state particle density reduces to the diagonal element of the right eigenmode, which is used as the characterization of the Liouvillian skin effect in single-particle systems in

Ref. [50]. Finally, we show that when the right eigenmode is written by the right eigenstate of the effective Hamiltonian, the right-state particle density gives the particle density, which is used as the characterization of the non-Hermitian skin effect in many-body systems in Refs. [84,85].

First, we define the following right-state particle density of the  $n$ th eigenmode of the Liouvillian superoperator  $\mathcal{L}$  to quantify the degree of localization of the eigenmode of the Liouvillian in many-body systems:

$$\Delta_l^{(n)} = \langle \langle J | c_l^\dagger c_l \otimes I | \rho_R^{(n)} \rangle \rangle = \langle \langle J | I \otimes c_l^\dagger c_l | \rho_R^{(n)} \rangle \rangle \quad (26)$$

with  $l$  denoting the set of  $j, \alpha$  and  $\sigma$ , i.e.,  $l = j\alpha\sigma$ . Here,  $|J\rangle\rangle$  is the identity operator defined by  $|J\rangle\rangle = \sum_j |j\rangle \otimes |j\rangle$ , and  $|\rho_R^{(n)}\rangle\rangle$  is the  $n$ th right eigenmode of the Liouvillian  $\mathcal{L}$  that satisfies the eigenvalue equation given in Eq. (14). We note that the right-state particle density is not identical to the ordinary particle density, which is observable and takes real values. Specifically, the right-state particle density is complex-valued, which is introduced to measure the degree of localization of eigenmodes in many-body systems.

Next, we show that the right-state particle density defined in Eq. (26) reduces to the diagonal element of the right eigenmode in the single-particle system. We note that the right-state particle density of the  $n$ th eigenmode given in Eq. (26) is expressed as

$$\Delta_l^{(n)} = \text{Tr}[c_l^\dagger c_l \rho_R^{(n)}], \quad (27)$$

where we have used the following relation:

$$\begin{aligned} \langle \langle J | A \otimes I | \rho_R^{(n)} \rangle \rangle &= \sum_{jkl} \langle j | \otimes \langle j | (A | k \rangle \otimes | l \rangle) \rho_{R,kl}^{(n)} \\ &= \sum_{jkl} \delta_{jl} \langle j | A | k \rangle \rho_{R,kl}^{(n)} \\ &= \text{Tr}[A \rho_R^{(n)}]. \end{aligned} \quad (28)$$

Now, we show that the definition given in Eq. (26) reduces to the diagonal element of the right eigenmode of the Liouvillian in the single-particle system. We take the single-particle basis  $|S\rangle$ , which is generated by applying the creation operator to the vacuum  $|\text{vac}\rangle$  as  $|S\rangle = c_S^\dagger |\text{vac}\rangle$  with  $S$  denoting the set of  $j, \alpha$  and  $\sigma$ , i.e.,  $S = j\alpha\sigma$ . Then the  $n$ th right eigenstate of the density matrix  $\rho_R^{(n)}$  is expanded by using the single-particle state  $|S\rangle$  as

$$\rho_R^{(n)} = \sum_{ST} \rho_{R,ST}^{(n)} |S\rangle \langle T|. \quad (29)$$

Here,  $\rho_{R,ST}^{(n)} \in \mathbb{C}$  is expansion coefficient. By substituting Eq. (29) into Eq. (27), we obtain

$$\Delta_S^{(n)} = \rho_{R,SS}^{(n)}. \quad (30)$$

Thus the right-state particle density defined in Eq. (27) is a generalization of the diagonal element of the right eigenmode, which measures the degree of localization of the eigenmode of the Liouvillian in the single-particle system.

Finally, we show that when the right eigenmode  $\rho_R^{(n)}$  is written by the right eigenstate of the effective Hamiltonian  $H_{\text{eff}}$  as  $|\varphi_R^{(n)}\rangle$ , the right-state particle density reduces to the particle density defined by  $n_l = \langle \varphi_R^{(n)} | c_l^\dagger c_l | \varphi_R^{(n)} \rangle$ , which is used



as the characterization of the skin mode in a non-Hermitian many-body system in Refs. [84,85]. We take the right and left eigenstates of the effective Hamiltonian as  $|\varphi_R^{(n)}\rangle$  and  $\langle\varphi_L^{(n)}|$ , which satisfy the eigenvalue equations

$$H_{\text{eff}}|\varphi_R^{(n)}\rangle = E_n|\varphi_R^{(n)}\rangle \quad (31)$$

and

$$\langle\varphi_L^{(n)}|H_{\text{eff}} = E_n\langle\varphi_L^{(n)}|, \quad (32)$$

respectively. When we take the right eigenmode  $\rho_R^{(n)}$  as

$$\rho_R^{(n)} = |\varphi_R^{(n)}\rangle\langle\varphi_R^{(n)}|, \quad (33)$$

the right-state particle density given in Eq. (27) becomes

$$\begin{aligned} \Delta_l^{(n)} &= \text{Tr}[\rho_R^{(n)} c_l^\dagger c_l] \\ &= \sum_m \langle\varphi_L^{(m)}|\varphi_R^{(n)}\rangle\langle\varphi_R^{(n)}|c_l^\dagger c_l|\varphi_R^{(m)}\rangle \\ &= \sum_m \delta_{mn} \langle\varphi_R^{(n)}|c_l^\dagger c_l|\varphi_R^{(n)}\rangle = n_l, \end{aligned} \quad (34)$$

where we have used the biorthogonal relation in the third equality [15]. Therefore, the right-state particle density is the generalization of the particle density  $n_l$ , which measures the degree of localization of the eigenstate in non-Hermitian many-body systems. In the following, we demonstrate that the right-state particle density  $\Delta_l^{(n)}$  exhibits localization near the edge. From Eqs. (30) and (34), we consider that the right-state particle density is the proper definition to measure the degree of localization of eigenmodes of the Liouvillian in many-body systems.

#### IV. NUMERICAL RESULTS

In this section, we demonstrate that interactions can induce the Liouvillian skin effect by analyzing the Falicov-Kimball model introduced in Sec. II. First, in the noninteracting case ( $U - i\gamma = 0$ ), we show that the Liouvillian skin effect is absent. Then, it is demonstrated that the complex-valued interaction  $U - i\gamma$  induces the Liouvillian skin effect. In the following discussion, we set  $t_h = 1$  as an energy unit.

##### A. Noninteracting case

First, we see that the Liouvillian skin effect is not observed in the noninteracting case ( $U = \gamma = 0$ ). In this case, the Liouvillian given in Eq. (11) becomes

$$\mathcal{L}_{\text{free}} = -i(H \otimes I - I \otimes H^T). \quad (35)$$

Because  $\mathcal{L}_{\text{free}}$  is skew-Hermitian, i.e.,  $\mathcal{L}_{\text{free}}^\dagger = -\mathcal{L}_{\text{free}}$ , its eigenvalues are purely imaginary or zero. In other words, all eigenvalues of the Liouvillian lie on the imaginary axis regardless of boundary conditions. As a result, the point-gap topology of the Liouvillian always becomes trivial because the winding number always takes zero. Correspondingly, the eigenvalues and the eigenmodes of the Liouvillian are not sensitive to the boundary conditions (for more details, see

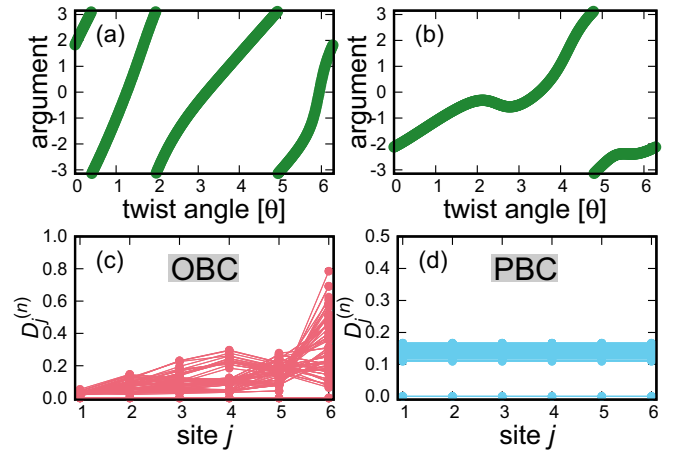


FIG. 2. (a) [(b)] Argument of  $\det[\mathcal{L}(\theta) - \Lambda_{\text{ref}}]$  for  $\Lambda_{\text{ref}} = -0.5 - 0.8i$  [ $\Lambda_{\text{ref}} = -0.3 - 0.2i$ ]. Panel (a) [(b)] shows that the winding number takes  $\nu = 3$  [ $\nu = 1$ ]. (c) [(d)]  $D_j^{(n)} = \sum_\alpha |\Delta_{j\alpha\uparrow}^{(n)}|$  computed from the right-state particle density under OBC (PBC). The parameters are set to be  $L = 6$ ,  $U = 0.5$ , and  $\gamma = 1.0$ . The configuration of fermions in the down-spin states is set to be  $\{n_\downarrow\} = \{1, \dots, 1\}$ . The density matrix is normalized as  $\sum_{ij} |\rho_{R,ij}^{(n)}|^2 = 1$ .

Appendix C). Therefore, the Liouvillian skin effect is absent for the noninteracting system.

##### B. Interacting case

Next, we demonstrate that the interaction  $U - i\gamma$  makes the point-gap topology nontrivial and induces the Liouvillian skin effect. Figures 2(a) and 2(b) display the  $\theta$  dependence of  $\det[\mathcal{L}(\theta) - \Lambda_{\text{ref}}]$ . We see that the winding number takes  $\nu = 3$  ( $\nu = 1$ ) for  $\Lambda_{\text{ref}} = -0.5 - 0.8i$  ( $\Lambda_{\text{ref}} = -0.3 - 0.2i$ ). Now, we analyze the emergence of skin modes by comparing the results under OBC with those under periodic boundary conditions (PBC). Figure 2(c) [2(d)] displays  $D_j^{(n)} = \sum_\alpha |\Delta_{j\alpha\uparrow}^{(n)}|$  for OBC (PBC). We note that the right-state particle density of the  $n$ th eigenmode  $\Delta_{j\alpha\uparrow}^{(n)}$  defined in Eq. (26) takes a complex value. Figure 2(c) indicates that the eigenmodes are localized at the right edge under OBC. In contrast, such a localization cannot be observed under PBC. These results demonstrate the emergence of skin modes of the Liouvillian. We also note that the sensitivity of the eigenvalues to boundary conditions is also observed, although it is smeared for small  $L$  (for more details, see Appendix D). With the above results (see Fig. 2), we conclude that interactions induce the Liouvillian skin effect, though the system is subject to homogeneous two-body losses. It should be noted that the Liouvillian skin effect occurs for  $U = 0$  and  $\gamma \neq 0$ . This skin effect is regarded as an interaction-induced skin effect since the nonzero  $\gamma$  results in a two-body interaction [see Eq. (9)].

Here, we comment on the relation between the winding number of the Liouvillian and that of the effective non-Hermitian Hamiltonian. As derived in Appendix E, we obtain the following relation for the winding number defined in Eq. (21):

$$\nu(\Lambda_{\text{ref}}) = \sum_j w(E_{\text{ref}} = E_j^* + i\Lambda_{\text{ref}}), \quad (36)$$

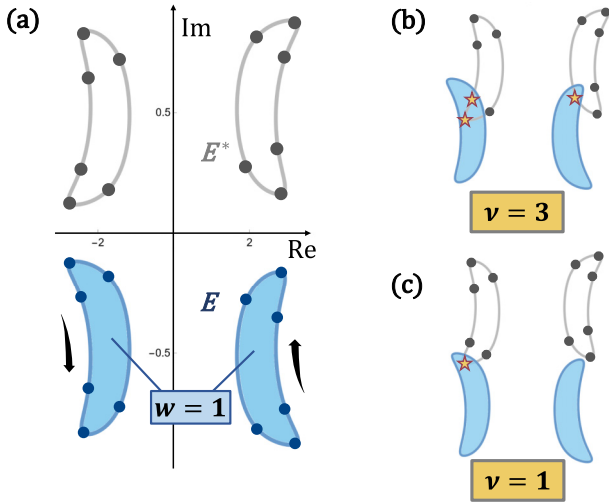


FIG. 3. Schematic figure that describes the relationship between the winding number  $\nu(\Lambda_{\text{ref}})$  and  $w(E_{\text{ref}})$  given in Eq. (36). (a) Eigenvalues of the effective Hamiltonian  $H_{\text{eff}}$  are indicated by blue dots.  $E^*$  is the complex conjugation of the eigenvalue  $E$  (gray dots). When  $E_{\text{ref}}$  is located inside the blue region, the winding number of the Hamiltonian equals 1, i.e.,  $w(E_{\text{ref}}) = 1$ . (b), (c) Schematic figure of the origin of the nontrivial winding number. The winding number  $\nu(\Lambda_{\text{ref}})$  equals the number of dots in the blue region indicated by the yellow star. The number of yellow stars in panel (b) [(c)] corresponds to the winding number  $\nu(\Lambda_{\text{ref}})$  in Fig. 2(a) [2(b)]. The constant shift  $i\Lambda_{\text{ref}}$  in Eq. (36) is set to be  $i\Lambda_{\text{ref}} = 0.8 - 0.5i$  and  $i\Lambda_{\text{ref}} = 0.2 - 0.3i$  for panels (b) and (c), respectively. The parameters are set to be  $L = 6$ ,  $U = 0.5$ ,  $\gamma = 1.0$ . The configuration of fermions in the down-spin states is set to be  $\{n_{\downarrow}\} = \{1, \dots, 1\}$ .

where  $w(E_{\text{ref}})$  is the winding number of the non-Hermitian Hamiltonian

$$w(E_{\text{ref}}) = \oint_0^{2\pi} \frac{d\theta}{2\pi i} \frac{d}{d\theta} \log \det[H_{\text{eff}}(\theta) - E_{\text{ref}}]. \quad (37)$$

Here,  $E_j$  denotes an eigenvalue of the non-Hermitian Hamiltonian  $H_{\text{eff}}$  defined in Eq. (9). Equation (36) indicates that the winding number of the Liouvillian  $\nu(\Lambda_{\text{ref}})$  can be computed from the winding number of the effective non-Hermitian Hamiltonian  $w(E_{\text{ref}})$  with this model. Here, we compute the winding number  $\nu(\Lambda_{\text{ref}})$  by making use of Eq. (36). First, we take the complex conjugate of the eigenvalue of the non-Hermitian Hamiltonian  $E_j^*$  [see Fig. 3(a)]. Then, we shift  $E_j^*$  by  $i\Lambda_{\text{ref}}$  and obtain  $E_{\text{ref}} = E_j^* + i\Lambda_{\text{ref}}$ . Because Eq. (36) indicates that the summation of  $w$  for all possible  $E_{\text{ref}} = E_j^* + i\Lambda_{\text{ref}}$  results in the winding number of the Liouvillian, we obtain  $\nu(\Lambda_{\text{ref}}) = 3$  [ $\nu(\Lambda_{\text{ref}}) = 1$ ] for  $\Lambda_{\text{ref}} = -0.5 - 0.8i$  ( $\Lambda_{\text{ref}} = -0.3 - 0.2i$ ) [see Figs. 3(b) and 3(c)]. We note that the winding number of the effective Hamiltonian takes one when  $E_{\text{ref}}$  is in the shaded region in Fig. 3(a). These results of the winding number  $\nu(\Lambda_{\text{ref}})$  are consistent with the results obtained by the direct computation of  $\nu(\Lambda_{\text{ref}})$  [see Figs. 2(a) and 2(b)].

### C. Dynamical properties

In this subsection, we show that the Liouvillian skin effect significantly affects the dynamics of the particle density. We

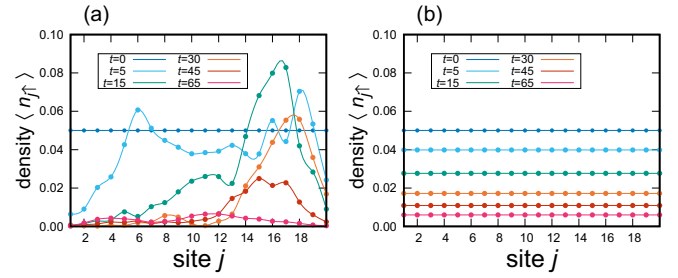


FIG. 4. (a) [(b)] Time evolution of the particle density under OBC (PBC). The parameters are set to be  $L = 20$ ,  $U = 0.1$ ,  $\gamma = 0.1$ . The configuration of fermions in the down-spin states is set to be  $\{n_{\downarrow}\} = \{1, \dots, 1\}$ . The particle is uniformly distributed in orbital  $a$  in the initial state. Only under OBC is the anomalous localization of the particle density observed.

assume that the total number of particles in the up-spin states is equal to one in the initial state, i.e.,

$$\langle \Psi(t=0) | N_{\uparrow} | \Psi(t=0) \rangle = 1, \quad (38)$$

where the wave function in the initial state  $|\Psi(t=0)\rangle$  reads

$$|\Psi(t=0)\rangle = \frac{1}{\sqrt{L}} \sum_{j=1}^L c_{ja\uparrow}^{\dagger} |\{n_{\downarrow}\}\rangle. \quad (39)$$

Here, we have assumed that the particle in orbital  $a$  is uniformly distributed in the initial state. The expectation value of the particle density in the up-spin state at time  $t$  is given by

$$\begin{aligned} \langle n_{j\uparrow}(t) \rangle &= \sum_{\alpha} \text{Tr}[n_{j\alpha\uparrow} \rho(t)] \\ &= \sum_{\alpha} \langle \langle J | c_{ja\uparrow}^{\dagger} c_{j\alpha\uparrow} \otimes I | \rho(t) \rangle \rangle. \end{aligned} \quad (40)$$

Then, the time evolution of the density matrix reads

$$|\rho(t)\rangle\rangle = e^{\mathcal{L}t} |\rho(t=0)\rangle\rangle. \quad (41)$$

By using the wave function in the initial state  $|\Psi(t=0)\rangle$ , the density matrix at time  $t=0$  is given by

$$|\rho(t=0)\rangle\rangle = |\Psi(t=0)\rangle \otimes |\Psi(t=0)\rangle. \quad (42)$$

Here,  $|\Psi(t=0)\rangle \otimes |\Psi(t=0)\rangle$  is defined by the following mapping:

$$\begin{aligned} |\Psi(t=0)\rangle \langle \Psi(t=0)| &= \sum_{ij} \Psi_{ij}(t=0) |i\rangle \langle j| \\ \mapsto |\Psi(t=0)\rangle \otimes |\Psi(t=0)\rangle &= \sum_{ij} \Psi_{ij}(t=0) |i\rangle \otimes |j\rangle, \end{aligned} \quad (43)$$

where  $\Psi_{ij}(t=0)$  is the matrix element of  $|\Psi(t=0)\rangle \langle \Psi(t=0)|$ , and  $|i\rangle \otimes |j\rangle$  is the element in the basis set given in Eq. (19). Now, we numerically calculate the expectation value of the particle density given in Eq. (40) considering Eqs. (41) and (42). We note that, since the initial state only has one fermion in an up-spin state, the dynamics is computed only from  $H_{\text{eff}}$  [114]. Figure 4 displays the time dependence of the expectation value  $\langle n_{j\uparrow}(t) \rangle$ . Under OBC, we see that the particle is accumulated near the right boundary, as shown in Fig. 4(a). In contrast, under PBC, we find that the particle

density decreases uniformly due to the dissipation, as shown in Fig. 4(b).

The above significant dependence of  $\langle n_{j\uparrow} \rangle$  on boundary conditions can be understood in terms of the right-state particle density of the  $n$ th eigenmode of the Liouvillian  $\Delta_{j\alpha\sigma}^{(n)}$ . First, we expand the initial density matrix  $|\rho(0)\rangle\rangle = \sum_n a_n |\rho_R^{(n)}\rangle\rangle$  by using an eigenmode of the Liouvillian. Then, by combining the eigenvalue equation of the Liouvillian given in Eq. (14) and the time evolution of the density matrix given in Eq. (41), we obtain the particle density at time  $t$  as

$$\langle n_{j\uparrow}(t) \rangle = \sum_{n,\alpha} e^{\Lambda_n t} a_n \Delta_{j\alpha\uparrow}^{(n)}. \quad (44)$$

Thus the particle accumulation quantitatively originates from the anomalous localization of  $\Delta_{j\alpha\uparrow}^{(n)}$  (see Fig. 2). Moreover, in the presence of the Liouvillian skin effect, we find the two-step relaxation process (see Appendix F).

We note that the particle accumulation due to the Liouvillian skin effect is different from the topological pumping [115,116]. In contrast to the topological pumping observed under the twisted boundary condition, the charge accumulation due to the skin modes is observed under OBC. We twist the boundary condition only for computation of the topological invariant [Eq. (21)].

## V. CONCLUSIONS

In this paper, by introducing two-body loss into the one-dimensional correlated system, we have demonstrated that complex-valued interactions induce the Liouvillian skin effect. Specifically, by introducing the winding number constructed by the Liouvillian superoperator, we have elucidated that interactions make the point-gap topology nontrivial. Moreover, we have seen that eigenvalues and eigenmodes of the Liouvillian exhibit extreme sensitivity to boundary conditions. As a result, we have observed the particle accumulation around the right edge in transient dynamics only under OBC, which is attributed to the emergence of the skin mode.

As two-body losses have already been introduced in ytterbium atoms by using photoassociation techniques [54,56], our results can be tested in ultracold atoms. The method to realize the Falicov-Kimball model is provided in Ref. [117] by introducing two species of atoms such as  $^{40}\text{K}$  and  $^6\text{Li}$ , where mobile and immobile atoms are coupled via an on-site interaction. We expect that the interaction-induced Liouvillian skin effect in our model can be observed in ultracold atoms.

Recently, classifications of the Liouvillian superoperator have been actively conducted [99,102]. When the Hamiltonian preserves the inversion symmetry, the winding number defined in Eq. (21), which characterizes the nontrivial topological phase of the Liouvillian, can be trivial. Last but not least, it is worthwhile to study the detailed relations between the symmetry and the topological number in a different Liouvillian, but we leave it for future work.

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## APPENDIX A: SYMMETRY CONSTRAINT ON THE WINDING NUMBER

In this Appendix, we discuss the relation between the symmetry of the Liouvillian and the winding number  $\nu(\Lambda_{\text{ref}})$ . As we will see below, breaking the inversion symmetry of the Hamiltonian is essential for the existence of the nonzero topological number. The winding number  $\nu(\Lambda_{\text{ref}})$  given in Eq. (21) becomes trivial when the Liouvillian superoperator satisfies

$$\mathcal{U}\mathcal{L}(-\theta)\mathcal{U}^\dagger = \mathcal{L}(\theta). \quad (\text{A1})$$

Here,  $\mathcal{U}$  is the unitary operator ( $\mathcal{U}\mathcal{U}^\dagger = \mathcal{U}^\dagger\mathcal{U} = 1$ ). We note that Eq. (A1) leads to  $\nu(\Lambda_{\text{ref}}) = -\nu(\Lambda_{\text{ref}})$ . This relation means that the point-gap topology of the Liouvillian is trivial,  $\nu(\Lambda_{\text{ref}}) = 0$ .

In the case of particle loss, this triviality [ $\nu(\Lambda_{\text{ref}}) = 0$ ] originates from the symmetry of the Hamiltonian. Since the eigenvalue of the Liouvillian is determined only by the effective Hamiltonian  $H_{\text{eff}}$  [111], the winding number given in Eq. (21) reduces to

$$\nu(\Lambda_{\text{ref}}) = \oint_0^{2\pi} \frac{d\theta}{2\pi i} \frac{d}{d\theta} \log \det[\mathcal{L}_0(\theta) - \Lambda_{\text{ref}}]. \quad (\text{A2})$$

If the effective Hamiltonian  $H_{\text{eff}}$  satisfies the following relation:

$$U H_{\text{eff}}(-\theta) U^\dagger = H_{\text{eff}}(\theta), \quad (\text{A3})$$

we can construct the unitary operator  $\mathcal{U}$  as

$$\mathcal{U} = U \otimes V, \quad (\text{A4})$$

where  $UU^\dagger = U^\dagger U = 1$ , and we have defined  $V = U^*$ . Due to the relation  $\mathcal{U}\mathcal{L}_0(-\theta)\mathcal{U}^\dagger = \mathcal{L}_0(\theta)$ , we find that the winding number Eq. (A2) becomes zero.

Now, we discuss whether the Liouvillian given in Eq. (11) satisfies the condition given in Eq. (A1). The effective Hamiltonian under twisted boundary conditions in real space is  $H_{\text{eff}}(\theta) = H_{\text{eff}}^{\text{bulk}} + H_{\text{eff}}^{\text{edge}}(\theta)$ , where  $H_{\text{eff}}^{\text{bulk}}$  is written as

$$H_{\text{eff}}^{\text{bulk}} = H_1 + H_2 + H_3 + H_4. \quad (\text{A5})$$

Here, we have defined

$$\begin{aligned} H_1 &= -2it_h \sum_{j=1}^L (c_{ja\uparrow}^\dagger c_{jb\uparrow} - \text{H.c.}), \\ H_2 &= -0.25t_h \sum_{j=1}^{L-1} (c_{j+1b\uparrow}^\dagger c_{ja\uparrow} - c_{j+1a\uparrow}^\dagger c_{jb\uparrow} + \text{H.c.}), \end{aligned}$$

$$\begin{aligned}
H_3 &= t_h \sum_{j=1}^{L-1} (c_{j+1a\uparrow}^\dagger c_{ja\uparrow} - c_{j+1b\uparrow}^\dagger c_{jb\uparrow} + \text{H.c.}), \\
H_4 &= (U - i\gamma) \sum_{j=1}^L n_{jb\uparrow} n_{jb\downarrow},
\end{aligned} \tag{A6}$$

and

$$\begin{aligned}
H_{\text{eff}}^{\text{edge}}(\theta) &= -0.25t_h(e^{i\theta} c_{1b\uparrow}^\dagger c_{La\uparrow} - e^{i\theta} c_{1a\uparrow}^\dagger c_{Lb\uparrow} + \text{H.c.}) \\
&\quad + t_h(e^{i\theta} c_{1a\uparrow}^\dagger c_{La\uparrow} - e^{i\theta} c_{1b\uparrow}^\dagger c_{Lb\uparrow} + \text{H.c.}).
\end{aligned} \tag{A7}$$

The first term  $H_1$  of the bulk Hamiltonian  $H_{\text{eff}}^{\text{bulk}}$  [i.e., the first term of the Bloch Hamiltonian in Eq. (3a) denoted by  $2t_h$ ] violates the condition Eq. (A1). In the absence of the first term, the effective Hamiltonian preserves the inversion symmetry defined by

$$PH_{\text{eff}}(-\theta)P^\dagger = H_{\text{eff}}(\theta), \tag{A8}$$

where the inversion operator  $P$  acts on the annihilation operator as  $Pc_{ja\sigma}P^\dagger = c_{L-(j-1)a\sigma}$ ,  $Pc_{jb\sigma}P^\dagger = -c_{L-(j-1)b\sigma}$  and satisfies  $P^\dagger P = PP^\dagger = \mathbf{1}$ . Then, we see that the inversion symmetry given in Eq. (A8) is nothing but the condition of the triviality of the winding number given in Eq. (A3). Therefore,

$$\begin{aligned}
\mathcal{L}^{H=0} &= \sum_{j,\alpha} \left[ L_{j,\alpha} \otimes L_{j,\alpha}^* - \frac{1}{2} (L_{j,\alpha}^\dagger L_{j,\alpha} \otimes I + I \otimes L_{j,\alpha}^T L_{j,\alpha}^*) \right] \\
&= \sum_{j=1}^L \left[ t_r b_{j+1}^\dagger b_j \otimes b_{j+1}^\dagger b_j + t_l b_j^\dagger b_{j+1} \otimes b_j^\dagger b_{j+1} - \frac{t_r + t_l}{2} (b_j^\dagger b_j \otimes I + I \otimes b_j^\dagger b_j) \right].
\end{aligned} \tag{B2}$$

When we impose twisted boundary conditions only on the ket space, the Liouvillian superoperator is expressed as

$$\mathcal{L}^{H=0}(\theta) = \mathcal{L}_{\text{bulk}}^{H=0} + t_r e^{i\theta} b_1^\dagger b_L \otimes b_1^\dagger b_L + t_l e^{-i\theta} b_L^\dagger b_1 \otimes b_L^\dagger b_1, \tag{B3}$$

where we have introduced the bulk term of the Liouvillian  $\mathcal{L}_{\text{bulk}}^{H=0}$  as

$$\mathcal{L}_{\text{bulk}}^{H=0} = \sum_{j=1}^{L-1} [t_r b_{j+1}^\dagger b_j \otimes b_{j+1}^\dagger b_j + t_l b_j^\dagger b_{j+1} \otimes b_j^\dagger b_{j+1}] - \sum_{j=1}^L \left[ \frac{t_r + t_l}{2} (b_j^\dagger b_j \otimes I + I \otimes b_j^\dagger b_j) \right]. \tag{B4}$$

In the following discussion, we focus on the single-particle diagonal subspace spanned by the basis  $\{|i\rangle \otimes |i\rangle\}_{i=1,\dots,L}$ . The matrix representation of the Liouvillian with respect to this basis is given by

$$\mathcal{L}^{H=0}(\theta) = \begin{pmatrix} -(t_l + t_r) & t_l & & t_r e^{i\theta} \\ t_r & \ddots & \ddots & \\ & \ddots & \ddots & t_l \\ t_l e^{-i\theta} & & t_r & -(t_l + t_r) \end{pmatrix}. \tag{B5}$$

In this subspace, the action of the Liouvillian  $\mathcal{L}^{H=0}(\theta)$  is identical to that of the following Hamiltonian in the single-particle system:

$$\begin{aligned}
H_{\text{HN}}(\theta) &= - \sum_{j=1}^L (t_l + t_r) c_j^\dagger c_j + \sum_{j=1}^{L-1} (t_l c_j^\dagger c_{j+1} + t_r c_{j+1}^\dagger c_j) \\
&\quad + t_l c_L^\dagger c_1 e^{-i\theta} + t_r c_1^\dagger c_L e^{i\theta}.
\end{aligned} \tag{B6}$$

the Liouvillian superoperator  $\mathcal{L}(\theta)$  given in Eq. (22) satisfies Eq. (A1) in the absence of  $H_1$ . The presence of  $H_1$  breaks the condition Eq. (A8), which leads to the violation of the condition Eq. (A1). Hence, the nonzero winding number originates from the property of the Liouvillian. In particular, the Hamiltonian breaks the inversion symmetry.

## APPENDIX B: TOPOLOGICAL CHARACTERIZATION OF THE LIOUVILLIAN SKIN EFFECT REPORTED IN REF. [50]

In this Appendix, we show that the winding number  $\nu(\Lambda_{\text{ref}})$  defined in Eq. (21) characterizes the Liouvillian skin effect reported in Ref. [50], which implies the validity of employing  $\nu(\Lambda_{\text{ref}})$  for characterizing the interaction-induced Liouvillian skin effect. We consider the bosonic systems and assume that the Lindblad operators are given by

$$\begin{aligned}
L_{j,l} &= \sqrt{t_l} b_j^\dagger b_{j+1}, \\
L_{j,r} &= \sqrt{t_r} b_{j+1}^\dagger b_j,
\end{aligned} \tag{B1}$$

which describe the stochastic hopping to the nearest-neighbor sites. Following the discussion in Ref. [50], we assume that the Hamiltonian of the systems is zero, i.e.,  $H = 0$ . The Liouvillian superoperator becomes

We note that the matrix representation of the Hamiltonian  $H_{\text{HN}}(\theta)$  with respect to the basis  $\{|i\rangle\}_{i=1,\dots,L}$  gives Eq. (B5). The Hamiltonian given in Eq. (B6) is nothing but the Hatano-Nelson model [118–120] under twisted boundary conditions.

Now, we calculate the winding number  $\nu(\Lambda_{\text{ref}})$  defined in Eq. (21) for the Liouvillian  $\mathcal{L}^{H=0}(\theta)$  in this subspace. First, we recover the translational invariance of the Hamiltonian by using the gauge transformation  $c_j \rightarrow c_j e^{-i\frac{j}{L}\theta}$  as

$$H_{\text{HN}}(\theta) = \sum_{j=1}^L [-(t_l + t_r) c_j^\dagger c_j + t_l e^{-i\frac{\theta}{L}} c_j^\dagger c_{j+1} + t_r e^{i\frac{\theta}{L}} c_{j+1}^\dagger c_j]. \tag{B7}$$

Then, we diagonalize Eq. (B7) as

$$H_{\text{HN}}(\theta) = \sum_k h_{\text{HN}} \left( k + \frac{\theta}{L} \right) c_k^\dagger c_k \tag{B8}$$



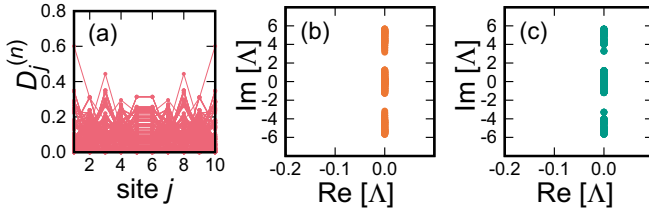


FIG. 5. (a) The right-state particle density of the Liouvillian under OBC. (b) [(c)] Eigenvalues of the Liouvillian under OBC (PBC) for the noninteracting case. The parameters are set to be  $L = 10$ ,  $U = \gamma = 0.0$ . The configuration of fermions in the down-spin states is set to be  $\{n_\downarrow\} = \{1, \dots, 1\}$ .

with

$$h_{\text{HN}}(k) = t_r e^{ik} + t_l e^{-ik} - (t_l + t_r). \quad (\text{B9})$$

In the translational invariant single-particle system, the many-body topological invariant of non-Hermitian systems reduces to the following topological invariant defined in the momentum space [84,121]:

$$W(\Lambda_{\text{ref}}) = \oint_0^{2\pi} \frac{dk}{2\pi i} \frac{d}{dk} \log \det[h_{\text{HN}}(k) - \Lambda_{\text{ref}}]. \quad (\text{B10})$$

Finally, in a similar way to Eq. (B10), we can compute the winding number  $\nu(\Lambda_{\text{ref}})$  defined in Eq. (21) for  $\mathcal{L}^{H=0}(\theta)$  given in Eq. (B5) as

$$\nu(\Lambda_{\text{ref}}) = \text{sgn}(t_r - t_l), \quad (\text{B11})$$

where we set  $\Lambda_{\text{ref}}$  inside the region enclosed by PBC spectrum. As shown in Ref. [50], the right-state particle density of the Liouvillian given in Eq. (26) exhibits the skin effect. Therefore, the Liouvillian skin effect demonstrated in Ref. [50] is characterized by the winding number defined in Eq. (21). This fact supports the use of  $\nu(\Lambda_{\text{ref}})$  as a characterization of the interaction-induced Liouvillian skin effect as shown in the main text. It should be noted that even for the presence of  $H$  in Ref. [50], we numerically confirm that the winding number takes a nonzero value.

#### APPENDIX C: ABSENCE OF THE LIOUVILLIAN SKIN EFFECT IN NONINTERACTING SYSTEMS

Here we numerically show that the Liouvillian skin effect is absent when the systems do not have interactions ( $U - i\gamma = 0$ ). In this case, the time evolution of the density matrix is described by the von Neumann equation. Figure 5(a) shows that the eigenmodes of the Liouvillian do not exhibit the skin effect for the noninteracting case. As mentioned in Sec. IV, all eigenvalues of the Liouvillian lie on the imaginary axis and are insensitive to boundary conditions [see Figs. 5(b) and 5(c)]. Thus, the Liouvillian skin effect does not occur in the noninteracting system.

#### APPENDIX D: SENSITIVITY OF EIGENVALUES OF THE LIOUVILLIAN TO BOUNDARY CONDITIONS

In this Appendix, we show that eigenvalues of the Liouvillian exhibit the sensitivity to boundary conditions. Under OBC as shown in Fig. 6(a), the eigenvalues form a linelike

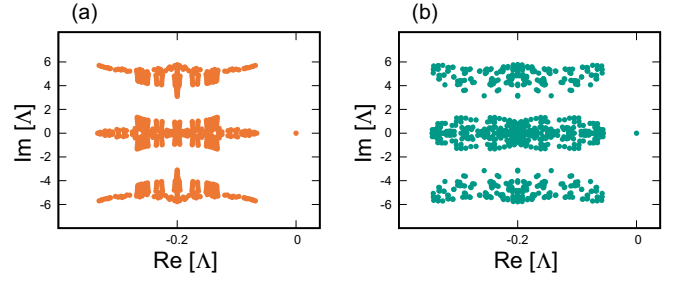


FIG. 6. (a) [(b)] Eigenvalues of the Liouvillian under OBC (PBC). The parameters are set to be  $L = 14$ ,  $U = 0.1$ , and  $\gamma = 0.2$ . The configuration of fermions in the down-spin states is set to be  $\{n_\downarrow\} = \{1, \dots, 1\}$ .

structure, which is in contrast to the case of PBC shown in Fig. 6(b). Such sensitivity is a signal of the Liouvillian skin effect. We note that since the steady state is  $N_\downarrow$ -fold degenerate regardless of boundary conditions, the eigenvalues corresponding to the steady state do not exhibit the Liouvillian skin effect.

#### APPENDIX E: DERIVATION OF THE RELATION BETWEEN WINDING NUMBERS GIVEN IN EQ. (36)

In this Appendix, we derive the relation between the winding number  $\nu(\Lambda_{\text{ref}})$  defined by the Liouvillian super-operator and  $w(E_{\text{ref}})$  defined by the Hamiltonian given in Eq. (36). First, we recall that in the case of particle losses, the Liouvillian takes the following block triangular structure:

$$\mathcal{L} = \begin{pmatrix} \mathcal{L}_0^{(N_\uparrow=1)}(\theta) & \\ & \mathcal{L}_0^{(N_\uparrow=0)}(\theta) \end{pmatrix}. \quad (\text{E1})$$

We note that, since the Lindblad operator given in Eq. (5) has no hopping term between site 1 and site  $L$ , the jump term is independent of  $\theta$ , i.e.,  $\mathcal{L}_J(\theta) = \mathcal{L}_J$ . For a block triangular matrix, the following relation holds:

$$\det \begin{pmatrix} A & \\ B & C \end{pmatrix} = \det A \det C. \quad (\text{E2})$$

Since  $\mathcal{L}^{(N_\uparrow=0)}(\theta)$  is independent of  $\theta$ , we obtain

$$\begin{aligned} d_\theta \log \det[\mathcal{L}(\theta) - \Lambda_{\text{ref}}] \\ &= d_\theta \log \det[\mathcal{L}_0^{(N_\uparrow=1)}(\theta) - \Lambda_{\text{ref}}] \\ &= d_\theta \log \det \mathcal{M}(\theta) \end{aligned} \quad (\text{E3})$$

for  $\Lambda_{\text{ref}} \neq 0$ , where

$$\mathcal{M}(\theta) = H_{\text{eff}}(\theta) \otimes I - I \otimes H_{\text{eff}}^* - i\Lambda_{\text{ref}} I \otimes I. \quad (\text{E4})$$

Then, we introduce  $\mathcal{N}(\theta)$  to diagonalize  $\mathcal{M}(\theta)$  defined by

$$\mathcal{N}(\theta) = S(\theta) \otimes T, \quad (\text{E5})$$

where  $T = S^*(\theta = 0)$ , and operator  $S(\theta)$ , which diagonalizes  $H_{\text{eff}}(\theta)$  as

$$S^{-1}(\theta) H_{\text{eff}}(\theta) S(\theta) = \text{diag}(E_1(\theta), \dots, E_{2L}(\theta)). \quad (\text{E6})$$

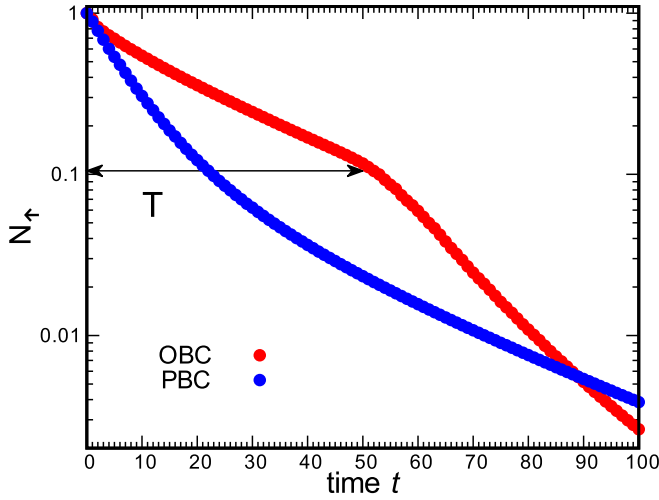


FIG. 7. Numerical result of the time dependence of the total number of the up-spin particles  $N_{\uparrow}(t)$ .  $U = 0.5$ ,  $\gamma = 0.1$ ,  $L = 60$ . The configuration of fermions in the down-spin states is set to be  $\{n_{\downarrow}\} = \{1, \dots, 1\}$ . In the initial state, the particle is distributed at the left side of the system in the orbital  $b$ . Time of transition  $T$  is denoted by the black arrow.

A straightforward calculation results in

$$\log \det[\mathcal{N}^{-1} \mathcal{M}(\theta) \mathcal{N}] = \sum_{i=1}^{2L} \log \det \prod_{j=1}^{2L} [E_j(\theta) - (E_i^* + i\Lambda_{\text{ref}})]. \quad (\text{E7})$$

Finally, we obtain the relation between  $\nu(\Lambda_{\text{ref}})$  and  $w(E_{\text{ref}})$  as

$$\oint_0^{2\pi} \frac{d\theta}{2\pi i} \frac{d}{d\theta} \log \det[\mathcal{L}(\theta) - \Lambda_{\text{ref}} I \otimes I] = \sum_j w(E_{\text{ref}} = E_j^* + i\Lambda_{\text{ref}}), \quad (\text{E8})$$

which is nothing but Eq. (36) in the main text.

#### APPENDIX F: SLOWING DOWN OF THE RELAXATION PROCESS UNDER OBC

There have been several studies that focus on the relaxation process in the presence of the Liouvillian skin effect [50,122,123]. In this Appendix, we show that the relaxation process slows down under OBC in a finite system. Figure 7 shows the time dependence of the total number of the particles in the up-spin state  $N_{\uparrow}(t)$ . By comparing the results under OBC and PBC, we find that the relaxation process is slowing down under OBC, which is originally discussed in Refs. [50]. Under OBC, the relaxation speed changes after time  $T$  in a finite system. We calculate the system size dependence of the time of the transition time  $T$ . Figure 8 indicates that the transition time  $T$  and systems size are proportional ( $T \propto L$ ). This fact implies that slowing down the relaxation process occurs even when we increase the system size. These results are consistent with the previous study of the dynamics with the Liouvillian skin effect [50].

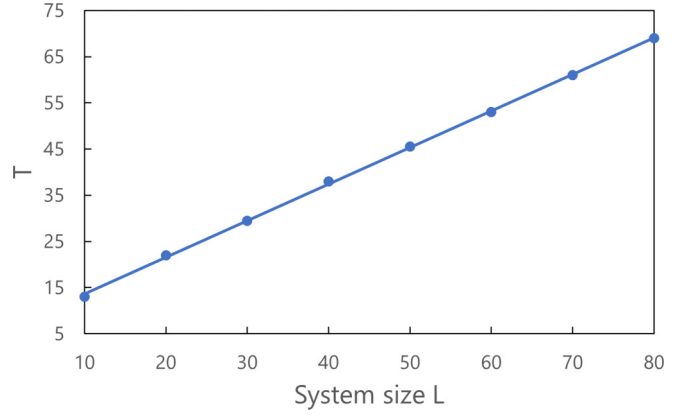


FIG. 8. The system size dependence of the transition time.  $U = 0.5$ ,  $\gamma = 0.1$ . In the initial state, the particle is distributed at the left side of the system in the orbital  $b$ .

#### APPENDIX G: RESULTS FOR OTHER CONFIGURATIONS OF DOWN-SPINS IN THE INITIAL STATE

In the main text, the configuration of fermions in the down-spin states is set to be  $\{n_{\downarrow}\} = \{1, \dots, 1\}$  in the initial state. In this Appendix, we show that the Liouvillian skin effect survives for other configurations of down-spins in the initial state. We set the configuration of fermions in the down-spin states to be  $\{n_{\downarrow}\} = \{1, 1, 1, 1, 0, 1\}$ . Then we numerically calculate the winding number given in Eq. (21). Figure 9(a) shows that the winding number takes three. Moreover, eigenmodes of the Liouvillian exhibit the skin effect under the OBC as shown in Fig. 9(b). We observe the dependence of eigenvalues on boundary conditions, which is similar to that presented in Appendix D. Therefore, the Liouvillian skin effect survives for other configurations of down-spins in the initial state.

#### APPENDIX H: CONTINUOUS DEFORMATION OF THE EFFECTIVE HAMILTONIAN

In this Appendix, we perform the continuous deformation of the effective Hamiltonian, which supplies an intuitive understanding of why the eigenstate is localized at the right edge. Non-Hermitian Hamiltonians  $H_0(k)$  and  $H_1(k)$  are defined to

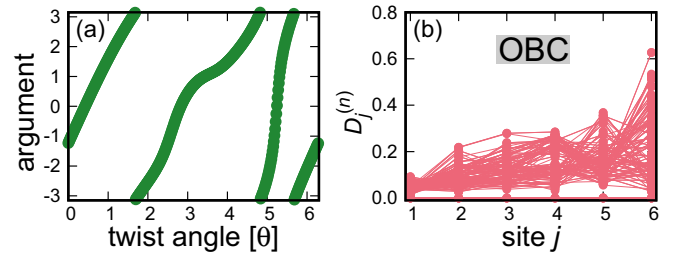


FIG. 9. (a) Argument of  $\det[\mathcal{L}(\theta) - \Lambda_{\text{ref}}]$ . (b) The right-state particle density of the Liouvillian under an OBC for other configuration cases of fermions in the down-spin states. The parameters are set to be  $L = 6$ ,  $U = 0.3$ ,  $\gamma = 0.7$ , and  $\Lambda_{\text{ref}} = -0.8 - 0.2i$ . The configuration of fermions in the down-spin states is set to be  $\{n_{\downarrow}\} = \{1, 1, 1, 1, 0, 1\}$ .

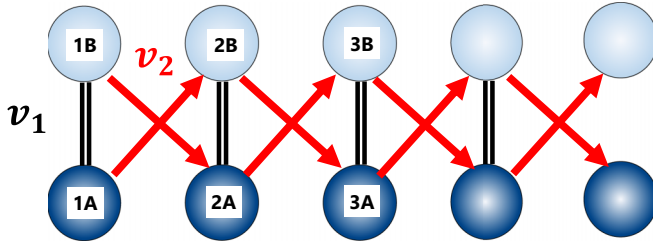


FIG. 10. Schematic figure of the Hamiltonian given in Eq. (H4).

be topologically equivalent if and only if there exists a Hamiltonian satisfying and maintaining both relevant symmetry and the point-gap  $\det[H_\lambda(k) - E_{\text{ref}}] \neq 0$  for all  $\lambda \in [0, 1]$  [16,17]. Here, we define the family of the non-Hermitian Hamiltonian as

$$h_\lambda(k) = \lambda h_{\text{asym}}(k) + (1 - \lambda) h_{\text{FK}}(k). \quad (\text{H1})$$

Here,  $h_{\text{FK}}$  and  $h_{\text{asym}}$  are the Bloch Hamiltonian of the Falicov-Kimball model with a complex-valued interaction and the asymmetric hopping Hamiltonian, which are explicitly written as

$$h_{\text{FK}}(k) = b_2(k)\sigma_2 + b_3(k)\sigma_3 + (U - i\gamma) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (\text{H2})$$

and

$$h_{\text{asym}} = (v_1 + v_2 e^{ik})\sigma_1 \quad (v_1, v_2 > 0), \quad (\text{H3})$$

respectively. The functions  $b_2(k)$  and  $b_3(k)$  are given in Eqs. (3a) and (3b) in the main text. Since the asymmetric hopping Hamiltonian  $h_{\text{asym}}$  is expressed as

$$H_{\text{asym}} = v_1 \sum_j (c_{jA}^\dagger c_{jB} + \text{H.c.}) + v_2 \sum_j (c_{j+1A}^\dagger c_{jB} + c_{j+1B}^\dagger c_{jA}) \quad (\text{H4})$$

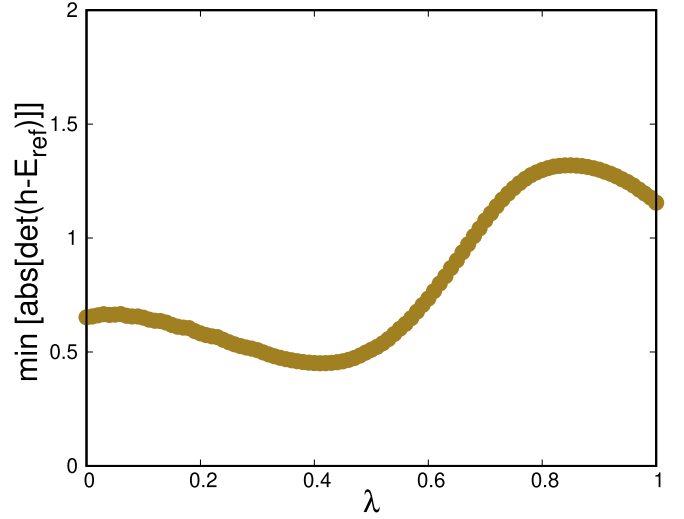


FIG. 11. The minimum value of  $|\det[h_\lambda(k) - E_{\text{ref}}]|$  as a function of  $\lambda$ . The parameters are set to be  $v_1 = 2$ ,  $v_2 = 1$ ,  $t = 1$ ,  $U = 0.1$ ,  $\gamma = 1.0$ ,  $E_{\text{ref}} = -2 - 0.5i$ .

in real space, the second term denotes the asymmetric hopping from left to right, which leads to the localization of the eigenstate at the right edge. The schematic illustration of the Hamiltonian is given in Fig. 10. Now we show that there exists a path to connect the Hamiltonians  $h_{\text{FK}}$  and  $h_{\text{asym}}$  without point-gap closing. Figure 11 shows the minimum value of the

$$|\det[h_\lambda(k) - E_{\text{ref}}]| \quad (\text{H5})$$

by varying the  $\lambda$  from 0 to 1. Remarkably, the point-gap at  $E_{\text{ref}}$  is always open, because  $|\det[h_\lambda - E_{\text{ref}}]|$  does not take the zero for any  $\lambda$ . Thus, the non-Hermitian Hamiltonian  $h_{\text{asym}}$  is topologically equivalent to the Falicov-Kimball Hamiltonian  $h_{\text{FK}}$ . It is worth noting that by changing  $k \longleftrightarrow -k$  in Eq. (H2), the eigenstate is localized on the left side.

- [1] C. L. Kane and E. J. Mele, Quantum spin Hall effect in graphene, *Phys. Rev. Lett.* **95**, 226801 (2005).
- [2] C. L. Kane and E. J. Mele,  $Z_2$  topological order and the quantum spin hall effect, *Phys. Rev. Lett.* **95**, 146802 (2005).
- [3] X.-L. Qi, T. L. Hughes, and S.-C. Zhang, Topological field theory of time-reversal invariant insulators, *Phys. Rev. B* **78**, 195424 (2008).
- [4] H. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang, and S.-C. Zhang, Topological insulators in  $\text{Bi}_2\text{Se}_3$ ,  $\text{Bi}_2\text{Te}_3$  and  $\text{Sb}_2\text{Te}_3$  with a single dirac cone on the surface, *Nat. Phys.* **5**, 438 (2009).
- [5] M. Z. Hasan and C. L. Kane, Colloquium: Topological insulators, *Rev. Mod. Phys.* **82**, 3045 (2010).
- [6] X.-L. Qi and S.-C. Zhang, Topological insulators and superconductors, *Rev. Mod. Phys.* **83**, 1057 (2011).
- [7] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Classification of topological insulators and superconductors in three spatial dimensions, *Phys. Rev. B* **78**, 195125 (2008).

- [8] S. Ryu, A. P. Schnyder, A. Furusaki, and A. W. W. Ludwig, Topological insulators and superconductors: Tenfold way and dimensional hierarchy, *New J. Phys.* **12**, 065010 (2010).
- [9] A. Kitaev, Periodic table for topological insulators and superconductors, *AIP Conf. Proc.* **1134**, 22 (2009).
- [10] M. Sato and Y. Ando, Topological superconductors: A review, *Rep. Prog. Phys.* **80**, 076501 (2017).
- [11] L. Fidkowski and A. Kitaev, Effects of interactions on the topological classification of free fermion systems, *Phys. Rev. B* **81**, 134509 (2010).
- [12] L. Fidkowski and A. Kitaev, Topological phases of fermions in one dimension, *Phys. Rev. B* **83**, 075103 (2011).
- [13] S. Raghu, X.-L. Qi, C. Honerkamp, and S.-C. Zhang, Topological mott insulators, *Phys. Rev. Lett.* **100**, 156401 (2008).
- [14] M. F. Lapa, J. C. Y. Teo, and T. L. Hughes, Interaction-enabled topological crystalline phases, *Phys. Rev. B* **93**, 115131 (2016).
- [15] Y. Ashida, Z. Gong, and M. Ueda, Non-Hermitian physics, *Adv. Phys.* **69**, 249 (2020).

- [16] Z. Gong, Y. Ashida, K. Kawabata, K. Takasan, S. Higashikawa, and M. Ueda, Topological phases of non-Hermitian systems, *Phys. Rev. X* **8**, 031079 (2018).
- [17] K. Kawabata, K. Shiozaki, M. Ueda, and M. Sato, Symmetry and topology in non-Hermitian physics, *Phys. Rev. X* **9**, 041015 (2019).
- [18] K. Esaki, M. Sato, K. Hasebe, and M. Kohmoto, Edge states and topological phases in non-Hermitian systems, *Phys. Rev. B* **84**, 205128 (2011).
- [19] F. K. Kunst, E. Edvardsson, J. C. Budich, and E. J. Bergholtz, Biorthogonal bulk-boundary correspondence in non-Hermitian systems, *Phys. Rev. Lett.* **121**, 026808 (2018).
- [20] E. J. Bergholtz, J. C. Budich, and F. K. Kunst, Exceptional topology of non-Hermitian systems, *Rev. Mod. Phys.* **93**, 015005 (2021).
- [21] N. Okuma and M. Sato, Non-Hermitian topological phenomena: A review, *Annu. Rev. Condens. Matter Phys.* **14**, 83 (2023).
- [22] K. Yokomizo and S. Murakami, Non-bloch band theory of non-Hermitian systems, *Phys. Rev. Lett.* **123**, 066404 (2019).
- [23] H. Shen, B. Zhen, and L. Fu, Topological band theory for non-Hermitian hamiltonians, *Phys. Rev. Lett.* **120**, 146402 (2018).
- [24] R. Lin, T. Tai, L. Li, and C. H. Lee, Topological non-Hermitian skin effect, *Front. Phys.* **18**, 53605 (2023).
- [25] V. Kozii and L. Fu, Non-Hermitian topological theory of finite-lifetime quasiparticles: Prediction of bulk fermi arc due to exceptional point, [arXiv:1708.05841](https://arxiv.org/abs/1708.05841).
- [26] T. Yoshida, R. Peters, N. Kawakami, and Y. Hatsugai, Symmetry-protected exceptional rings in two-dimensional correlated systems with chiral symmetry, *Phys. Rev. B* **99**, 121101(R) (2019).
- [27] T. Yoshida, R. Peters, and N. Kawakami, Non-Hermitian perspective of the band structure in heavy-fermion systems, *Phys. Rev. B* **98**, 035141 (2018).
- [28] S. Yao, F. Song, and Z. Wang, Non-Hermitian chern bands, *Phys. Rev. Lett.* **121**, 136802 (2018).
- [29] J. C. Budich, J. Carlström, F. K. Kunst, and E. J. Bergholtz, Symmetry-protected nodal phases in non-Hermitian systems, *Phys. Rev. B* **99**, 041406(R) (2019).
- [30] R. Okugawa and T. Yokoyama, Topological exceptional surfaces in non-Hermitian systems with parity-time and parity-particle-hole symmetries, *Phys. Rev. B* **99**, 041202(R) (2019).
- [31] E. Edvardsson, F. K. Kunst, and E. J. Bergholtz, Non-Hermitian extensions of higher-order topological phases and their biorthogonal bulk-boundary correspondence, *Phys. Rev. B* **99**, 081302(R) (2019).
- [32] J. Carlström, M. Stålhammar, J. C. Budich, and E. J. Bergholtz, Knotted non-Hermitian metals, *Phys. Rev. B* **99**, 161115(R) (2019).
- [33] D. S. Borgnia, A. J. Kruchkov, and R.-J. Slager, Non-Hermitian boundary modes and topology, *Phys. Rev. Lett.* **124**, 056802 (2020).
- [34] S. Yao and Z. Wang, Edge states and topological invariants of non-Hermitian systems, *Phys. Rev. Lett.* **121**, 086803 (2018).
- [35] C. H. Lee and R. Thomale, Anatomy of skin modes and topology in non-Hermitian systems, *Phys. Rev. B* **99**, 201103(R) (2019).
- [36] N. Okuma, K. Kawabata, K. Shiozaki, and M. Sato, Topological origin of non-Hermitian skin effects, *Phys. Rev. Lett.* **124**, 086801 (2020).
- [37] K. Zhang, Z. Yang, and C. Fang, Correspondence between winding numbers and skin modes in non-Hermitian systems, *Phys. Rev. Lett.* **125**, 126402 (2020).
- [38] K. Kawabata, M. Sato, and K. Shiozaki, Higher-order non-Hermitian skin effect, *Phys. Rev. B* **102**, 205118 (2020).
- [39] F. Song, S. Yao, and Z. Wang, Non-Hermitian skin effect and chiral damping in open quantum systems, *Phys. Rev. Lett.* **123**, 170401 (2019).
- [40] L. Li, C. H. Lee, S. Mu, and J. Gong, Critical non-Hermitian skin effect, *Nat. Commun.* **11**, 5491 (2020).
- [41] S. Longhi, Probing non-Hermitian skin effect and non-bloch phase transitions, *Phys. Rev. Res.* **1**, 023013 (2019).
- [42] K. Kawabata, T. Numasawa, and S. Ryu, Entanglement phase transition induced by the non-Hermitian skin effect, *Phys. Rev. X* **13**, 021007 (2023).
- [43] R. Okugawa, R. Takahashi, and K. Yokomizo, Second-order topological non-Hermitian skin effects, *Phys. Rev. B* **102**, 241202(R) (2020).
- [44] T. Yoshida, T. Mizoguchi, and Y. Hatsugai, Mirror skin effect and its electric circuit simulation, *Phys. Rev. Res.* **2**, 022062(R) (2020).
- [45] Q. Liang, D. Xie, Z. Dong, H. Li, H. Li, B. Gadway, W. Yi, and B. Yan, Dynamic signatures of non-Hermitian skin effect and topology in ultracold atoms, *Phys. Rev. Lett.* **129**, 070401 (2022).
- [46] T. Hofmann, T. Helbig, F. Schindler, N. Salgo, M. Brzezińska, M. Greiter, T. Kiessling, D. Wolf, A. Vollhardt, A. Kabašić, C. H. Lee, A. Bilušić, R. Thomale, and T. Neupert, Reciprocal skin effect and its realization in a topoelectrical circuit, *Phys. Rev. Res.* **2**, 023265 (2020).
- [47] L. Xiao, T. Deng, K. Wang, G. Zhu, Z. Wang, W. Yi, and P. Xue, Non-Hermitian bulk-boundary correspondence in quantum dynamics, *Nat. Phys.* **16**, 761 (2020).
- [48] A. Ghatak, M. Brandenbourger, J. van Wezel, and C. Coullais, Observation of non-Hermitian topology and its bulk-edge correspondence in an active mechanical metamaterial, *Proc. Natl. Acad. Sci. USA* **117**, 29561 (2020).
- [49] F. Yang, Q.-D. Jiang, and E. J. Bergholtz, Liouvillian skin effect in an exactly solvable model, *Phys. Rev. Res.* **4**, 023160 (2022).
- [50] T. Haga, M. Nakagawa, R. Hamazaki, and M. Ueda, Liouvillian skin effect: Slowing down of relaxation processes without gap closing, *Phys. Rev. Lett.* **127**, 070402 (2021).
- [51] J.-S. Pan, L. Li, and J. Gong, Point-gap topology with complete bulk-boundary correspondence and anomalous amplification in the fock space of dissipative quantum systems, *Phys. Rev. B* **103**, 205425 (2021).
- [52] C.-H. Liu, K. Zhang, Z. Yang, and S. Chen, Helical damping and dynamical critical skin effect in open quantum systems, *Phys. Rev. Res.* **2**, 043167 (2020).
- [53] T. Mori and T. Shirai, Resolving a discrepancy between liouvillian gap and relaxation time in boundary-dissipated quantum many-body systems, *Phys. Rev. Lett.* **125**, 230604 (2020).
- [54] K. Honda, S. Taie, Y. Takasu, N. Nishizawa, M. Nakagawa, and Y. Takahashi, Observation of the sign reversal of the



- magnetic correlation in a driven-dissipative fermi gas in double wells, *Phys. Rev. Lett.* **130**, 063001 (2023).
- [55] T. Tomita, S. Nakajima, Y. Takasu, and Y. Takahashi, Dissipative bose-hubbard system with intrinsic two-body loss, *Phys. Rev. A* **99**, 031601(R) (2019).
- [56] T. Tomita, S. Nakajima, I. Danshita, Y. Takasu, and Y. Takahashi, Observation of the mott insulator to superfluid crossover of a driven-dissipative bose-hubbard system, *Sci. Adv.* **3**, e1701513 (2017).
- [57] N. Syassen, D. M. Bauer, M. Lettner, T. Volz, D. Dietze, J. J. García-Ripoll, J. I. Cirac, G. Rempe, and S. Dürr, Strong dissipation inhibits losses and induces correlations in cold molecular gases, *Science* **320**, 1329 (2008).
- [58] G. Barontini, R. Labouvie, F. Stubenrauch, A. Vogler, V. Guarrera, and H. Ott, Controlling the dynamics of an open many-body quantum system with localized dissipation, *Phys. Rev. Lett.* **110**, 035302 (2013).
- [59] R. Labouvie, B. Santra, S. Heun, S. Wimberger, and H. Ott, Negative differential conductivity in an interacting quantum Gas, *Phys. Rev. Lett.* **115**, 050601 (2015).
- [60] K. Sponselee, L. Freystatzky, B. Abeln, M. Diem, B. Hundt, A. Kochanek, T. Ponath, B. Santra, L. Mathey, K. Sengstock, and C. Becker, Dynamics of ultracold quantum gases in the dissipative fermi-hubbard model, *Quantum Sci. Technol.* **4**, 014002 (2018).
- [61] R. Bouganne, M. Bosch Aguilera, A. Ghermaoui, J. Beugnon, and F. Gerbier, Anomalous decay of coherence in a dissipative many-body system, *Nat. Phys.* **16**, 21 (2020).
- [62] B. Yan, S. A. Moses, B. Gadway, J. P. Covey, K. R. A. Hazzard, A. M. Rey, D. S. Jin, and J. Ye, Observation of dipolar spin-exchange interactions with lattice-confined polar molecules, *Nature (London)* **501**, 521 (2013).
- [63] K. Yamamoto, Y. Ashida, and N. Kawakami, Rectification in nonequilibrium steady states of open many-body systems, *Phys. Rev. Res.* **2**, 043343 (2020).
- [64] N. Yoshioka and R. Hamazaki, Constructing neural stationary states for open quantum many-body systems, *Phys. Rev. B* **99**, 214306 (2019).
- [65] T. Haga, M. Nakagawa, R. Hamazaki, and M. Ueda, Quasiparticles of decoherence processes in open quantum many-body systems: Incoherentons, *arXiv:2211.14991*.
- [66] R. Hamazaki, M. Nakagawa, T. Haga, and M. Ueda, Lindbladian many-body localization, *arXiv:2206.02984*.
- [67] M. Nakagawa, N. Tsuji, N. Kawakami, and M. Ueda, Dynamical sign reversal of magnetic correlations in dissipative hubbard models, *Phys. Rev. Lett.* **124**, 147203 (2020).
- [68] K. Yamamoto, M. Nakagawa, M. Tezuka, M. Ueda, and N. Kawakami, Universal properties of dissipative tomonaga-luttinger liquids: Case study of a non-Hermitian XXZ spin chain, *Phys. Rev. B* **105**, 205125 (2022).
- [69] K. Yamamoto and N. Kawakami, Universal description of dissipative tomonaga-luttinger liquids with  $SU(n)$  spin symmetry: Exact spectrum and critical exponents, *Phys. Rev. B* **107**, 045110 (2023).
- [70] L. Rosso, A. Biella, J. De Nardis, and L. Mazza, Dynamical theory for one-dimensional fermions with strong two-body losses: Universal non-Hermitian zeno physics and spin-charge separation, *Phys. Rev. A* **107**, 013303 (2023).
- [71] L. Rosso, A. Biella, and L. Mazza, The one-dimensional Bose gas with strong two-body losses: the effect of the harmonic confinement, *SciPost Phys.* **12**, 044 (2022).
- [72] B. Zhu, B. Gadway, M. Foss-Feig, J. Schachenmayer, M. L. Wall, K. R. A. Hazzard, B. Yan, S. A. Moses, J. P. Covey, D. S. Jin, J. Ye, M. Holland, and A. M. Rey, Suppressing the loss of ultracold molecules Bia the continuous quantum zeno effect, *Phys. Rev. Lett.* **112**, 070404 (2014).
- [73] J. J. García-Ripoll, S. Dürr, N. Syassen, D. M. Bauer, M. Lettner, G. Rempe, and J. I. Cirac, Dissipation-induced hard-core Boson gas in an optical lattice, *New J. Phys.* **11**, 013053 (2009).
- [74] Y. Ashida, S. Furukawa, and M. Ueda, Quantum critical behavior influenced by measurement backaction in ultracold gases, *Phys. Rev. A* **94**, 053615 (2016).
- [75] G. Peretto, F. Carollo, J. P. Garrahan, and I. Lesanovsky, Reaction-limited quantum reaction-diffusion dynamics, *Phys. Rev. Lett.* **130**, 210402 (2023).
- [76] G. Peretto, F. Carollo, J. P. Garrahan, and I. Lesanovsky, Quantum reaction-limited reaction-diffusion dynamics of annihilation processes, *arXiv:2305.06944*.
- [77] K. Yamamoto, M. Nakagawa, K. Adachi, K. Takasan, M. Ueda, and N. Kawakami, Theory of non-Hermitian fermionic superfluidity with a complex-valued interaction, *Phys. Rev. Lett.* **123**, 123601 (2019).
- [78] K. Yamamoto, M. Nakagawa, N. Tsuji, M. Ueda, and N. Kawakami, Collective excitations and nonequilibrium phase transition in dissipative fermionic superfluids, *Phys. Rev. Lett.* **127**, 055301 (2021).
- [79] M. Nakagawa, N. Kawakami, and M. Ueda, Non-Hermitian kondo effect in ultracold alkaline-earth atoms, *Phys. Rev. Lett.* **121**, 203001 (2018).
- [80] T. Yoshida and Y. Hatsugai, Fate of exceptional points under interactions: Reduction of topological classifications, *Phys. Rev. B* **107**, 075118 (2023).
- [81] T. Yoshida, K. Kudo, and Y. Hatsugai, Non-Hermitian fractional quantum hall states, *Sci. Rep.* **9**, 16895 (2019).
- [82] T. Yoshida and Y. Hatsugai, Reduction of one-dimensional non-Hermitian point-gap topology by interactions, *Phys. Rev. B* **106**, 205147 (2022).
- [83] T. Yoshida and Y. Hatsugai, Correlation effects on non-Hermitian point-gap topology in zero dimension: Reduction of topological classification, *Phys. Rev. B* **104**, 075106 (2021).
- [84] K. Kawabata, K. Shiozaki, and S. Ryu, Many-body topology of non-Hermitian systems, *Phys. Rev. B* **105**, 165137 (2022).
- [85] W. N. Fugno and T. Ozawa, Interaction-induced non-Hermitian topological phases from a dynamical gauge field, *Phys. Rev. Lett.* **129**, 180401 (2022).
- [86] S.-B. Zhang, M. M. Denner, T. C. V. Bzdušek, M. A. Sentef, and T. Neupert, Symmetry breaking and spectral structure of the interacting Hatano-Nelson model, *Phys. Rev. B* **106**, L121102 (2022).
- [87] T. Guo, K. Kawabata, R. Nakai, and S. Ryu, Non-Hermitian boost deformation, *Phys. Rev. B* **108**, 075108 (2023).
- [88] Y.-C. Wang, K. Suthar, H. H. Jen, Y.-T. Hsu, and J.-S. You, Non-Hermitian skin effects on many-body localized and thermal phases, *Phys. Rev. B* **107**, L220205 (2023).
- [89] L. Mao, Y. Hao, and L. Pan, Non-Hermitian skin effect in one-dimensional interacting Bose gas, *Phys. Rev. A* **107**, 043315 (2023).

- [90] T. Micallo, C. Lehmann, and J. C. Budich, Correlation-induced sensitivity and non-Hermitian skin effect of quasiparticles, [arXiv:2302.00019](#).
- [91] D. J. Luitz and F. Piazza, Exceptional points and the topology of quantum many-body spectra, *Phys. Rev. Res.* **1**, 033051 (2019).
- [92] T. Liu, J. J. He, T. Yoshida, Z.-L. Xiang, and F. Nori, Non-Hermitian topological mott insulators in one-dimensional fermionic superlattices, *Phys. Rev. B* **102**, 235151 (2020).
- [93] C. H. Lee, Many-body topological and skin states without open boundaries, *Phys. Rev. B* **104**, 195102 (2021).
- [94] R. Shen and C. H. Lee, Non-Hermitian skin clusters from strong interactions, *Commun. Phys.* **5**, 238 (2022).
- [95] F. Alsallom, L. Herviou, O. V. Yazyev, and M. Brzezińska, Fate of the non-Hermitian skin effect in many-body fermionic systems, *Phys. Rev. Res.* **4**, 033122 (2022).
- [96] S. Tsubota, H. Yang, Y. Akagi, and H. Katsura, Symmetry-protected quantization of complex Berry phases in non-Hermitian many-body systems, *Phys. Rev. B* **105**, L201113 (2022).
- [97] C. Wang, T.-C. Yi, J. Li, and R. Mondaini, Non-Hermitian haldane-hubbard model: Effective description of one and two-body dissipations, *Phys. Rev. B* **108**, 085134 (2023).
- [98] A. J. Daley, Quantum trajectories and open many-body quantum systems, *Adv. Phys.* **63**, 77 (2014).
- [99] S. Lieu, M. McGinley, and N. R. Cooper, Tenfold way for quadratic lindbladans, *Phys. Rev. Lett.* **124**, 040401 (2020).
- [100] T. Yoshida, K. Kudo, H. Katsura, and Y. Hatsugai, Fate of fractional quantum hall states in open quantum systems: Characterization of correlated topological states for the full liouvillian, *Phys. Rev. Res.* **2**, 033428 (2020).
- [101] M. Kawasaki, K. Mochizuki, and H. Obuse, Topological phases protected by shifted sublattice symmetry in dissipative quantum systems, *Phys. Rev. B* **106**, 035408 (2022).
- [102] K. Kawabata, A. Kulkarni, J. Li, T. Numasawa, and S. Ryu, Symmetry of open quantum systems: Classification of dissipative quantum chaos, *PRX Quantum* **4**, 030328 (2023).
- [103] H. Li, H. Wu, W. Zheng, and W. Yi, Many-body non-Hermitian skin effect under dynamic gauge coupling, *Phys. Rev. Res.* **5**, 033173 (2023).
- [104] L. M. Falicov and J. C. Kimball, Simple model for semiconductor-metal transitions:  $\text{SmB}_6$  and transition-metal oxides, *Phys. Rev. Lett.* **22**, 997 (1969).
- [105] We note that hopping of down-spin states excluded, which simplifies the computation of eigenstates and eigenvalues of  $H_{\text{eff}}$  [see Eq. (9)]. Introducing the hopping of down-spin states may change the eigenvalues. We note, however, that the topological properties remain unchanged as long as the point-gap opens.
- [106] G. Lindblad, On the generators of quantum dynamical semigroups, *Commun. Math. Phys.* **48**, 119 (1976).
- [107] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Completely positive dynamical semigroups of  $N$ -level systems, *J. Math. Phys.* **17**, 821 (1976).
- [108] The orbital dependence of the anti-Hermitian part is essential for the interaction-induced Liouvillian skin effect. As a simple example, we introduce interaction  $U$  and  $\gamma$  only to orbital  $b$ . We consider that introducing complex-valued interaction does not change the topology as long as the point-gap opens. We also note that by fixing the configuration of down-spins, the effective Hamiltonian takes a quadratic form in the subspace.
- [109] N. Shibata and H. Katsura, Dissipative spin chain as a non-Hermitian kitaev ladder, *Phys. Rev. B* **99**, 174303 (2019).
- [110] We note that  $c$ 's satisfy the anticommutation relation with each other as well as  $\tilde{c}$ 's. Here, we denote the operator marked with a tilde as acting on the bra space. However,  $c_{ja\sigma}$  and  $\tilde{c}_{j'a'\sigma'}$  commute  $[c_{ja\sigma}, \tilde{c}_{j'a'\sigma'}] = 0$ . We note, however, that the following results can be obtained by introducing  $d_{ja\sigma} = c_{ja\sigma}$  and  $d_{j'a'\sigma'} = P_{\text{fK}} \tilde{c}_{j'a'\sigma'}$  with  $P_{\text{fK}} = e^{i\pi \sum_{ja\sigma} c_{ja\sigma}^\dagger c_{ja\sigma}}$ .
- [111] J. M. Torres, Closed-form solution of lindblad master equations without gain, *Phys. Rev. A* **89**, 052133 (2014).
- [112] M. Nakagawa, N. Kawakami, and M. Ueda, Exact liouvillian spectrum of a one-dimensional dissipative hubbard model, *Phys. Rev. Lett.* **126**, 110404 (2021).
- [113] In the case of the non-Hermitian skin effect in many-body systems, the rigorous proof of the relation between the nonzero topological number and the emergence of the skin effect has not been clarified. Some research has shown this relation by analyzing the specific model. Similarly, we demonstrate that our topological number takes a nontrivial value corresponding to the Liouvillian skin effect in many-body systems by analyzing the Falicov-Kimball model in the main text.
- [114] H. Yoshida and H. Katsura, Liouvillian gap and single spin-flip dynamics in the dissipative Fermi-Hubbard model, *Phys. Rev. A* **107**, 033332 (2023).
- [115] M. Lohse, C. Schweizer, O. Zilberberg, M. Aidelsburger, and I. Bloch, A thouless quantum pump with ultracold Bosonic atoms in an optical superlattice, *Nat. Phys.* **12**, 350 (2016).
- [116] S. Nakajima, T. Tomita, S. Taie, T. Ichinose, H. Ozawa, L. Wang, M. Troyer, and Y. Takahashi, Topological thouless pumping of ultracold Fermions, *Nat. Phys.* **12**, 296 (2016).
- [117] D. Semmler, K. Byczuk, and W. Hofstetter, Mott-hubbard and anderson metal-insulator transitions in correlated lattice fermions with binary disorder, *Phys. Rev. B* **81**, 115111 (2010).
- [118] N. Hatano and D. R. Nelson, Localization transitions in non-Hermitian quantum mechanics, *Phys. Rev. Lett.* **77**, 570 (1996).
- [119] N. Hatano and D. R. Nelson, Vortex pinning and non-Hermitian quantum mechanics, *Phys. Rev. B* **56**, 8651 (1997).
- [120] N. Hatano and D. R. Nelson, Non-Hermitian delocalization and eigenfunctions, *Phys. Rev. B* **58**, 8384 (1998).
- [121] The contradiction of the sign in Eq. (17) in Ref. [84] can be lifted by identifying  $\phi = -\theta$ .
- [122] Z. Wang, Y. Lu, Y. Peng, R. Qi, Y. Wang, and J. Jie, Accelerating relaxation dynamics in open quantum systems with liouvillian skin effect, *Phys. Rev. B* **108**, 054313 (2023).
- [123] S. E. Begg and R. Hanai, Universality in open quantum spin chains with non-reciprocity, [arXiv:2307.03714](#).