

Topologically protected generation of spatiotemporal optical vortices with nonlocal spatial mirror symmetry breaking metasurface

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Recently, nonlocal spatial mirror symmetry breaking metasurfaces have been proposed to generate spatiotemporal optical vortices (STOVs), which carry transverse orbital angular momenta. Here, we investigate the topological property of the STOV generator and show that spatial mirror symmetry breaking introduces a synthetic parameter dimension associated with the metasurface geometry. Furthermore, we demonstrate that there are well-defined vortices emerging with a synthetic parameter dimension, which can topologically protect the STOV generation robustly against structural perturbations and disorders. Since the vortices are always “created” or “annihilated” together in pairs of opposite charges in the k_x - ω domain, the total topological charge of these vortices is a conserved quantity. Our studies not only provide a topological perspective for STOV generation but also lay a solid foundation for potential applications of STOV metasurfaces integrated with other optoelectronic devices, because of their robust immunity to fabrication defects.

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I. INTRODUCTION

Since the intriguing recognition that optical phase singularity [1] can lead to longitudinal orbital angular momentum (OAM) [2–4], the studies of optical field singularities have been broadly extended over spatial dimensions. Recently, intense interest has been attracted to explore optical vortices in the spatiotemporal domain [5–26]. In particular, spatiotemporal optical vortices (STOVs), as an optical pulse, can be tilted with respect to the propagation direction and exhibit transverse components of OAM unique in the spatiotemporal domain [7–9]. With such transverse OAM, all the family members of the optical angular momentum are united from both spin and orbital perspectives and with both transverse and longitudinal components.

Although transverse OAM offers a new degree of freedom and potential advances, it is a challenge to generate STOVs due to the requirement of neat optical manipulations associated with both spatial and temporal domains, simultaneously. By using pulse shapers, the spatiotemporal control method was first experimentally demonstrated to generate STOVs in free space with bulk optical systems [10–17]. Alternatively, much more compact than pulse shapers, nonlocal metasurfaces were proposed to create phase singularities in the spatiotemporal domain by breaking spatial mirror symmetry [18,19]. Moreover, we showed that the phase singularity can be used to realize differentiation computation in the spatiotemporal domain [19]. In analogy to the image processing of edge detection in the spatial domain [27–39], we proposed spatiotemporal differentiation with the generated STOVs to

detect the sharp changes of pulse envelopes, in both spatial and temporal dimensions [19].

In this paper, we move a step further to investigate the topological properties of STOV metasurfaces. We show that spatial mirror symmetry breaking introduces a synthetic parameter dimension [40] associated with the metasurface geometry. More importantly, STOV generation can be topologically protected by a vortex configuration emerging with the synthetic parameter dimension, which provides robustness against structural perturbations and disorders. In fact, fabrication tolerance is of great importance for potential applications when the STOV devices are integrated with other optoelectronic devices. Originally, such robust topological protection was discovered in solid-state electronic systems [41–46] and has been extended to photonics [47–50]. In particular, nontrivial photonic edge states for photonic crystals were greatly developed for robust light transmission [51–68]. As a key area of modern optical research, topological photonics fundamentally deepens the understanding of light interference, diffraction, and scattering. Moreover, topological protections have been explored for high-performance devices against fabrication defects, such as topological delay lines [55], robust lasing [69,70], infinite Q -factor resonators with bound states in a continuum [71,72], polarization conversion [73,74], and spatial field transformation for analog computing [37,75]. In this context, our studies not only provide a topological perspective for STOV generation, but also lay a solid foundation for STOV metasurfaces, because of their robust immunity to perturbations and disorders. Since the synthetic parameter dimensions can be easily extended to high dimensions by including more geometry parameters, our studies also indicate that STOV metasurfaces are potentially favorable platforms to explore high-dimensional topological physics.

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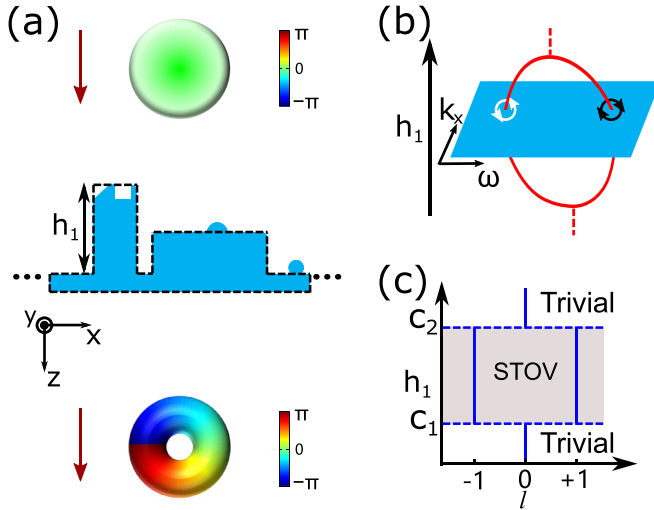


FIG. 1. (a) Schematic of topological protection for STOV generation with the metasurface, a one-dimensional periodic grating (the dashed line) perturbed with random defects along x with spatial mirror symmetry breaking. The detailed geometry of the metasurface is described in Sec. I of the Supplemental Material [76]. The transmitted pulse corresponds to a STOV carrying a transverse orbital angular momentum for an incident pulse with both Gaussian envelopes in the spatial and temporal domains. (b) A closed path nodal line in the three-dimensional synthetic parameter space of k_x , ω , and h_1 . The white and black circles in the cross-section k_x - ω plane indicate the opposite topological charges, and the total topological charge of these vortices is a conserved quantity. (c) Nature of the topological protection for STOV generation: The topological phase transition emerges from the trivial cases with the topological charge $l = 0$ to the nontrivial STOV one, where c_1 and c_2 correspond to the critical values of h_1 .

II. PHASE TRANSITION WITH SPATIAL MIRROR SYMMETRY BREAKING

In order to elucidate the topological protection, we begin by briefly discussing the physical motivation for breaking the spatial mirror symmetry for STOV generation [19]. We first consider a metasurface consisting of two rods in each period etched on a silicon substrate, with a one-dimensional grating structure along x [Fig. 1(a)]. We suppose that the metasurface has a spatial mirror symmetry, so when a pulse with both Gaussian envelopes in the spatial and temporal domains impinges on the structure at normal incidence, due to the mirror symmetry, the phase distribution of the transmitted pulse must also be symmetric about the mirror plane. As a result, there is no phase singularity in the spatiotemporal dimension. Therefore, in order to generate STOVs at normal incidence, it is necessary to break the mirror symmetry of the structure to achieve asymmetric phase modulation and create a phase singularity. In fact, the spatial mirror symmetry breaking can be extended to more than one plane, e.g., by designing a two-dimensional asymmetric grating.

In this way, we next investigate the impact of spatial symmetry breaking, by constructing a synthetic dimension with the geometry parameter h_1 , the height of the left rod in the unit cell. We analyze the phase modulation of the

metasurface by considering an incident plane wave polarized in the y direction $E_{\text{in}} = \tilde{s}_{\text{in}} \exp(ik_x x + ik_z z - i\omega t)$, where \tilde{s}_{in} is the incident amplitude, and k_x and k_z are the wave-vector components parallel and perpendicular to the structure interface, respectively. Due to the phase match condition for grating diffraction, in order to simplify the analysis, here we consider the case where only the zeroth-order diffraction light is transmitted with the same wave-vector component k_x , $E_{\text{trans}} = \tilde{s}_{\text{trans}} \exp(ik_x x + ik_z z - i\omega t)$, where \tilde{s}_{trans} is the transmitted amplitude. Therefore, the spatiotemporal transformation by the metasurface is defined by a transmission spectrum function $T(k_x, \omega) = \tilde{s}_{\text{trans}}(k_x, \omega)/\tilde{s}_{\text{in}}(k_x, \omega)$.

We expect that due to spatial symmetry breaking, the phase singularities can be illustrated by the phase distribution of $T(k_x, \omega)$. Indeed, Figs. 2(a)–2(e) show the phase distributions of the transmission spectrum functions T by gradually increasing h_1 , which exhibit different topological features. The detailed geometry of the metasurface is described in Sec. I of the Supplemental Material [76]. We numerically calculate the transmission spectrum function T by the finite-element method using the commercial full-wave software package COMSOL. In the simulation, the material dispersion has been taken into account, and the optical constants of silicon are as referred to in the experimental data of Ref. [77]. For $h_1 < 230$ nm, there is no phase singularity in the transmission spectrum function [Fig. 2(a)], which corresponds to the trivial case. However, due to spatial mirror symmetry breaking, for h_1 around 238.5 nm, there is a topological phase transition from the trivial case to the topological STOV one. For example, when $h_1 = 240$ nm, Fig. 2(b) shows the two phase singularities are with the topological charges $l = \pm 1$ at $\omega = 0.8641 \times 2\pi c/p$ and $\omega = 0.8678 \times 2\pi c/p$. By further increasing h_1 over 388.7 nm, the phase singularities disappear because the impact of the spatial symmetry breaking becomes weak, as the left rod is much taller than the other and dominates the symmetric property of the metasurface. We note that the vortices with opposite handedness are the fundamental “charges” in the k_x - ω dimensions. Since they are always “created” or “annihilated” together in pairs of opposite charges, we identify that the total topological charge of these vortices is a conserved quantity. The change of the phase distribution demonstrates that spatial symmetry breaking leads to the transition from the trivial cases to the topological STOV one in the k_x - ω domain [Fig. 1(c)].

In order to illustrate the topological phase transition more clearly, Fig. 2(f) shows the nodal lines in the synthetic dimension with the geometry parameter h_1 . These nodal lines are the intersection of $\text{Re}(T) = 0$ and $\text{Im}(T) = 0$ (cf. Sec. II in the Supplemental Material [76]). More importantly, the topological structure is similar to the optical vortices of the scalar field in real space [1]. Therefore, when a plane with a fixed value h_1 has intersections with the nodal lines in the synthetic dimension, two vortices with opposite handedness appear in the cross-section plane [Fig. 1(b)]. In general, the geometry parameter h_1 as the order parameter has two critical values c_1 and c_2 , during the transition from the trivial cases to the topological STOV ones, and the nodal line with a ring structure is a topological feature as shown in Fig. 1(b).

We also note that due to the crossovers of nodal lines, for $h_1 > 248$ nm, the k_x - ω plane has two more interactions

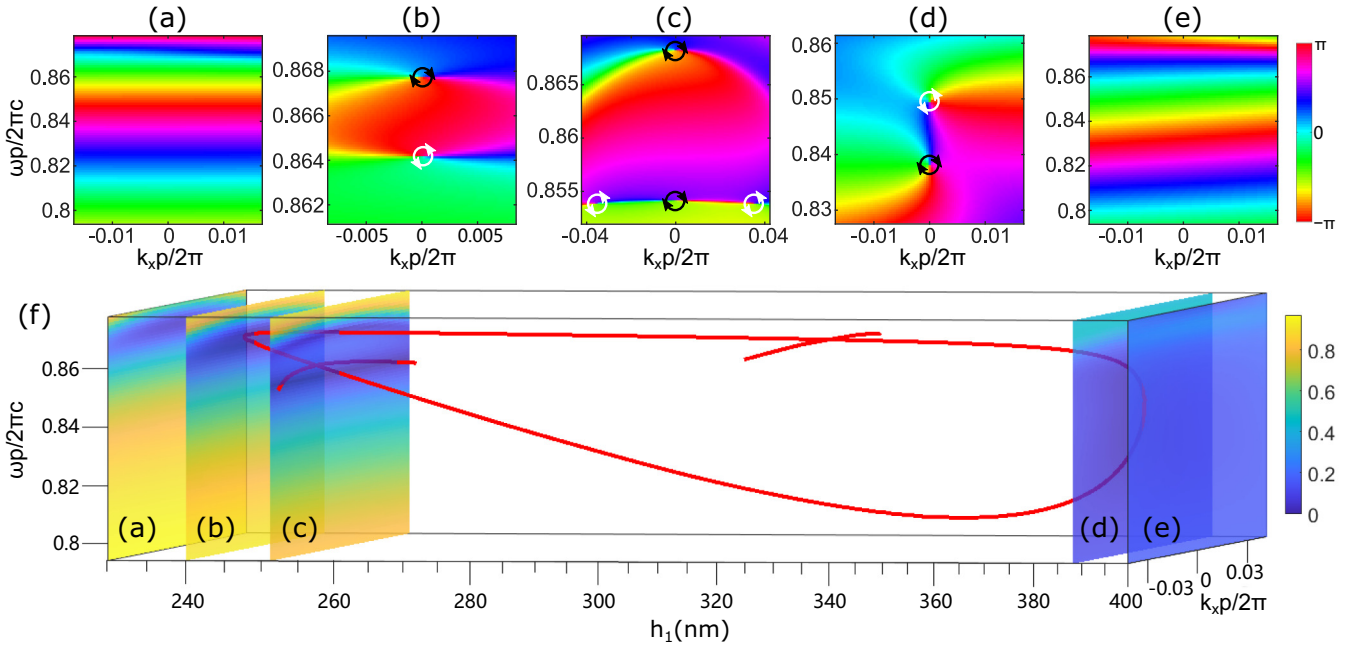


FIG. 2. [(a)–(e)] The phase distributions of transmission spectrum functions T at $h_1 = 230, 240, 251.3, 388.3,$ and 400 nm, respectively. (f) The nodal line in the three-dimensional synthetic parameter space of the structure. The insets are the amplitude distributions corresponding to (a)–(e).

with the nodal lines. For example, Fig. 2(c) shows the phase distribution of the transmission spectrum function for $h_1 = 251.3$ nm, where two additional vortices are created. Even when the nodal lines cross over, the total topological charges of these vortices are conserved [Fig. 2(c)]. Due to the conservation law and the additional vortices at $k_x \neq 0$ having the same handedness, Fig. 2(c) shows that the handedness of the vortex at $\omega = 0.854 \times 2\pi c/p$ and $k_x = 0$ has to change from $l = +1$ to $l = -1$ in comparison with Fig. 2(b). In the same situation, due to the crossover of nodal lines around $h_1 = 330$ nm, Fig. 2(d) shows that the vortex handedness at $\omega = 0.849 \times 2\pi c/p$ and $k_x = 0$ changes from $l = -1$ to $l = +1$. The details of the transmission spectrum functions near the crossovers of the nodal lines for $h_1 = 251.3$ nm and $h_1 = 324.5$ nm are shown in Fig. 3. Finally, the two vortices are annihilated as shown in Fig. 2(e) at $h_1 = 400$ nm.

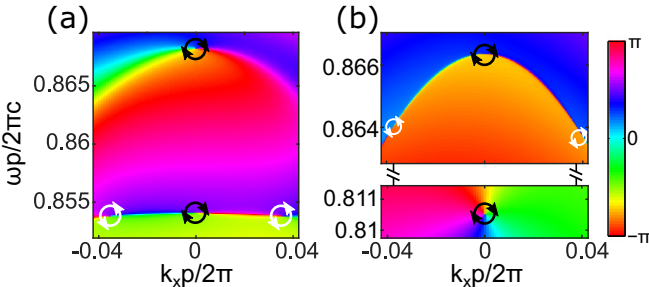


FIG. 3. [(a),(b)] The phase distributions of transmission spectrum functions T near the crossovers of the nodal lines for $h_1 = 251.3$ nm and $h_1 = 324.5$ nm, respectively.

III. TOPOLOGICALLY PROTECTED SPATIOTEMPORAL VORTEX GENERATION

Next we show that the topologically protected vortex in the k_x - ω domain can be transferred to the spatiotemporal domain and thus generates the topologically protected STOV. We suppose a vortex with charge number l is located at (ω_0, k_x^0) in the k_x - ω domain, so we consider an incident (transmitted) pulse with central frequency ω_0 and transverse wave-vector component k_x^0 as $E_{\text{in(trans)}}(x, t) = s_{\text{in(trans)}}(x, t) \exp(ik_x^0 x + ik_z^0 z - i\omega_0 t)$. Here, $s_{\text{in}}(x, t)$ and $s_{\text{trans}}(x, t)$ are the spatiotemporal envelope amplitudes of the incident and transmitted fields, respectively. In order to specifically depict the transformation between the incident and transmitted pulses while propagating through the STOV generator, we decompose the incident (transmitted) pulse envelope into a series of plane waves by Fourier transform $s_{\text{in(trans)}}(x, t) = \iint \tilde{s}_{\text{in(trans)}}(K_x, \Omega) \exp(iK_x x - i\Omega t) dK_x d\Omega$, where $K_x = k_x - k_x^0$ is the wave-vector component shifted along the x direction, and $\Omega = \omega - \omega_0$ is the sideband angular frequency from the center one ω_0 . In the polar coordinate, the expression is converted to $s_{\text{in(trans)}}(r, \theta) = \int_0^\infty \int_0^{2\pi} \tilde{s}_{\text{in(trans)}}(\rho, \chi) \exp[i\rho r \cos(\chi + \theta)] \rho d\rho d\chi$, where $r = \sqrt{x^2 + c^2 t^2}$, $\theta = \tan^{-1}(ct/x)$ and $\rho = \sqrt{K_x^2 + \Omega^2/c^2}$, $\chi = \tan^{-1}(\Omega/cK_x)$, and c is the group velocity of light. According to the restriction of the winding number around the vortex, within the small enough region, the amplitude of the transmission spectrum function can be written as $f(\rho)$, and without loss of generality T can be expressed as

$$T(\rho, \chi) = f(\rho)(Ae^{il\chi} + Be^{-il\chi}), \quad (1)$$

where l is the winding number of the vortex, and A and B are the two constant parameters, respectively. We note that when

the phase singularity is anisotropic, both A and B are nonzero. The overall topological charge is l if $|A|$ is larger than $|B|$, and $-l$ otherwise.

We suppose that an incident pulse with zero topological charge $\tilde{s}_{\text{in}}(\rho)$ impinges on the metasurface, so the transmitted pulse envelope can be expressed as

$$\begin{aligned} s_{\text{trans}}(r, \theta) &= \iint \tilde{s}_{\text{in}}(\rho) f(\rho) (Ae^{il\chi} + Be^{-il\chi}) \cdot e^{i(K_x x - \Omega t)} dK_x d\Omega \\ &= \int_0^\infty \tilde{s}_{\text{in}}(\rho) f(\rho) \rho \int_0^{2\pi} Ae^{il\chi} e^{i(\rho \cos \chi \cdot r \cos \theta - \rho \sin \chi \cdot r \sin \theta)} + Be^{-il\chi} e^{i(\rho \cos \chi \cdot r \cos \theta - \rho \sin \chi \cdot r \sin \theta)} d\chi d\rho \\ &= \int_0^\infty \tilde{s}_{\text{in}}(\rho) f(\rho) \rho \left[Ae^{-il\theta} \int_0^{2\pi} e^{i[-l(-\chi-\theta)+\rho r \cos(\chi+\theta)]} d\chi + Be^{il\theta} \int_0^{2\pi} e^{i[-l(\chi+\theta)+\rho r \cos(\chi+\theta)]} d\chi \right] d\rho. \end{aligned} \quad (2)$$

Taking $\eta = -\chi - \theta + \frac{\pi}{2}$ and $\tau = \chi + \theta + \frac{\pi}{2}$, then $d\eta = -d\chi$ and $d\tau = d\chi$, by periodicity, the integral of $d\chi$ becomes

$$\begin{aligned} & Ae^{-il\theta} \int_0^{2\pi} e^{i[-l(-\chi-\theta)+\rho r \cos(\chi+\theta)]} d\chi + Be^{il\theta} \int_0^{2\pi} e^{i[-l(\chi+\theta)+\rho r \cos(\chi+\theta)]} d\chi \\ &= -Ae^{-il\theta} \int_{-\theta+\frac{\pi}{2}}^{-2\pi-\theta+\frac{\pi}{2}} e^{i[-l(\eta-\frac{\pi}{2})+\rho r \cos(\frac{\pi}{2}-\eta)]} d\eta + Be^{il\theta} \int_{\theta+\frac{\pi}{2}}^{2\pi+\theta+\frac{\pi}{2}} e^{i[-l(\tau-\frac{\pi}{2})+\rho r \cos(\tau-\frac{\pi}{2})]} d\tau \\ &= -i^l Ae^{-il\theta} \int_0^{-2\pi} e^{i[-l\eta+\rho r \sin(\eta)]} d\eta + i^l Be^{il\theta} \int_0^{2\pi} e^{i[-l\tau+\rho r \sin(\tau)]} d\tau \\ &= i^l Ae^{-il\theta} \int_0^{2\pi} e^{i[-l\eta+\rho r \sin(\eta)]} d\eta + i^l Be^{il\theta} \int_0^{2\pi} e^{i[-l\tau+\rho r \sin(\tau)]} d\tau \\ &= 2\pi i^l (Ae^{-il\theta} + Be^{il\theta}) J_l(\rho r), \end{aligned} \quad (3)$$

where $J_l(\rho r) = \frac{1}{2\pi} \int_0^{2\pi} e^{i[-l\gamma+\rho r \sin(\gamma)]} d\gamma$ is the l th-order Bessel function of the first kind. Therefore, the transmitted envelope can be expressed as

$$s_{\text{trans}}(r, \theta) = 2\pi i^l (Ae^{-il\theta} + Be^{il\theta}) \int_0^\infty \tilde{s}_{\text{in}}(\rho) f(\rho) J_l(\rho r) \rho d\rho. \quad (4)$$

Clearly, Eq. (4) shows that the transmitted pulse has a vortex with a nonzero topological charge in the spatiotemporal domain, but with the opposite one in the k_x - ω domain.

Next we further show that STOV generation with the metasurface can be topologically protected by the vortices in the k_x - ω domain. When the vortices are distanced from the crossovers of nodal lines and the critical points around the topological phase transition, small changes to the metasurface geometry in real space can be treated as perturbations to the vortex at (ω_0, k_x^0) . The strength of the topological protection can be evaluated by the distance to its nearest vortex,

$$\Delta = \sqrt{(\omega_n - \omega_0)^2 + c^2(k_x^n - k_x^0)^2}, \quad (5)$$

where ω_n and k_x^n are the frequency and the wave-vector component of the nearest vortex. When Δ is large enough, the vortices with $l = \pm 1$ are stable due to topological protection from the perturbation: Since the continuous structural perturbation only continuously perturbs the contours $\text{Re}(T) = 0$ and $\text{Im}(T) = 0$, the intersection of the two contours still gives vortices in the k_x - ω domain. On the other hand, when the vortices are close to the crossovers of nodal lines or the critical points of the transitions, intrinsically, these vortices are vulnerable and cannot be robust to perturbations, and it is meaningless to define the protection strength for these vulnerable vortices.

To demonstrate the robust generation of STOV against structural perturbations and disorders, we randomly generate some defects shown in Fig. 4(a), where $h_1 = 340$ nm and the topological protection strength $\Delta = 0.059 \times 2\pi c/p$. The defects are randomly positioned, including partial convexities, concavities, and various kinds of chamfers (for the detailed geometry, cf. Sec. I of Supplemental Material [76]). We numerically calculate the amplitude and phase distributions of the transmission spectrum function T [Figs. 4(b) and 4(c)]

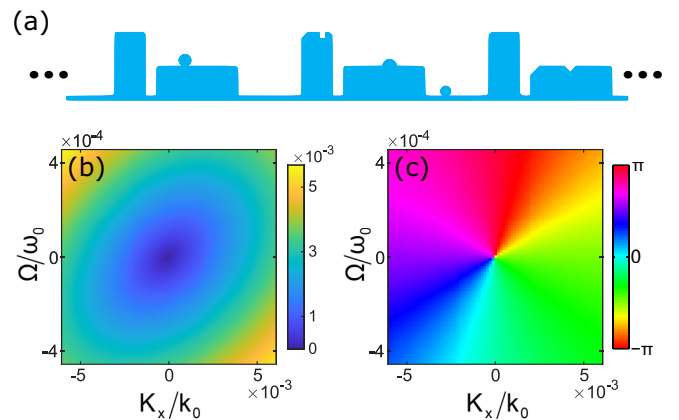


FIG. 4. The transmission spectrum function of the STOV generator with random defects. (a) The unit cell of the periodic STOV metasurface with random defects. (b) Amplitude and (c) phase distributions of the transmission spectrum function with respect to K_x and Ω , where the vortex in the k_x - ω domain is at $\omega_0 = 0.798 \times 2\pi c/p$ and $k_x^0 = -0.0288k_0$.

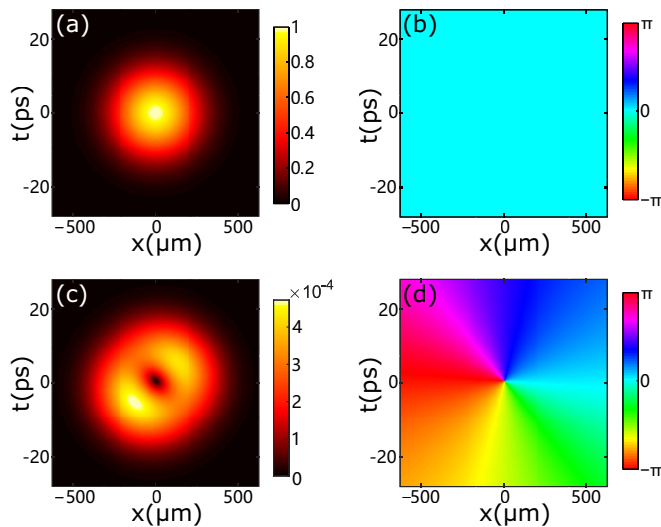


FIG. 5. Generation of a STOV by the metasurface with random defects for a Gaussian-enveloped incident pulse. (a) Amplitude and (b) phase distributions of obliquely incident pulse with Gaussian envelopes in both spatial and temporal domains. (c) Amplitude and (d) phase distributions of the transmitted one. The transmitted pulse with transverse OAM has a phase singularity, leading to the zero amplitude at the pulse center.

for the zeroth-order diffraction light. Indeed, even with such perturbations, the phase distribution of T exhibits that the vortex in the k_x - ω domain still survives, which leads to a zero amplitude at $\omega_0 = 0.798 \times 2\pi c/p$ and $k_x^0 = -0.0288k_0$, where $k_0 = \omega_0/c$. Clearly, it indicates the topologically protected robustness of the STOV metasurface.

Next we simulate an incident pulse with Gaussian envelopes in both spatial and temporal domains to demonstrate the STOV generation. Figures 5(a) and 5(b) show the amplitude and phase distributions of the incident pulse envelope,

where the beam waist and pulse width are $502 \mu\text{m}$ and 22ps within the ranges of spatial and temporal bandwidths shown in Fig. 4. After passing through the metasurface, we simulate the transmitted pulse by the Fourier transform method, with the transmission spectrum function in Fig. 4. The amplitude and phase distributions of the transmitted pulse envelope are shown in Figs. 5(c) and 5(d), respectively. Indeed, they show that with the topological protection, the transmitted pulse is still a STOV with zero amplitude at the pulse center [Fig. 5(c)] and has a phase singularity [Fig. 5(d)] in the spatiotemporal domain, which successfully immunizes against the defects in the metasurface.

IV. CONCLUSION

We investigate the topological property of a STOV metasurface in a synthetic parameter dimension associated with the metasurface geometry. Since STOV metasurfaces are much more compact than the pulse shapers, their robust immunity to fabrication defects paves the way forward to generating and modulating the transverse OAM with integrated devices. We also note that here the chosen geometry parameter h_1 is not particular, and there are plenty of the other geometry parameters which can realize the STOVs. Therefore, the nodal line discussed in the present paper is transformed to a hyperplane in high-dimensional synthetic space. It is intriguing to explore the topological structure in such a synthetic space, associated with frequency and wave vector. We believe that there are high-order vortices emerging in the synthetic parameter dimension, which could be important in the applications of STOVs.

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