Nascent vortices in current-carrying hybrid superconducting bridge

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We theoretically show that in a current-carrying hybrid superconductor (S)/normal metal (N) bridge there is a state with spatial oscillations of modulus of superconducting order parameter Δ along the bridge and *zero vorticity*. This stationary state is realized at large currents, when proximity-induced superconductivity in the N layer is suppressed. With an increase of the current the number of oscillations of Δ increases, which leads to oscillations of differential resistance or kinetic inductance of an SN bridge with normal or superconducting leads. At current the exceeding critical current I_c, this spatially oscillating state transforms to periodically in time moving the vortex chain across the SN bridge. Because of these properties, we call such a state a nascent vortex state.

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I. INTRODUCTION

A superconductor placed in a magnetic field or carrying transport current can be in either in a vortex (mixed) state or a vortex-free (Meissner) state. In the vortex-free state superconducting properties (for example magnetization, critical current, or resistance) vary monotonically with a change of the magnetic field. On the contrary, in the vortex state superconducting properties may change nonmonotonically as functions of the magnetic field or current. The most familiar example is the Little-Parks oscillations of the resistance of the hollow superconducting cylinder as a function of the magnetic field, which are connected with the change of the vorticity or the number of vortices trapped by the cylinder one by one [1]. Another example is the superconducting disk with the radius about of the superconducting coherence length ξ, where magnetization changes nonmonotonically with the magnetic field due to the discrete character of the vortex entry and exit [2]. In a narrow superconducting strip or a small size superconductor, entry of the vortex chain or vortex leads to nonmonotonic dependence of the resistance [3,4] or critical current [5–10] on the magnetic field.

Here we present the superconducting system, where despite *zero vorticity* there is nonmonotonic dependence of transport properties as function of the applied *current*. This is a hybrid superconductor (S)/normal metal (N) bridge connected with normal or superconducting leads (see Fig. 1). We find that at large currents, when proximity induced in the N layer superconducting order parameter $\Psi = \Delta e^{i\phi}$ is strongly suppressed, in the SN bridge there is a state with spatial oscillations of Δ along the bridge. With increasing of the current the number of oscillations increases and it resembles the increase of the number of vortices in an ordinary superconducting bridge or strip with an increase of the external magnetic field. However, the vorticity $N = \oint \nabla\phi dl/2\pi$ in the SN bridge is equal to zero up to currents close to the critical current I_c. Moreover, the change of the number of oscillations of Δ leads to oscillations of differential resistance or kinetic inductance of the SN bridge connected with normal or superconducting leads. At current $I \gtrsim I_c$ this state transforms to periodically in time moving the vortex chain across the SN bridge with the number of vortices equal to the number of oscillations of Δ [the number of minima in dependence $\Delta(x)$] at $I \lesssim I_c$. For this state we adopt the name nascent vortex/vortices state, which was used previously for physically similar objects which presumably may exist in a mixed state of type II superconducting slab near its surface [11]. The main difference between a nascent vortex and an ordinary vortex is in zero vorticity, leading to the absence of the point where $\Delta \to 0$ (in the center of the vortex core) and circulating currents around this point. A nascent vortex has elongated along the thickness of the SN bridge "core," which is a region with locally smaller Δ.

The structure of our paper as follows. In Sec. II we present our model. In Secs. III and IV we show our results for the SN bridge with normal and superconducting leads, respectively. In Sec. V we discuss our result and its relation with previous works, and in Sec. VI we present our conclusions.

II. MODEL

To calculate superconducting and transport properties of the SN bridge either with normal or superconducting leads, we use the two-dimensional time-dependent Ginzburg-Landau equation for the superconducting order parameter $\Psi = \Delta e^{i\phi}$,

$$\frac{\pi\hbar}{8k_BT}\left(\frac{\partial}{\partial t}+i\frac{2e\varphi}{\hbar}\right)\Psi = \frac{\pi\hbar D_{\rm S,N}}{8k_BT}\left(\nabla+i\frac{2\pi A}{\Phi_0}\right)^2\Psi+\left(1-\frac{T}{T_{\rm cS,cN}}-\frac{\Delta^2}{\Delta_{\rm GL}^2}\right)\Psi, \tag{1}$$

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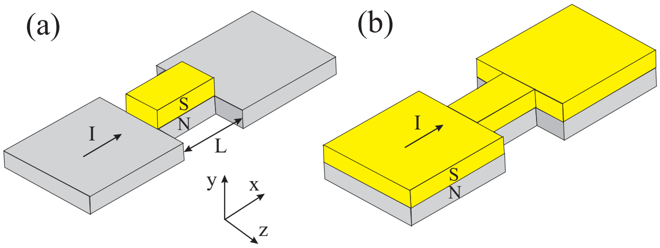

FIG. 1. (a) SN bridge with normal leads. (b) SN bridge with superconducting leads.

and the current conservation law

$$\mathrm{div}\, j = \mathrm{div}\,(j_s + j_n) = \mathrm{div}\left(-\frac{\sigma_{S,N}}{e}\frac{\pi|\Delta|^2 q}{4k_B T} - \sigma_{S,N}\nabla\varphi\right) = 0, \quad (2)$$

where j_s is a superconducting current density and j_n is a normal current density, $D_{S,N}$ is a diffusion coefficient in S and N layers, respectively, $\sigma_{S,N} = 2e^2 D_{S,N} N(0)$ is a conductivity in the corresponding layer [$N(0)$ is a density of states of electrons on the Fermi level, which is chosen to be the same in S and N layers to reduce the number of free parameters], $\Delta_{GL} = 3.06 k_B T_{cS,cN}$, $q = (\nabla\phi + 2\pi A/\Phi_0)$ ($\Phi_0 = \pi\hbar c/|e|$ is a magnetic flux quantum), A is a vector potential (we put $A = 0$ for the SN bridge with normal leads), and φ is a electrostatic potential. The N layer is modeled as a superconductor, having critical temperature $T_{cN} < T_{cS}$ while temperature is chosen from the interval $T_{cN} < T < T_{cS}$. We are interested in stationary solution of Eq. (1), when $\partial\Delta/\partial t \to 0$. On the SN interface we use the boundary conditions $D_S d\Psi/dy = D_N d\Psi/dy$ and $D_S d\varphi/dy = D_N d\varphi/dy$, and on boundaries with vacuum we use $d\Psi/dn = 0$ and $d\varphi/dn = 0$. In the place of contact of the SN bridge with normal leads, we use the following boundary conditions: $\Psi = 0$ and $d\varphi/dx = -j/\sigma_N$, where j is the applied transport current density.

In the case of the SN bridge with superconducting leads there is a problem with "injection" of supercurrent in the bridge. We solve it by using periodical boundary conditions in the x direction: $\Psi(x = -L/2) = \Psi(x = L/2)$, and we choose $\varphi = 0$ because in this situation there is no normal current in the superconducting state. A nonzero superconducting current is controlled by the value of the spatially independent vector potential $\mathbf{A} = (A_x, 0, 0)$—we vary A_x from zero up to the maximal value at which the supercurrent in the bridge reaches I_c. A locally larger value of Δ near the ends of the SN bridge (due to locally smaller j_s in the leads) we model by locally larger T_{cN} in the finite region near the bridge ends located at $x = \pm L/2$.

In our model we assume no dependence on transverse coordinates (z direction in Fig. 1) and we solve two-dimensional equations (along the x and y directions in Fig. 1). The current is normalized in units of depairing current I_{dep} of a single S layer with thickness d_S, length is in units of $\xi_c = (\hbar D_S/k_B T_{cS})^{1/2}$, and voltage is in units of $V_0 = k_B T_{cS}/|e|$.

III. SN BRIDGE WITH NORMAL LEADS

In Fig. 2(a) we present the spatial dependence of Δ along the SN bridge having normal leads on the boundary of the N layer with vacuum, calculated at different currents (parameters of the SN bridge as follows: $D_N/D_S = 50$, $T/T_{cS} = 0.7$, $T/T_{cN} = 1.2$, $d_S = d_N = 5\xi_c$, and $L = 120\xi_c$). When the current exceeds some value (about $0.4I_{dep}$ at chosen parameters), the superconductivity in the N layer becomes strongly suppressed due to the depairing effect of the supervelocity $v \sim q$, which is stronger in the N layer than in the S layer due to large differences in diffusion coefficients. It is accompanied by the appearance of spatial oscillations of Δ along the bridge [see Figs. 2(a) and 2(b)]. The number of minima of $\Delta(x)$ increases with increasing of the current and it resembles the increase of the number of vortices in the ordinary superconducting strip or bridge with an increase of the external magnetic field.

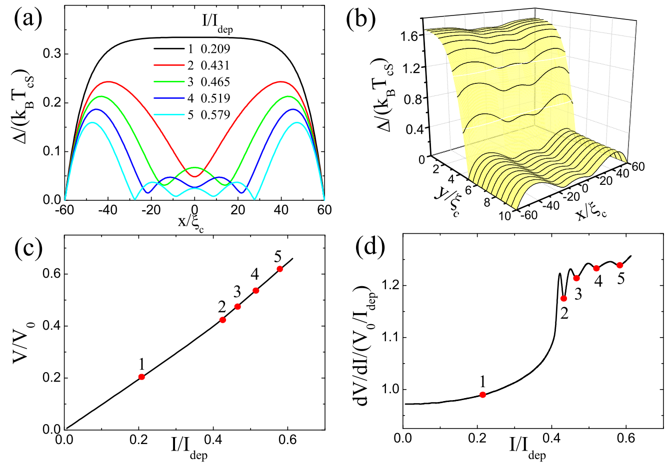

FIG. 2. (a) Current-dependent spatial variation of Δ along the SN bridge with normal leads on the boundary of the N layer with vacuum. (b) Example of variation of Δ across thickness and length of the SN bridge ($I = 0.465I_{dep}$). (c) Voltage-current characteristic of the SN bridge up to $I = I_c$. Resistance is finite due to penetration of the electric field to the bridge from normal leads. (d) Dependence of differential resistance of the SN bridge on the current.

Change of $\Delta(x, y)$ influences the voltage response [see Fig. 2(c)], which is well visible in the dependence of the differential resistance on the current [see Fig. 2(d)]. For our parameters, the electric field penetrates on the finite length L_E to the bridge (it changes from $L_E \sim 13\xi_c$ at small currents up to $L_E \sim 17\xi_c$ at $I = I_c$), which provides its finite resistance. We find that every oscillation of the dependence $dV/dI(I)$ is connected with the appearance of additional minima of the dependence $\Delta(x)$ and it looks similar to Little-Parks oscillations of resistance of the hollow superconducting cylinder with the change of the number of vortices. At $I > I_c$ in-plane vortices enter the SN bridge from the side of the N layer via the local minima of $\Delta(x)$ and pass through the bridge [their number is equal to the number of minima of $\Delta(x)$].

All these results allow us to call the found spatially oscillating state a nascent vortex state where the number of minima of $\Delta(x)$ corresponds to the number of nascent vortices. In comparison with the ordinary vortex, the nascent vortex has vorticity $N = 0$, finite Δ, and is elongated along the thickness of the SN bridge core [region with locally smaller Δ—see Fig. 2(b)].

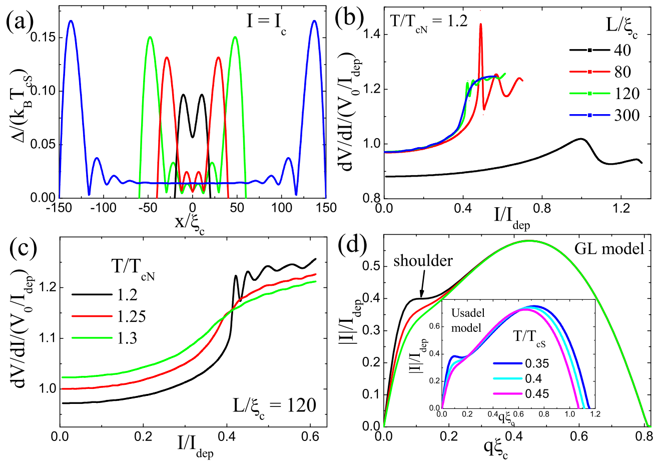

FIG. 3. (a) Spatial variation of Δ along the SN bridge with normal leads on the boundary of the N layer with vacuum, shown for bridges with different lengths (other parameters are the same as those in Fig. 2) at $I = I_c$. (b) Dependence of differential resistance of the same bridges on current. (c) Dependence of differential resistance on current for the SN bridge with a length of $L = 120\xi_c$ at different T_{cN} values and fixed $T = 0.7T_{cS}$. (d) Dependence of supercurrent on supervelocity calculated for the SN bilayer in the framework of the one-dimensional Ginzburg-Landau model at the same T/T_{cN} as in panel (c). In the inset we show the dependence $I(q)$ calculated in the framework of the one-dimensional Usadel model at fixed $T_{cN} = 0.15T_{cS}$ and different T values (ratio D_N/D_S, d_N, and d_S are the same as those in the GL and Usadel models; the numerical method and the Usadel equation are presented in Ref. [12]).

We should note that at $I > 0.59I_{dep} \sim I_c$ in the SN bridge there are two ordinary in-plane vortices which are located nearest to the ends of the bridge minima of $\Delta(x)$. Each of them has vorticity $N = 1$ and a core with $\Delta \to 0$ in one point. Their appearance does not add new features to the dependence $dV/dI(I)$, and their entry and exit to/from the SN bridge with the change of the current are reversible, also like the entry or exit of nascent vortices. Our calculations show that their existence is not a universal property, and they appear in the case of a relatively large proximity-induced Δ in the N layer, a large ratio $D_N/D_S \gg 1$, and currents close to I_c.

Figures 3(a) and 3(b) demonstrate the dependence of the effect on the length of the bridge. First of all one can notice the edge character of the found effect—with an increase of L the amplitudes of oscillations rapidly decay far from the bridge ends. It seems that larger Δ near the bridge ends plays a decisive role in the appearance of a nascent vortex (Δ is larger near the bridge ends because the current is mainly a normal one there and the superconducting current is small). In Sec. IV we demonstrate the importance of the gradient of Δ along the bridge for the appearance of this spatially oscillating state.

From Figs. 3(a) and 3(b) it is also seen that with increasing of L nascent vortices have smaller influence on the differential resistance (the amplitude and the period of oscillation of dV/dI decrease) despite their presence. This behavior resembles ordinary finite-size superconductors (rings, disks, and narrow strips) where the amplitude and the period of oscillations of resistance, critical current, or magnetization as functions of the magnetic field go down with an increase of the sample size.

We also have studied how the observed effect depends on the strength of the proximity-induced Δ in the N layer. With an increasing ratio T/T_{cN} (i.e., with decreasing T_{cN} at fixed T), Δ in the N layer decreases and it diminishes its influence on transport properties of the SN bridge. As a result, despite the presence of the spatially oscillating state, it becomes much less "visible" in dependence $dV/dI(I)$ [see Fig. 3(c)].

In general, we find that nascent vortices have noticeable impact on the dependence $dV/dI(I)$ for short SN bridges with a length of about several periods of oscillations of Δ [one period equal to the size of the nascent vortex core is roughly $\sim\xi_N \sim (\hbar D_N/k_B T)^{1/2}$]. Besides the SN bilayer should have such a parameter when on the dependence $I(q)$ there is a "shoulder" [see Fig. 3(d)]. Its presence indicates the considerable contribution of the N layer to transport properties of the SN bilayer at small currents and the suppression of superconductivity in the N layer before it becomes suppressed in the S layer. In the inset in Fig. 3(d) we show the dependence $I(q)$ calculated in the framework of the Usadel model at different temperatures and fixed T_{cN}. This result demonstrates that similar $I(q)$ follows from microscopic theory and in the experiment one may vary temperature to find the proper $I(q)$ where the effect of nascent vortices is expected to be the strongest one.

Nascent vortices are absent or exist in a narrow current interval when the ratio $D_N/D_S \lesssim 10$. It occurs due to the following reasons. At first, critical supervelocities in S and N layers occur close to each other and close to the supervelocity corresponding to maxima in the dependence $I(q)$. It considerably decreases the interval of currents where such a state can exist. Second, in this case there is a smaller gradient of Δ near the SN interface and a larger value of Δ in the N layer than in the SN bridge with $D_N/D_S \gg 10$. It decreases the vortex pinning on the SN interface. Indeed, the vortex energy $E_v \sim \Phi_0^2/\lambda^2 \sim \Delta/D$ and it is larger in the S layer than in the N layer because of the much smaller Δ in the N layer, but this difference decreases with the decreasing ratio D_N/D_S. We find that when $D_N/D_S \lesssim 10$ the vortex motion starts immediately after suppression of the proximity-induced superconductivity in the N layer. Phase-slip centers and lines in our system are not realized due to the large gradient of Δ over the bridge thickness.

IV. SN BRIDGE WITH SUPERCONDUCTING LEADS

Similar nascent vortices do exist in the SN bridge with superconducting leads at large currents [see Fig. 4(a)]. We choose the same parameters as for the SN bridge in Fig. 2 and put $T/T_{cN} = 0.8$ in the N layer on the distance $5\xi_c$ near each bridge end to model larger values of Δ in the superconducting leads (see discussion of the model in Sec. II). Here the change of the number of nascent vortices with current results in oscillations of kinetic inductance of the SN bridge, $L_k(I) \sim d\bar{q}/dI$ [see Fig. 4(b)]. Similarly to the SN bridge with normal leads, here also there are ordinary in-plane vortices at $I \simeq I_c$. In the current interval $0.504 < I/I_{dep} < 0.563$ there are two vortices located in the minima of $\Delta(x)$, nearest to the bridge ends. By approaching to I_c, an additional pair of

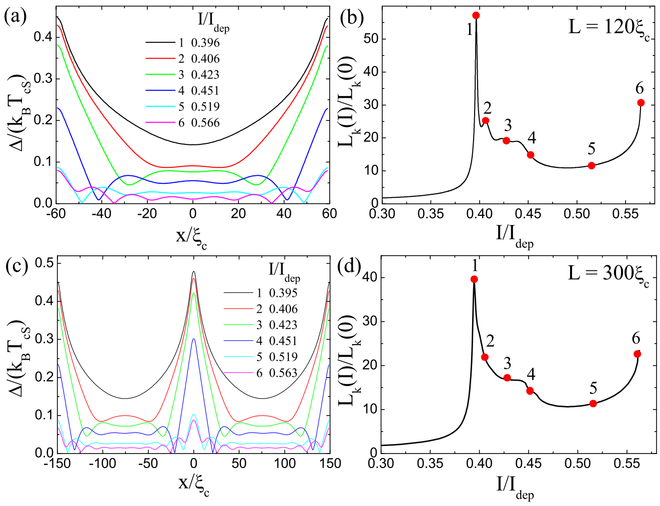

FIG. 4. (a) Current-dependent spatial variation of Δ along the SN bridge with superconducting leads on the boundary of the N layer with vacuum (bridge parameters are the same as those in Fig. 2, $L = 120\xi_c$). (b) Dependence of the bridge kinetic inductance $L_k \sim d\bar{q}/dI$ ($\bar{q}$ is an average along the bridge's supervelocity) on current. The peak at $I \simeq 0.396 I_{\rm dep}$ (symbol 1) is related to the suppression of superconductivity in the N layer. Small peaks at larger currents are related to the change of the number of nascent vortices. At $I \to 0.565 I_{\rm dep} = I_c$ (symbol 6), there is an increase of L_k due to the suppression of superconductivity in the S layer. In panels (c) and (d), we show similar results for the SN bridge with $L = 300\xi_c$ and additionally enhanced superconductivity (realized by the increase of $T_{\rm cN}$) in the middle of the bridge center.

in-plane vortices enter the bridge in the second from its end's minima of $\Delta(x)$.

Note that for the *homogenous* (without local variation of $T_{\rm cN}$) SN bridge the state with spatial oscillations of Δ does not exist. Up to $I = I_c$, corresponding to the maximal value of the current on the dependence $I(q)$, $\Delta(x) = $ const, and at $I > I_c$ the chain of in-plane vortices enters the SN bridge from the side of the N layer with an intervortex distance close to the period of the spatially oscillating state for the inhomogenous bridge and passes through it. This result confirms the need for an intrinsic gradient of Δ along the bridge to have nascent vortices. To prove it, in addition to enhancement of $T_{\rm cN}$ at the ends of the bridge we also enhance locally $T_{\rm cN}$ in the middle of the bridge. Indeed, we observe oscillation of Δ both near the ends and in the middle of the bridge [see Fig. 4(c)]. Moreover a qualitatively similar result is obtained in the case of a local *decrease* of $T_{\rm cN}$ (results are not presented here). Visually oscillations of Δ near such a "defect" are reminiscent of Friedel oscillations of electron density near a charged defect [13]; however, in our SN bridge they are not connected with interference of the electron wave function.

V. DISCUSSION

Somewhere a similar spatially oscillating state, which is a chain of nuclei for following the vortex chain entry, arises in the problem with the stability of the vortex-free state for the superconducting slab placed in parallel to its surface magnetic field [14–16] or current-carrying superconducting strip [17]. This state appears at the field/current *above* the critical field/current (H_s/I_s) when the edge barrier for the vortex entry is suppressed. Its period depends on the magnetic field/current and the vorticity is equal to zero. But it is a transient time-dependent state between the Meissner and mixed states and it evolves to a vortex chain entering the superconductor. In Ref. [11] it was supposed that such a state may exist in the *mixed* state of the superconducting slab near its surface, it could be *stationary,* and it was named a nascent vortex state. It was phenomenologically introduced to explain the results of the experiment [11]. However, as far as we know there are no calculations which would confirm its existence in that system.

The nucleus for vortex entry with zero vorticity exists at the field/current *less* than H_s/I_s and it corresponds to the saddle-point state of the system. It is a region with locally suppressed Δ near the edge of the superconducting disk placed in the out-of-plane magnetic field [18,19] or at the edge of the current-currying strip [20,21]. Note that only one such a nucleus may simultaneously exist because the probability for its appearance depends exponentially on its energy. It is also a transient state and its evolution in time leads to vortex entry to the superconductor. Only in the special case of a small size superconducting ring with constriction may such a nucleus be stabilized in time as has been shown experimentally in Ref. [22] and it also may be called a nascent vortex. This result can be interpreted as stabilization in time of the phase-slip nucleus from the well-known problem about the saddle-point state in current-carrying superconducting wire considered by Langer and Ambegaokar in 1967 [23].

It occurs that in our SN bridge it is possible to stabilize in time a similar nucleus of the vortex (chain of vortex nuclei). At large current, superconductivity in the N layer is suppressed and it behaves as a weak place through which vortices may preferably enter the SN bridge. But a large difference in Δ in the S and N layers, following from a large difference in D, does not allow vortices to enter and/or pass across the SN bridge, leading to the appearance of nascent vortices. The gradient of Δ *along* the bridge provides nonequivalence for vortex entry points, and nascent vortices appear one by one in finite intervals of currents, which can be seen via their impact on differential resistance or kinetic inductance. This longitudinal gradient is an important factor for realization of nascent vortices because in its absent they do not exist, as our results show for a uniform SN bridge with periodical boundary conditions. At $I > I_c$ the nascent vortices develop to ordinary vortices moving across the SN bridge from the N layer to the S layer.

In a uniform superconducting slab in the mixed state, there is also a gradient of Δ along the slab surface due to interior vortices. However, there should still be a large gradient of Δ inside the superconductor near its surface like in the SN bilayer with different $D_{\rm S,N}$ and $T_{\rm cS,cN}$ values. The absence of such a gradient for an intrinsically uniform superconducting slab makes doubtful the realization of stationary nascent vortices in this system.

Experimental verification of the predicted phenomena requires the use of extremely dirty superconductors like NbN, MoSi, and NbTiN with large resistivity in the normal state of $\rho \sim 100$–$200\ \mu\Omega$ cm, the diffusion coefficient $D \simeq 0.5$

cm^2/s, and $T_c \lesssim 10$ K. The N layer may be made of a low-temperature superconductor like Al ($T_c \sim 1.3$ K, $\rho(4K) \simeq$ 3–4 μΩ cm) or normal metals like Au and Cu having low resistivity. In the last case, it is possible to find parameters (d_S, d_N, temperature) when the dependence $I(q)$ has the needed shape with a "shoulder". The optimal length of the SN bridge, when nascent vortices are well visible on the dependence $dV/dI(I)$ or $L_k(I)$ is $L \lesssim 100\xi_c \sim 1$ μm for the abovementioned superconductors. Besides of transport measurements one may use a scanning tunnel microscope to measure locally the spatial oscillations of the density of states along the SN bridge which have periods of several dozens of nanometers. In this case, there is no upper limit for the length of the bridge, and one can observe these oscillations near bridge ends. In measurements one may tune the temperature to find the dependence $L_k(I)$ with a peak at $I < I_c$ [symbol 1 in Fig. 4(b)], which is a fingerprint of the needed $I(q)$.

VI. CONCLUSION

We theoretically find that in a current-carrying hybrid superconductor (S)/normal metal (N) bridge there is a stationary state with spatial oscillations of the modulus of the superconducting order parameter Δ along the bridge and *zero vorticity*—a nascent vortex state. It is realized at large currents, when proximity-induced superconductivity in the N layer is suppressed. To have a nascent vortex state one needs gradients of superconducting properties (Δ) both along and inside of the superconducting system. In the SN bridge this is provided by the presence of normal or superconducting leads, a large difference in diffusion coefficients $D_{S,N}$, and different critical temperatures of the S and N layers. For a relatively short SN bridge made of the SN bilayer having a "shoulder" on its current-velocity dependence, the change of the number of nascent vortices is accompanied by noticeable oscillations of differential resistance or kinetic inductance with a change of the current. Above critical current, stationary nascent vortices transform to moving ordinary vortices.

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