


Chiral and nodal superconductors in the t - J model with valley contrasting flux on a triangular moiré lattice

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(Received 4 October 2022; revised 18 September 2023; accepted 20 September 2023; published 6 October 2023)

Recent experimental progress has made it possible to simulate spin-1/2 Hubbard model on triangular lattice in moiré materials formed by transition metal dichalcogenide (TMD) heterobilayer or homobilayer. In twisted TMD homobilayer, a vertical electric field can induce a valley contrasting flux in the hopping term. In this paper we study possible superconductors from a t - J model with valley contrasting flux Φ using the slave boson mean-field theory. We obtain a phase diagram with doping x and Φ . A finite Φ breaks spin rotation symmetry and the pairing symmetry is a superposition of spin singlet $d - id$ and spin triplet $p + ip$. There are two topological phase transitions when tuning Φ from 0 to π , with three Dirac nodes at one transition and one single Dirac node at the other transition. We also discuss the effects of Van Hove singularity and a three-site correlated hopping term on the pairing strength. Lastly, we demonstrate that a small anisotropy term breaking the C_3 rotation can lead to a time-reversal invariant nodal superconductor connected to the $d_{x^2-y^2}$ superconductor on square lattice.

DOI: [10.1103/PhysRevB.108.155111](https://doi.org/10.1103/PhysRevB.108.155111)

I. INTRODUCTION

Superconductivity from doping a Mott insulator has been intensively studied in the last several decades after the discovery of the high- T_c superconductor in cuprates [1]. Recently, moiré superlattices emerge to be a wonderful platform to simulate strongly correlated physics [2–22]. Superconductivity has been experimentally reported in twisted bilayer graphene and a twisted multilayer graphene system with alternating twist angles [5,23–25]. However, the mechanism of the superconductivity there is still under debate. Theoretical study in twisted bilayer graphene is hard due to the lack of a simple lattice model which is obstructed by the fragile topology [26]. On the other hand, moiré superlattice based on transition metal dichalcogenides (TMDs) is believed to be described by a simple Hubbard model [27] similar to that of the cuprate. This offers an opportunity to study superconductivity in the Hubbard model or the t - J model on triangular lattice.

Moiré superlattice can be formed by twisting TMD homobilayer or heterobilayer. In both cases the resulting Hubbard model has two flavors from the two valleys, which are locked to the spins due to the strong Ising spin-orbit coupling of the valence band. In the heterobilayer case, it was shown that there is a good SU(2) spin rotation symmetry in the valley (spin) space [27]. Thus the physics of doping the Mott insulator at $\nu_T = 1$ is captured by the standard t - J model. In contrast, for the twisting TMD homobilayer [28], the low-energy model has a valley contrasting flux Φ induced by a displacement field in the z direction [30,31]. The valley contrasting flux originates from the inversion symmetry breaking within each valley and is known to exist also in graphene moiré systems [32]. In such a case, the SU(2) spin rotation symmetry is broken down to U(1) except at $\Phi = 0 \bmod 2\pi$. The t - J model with $\Phi = 0$ has been found to host a $d - id$ superconductor by the slave boson mean-field theory [33]. But the fate at generic Φ is unknown, although there are already a few studies at the weak-coupling limit [34–36]. For

application to twisted TMD homobilayer, it is desirable to obtain a phase diagram of (n, Φ) given that both the density n and the flux Φ can be conveniently tuned in the dual gated sample.

In this paper we perform a slave boson mean-field study of the t - J model with a generic valley contrasting flux Φ . The flux Φ enters both the hopping term and the J term through a Dzyaloshinskii-Moriya interaction. At finite Φ , we find that the $d - id$ spin-singlet pairing is mixed with a $p + ip$ spin-triplet pairing. By changing Φ at fixed density n , we find two topological phase transitions between Chern number $|C| = 2$ and $|C| = 1$ through three Dirac nodes and one Dirac node, respectively. We also notice the correlations between the density of states (DOS) and the pairing strength. Pairing is stronger at the Van Hove singularity. Chiral superconductivity has been observed in numerical simulations in the SU(2) symmetric limit [37–39]; our study demonstrates a route to tune a topological phase transition through changing the valley contrasting flux. We also study the effect of strain which breaks the C_3 rotation symmetry. We find that a small strain favors a nodal superconductor with the same symmetry as the $d_{x^2-y^2}$ pairing in cuprates. This suggests that a nematic nodal superconductor is a strong competing state. Indeed, a recent numerical simulation shows that adding a J_2 term can lead to a transition from the chiral superconductor to the nematic nodal superconductor [39].

II. MODEL

The twisted TMD homobilayer can be described by a generalized triangular-lattice Hubbard model [30]:

$$\begin{aligned}
 H = & - \sum_{\langle ij \rangle, s} (t e^{i\phi_{ij}} c_{i,s}^\dagger c_{j,s} + \text{H.c.}) \\
 & + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i (n_{i\uparrow} + n_{i\downarrow}), \quad (1)
 \end{aligned}$$

where $s = \uparrow, \downarrow$ labels the valley index which is locked to the spin. $\langle ij \rangle$ represents the nearest-neighbor bond. We have $\phi_{ij}^\uparrow = -\phi_{ij}^\downarrow$ due to the time-reversal symmetry. As illustrated in Fig. 1, $\phi_{ij}^\uparrow = \pm\phi$ depending on the direction of the bond. There is a valley contrasting flux $\Phi = 3\phi$ and $-\Phi$ in the two types of triangles for one valley. We label σ_a as Pauli matrices in the valley space. We have time-reversal symmetry \mathcal{T} acting as $\sigma_x \mathcal{K}$, where \mathcal{K} is the complex conjugate. The mirror reflection operators \mathcal{M}_x and \mathcal{M}_y act as $c_s(x, y) \rightarrow c_s(x, -y)$ and $c_s(x, y) \rightarrow (\sigma_x)_{ss'} c_{s'}(-x, y)$, respectively. C_6 around site \mathbf{x} acts as $c_s(\mathbf{x}) \rightarrow (\sigma_x)_{ss'} c_{s'}(C_6 \mathbf{x})$. Two basis vectors of the lattice are $\mathbf{a}_1 = (1, 0)$ and $\mathbf{a}_2 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$. The corresponding reciprocal basis vectors are $\mathbf{b}_1 = (2\pi, \frac{2\pi}{\sqrt{3}})$ and $\mathbf{b}_2 = (0, \frac{4\pi}{\sqrt{3}})$. ϕ and $\phi + \frac{2\pi}{3}$ are gauge equivalent upon a transformation: $c_s(\mathbf{x}) \rightarrow (e^{i(\mathbf{x}/3) \cdot (\mathbf{b}_1 + \mathbf{b}_2) \sigma_z})_{ss'} c_{s'}(\mathbf{x})$. We can combine a particle-hole transformation with the gauge transformation: $c_s(\mathbf{x}) \rightarrow (e^{i(\mathbf{x}/6) \cdot (\mathbf{b}_1 + \mathbf{b}_2) \sigma_z} \sigma_x)_{ss'} c_{s'}^\dagger(\mathbf{x})$, which maps Φ to $\Phi + \pi$ and the density n to $2 - n$ [34].

In the strong-coupling limit $t \ll U$ and filling factor $n < 1$, we can use the standard t/U expansion [40] to obtain a t - J model [30]:

$$H = - \sum_{\langle ij \rangle, s} P (t e^{i\phi_{ij}^\uparrow} c_{i,s}^\dagger c_{j,s} + \text{H.c.}) P + J \sum_{\langle ij \rangle} \left[S_i^z S_j^z + \cos(2\phi_{ij}^\uparrow) \sum_{\alpha=x,y} S_i^\alpha S_j^\alpha + \sin(2\phi_{ij}^\uparrow) \times (S_i^x S_j^y - S_i^y S_j^x) - \frac{1}{4} n_i n_j \right] - \mu \sum_i (n_{i\uparrow} + n_{i\downarrow}), \quad (2)$$

where $J = \frac{4t^2}{U}$ and P is the projection operator which forbids double occupancy. For $\phi \neq 0$, the system only has a U(1) spin rotation symmetry generated by σ_z . At $n = 1$, the ground state is an XY ferromagnetic phase for $|\phi| \in (\frac{\pi}{3}, \frac{2\pi}{3})$, 120° AF⁺ phase for $\phi \in (0, \frac{\pi}{3}) \cup (\pi, \frac{4\pi}{3})$ and 120° AF⁻ phase for $\phi \in (\frac{2\pi}{3}, \pi) \cup (\frac{5\pi}{3}, 2\pi)$ [30]. The ground state remains unexplored at finite doping with $n = 1 - x$. We expect that the magnetic order is suppressed by the doping [33]. The major focus of this paper is to explore the possibility of superconducting phase in the small doping regimes. For the $n > 1$ case, we can apply the particle-hole transformation to map the model to the $n < 1$ model above, but with a change of the flux $\phi \rightarrow \phi + \pi$.

III. SLAVE BOSON MEAN-FIELD THEORY

Since it is difficult to handle the projection operator directly, we use the U(1) slave boson theory [1,41] to deal with the model. We focus our discussion in the region $n < 1$, while the $n > 1$ regime can be obtained by the particle-hole transformation. The electron operator can be represented as $c_{i,s}^\dagger = f_{i,s}^\dagger b_i$. By decoupling the interaction part in the hopping and pairing channel, we use the following mean-field

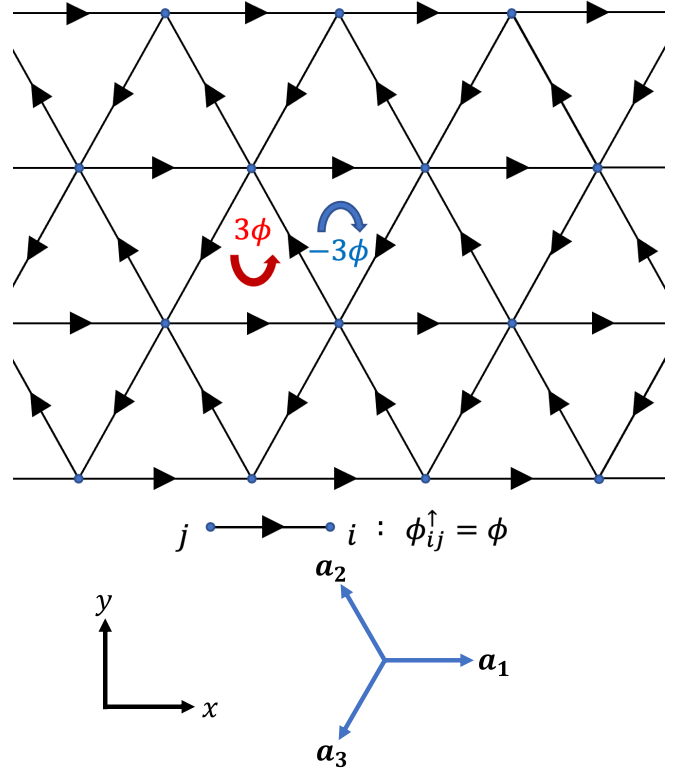


FIG. 1. Illustration of the flux pattern for the valley \uparrow . The other valley is related by time-reversal symmetry and has opposite flux pattern.

Hamiltonian to perform the calculation:

$$H_{\text{MF}} = - \sum_{\langle ij \rangle, s} (t e^{i\phi_{ij}^\uparrow} \langle b_i b_j^\dagger \rangle f_{i,s}^\dagger f_{j,s} + \text{H.c.}) - \frac{J}{8} \sum_{\langle ij \rangle, s} [(\chi_{ij,s}^* + 2e^{i2\phi_{ij}^\uparrow} \chi_{ij,-s}^*) f_{i,s}^\dagger f_{j,s} + \text{H.c.}] + \frac{J}{8} \sum_{\langle ij \rangle} [(\Delta_{ij} + 2\Delta_{ji} e^{i2\phi_{ij}^\uparrow}) f_{i,\uparrow}^\dagger f_{j,\downarrow}^\dagger + (i \leftrightarrow j) + \text{H.c.}] - \mu \sum_i (n_{i\uparrow} + n_{i\downarrow}), \quad (3)$$

with $\chi_{ij,s} = 2 \langle f_{i,s}^\dagger f_{j,s} \rangle$ and $\Delta_{ij} = 2 \langle f_{i\uparrow} f_{j\downarrow} \rangle$. The chemical potential μ is tuned to make $\frac{1}{N} \sum_{i,s} \langle n_{i,s} \rangle = 1 - x$, where x is the doping level. We ignore the magnetic order since it is suppressed on a doping level greater than a critical value x_c [33]. Moreover, we assume that the system respects the translation symmetry, then the bosons are condensed and satisfy $\langle b_i b_j^\dagger \rangle = \langle b_j b_i \rangle = x$ [33]. We can simply take $\langle b \rangle = \sqrt{x}$.

As for the $n > 1$ case, we can apply a particle-hole transformation $c_{i,s} \rightarrow c_{i,-s}^\dagger$ on the original Hamiltonian and apply the same procedure. By solving the self-consistent equation, we obtained the order parameter dependence on ϕ and n . Moreover, we remove the regime $0.95 < n < 1.05$ by assuming the superconducting order is suppressed. The result is shown in Fig. 2. We found that the superconducting order is mixed by p wave and d wave. Moreover, the pairing strength

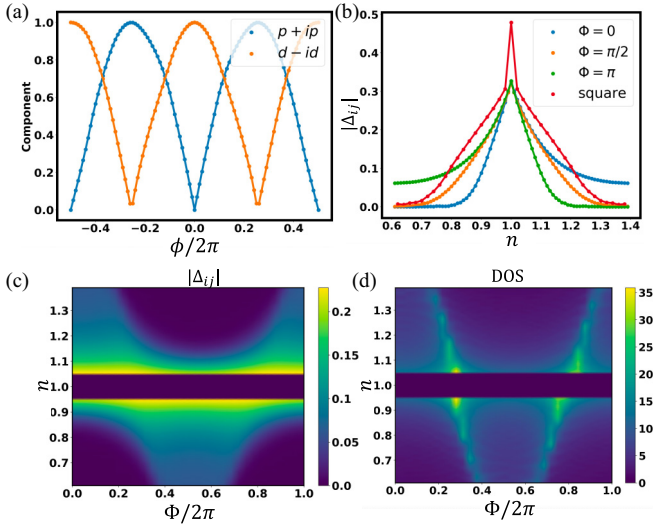


FIG. 2. $t/J = 3$. (a) Dependence of $p + ip$ and $d - id$ pairing's component on ϕ at $n = 0.9$. (b) Dependence of $|\Delta_{ij}|$ on the density n while flux $\Phi = 0, \frac{\pi}{2}$ on triangular lattice and $\Phi = 0$ on square lattice. (c) Dependence of $|\Delta_{ij}|$ on Φ and n . (d) Dependence of DOS on Φ and n . In (c) and (d), the region $0.95 < n < 1.05$ is removed because the superconductivity is expected to be suppressed because the slave boson condensation $\langle b \rangle = \sqrt{x}$ is weak in this regime.

is enhanced near the Van Hove singularity, which depends on the value of ϕ .

IV. p WAVE VS d WAVE

The special case with $\Phi = 0$ has been studied before [33]. There $\Delta_{ij} = \Delta e^{\pm i2\theta_{ij}}$ is in the $d_{x^2-y^2} \pm id_{xy}$ pairing symmetry, where $\Delta = |\Delta_{ij}|$ does not depend on the bond direction. With a finite Φ , the spin rotation symmetry is broken down to $U(1)$ generated by the S_z rotation. Therefore, the relation $\Delta_{ij} = \Delta_{ji}$ would not hold anymore and generically the pairing symmetry is a superposition of the spin-triplet p wave and the spin-singlet d wave. Δ_{ij} and Δ_{ji} have the same magnitude but different phases. Since Δ_{ij} respects C_3 symmetry instead of C_6 symmetry, the angular momentum is defined mod 3. The $p \pm ip$ and $d \mp id$ pairings are mixed, and we can define their components as $\Delta_p = \frac{1}{2|\Delta_{ij}|}|\Delta_{ij} - \Delta_{ji}|$ and $\Delta_d = \frac{1}{2|\Delta_{ij}|}|\Delta_{ij} + \Delta_{ji}|$. The components' dependence on ϕ is shown in Fig. 2(a).

We note that the superconducting order breaks the time-reversal symmetry \mathcal{T} and mirror reflection symmetry $\mathcal{M}_x(\mathcal{M}_y)$, but satisfied the combined mirror time-reversal $\mathcal{M}_x\mathcal{T}(\mathcal{M}_y\mathcal{T})$ symmetry combined with a $U(1)$ transformation.

V. PAIRING STRENGTH

We study the dependence of $|\Delta_{ij}|$ on different values of n and Φ . As shown in Figs. 2(b) and 2(c), the superconducting gap $|\Delta_{ij}|$ is larger at electron doping than hole doping when $0 \leq \Phi < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \Phi \leq 2\pi$, and smaller when $\frac{\pi}{2} < \Phi < \frac{3\pi}{2}$. The reason is that the density of states (DOS) of the free dispersion does not respect particle-hole symmetry, and

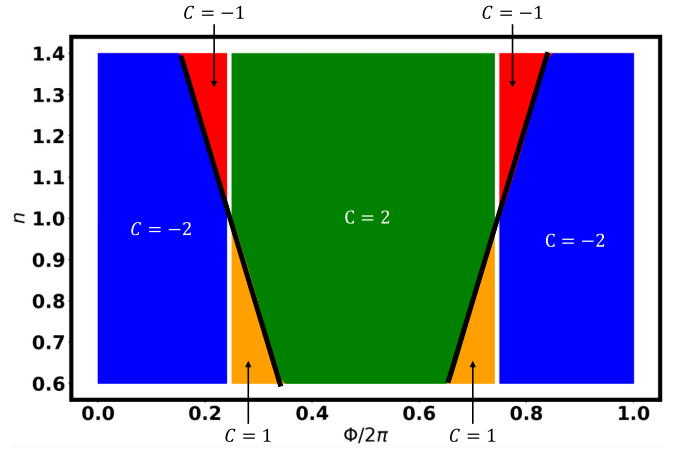


FIG. 3. $t/J = 3$. Dependence of Chern number on n and Φ for fixed chirality. The black and white line corresponds to one and three Dirac nodes in the dispersion plot.

the Van Hove peak locates at $n > 1$ for the former case and $n < 1$ for the latter case. The apparent correlation between the pairing strength and the DOS in Fig. 2(d) suggest that we should look for superconductors close to the Van Hove singularity, which is tunable by gating in the twisted TMD homobilayer.

VI. TOPOLOGICAL PROPERTY

The chiral superconductor is known to host a nonzero Chern number, which could be calculated via [42]

$$C = \frac{1}{4\pi} \int_{\text{BZ}} d^2\mathbf{k} [\hat{\mathbf{m}} \cdot (\partial_{k_x} \hat{\mathbf{m}} \times \partial_{k_y} \hat{\mathbf{m}})], \quad (4)$$

where $\hat{\mathbf{m}} = \frac{1}{\sqrt{\epsilon_k^2 + |\Delta_k|^2}}(\text{Re}\Delta_k, \text{Im}\Delta_k, \epsilon_k)$. The definition is equivalent to the winding number of Δ_k as k moves around the Fermi surface in the anticlockwise direction. We can also calculate the winding number by counting the number of Δ_k 's zero points inside the Fermi sphere; the formula is $C = \#_1 - \#_2$, where $\#_1$ and $\#_2$ represent the number of saddles and sources, respectively. The phase diagram is shown in Fig. 3 provided that the chirality is fixed. The change in Chern number equals to the number of Dirac nodes at the transition point. There are two kinds of transitions, which corresponds to one Dirac node (at κ point) and three Dirac nodes (inside the mini Brillouin zone) closing the gap, respectively. For details, please see Appendix A.

VII. EFFECTS OF ϕ

The phase of hopping parameters ϕ_{ij}^s plays two roles in our model: (1) It can produce an effective magnetic flux in the t term. (2) It enters the spin-spin coupling J term. The first one changes the distribution of DOS and the second one changes the form of pairing. To study these two effects in detail, we change the kinetic term and the interaction term in Eq. (2) to the conventional t - J model's form, which are named \hat{t} - J model and t - \hat{J} model, respectively. The concrete form of their Hamiltonian can be found in Appendix B. We note that ϕ and $\phi + \frac{2\pi}{3}$ could no longer be related via a gauge transformation

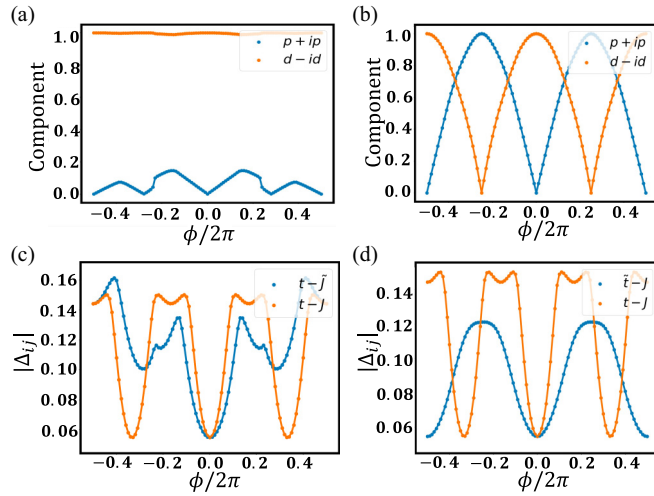


FIG. 4. $t/J = 3$. (a), (b) Components of $p + ip$ and $d - id$ at $n = 0.9$ for the $t\text{-}\tilde{J}$ model and the $\tilde{t}\text{-}J$ model, respectively. (c), (d) The superconducting gap $|\Delta_{ij}|$ at $n = 0.88$ for the $t\text{-}\tilde{J}$ model and the $\tilde{t}\text{-}J$ model, respectively.

here. In the $t\text{-}\tilde{J}$ model, we find that the superconducting order parameter returns to nearly pure d wave pairing shown in Fig. 4(a), but the hopping term still acquires a phase based on the mean-field calculation. As shown in Fig. 4(b), the components' dependence on ϕ in the $\tilde{t}\text{-}J$ model is nearly the same as the original model. Therefore, we can conclude that the pairing symmetry is decided by the J term as expected. In the $\tilde{t}\text{-}J$ model, the phase of the hopping parameter is found to be relatively small, especially at high doping levels. It makes the effective flux become smaller than the original Hamiltonian, causing the superconducting gap to become less dependent on ϕ than the one in the $t\text{-}\tilde{J}$ model, as shown in Figs. 4(c) and 4(d).

VIII. NEMATIC NODAL SUPERCONDUCTOR FROM STRAIN

So far we have only observed a gapped chiral superconductor respecting C_3 symmetry. One may expect that a nematic superconductor with mixing between $l = 1$ and $l = -1$ angular momentum channels is a competing state. In the following we show that a small strain can easily favor a nematic nodal superconductor. We consider the effects of strain by multiplying a factor $(1 - \alpha)$ on the hopping parameter $t e^{i\phi_{ij}^s}$ on the bond along the x axis, i.e., $t_{i\pm a_1, i}$, where α implies the stress intensity. Since $J_{ij} = \frac{t_{ij}^s}{U}$, the J term needs to be multiplied by a factor $(1 - \alpha)^2$ along the x axis. When $\alpha = 1$, the kinetic and the interaction terms are nonzero on four bonds. Then the Hamiltonian is mathematically equivalent to the one on the square lattice, where the pairing symmetry is known to be $d_{x^2-y^2}$. Therefore, the time-reversal symmetry and mirror reflection symmetry can be restored by sufficiently large strain. During the calculation, the pairing parameter $\Delta_{i\pm a_1, i}$ is found to decay very fast when α grows as shown in Fig. 5(c). It shows that the $d_{x^2-y^2}$ superconductor in the square lattice can be reached by small α . We use $1 - \langle \mathcal{T} \rangle$ and $1 - \langle \mathcal{M}_y \rangle$ to label the symmetry of the ground states. Here $\langle \mathcal{T} \rangle$ is defined

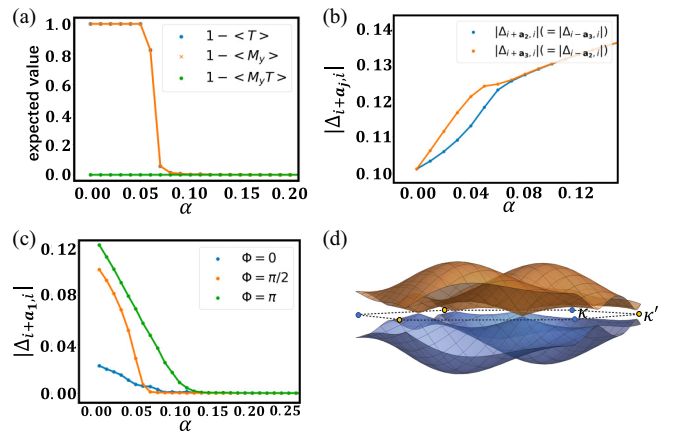


FIG. 5. $t/J = 3$, $n = 0.85$. (a) The expected value of the time-reversal operator, the mirror reflection operator, and the mirror time-reversal operator as α changes for flux $\Phi = \frac{\pi}{2}$. (b) Dependence of $|\Delta_{i+a_2, i}| (= |\Delta_{i-a_3, i}|)$ and $|\Delta_{i+a_3, i}| (= |\Delta_{i-a_2, i}|)$ on α for $\Phi = \frac{\pi}{2}$ with the chirality fixed. (c) Dependence of $|\Delta_{i+a_1, i}|$ on α for flux $\Phi = 0, \frac{\pi}{2}, \pi$. (d) The dispersion plot for $\Phi = \frac{\pi}{2}$, $\alpha = 0.15$; the gap closes at four nodal points in the first Brillouin zone.

by the inner product of the ground states of H and $\mathcal{T}H\mathcal{T}^{-1}$, and similar for \mathcal{M}_y . Since the Hamiltonian has a global $U(1)$ symmetry, $\langle \mathcal{T} \rangle$ and $\langle \mathcal{M}_y \rangle$ will change as the global phase θ changes. Therefore, we need to choose the appropriate θ to maximize the expected value to represent the symmetry faithfully. As shown in Fig. 5(a), the time-reversal symmetry and mirror reflection symmetry can be restored by a small strain α_c . Moreover, we find that $|\Delta_{i+a_2, i}| \neq |\Delta_{i+a_3, i}|$ if $0 < \alpha < \alpha_c$ as shown in Fig. 5(b). In Fig. 5(d), we show that there are four nodes in the resulting superconductor phase under strain. This calculation demonstrates that a nodal nematic superconductor is indeed a strong competing state and can be favored by a small strain. It is interesting to also study the possibility that the C_3 symmetry is spontaneously broken by additional terms.

IX. THE EFFECT OF THREE-SITE HOPPING TERM

In addition to Eq. (2), perturbation theory also gives a three-site hopping term [43], which can be written as

$$H_{t_3} = -t_3 \sum_{\langle ijk \rangle, s} P(n_{j,-s} c_{k,s}^\dagger c_{i,s} e^{i2\phi_{kj}^s} - c_{k,s}^\dagger c_{j,-s}^\dagger c_{j,s} c_{i,-s} + \text{H.c.}) P, \quad (5)$$

where $t_3 = \frac{J}{4}$. Here we suppose t_3 can be changed independently in order to discuss this term's effect. By applying the slave boson approach mentioned above, we can read off this term as

$$H_{t_3} = -t_3 \sum_{\langle ijk \rangle, s} (b_k b_i^\dagger f_{j,-s}^\dagger f_{j,-s} f_{k,s}^\dagger f_{k,s} f_{i,s} e^{i2\phi_{kj}^s} - b_k b_i^\dagger f_{k,s}^\dagger f_{j,-s}^\dagger f_{j,-s} f_{j,s} f_{i,-s} + \text{H.c.}), \quad (6)$$

where $b_k b_i^\dagger \approx \langle b_k b_i^\dagger \rangle = x$ is the doping level, so this term's effect increases as the doping level increases. Indeed, Eq. (6)

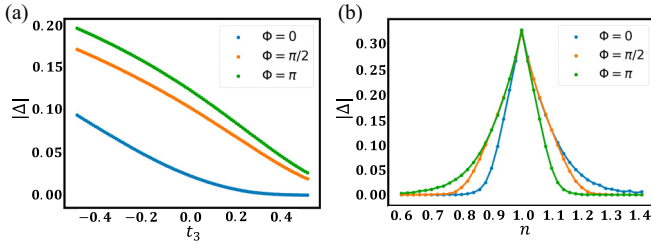


FIG. 6. $t/J = 3$. (a) Dependence of $|\Delta_{ij}|$ on t_3 for $n = 0.85$. (b) Dependence of $|\Delta_{ij}|$ on n for $t_3 = 0.25J$.

can be decoupled in the pairing channel [43] as

$$H_{t_3} = xt_3 \sum_{\langle ij \rangle} [\Delta_{ij} (\Delta_{ji}^* e^{i2\phi_{ij}^\dagger} + \Delta_{ij}^*) + \text{H.c.}]. \quad (7)$$

So we can expect that a positive value of t_3 can increase the total energy, leading to a decrease in $|\Delta_{ij}|$ by solving the self-consistent equation. The mean-field calculation validates our expectation as shown in Fig. 6(a). For different value of Φ , the pairing strength $|\Delta_{ij}|$ always decreases as t_3 increases. Moreover, the existence of Eq. (7) does not violate the particle-hole symmetry at $\Phi = \frac{\pi}{2}$ as shown in Fig. 6(b).

X. SUMMARY

In conclusion, we use a slave boson method to investigate a t - J model with valley contrasting flux in the context of twisted TMD homobilayer. We show that the superconducting order parameter is a mixture of $p - ip$ and $d + id$ pairing when $\phi \neq 0$. We notice two topological phase transitions with jump of Chern numbers by tuning the valley contrasting flux Φ , which is controlled by the vertical displacement field in the TMD homobilayer. The pairing strength correlates with the density of states, suggesting that we should search for superconductors near the Van Hove singularity. Finally, we find that a small strain can favor a nematic nodal superconductor similar to the $d_{x^2-y^2}$ pairing on the square lattice.

ACKNOWLEDGMENT

Y.-H.Z. was supported by the National Science Foundation under Grant No. DMR2237031.

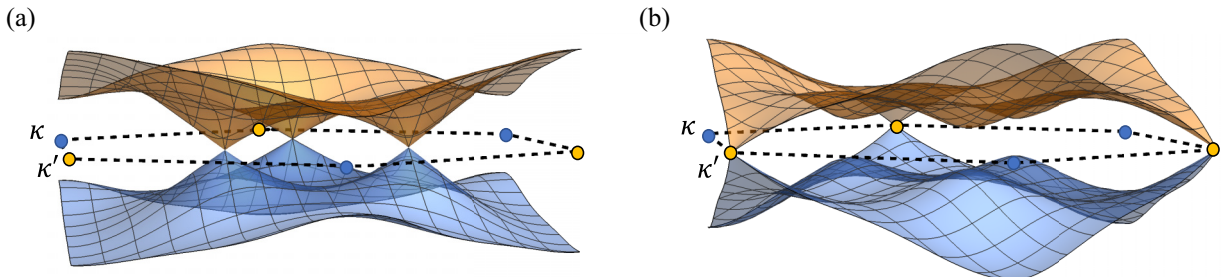


FIG. 7. (a) Illustration of the three Dirac nodes inside the first Brillouin zone. (b) Illustration of the Dirac node at κ' .

APPENDIX A: TOPOLOGICAL TRANSITIONS

In the main text, we observe that the Chern number depends on Φ and n . There are two kinds of topological transitions, which corresponds to one Dirac node and three Dirac nodes, respectively. The sketch of the dispersion plot for two different situations is shown in Fig. 7.

APPENDIX B: HAMILTONIAN OF \tilde{t} - J MODEL AND t - \tilde{J} MODEL

The concrete form of the \tilde{t} - J model and the t - \tilde{J} can be written as

$$\begin{aligned} H_{\tilde{t}-J} &= - \sum_{\langle ij \rangle, s} P (t c_{i,s}^\dagger c_{j,s} + \text{H.c.}) P \\ &+ J \sum_{\langle ij \rangle} \left[S_i^z S_j^z + \cos(2\phi_{ij}^\dagger) \sum_{\alpha=x,y} S_i^\alpha S_j^\alpha \right. \\ &\quad \left. + \sin(2\phi_{ij}^\dagger) (S_i^x S_j^y - S_i^y S_j^x) - \frac{1}{4} n_i n_j \right] \\ &- \mu \sum_i (n_{i\uparrow} + n_{i\downarrow}), \\ H_{t-\tilde{J}} &= - \sum_{\langle ij \rangle, s} P (t e^{i\phi_{ij}^\dagger} c_{i,s}^\dagger c_{j,s} + \text{H.c.}) P \\ &+ J \sum_{\langle ij \rangle} \left(\sum_{\alpha=x,y,z} S_i^\alpha S_j^\alpha - \frac{1}{4} n_i n_j \right) \\ &- \mu \sum_i (n_{i\uparrow} + n_{i\downarrow}). \end{aligned} \quad (\text{B1})$$

APPENDIX C: STABILITY OF THE DIRAC NODES

In the main text, we show that the Chern number has two kinds of transitions while varying the parameters. Here we demonstrate that the transitions will remain the same type under perturbations respecting C_3 symmetry. This fact can be verified by calculating the number of Dirac nodes at the transition point. The Dirac node corresponds to the point in the first Brillouin zone where $\Delta(\mathbf{k})$ and $\epsilon(\mathbf{k})$ vanish simultaneously. In our system, the C_3 symmetry in the Hamiltonian requires that $\Delta(C_3 \mathbf{k}) = \Delta(\mathbf{k})$ and $\epsilon(C_3 \mathbf{k}) = \epsilon(\mathbf{k})$. Therefore, at the transition points that the Chern number only change by 1, the Dirac node locates at Γ , κ or κ' . The superconducting

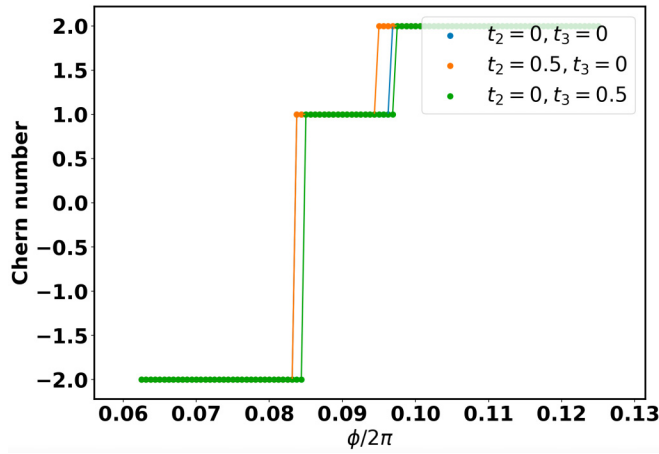


FIG. 8. Dependence of the Chern number on ϕ at $n = 0.85$.

order parameter in k space can be written as

$$\Delta(\mathbf{k}) = 2|\Delta_{ij}| \sum_{j=1,2,3} e^{i(2\pi/3)(j-1)} \cos(\mathbf{k} \cdot \mathbf{a}_j - \phi_{\text{SC}}), \quad (\text{C1})$$

where ϕ_{SC} is defined by the relation $\Delta_{i+\mathbf{a}_j,i} = e^{-i2\phi_{\text{SC}}} \Delta_{i-\mathbf{a}_j,i}$. The expression around Γ , κ , or κ' can be expanded as

$$\Delta(\mathbf{k}) = 3|\Delta_{ij}|\beta(k_x + ik_y), \quad (\text{C2})$$

where $\beta = \sin \phi_{\text{SC}}$, $\sin(\phi_{\text{SC}} + \frac{2\pi}{3})$, and $\sin(\phi_{\text{SC}} - \frac{2\pi}{3})$ corresponds to Γ , κ , and κ' , respectively. Any rotation around the z axis is equivalent to a U(1) gauge transform. The solutions of $\Delta(\mathbf{k}) = 0$ can only be shifted by adding a constant, which breaks the z -rotation symmetry. Therefore, we can conclude that the existence of the Dirac node at Γ , κ , or κ' is robust under perturbations respecting C_3 symmetry. This can be illustrated by considering the three-site term, Eq. (6), and

next-nearest-neighbor (NNN) hopping:

$$H_{t_2} = -t_2 \sum_{\langle\langle ij \rangle\rangle, s} P(c_{i,s}^\dagger c_{j,s} + \text{H.c.})P, \quad (\text{C3})$$

where $\langle\langle ij \rangle\rangle$ represents NNN sites. As shown in Fig. 8, the transition of the Chern number remains the same pattern.

APPENDIX D: SYMMETRY TRANSFORMATION ON THE ORDER PARAMETERS

We note that the order parameters in Eq. (3) have 12 degrees of freedom, in which 6 corresponds to the pairing in six bonds; they are $\Delta_{i\pm\mathbf{a}_j,i}$, $j = 1, 2, 3$, where $\mathbf{a}_1 = (1, 0)$, $\mathbf{a}_2 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$, $\mathbf{a}_3 = (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$, and $\mathbf{a}_{j+3} = \mathbf{a}_j$. The remaining $6 = 3 + 3$ corresponds to the hopping parameters in spin-up and spin-down channels; they are $\chi_{i+\mathbf{a}_j,i,\uparrow}$ and $\chi_{i+\mathbf{a}_j,i,\downarrow}$, $j = 1, 2, 3$. Under C_6 rotation, the order parameters transform as $\Delta_{i+\mathbf{a}_j,i} \rightarrow -\Delta_{i+\mathbf{a}_{j-1},i}$, $\chi_{i+\mathbf{a}_j,i,s} \rightarrow \chi_{i-\mathbf{a}_{j-1},i,-s}$. Under time-reversal transformation, the order parameters transform as $\Delta_{i\pm\mathbf{a}_j,i} \rightarrow -\Delta_{i\mp\mathbf{a}_j,i}^*$ and $\chi_{i\pm\mathbf{a}_j,i,s} \rightarrow \chi_{i\mp\mathbf{a}_j,i,-s}^*$. Under mirror reflection transformation about the x axis, the order parameters transform as $\Delta_{i\pm\mathbf{a}_j,i} \rightarrow \Delta_{i\pm\mathbf{a}_5-j,i}$ and $\chi_{i\pm\mathbf{a}_j,i,s} \rightarrow \chi_{i\pm\mathbf{a}_5-j,i,s}$. Under mirror reflection transformation about the y axis, the order parameters transform as $\Delta_{i\pm\mathbf{a}_j,i} \rightarrow -\Delta_{i\pm\mathbf{a}_5-j,i}$ and $\chi_{i\pm\mathbf{a}_j,i,s} \rightarrow \chi_{i\mp\mathbf{a}_5-j,i,-s}$.

From the above analysis, one can verify that under mirror time-reversal transformation about the x axis, the order parameters transform as $\Delta_{i\pm\mathbf{a}_j,i} \rightarrow -\Delta_{i\mp\mathbf{a}_5-j,i}^*$ and $\chi_{i\pm\mathbf{a}_j,i,s} \rightarrow \chi_{i\mp\mathbf{a}_5-j,i,-s}^*$. Under mirror time-reversal transformation about the y axis, the order parameters transform as $\Delta_{i\pm\mathbf{a}_j,i} \rightarrow \Delta_{i\mp\mathbf{a}_5-j,i}^*$ and $\chi_{i\pm\mathbf{a}_j,i,s} \rightarrow \chi_{i\mp\mathbf{a}_5-j,i,-s}^*$. The mean-field results give that the order parameters are gauge equivalent to the ones before the symmetry transformation.

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