# Quantum oscillations and transport evidence of topological bands in La<sub>3</sub>MgBi<sub>5</sub> single crystals

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In this work, we synthesized single crystals of  $La<sub>3</sub>MgBi<sub>5</sub>$  and systematically investigated their structure, magnetism, and transport properties. La<sub>3</sub>MgBi<sub>5</sub> belongs to the  $A_3MX_5$  family ( $A =$  Ca-Ba or rare earth,  $M =$ transition materials or  $Mg$ ,  $X =$  chalcogen or pnictogen) and has a quasi-one-dimensional structure. Our magnetic and magnetoresistance measurements show the presence of de Haas–van Alphen (dHvA) and Shubnikov–de Haas (SdH) oscillations. Analysis of these quantum oscillation spectra reveals two major oscillation frequencies  $(F_\alpha, F_\beta)$  with corresponding light effective mass and the nearly  $\pi$  Berry phase for  $F_\alpha$ , suggesting the nontrivial band topology in La<sub>3</sub>MgBi<sub>5</sub>, which is further supported by our first-principles calculations. Besides, La<sub>3</sub>MgBi<sub>5</sub> manifests a large nonsaturating magnetoresistance and a nonlinear Hall effect at low temperatures. Our results show that La<sub>3</sub>MgBi<sub>5</sub> is a topological nontrivial material and a member of the  $A_3MX_5$  family exhibiting topological properties, providing a platform for further investigation of topological fermionic properties in this family.

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#### **I. INTRODUCTION**

Topological materials, such as Dirac semimetals and Weyl semimetals, have attracted much attention due to their exotic properties and potential applications. Three-dimensional (3D) Dirac semimetals were predicted and discovered in Na<sub>3</sub>Bi and  $Cd<sub>3</sub>As<sub>2</sub>$  [\[1–4\]](#page-6-0). Subsequently, the Weyl semimetals [\[5–8\]](#page-6-0) and the nodal-line semimetals  $[9,10]$  have further expanded the range of topological materials. Up to now, several families of topological material candidates have been extensively investigated, such as the " $LnMPn_2$ " ( $Ln =$  rare earth, Ca-Ba;  $M = Mn$ , Zn;  $Pn = Sb$ , Bi) system, "*WHM*" (*W* = Zr, Hf or rare earth;  $H =$  Si-Sn, Sb;  $M =$  O-Te) family, and materials with kagome lattice. Many exotic phenomena has been ob-served, such as tunable Weyl and Dirac states in CeSbTe [\[11\]](#page-6-0), quantum Hall effect in bulk antiferromagnet EuMnBi<sub>2</sub> [\[12\]](#page-6-0), and nontrivial topological electronic structure in the kagome superconductor  $CsV_3Sb_5$  [\[13,14\]](#page-6-0). However, many other topological material candidate families remain unexplored, and it is likely that these families could hold great potential for discovering new and intriguing properties that have yet to be uncovered.

The  $A_3MX_5$  family ( $A = Ca-Ba$  or rare earth,  $M =$ transition materials or  $Mg$ ,  $X =$  chalcogen or pnictogen) have received considerable attention due to their strong

1D structural characteristic. These materials have shown many interesting magnetic and superconducting phenomena, such as high-pressure-induced superconductivity in  $Ba<sub>3</sub>TiTe<sub>5</sub>$ [\[15\]](#page-6-0) and  $Ba_9Fe_3Te_{15}$  [\[16,17\]](#page-6-0), rare ferrotoroidicity with 1D spin chains in  $Ba_6Cr_2S_{10}$  [\[18\]](#page-7-0), and complex magnetism in La<sub>3</sub>MnAs<sub>5</sub> [\[19\]](#page-7-0) and Ce<sub>3</sub>TiSb<sub>5</sub> [\[20\]](#page-7-0). Recently,  $A_3MBi_5$  compounds have been reported as candidates for topological materials [\[21\]](#page-7-0). However, the relevant transport properties of  $A_3MBi_5$  have been rarely explored and the nontrivial topological band has not been experimentally confirmed.

In this work, we have successfully grown single crystals of  $La<sub>3</sub>MgBi<sub>5</sub>$  and investigated its magnetic and electrical properties. While the crystal structure of  $La<sub>3</sub>MgBi<sub>5</sub>$  has been confirmed experimentally before, the magnetic and electrical properties of  $La<sub>3</sub>MgBi<sub>5</sub>$  have not been reported [\[22\]](#page-7-0). Our experimental results reveal the presence of de Haas–van Alphen (dHvA) and Shubnikov–de Haas (SdH) oscillations in magnetization and magnetoresistance measurements, respectively. Analysis of these oscillations demonstrates that  $La<sub>3</sub>MgBi<sub>5</sub>$ has nearly massless relativistic fermions. Besides, our firstprinciples calculations support the nontrivial band topology in  $La<sub>3</sub>MgBi<sub>5</sub>$ , confirming that it is a topologically nontrivial material. These findings pave a way for further exploration of topological properties in the  $A_3MX_5$  family.

## **II. EXPERIMENT DETAILS**

Single crystals of  $La<sub>3</sub>MgBi<sub>5</sub>$  were grown by the flux method. Starting materials of La (ingot, 99.9%, Purui Advanced Material Technology Co. Ltd., Beijing, China), Mg (slug, 99.95%, Alfa Aesar), and Bi (pills, 99.999%,

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FIG. 1. Structure of La<sub>3</sub>MgBi<sub>5</sub>. (a),(c) The schematic crystalline structure of La<sub>3</sub>MgBi<sub>5</sub> along *c* axis and *a* axis, respectively. (b) A photograph of a typical La<sub>3</sub>MgBi<sub>5</sub> single crystal used for dHvA measurement and the schematic configuration for angular ( $\theta$ ) dependent measurement of magnetization (the red square of  $1 \times 1$  mm indicates the size of the crystal). (d) Three types of 1D motifs in La<sub>3</sub>MgBi<sub>5</sub>: The separate 1D MgBi<sub>6</sub> face-sharing octahedral chain, 1D Bi chain, and La zigzag chain. (e),(f) The calculated band structures without and with the consideration of spin-orbit coupling; the red lines are the band projection of the Bi@4*d pz* orbital that exhibits a substantial slope.

Alfa Aesar) with a mole ratio of 1: 4: 6 were mixed and placed into an alumina crucible, which was then sealed in a fully evacuated quartz tube. The tube was heated to 1173 K over 10 h and maintained for 1 d. It was then slowly cooled to 923 K at a rate of 2 K/h followed by centrifuging. Rodlike crystals were obtained, as shown in Fig.  $1(b)$ .

Single-crystal x-ray diffraction (XRD) was conducted on a Bruker D8 Venture diffractometer at 274 K using Mo *Kα* radiation ( $\lambda = 0.71073\text{\AA}$ ). Crystal structure was refined by the full-matrix least-squares method on  $F<sup>2</sup>$  using the SHELXL-2018/3 program. The electrical transport properties were measured in a physical properties measurement system (PPMS, 16 T, Quantum Design) using the standard four-probe technique. Three longer and thinner crystals, SR1, SR2, and SR3, were used for the SdH oscillation measurements. The magnetization measurements were performed on a magnetic property measurement system (MPMS, 7 T, Quantum Design) and the PPMS (9 T, Quantum Design) with a vibrating sample magnetometer (VSM) option. Four different crystals SM1, SM2, SM3, and SM4 were used for the measurement of dHvA oscillations. Each of the crystals is about 40 mg. The specific heat data were measured in the PPMS using its specific heat option with a thermal relaxation method.

# **III. RESULTS AND DISCUSSION**

Analysis of the single-crystal XRD data of  $La<sub>3</sub>MgBi<sub>5</sub>$ confirms that  $La<sub>3</sub>MgBi<sub>5</sub>$  has an anti-Hf<sub>5</sub>Sn<sub>3</sub>Cu hexagonal structure with space group  $P6_3/mcm$  (no. 193) which has previously been reported for this compound [\[22\]](#page-7-0). This structure belongs to the  $A_3MX_5$  family ( $A = Ca$ -Ba or rare earth,  $M =$ transition materials or  $Mg$ ,  $X =$  chalcogen or pnictogen). The refined lattice parameters for La<sub>3</sub>MgBi<sub>5</sub> are  $a = 9.7792(2)$  Å and  $c = 6.5472(2)$  Å. The detailed crystallographic data, refined atomic positions, and displacement parameters obtained from the single-crystal XRD data are summarized in Supplemental Material Tables S1, S2, and S3 [\[23\]](#page-7-0).

Figure 1 depicts the crystal structure of  $La<sub>3</sub>MgBi<sub>5</sub>$ , which comprises 1D chains of Bi, 1D chains of face-sharing MgBi<sub>6</sub> octahedra, and zigzag chains of La atoms along the *c* axis. The bismuth atoms in this structure occupy two different Wyckoff positions: the (6*g*) site for Bi1 and the (4*d*) site for Bi2. To gain insight into its electronic structure, we performed the first-principles calculations by using the generalized gradient approximation within the Vienna Ab initio Simulation Package (VASP) [\[24,25\]](#page-7-0). Figures  $1(e)$  and  $1(f)$  show the band structure calculated with and without spin-orbit coupling (SOC). The band structure in Fig.  $1(e)$  without SOC displays two band crossings near the Fermi level along the *L-M* direction, forming a nodal ring on the (210) plane that is protected by mirror symmetry. The plus and minus signs indicate the eigenvalue of the mirror symmetry operation. The nodal ring is mainly composed of the  $p_x$  and  $p_y$  orbitals of Bi atoms in the face-sharing  $MgBi_6$  octahedra. However, when taking the SOC interaction into account, the nodal ring becomes fully gapped. Nonetheless, a small energy gap is observed in the *L-M* direction [indicated by a red square in Fig. 1(f)], which can be considered a massive Dirac cone with a 5-meV gap and gives rise to the  $\pi$  phase detected in dHvA oscillation. Besides, the band projection of the Bi@4*d pz* orbital between Г and *A* exhibits a substantial slope [red line in Figs. 1(e) and  $1(f)$ ], indicating a large Fermi velocity and potentially favorable thermal conductivity. Further experiments are needed to explore the thermal properties of  $La<sub>3</sub>MgBi<sub>5</sub>$ .

Figure [2\(a\)](#page-2-0) presents the isothermal magnetization *M-B* curves at different temperatures measured on SM2 crystal for *B* // *a*, which exhibits strong dHvA oscillations superimposed on a paramagnetic background [similar dHvA oscillations

<span id="page-2-0"></span>

FIG. 2. (a) Isothermal magnetization for La<sub>3</sub>MgB<sub>i5</sub> at different temperatures when the magnetic field is parallel to *a* axis. (b) The oscillatory components of magnetization. (c) The fits of the FFT amplitudes of  $F_\alpha$  and  $F_\beta$  to the temperature damping factor  $R_\text{T}$  by the LK formula. (d) The corresponding FFT spectrum of the oscillatory component of the dHvA oscillations at various temperatures. (e) Multiband LK fit of dHvA oscillations at 2K. (f) The Landau level (LL) index fan diagram for  $\alpha$  band derived from the oscillatory magnetization at 2 K.

were observed in SM1, SM3, and SM4 (Fig. S3 [\[23\]](#page-7-0))]. In contrast, the *M-B* curves for *B* // *c* show diamagnetic susceptibility without oscillation (Fig. S2 [\[23\]](#page-7-0)). The oscillations in Fig.  $2(a)$ persist down to a field as low as 2 T at 2 K and can be observed up to 20 K. By subtracting the paramagnetic background, we can extract the oscillatory component of  $\Delta M$  versus  $1/B$ as shown in Fig.  $2(b)$ . From the fast Fourier transformation (FFT) analyses of the oscillatory magnetization  $\Delta M$ , we have derived two oscillation frequencies of 27.29 T ( $F_{\alpha}$ ) and 93.58 T ( $F_\beta$ ) [Fig. 2(d)]. The  $F_\alpha$  component has a much greater oscillation amplitude than the  $F_\beta$  component. Based on the FFT results, we analyzed the extremal cross-sectional area of the FS using the Onsager relation  $F = (\Phi_0/2\pi^2) A_F$ , where  $\Phi_0 = h/2e$  is the magnetic flux quantum. The resulting  $A_F$ values are  $0.23 \times 10^{-2} \text{ Å}^{-2}$  for  $F_{\alpha}$  and  $0.89 \times 10^{-2} \text{Å}^{-2}$  for  $F_\beta$ , respectively.

In general, the dHvA oscillation of a Dirac system can be described by the Lifshitz-Kosevich (LK) formula [\[26\]](#page-7-0), taking the Berry phase into account [\[27,28\]](#page-7-0):

$$
\Delta M \propto -B^{1/2} R_{\rm T} R_{\rm D} R_{\rm S} \sin \left[2\pi \left(\frac{\rm F}{\rm B} - \gamma \ - \ \delta\right)\right],\tag{1}
$$

where  $R_T = KT\mu/B/\sinh(KT\mu/B), \quad \mu = m^*/m_e, \quad K =$  $2\pi^2 k_B m_e / (\hbar/e) \approx 14.69 \text{ T/K}, R_D = \exp(-KT_D \mu/B).$  *T*<sub>D</sub> is the Dingle temperature and  $R<sub>S</sub>$  is the spin reduction factor due to Zeeman splitting. The oscillation of  $\Delta M$  is described by sin  $[2\pi(F/B-\gamma-\delta)]$ , in which  $\gamma = 1/2 - \varphi_B/2\pi$  and  $\varphi_B$ is the Berry phase.  $\delta$  is an extra phase factor that depends on the dimensionality of the Fermi surface and takes the value 0 or  $\pm 1/8$  for two and three dimensions, respectively, with the sign depending on whether the probed extreme cross-section area of the Fermi surface is maximal or minimal. The fit of the temperature-dependent amplitude to the damping factor

*R*<sub>T</sub> yields  $m_{\alpha}^{*} = 0.086m_0$  and  $m_{\beta}^{*} = 0.1608m_0$ , as shown in Fig.  $2(c)$ . The obtained nearly zero effective mass is comparable with other topological materials such as PtBi<sub>2</sub> [\[29\]](#page-7-0), PdTe<sub>2</sub> [\[30\]](#page-7-0), and Cd<sub>3</sub>As<sub>2</sub> [\[31\]](#page-7-0). To further investigate the relativistic fermions, we fit the oscillation pattern at 2 K by the multiband LK formula with fixed frequency ( $F_\alpha = 27.29$  T,  $F_\beta = 93.58 \text{ T}$ ) and fixed effective mass  $(m_\alpha^* = 0.086 m_0,$  $m_{\beta}^{*} = 0.1608m_0$ ) [Fig. 2(e)]. The fitted  $T_D = 3.455 \text{ K}$  for  $F_{\alpha}$ , and the quantum relaxation time corresponding to the frequency is  $\tau_q = \hbar/(2\pi k_B T_D) = 3.52 \times 10^{-13}$ s, from which the quantum mobility  $\mu_q = e\tau_q/m^*$  is estimated to be  $7196 \text{cm}^2/\text{V}$  s (see Table [I\)](#page-3-0). In addition to the light effective mass and high quantum mobility, the Berry phase close to  $\pi$  is another important characteristic of the topological nontrivial band. The LK fit yields a factor  $-\gamma - \delta$  of 1.04 for  $F_\alpha$ , from which the Berry phase of  $(1.54 + \delta) \times 2\pi$  can be derived. Here we take  $\delta = 1/8$ , since the  $F_{\alpha}$  is a 3D hole pocket as discussed below and the probe  $F_\alpha$  oscillation should correspond to a maximal cross section. Thus, the Berry phase is 1.33 $\pi$ , which is close to  $\pi$  and implies a nontrivial Berry phase. The Dingle temperature, relation time, quantum mobility, and Berry phase obtained by LK fitting for  $F_\beta$  are summarized in Table [I.](#page-3-0) Note that  $\delta$  should be taken as 1/8 for the  $F_\beta$  corresponding to the minimal cross section of a 3D electron pocket, which will be discussed later.

In addition to the LK fitting, the Berry phase can also be extracted by the Landau level (LL) index fan diagram. Because the DOS ( $E_F$ )  $\propto dM/dB$  [\[26\]](#page-7-0), the oscillation peaks of  $\Delta M$  correspond to the Landau indices of  $n + 1/4$ , and the oscillation valleys correspond to *n*-1/4. However, the Landau indices of  $F_\beta$  are far from the quantum limit, resulting in a relatively large uncertainty in determining the intercept. We constructed an LL index fan diagram for the oscillations of

		F(T)	$m^*/m_0$	$T_{\rm D}$ (K)	$\mu_q$ (cm <sup>2</sup> V <sup>-1</sup> s <sup>-1</sup> )	$\varphi_{\rm B}$
dHvA	$\alpha$ band	27.29	0.086	3.455	7196	$1.33\pi \pm 0.0014\pi$
$(B \mid a)$	$\beta$ band	93.58	0.1608	6.44	2056	$1.7596\pi \pm 0.0418\pi$
SdH	$\alpha$ band	31.997	0.108			$1.3544\pi \pm 0.0344\pi$
(B//ab)	$\beta$ band	101.32	0.175			$1.4364\pi \pm 0.344\pi$ or 0.9364 $\pm$ 0.344 $\pi$
	$\eta$ band	391.96	0.245			

<span id="page-3-0"></span>TABLE I. Several characteristic parameters derived from the dHvA and SdH oscillations. Oscillation frequency *F*; effective mass *m*∗; Dingle temperature  $T_D$ ; quantum mobility  $\mu_q$ ; Berry phase  $\varphi_B$ .

 $F_{\alpha} = 27.29$  T, as shown in Fig. [2\(f\).](#page-2-0) The slope of the linear fit of the LL indices is 27.03 T, which is nearly the same as the value obtained from FFT (27.29 T), indicating the reliability of linear fit in the LL fan diagram. The intercept derived from the linear extrapolation is  $(0.59 \pm 0.009024)$ , corresponding to a Berry phase of  $1.43\pi \pm 0.018\pi$  which is consistent with the result obtained from LK fitting. However, it should be noted that a finite Berry phase deviating from the exact value of  $\pi$  can also occur as a result of the deviation from an ideal linear dispersion [\[32\]](#page-7-0). In this material, the presence of the SOC gives rise to a narrow energy gap at the Dirac point, leading to the berry phase that slightly deviates from  $\pi$ .

To investigate the anisotropy of the electronic structure of  $La<sub>3</sub>MgBi<sub>5</sub>$ , we performed the dHvA experiments with the magnetic field being rotated from *B* // *a* to *B* // *c* at 2 K [Fig.  $3(a)$ ]. The magnetization gradually changes from paramagnetic to diamagnetic with the  $\theta$  increasing from 0° to 90 ° [Fig. 3(a)]. After subtracting a smooth background, we obtained the oscillation pattern of  $\Delta M$  versus  $1/B$  curves at various angles, which was gradually suppressed with the increasing  $\theta$  and became nonobservable when  $\theta$  reached 49<sup>o</sup>, as shown in Fig.  $3(b)$ . From the FFT analyses [Fig.  $3(c)$ ], we determined the angular dependence of the oscillation frequencies. A new frequency  $F_\alpha'$  emerged with the increasing the angle [see Fig.  $3(c)$ ]. The angular dependence of oscillation frequencies [Fig. 3(d)] suggests complicated Fermi surfaces, which is confirmed by our first-principles calculations.

In addition to the dHvA oscillations, we conducted transport measurements on  $La<sub>3</sub>MgBi<sub>5</sub>$  to obtain further information associated with the relativistic fermions. As depicted in Fig. [4\(a\),](#page-4-0) temperature-dependent resistivities indicate metallic behavior for both  $I//c(\rho_{xx})$  and  $I//ab(\rho_{zz})$ . The residual resistivity ratio (RRR) defined as  $\rho$  (300 K)/ $\rho$  (2 K) is 36 for  $\rho_{xx}$ . The  $\rho_{zz}/\rho_{xx}$  ratio increases with decreasing temperature and reaches approximately 20 at 2 K [inset to Fig.  $4(a)$ ], which reflects the anisotropic Fermi surface in  $La<sub>3</sub>MgBi<sub>5</sub>$ . Figure  $4(b)$ shows the temperature dependence of the  $\rho_{xx}$  measured under zero fields, 9 and 16 T. A positive magnetoresistance (MR) is observed at lower temperatures. For  $B = 16$  T, the  $\rho_{xx}$  curve shows a minimum at around 25 K and slightly increases below 25 K. Similar behaviors have also been observed in other topological materials such as TaP  $[33]$ , NbP  $[34]$ , WTe<sub>2</sub>  $[6]$ and TaAs [\[35\]](#page-7-0), which were explained with the Kohler's rule [\[36\]](#page-7-0). Figure [4\(c\)](#page-4-0) presents the field dependence of  $\rho_{xx}$  (*B*) up to 16 T measured in SR2 at various temperatures.  $La<sub>3</sub>MgBi<sub>5</sub>$ exhibits a large positive MR without saturation  $(+600\%)$  at  $T = 2$  K [inset of Fig. [4\(c\)\]](#page-4-0). Figure [4\(f\)](#page-4-0) shows the numerical simulations for the magnetic field dependence of MR. The calculated  $\rho_{xx}^* \tau$  shows a nonsaturating  $(B\tau)^{1.5}$  dependence when *B*//*ab*, which is quite consistent with the experimental results of MR  $\propto B^{1.55}$ . The magnetoresistance shown in Fig. [4\(f\)](#page-4-0) was calculated using the WANNIERTOOLS package [\[37\]](#page-7-0) based on the Boltzmann transport equation [\[38,39\]](#page-7-0) and the Wannier function techniques [\[40\]](#page-7-0). The calculation was carried out on



FIG. 3. (a) Isothermal magnetization of La<sub>3</sub>MgBi<sub>5</sub> at different angles for  $T = 2 K$ . (b) The oscillatory components of magnetization at 2 K under different magnetic field orientations. (c) The corresponding FFT spectrum of the oscillatory component of the dHvA oscillations at various angles. (d) The angular dependence of dHvA oscillation frequencies  $F_{\alpha}$ ,  $F'_{\alpha}$ ,  $F_{\beta}$ .

<span id="page-4-0"></span>

FIG. 4. (a) Temperature dependence of resistivity  $\rho_{xx}$  (*I*//*c*) and  $\rho_{zz}$  (*I*//*ab* plane). Inset: Temperature dependence of the radio of  $\rho_{xx}/\rho_{xx}$ . (b) Temperature dependence of  $\rho_{xx}(T)$  with various fields. (c) Field dependence of  $\rho_{xx}(B)$  at various temperatures. The inset shows an enlarged view of the MR above 10 T. (d) The oscillatory components of magnetoresistance. (e),(f) Black lines show the measured (e) and the calculated (f) MR at 2 K as a function of the magnetic field. Red lines are the fitting data as a power function of the magnetic field.

a  $51 \times 51 \times 51$  *k* mesh and the charge carriers are assumed to have the same relaxation time  $\tau$ . The agreement between calculation and experiment indicates that the MR can be fully explained by the semiclassical theory while the topological properties, such as the massive Dirac point, have no contribution.

In the high-field range, the  $\rho_{xx}$  (*B*) curves show the SdH oscillations which decay with increasing temperatures. (This phenomenon is also reproduced for samples SR1 and SR3, as shown in Supplemental Material Fig. S4 [\[23\]](#page-7-0).) However, compared with the strong dHvA oscillations, the SdH oscillations of  $La<sub>3</sub>MgBi<sub>5</sub>$  are relatively weak. It could be attributed to a strong background in MR, as has been discussed in PdTe<sub>2</sub> [\[30\]](#page-7-0) and  $BaZnBi<sub>2</sub>$  [\[41\]](#page-7-0). After subtracting a smooth background, the oscillatory parts  $\Delta \rho_{xx}$  as a function of  $1/B$  is shown in Fig. 4(d). The FFT of the SdH oscillations yields three major frequencies,  $F_{\alpha} = 31.997$  T,  $F_{\beta} = 101.32$  T,  $F_{\eta} = 391.96$  T, and a harmonic frequency of  $2F_\alpha$  [Fig. 5(a)]. The frequency of  $F_\alpha$  and  $F_\beta$  are close to the results measured by the dHvA oscillations. However, the  $F_\eta$  was not detected in the dHvA results, possibly due to the wider range of magnetic fields applied for MR measurement. Similarly, we obtained the effect mass  $m_{\alpha}^{*} = 0.108m_0$ ,  $m_{\beta}^{*} = 0.175m_0$ , and  $m_{\eta}^{*} = 0.247m_0$  by fitting the temperature dependence of the FFT amplitudes to *R*<sub>T</sub> [inset to Fig. 5(a)]. The  $F_{\alpha}$ ,  $F_{\beta}$ ,  $m_{\alpha}^{*}$ , and  $m_{\beta}^{*}$  obtained from the SdH oscillations are slightly larger than those from dHvA oscillations. One of the reasons is that SdH oscillations are affected by the scattering mechanism, thus deviating from the prediction by the LK theory. However, the dHvA oscillation is not impacted by the scatting mechanism and can be well interpreted by LK theory [\[10,](#page-6-0)[29,42,43\]](#page-7-0).

In SdH oscillations, the oscillatory part of the conductivity follows  $\sigma_{xx} \propto \cos[2\pi (F/B-\gamma)]$ , where  $\gamma = 1/2 - \varphi_B/2\pi$ .

The minima of the oscillations in  $\sigma_{xx}$  are assigned to the Landau indices  $n$  while the Berry phases  $\varphi_B$  can be determined from analysis of the LL fan diagram [\[26\]](#page-7-0). Since the  $\sigma_{xx}$  can be obtained from the longitudinal resistivity  $\rho_{xx}$  and the Hall resistivity  $\rho_{yx}$  using  $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{yx}^2)$ , the minima of  $\rho_{xx}$  is used for indexing the integer *n* when  $\rho_{xx} \ll \rho_{yx}$  and the maxima of  $\rho_{xx}$  is used for indexing the integer *n* when  $\rho_{xx} \gg \rho_{yx}$ 



FIG. 5. (a) The corresponding FFT spectrum of the oscillatory component of the SdH oscillations at various temperatures. Inset: The fits of the FFT amplitudes of  $F_{\alpha}$ ,  $F_{\beta}$ , and  $F_{\eta}$  to the temperature damping factor  $R_T$ . (b),(c) The Landau level (LL) index fan diagram for  $\alpha$  band and  $\beta$  band derived from the SdH oscillations at 2 K, respectively. (d) Field dependence of Hall resistivity  $\rho_{yx}(B)$  at several different temperatures; the black line is the calculated Hall resistance.



FIG. 6. (a)–(c) The Fermi surfaces with SOC included. (a) The hole pocket referred as *h* pocket. The red box shows the enlarged pocket, where the green line in the upper box and the red line in the lower box indicate the extreme orbit of  $F_{\alpha}$ , and  $F_{\alpha}$ , respectively. The yellow line and green spot indicate the nodal ring without SOC and massive Dirac point with SOC, respectively. (b) The electron pockets referred as *e*1 pocket with the red box showing the enlarged pocket in the *H-K* direction. The blue line on the hexagonal hollow pocket shows the extreme orbit of  $F_\beta$  ( $\theta = 0^\circ$ ). (c) The electron pocket referred as *e*2 pocket. (d) The angular dependence of calculated dHvA oscillation frequencies below 160 T.  $F_\alpha$ ,  $F_\alpha$  contributed from *h* pocket are denoted with red spots while  $F_\beta$ ,  $F_\gamma$  contributed from *e*1 pocket are denoted with blue spots.

[\[44\]](#page-7-0). In our study, the  $\rho_{xx}$  is about ten times larger than  $\rho_{xy}$ at 2K, indicating that  $\rho_{xx} \approx 1/\sigma_{xx}$  and the Landau indices of *n* should be assigned to maxima in  $\Delta \rho_{xx}$ . Figures [5\(b\)](#page-4-0) and [5\(c\)](#page-4-0) show the LL fan diagram for  $F_\alpha$  and  $F_\beta$ , respectively. The intercept obtained from the extrapolation of the linear fit in the fan diagram is  $0.4478 \pm 0.01734$  for  $F_\alpha$  and  $0.4068 \pm$ 0.172 for  $F_\beta$ . In SdH oscillation measurements, the direction of the magnetic field in the *ab* plane is uncertain because the samples are very thin. Consequently, the extremal orbit of the electron pocket corresponding to  $F_\beta$  can be either maximal or minimal, depending on the magnetic field direction. This uncertainty leads to  $\delta$  being either  $+1/8$  or  $-1/8$  for *F*<sub>β</sub>, resulting in an uncertain phase of 1.4364 $\pi \pm 0.344\pi$  or  $0.9364\pi \pm 0.344\pi$  for  $F_\beta$ . In contrast, the extremal orbit of the  $F_\alpha$  pocket always corresponds to a maximal cross section as the magnetic field changes in the *ab* plane. Therefore, for *F*<sub>α</sub>, we have  $\varphi_B = 1.3544\pi \pm 0.0344\pi$  [\[14\]](#page-6-0). Moreover, the fitted oscillation frequency is 33.14 T for  $F_\alpha$  and 105.265 T for  $F_\beta$ , which are close to the results from the FFT, indicating the reliability of linear fit in the LL fan diagram. The result for  $F_{\alpha}$  is consistent with dHvA oscillations indicating a nontrivial Berry phase. However, the large fitting errors and undetermined field directions prevent us from obtaining a relatively accurate  $\varphi_B$  for  $F_\beta$ . Furthermore, the lowest Landau index for  $F_n$  is also far from the quantum limit, making it challenging to determine its intercept with certainty.

The Hall resistivity  $\rho_{vx}$  of La<sub>3</sub>MgBi<sub>5</sub>, as shown in Fig. [5\(d\),](#page-4-0) exhibits nonlinear behavior up to 200 K (Fig.  $S5$  [\[23\]](#page-7-0)), suggesting the presence of multiple kinds of carriers with different densities and mobilities. To calculate the carrier densities and mobilities of  $La<sub>3</sub>MgBi<sub>5</sub>$ , we used a three-band model [\[45,46\]](#page-7-0),

$$
\sigma_{xx}(B) = e \left( \frac{n_1 \mu_1}{1 + \mu_1^2 B^2} + \frac{n_2 \mu_2}{1 + \mu_2^2 B^2} + \frac{n_3 \mu_3}{1 + \mu_3^2 B^2} \right), \tag{2}
$$

$$
\sigma_{xy}(B) = eB\left(\frac{n_1\mu_1^2}{1+\mu_1^2B^2} + \frac{n_2\mu_2^2}{1+\mu_2^2B^2} + \frac{n_3\mu_3^2}{1+\mu_3^2B^2}\right),\tag{3}
$$

to fit the conductivity  $\sigma_{xx}$  and Hall conductivity  $\sigma_{xy} =$  $\rho_{yx}/(\rho_{xx}^2 + \rho_{yx}^2)$  at  $T = 2$  K. (The fitting results are shown in Supplemental Material Fig. S7 [\[23\]](#page-7-0).) The fitting indicates that the carrier densities of  $La<sub>3</sub>MgBi<sub>5</sub>$  are  $n_{e1} =$ 1.14 × 10<sup>21</sup>cm<sup>-3</sup>,  $n_{e2} = 1.9 \times 10^{20}$ cm<sup>-3</sup>, and  $n_h = 6.55 \times$  $10^{20}$ cm<sup>-3</sup> at 2 K. The carrier mobilities obtained from the fitting are  $\mu_{e1} = 897 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1}$ ,  $\mu_{e2} = 4743 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1}$ , and  $\mu_h = 1843 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1}$ . It should be pointed out that the three-band model cannot precisely describe the the conductivity and Hall resistivity of  $La<sub>3</sub>MgBi<sub>5</sub>$ . However, if all bands are considered, it would be difficult to obtain meaningful information due to a large number of fitting parameters. Additionally, the Hall resistivity is calculated using the Boltzmann transport equation [\[38,39\]](#page-7-0) and the Wannier function technique [\[37\]](#page-7-0), based on first-principles calculations. The resulting simulated Hall resistivity curve is depicted as the black line in Fig.  $5(d)$ , considering a relaxation time of 0.25 ps. Remarkably, this simulation aligns well with the experimental measurements, demonstrating multicarrier behavior.

Figures  $6(a)$ – $6(c)$  depict the Fermi surfaces with SOC, comprising one hole pocket (*h* pocket) and two electron pockets (*e*1 pocket and *e*2 pocket). The inset of Fig. 6(a) shows the location of the nodal ring and massive Dirac cone, denoted by a yellow circle and green spot, respectively. Using the SKEAF package [\[47\]](#page-7-0), we calculated the angular dependence of dHvA oscillation frequencies, which are shown in Fig. 6(d) and exhibit good agreement with experimental results. Note that we only display dHvA oscillation frequencies below 160 T as detecting higher frequencies in experiments necessitates a stronger magnetic field, lower temperature, and cleaner sample.

Assisted by the calculations, we can determine the origin of the dHvA frequencies detected in experiments. The  $F_\beta$ frequency is contributed by the extremal orbit on the hexagonal hollow pockets in Fig.  $6(b)$ , with the extremal orbit of  $F_{\beta}$  ( $\theta = 0^{\circ}$ ) indicated by a blue line. The electron pockets in the  $K-H$  direction [enlarged in the red box in Fig.  $6(b)$ ] contribute to  $F_\gamma$ . The smaller hole pockets in the *L-M* direction shown in Fig. 6(a) give rise to the  $F_\alpha$  and  $F_{\alpha'}$  frequencies. The hole pockets can be classified into two types based on the angle between their located plane and the magnetic field when  $\theta < 90^\circ$ . Consequently,  $F_\alpha(\theta < 90^\circ)$  is different from  $(\theta < 90^\circ)$  before  $\theta$  increases to 90°. Moreover, the extremal orbit of  $F_{\alpha/\alpha}(\theta = 0^{\circ})$  wraps around the massive Dirac point <span id="page-6-0"></span>and then deviates from the point with increasing  $\theta$ , resulting in the decreasing amplitude of  $F_{\alpha/\alpha}$ , as shown in Fig. [3\(c\).](#page-3-0)

#### **IV. CONCLUSIONS**

We have successfully grown high-quality single crystals of La<sub>3</sub>MgBi<sub>5</sub>, a member of the  $A_3$ *MX*<sub>5</sub> family, and performed systematic measurements for magnetization and electrical transport properties. Our results show that  $La<sub>3</sub>MgBi<sub>5</sub>$  has anisotropic magnetic susceptibility and displays the dHvA oscillations in the case of *B* // *a*. Analysis of the oscillation patterns reveals high quantum mobilities, light effective masses, and nontrivial Berry phase for the relativistic fermions. Additionally,  $La<sub>3</sub>MgBi<sub>5</sub>$  shows large positive MR without saturation and the SdH oscillations. The nontrivial Berry phase can also be obtained by SdH oscillations. Large nonsaturating magnetoresistance can be explained by the Boltzmann transport model. Furthermore, the nonlinear Hall effect at low temperature indicates that  $La<sub>3</sub>MgBi<sub>5</sub>$  is a multiband system. Our experimental results agree with the first-principles calculations, indicating that  $La<sub>3</sub>MgBi<sub>5</sub>$  is an interesting material

- [1] T. Liang, Q. Gibson, M. N. Ali, M. Liu, R. J. Cava, and N. P. Ong, Ultrahigh mobility and giant magnetoresistance in the dirac semimetal  $Cd<sub>3</sub>As<sub>2</sub>$ , Nat. Mater. **14**, 280 (2015).
- [2] Z. K. Liu, J. Jiang, B. Zhou, Z. J. Wang, Y. Zhang, H. M. Weng, D. Prabhakaran, S.-K. Mo, H. Peng, P. Dudin *et al.*, A stable three-dimensional topological dirac semimetal  $Cd<sub>3</sub>As<sub>2</sub>$ , Nat. Mater. **13**[, 677 \(2014\).](https://doi.org/10.1038/nmat3990)
- [3] Z. K. Liu, B. Zhou, Y. Zhang, Z. J. Wang, H. M. Weng, D. Prabhakaran, S.-K. Mo, Z. X. Shen, Z. Fang, X. Dai *et al.*, Discovery of a three-dimensional topological dirac semimetal, Na3Bi, Science **343**[, 864 \(2014\).](https://doi.org/10.1126/science.1245085)
- [4] Z. Wang, Y. Sun, X.-Q. Chen, C. Franchini, G. Xu, H. Weng, X. Dai, and Z. Fang, Dirac semimetal and topological phase transitions in  $A_3B$ i (A = Na, K, Rb), Phys. Rev. B 85, 195320 (2012).
- [5] B. Q. Lv, H. M. Weng, B. B. Fu, X. P. Wang, H. Miao, J. Ma, P. Richard, X. C. Huang, L. X. Zhao, G. F. Chen *et al.*, [Experimental Discovery of Weyl Semimetal TaAs,](https://doi.org/10.1103/PhysRevX.5.031013) Phys. Rev. X **5**, 031013 (2015).
- [6] M. N. Ali, J. Xiong, S. Flynn, J. Tao, Q. D. Gibson, L. M. Schoop, T. Liang, N. Haldolaarachchige, M. Hirschberger, N. P. Ong *et al.*, Large, non-saturating magnetoresistance in WTe<sub>2</sub>, [Nature \(London\)](https://doi.org/10.1038/nature13763) **514**, 205 (2014).
- [7] N. Morali, R. Batabyal, P. K. Nag, E. Liu, Q. Xu, Y. Sun, B. Yan, C. Felser, N. Avraham, and H. Beidenkopf, Fermi-Arc diversity on surface terminations of the magnetic weyl semimetal Co<sub>3</sub>Sn<sub>2</sub>S<sub>2</sub>, Science 365[, 1286 \(2019\).](https://doi.org/10.1126/science.aav2334)
- [8] C. Liu, J. Shen, J. Gao, C. Yi, D. Liu, T. Xie, L. Yang, S. Danilkin, G. Deng, W. Wang *et al.*, Spin excitations and spin wave gap in the ferromagnetic weyl semimetal  $Co<sub>3</sub>Sn<sub>2</sub>S<sub>2</sub>$ , Sci. China Phys. Mech. Astron. **64**, 217062 (2021).
- [9] L. M. Schoop, M. N. Ali, C. Straßer, A. Topp, A. Varykhalov, D. Marchenko, V. Duppel, S. S. P. Parkin, B. V. Lotsch, and C. R. Ast, Dirac cone protected by non-symmorphic symmetry

for observing topological phases. Moreover, as  $La<sub>3</sub>MgBi<sub>5</sub>$  is a member of the extensive  $A_3MX_5$  family, our research may inspire further investigations into magnetic compounds (when  $A =$  magnetic rare earth elements or  $M =$  magnetic transition metals) within this family. If such compounds also exhibit relativistic fermions, they could present exciting opportunities to explore the intricate interplay between complex magnetism and relativistic fermions.

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[and three-dimensional dirac line node in ZrSiS,](https://doi.org/10.1038/ncomms11696) Nat. Commun. **7**, 11696 (2016).

- [10] J. Hu, Z. Tang, J. Liu, Y. Zhu, J. Wei, and Z. Mao, Nearly massless dirac fermions and strong zeeman splitting in the nodal-line semimetal ZrSiS probed by de Haas–van Alphen quantum oscillations, Phys. Rev. B **96**[, 045127 \(2017\).](https://doi.org/10.1103/PhysRevB.96.045127)
- [11] L. M. Schoop, A. Topp, J. Lippmann, F. Orlandi, L. Müchler, M. G. Vergniory, Y. Sun, A. W. Rost, V. Duppel, M. Krivenkov *et al.*, Tunable weyl and dirac states in the nonsymmorphic compound CeSbTe, Sci. Adv. **4**[, eaar2317 \(2018\).](https://doi.org/10.1126/sciadv.aar2317)
- [12] H. Masuda, H. Sakai, M. Tokunaga, Y. Yamasaki, A. Miyake, J. Shiogai, S. Nakamura, S. Awaji, A. Tsukazaki, H. Nakao *et*  $al.$ , Quantum hall effect in a bulk antiferromagnet EuMnBi<sub>2</sub> with magnetically confined two-dimensional dirac fermions, Sci. Adv. **2**[, e1501117 \(2016\).](https://doi.org/10.1126/sciadv.1501117)
- [13] Y. Hu, S. M. L. Teicher, B. R. Ortiz, Y. Luo, S. Peng, L. Huai, J. Ma, N. C. Plumb, S. D. Wilson, J. He *et al.*, Topological surface states and flat bands in the kagome superconductor  $CsV<sub>3</sub>Sb<sub>5</sub>$ , Sci. Bull. **67**[, 495 \(2022\).](https://doi.org/10.1016/j.scib.2021.11.026)
- [14] Y. Fu, N. Zhao, Z. Chen, Q. Yin, Z. Tu, C. Gong, C. Xi, X. Zhu, Y. Sun, K. Liu *et al.*, Quantum Transport Evidence of Topological Band Structures of Kagome Superconductor CsV<sub>3</sub>Sb<sub>5</sub>, Phys. Rev. Lett. **127**[, 207002 \(2021\).](https://doi.org/10.1103/PhysRevLett.127.207002)
- [15] J. Zhang, Y. Jia, X. Wang, Z. Li, L. Duan, W. Li, J. Zhao, L. Cao, G. Dai, Z. Deng *et al.*, A new quasi-one-dimensional compound  $Ba<sub>3</sub>TiTe<sub>5</sub>$  [and superconductivity induced by pressure,](https://doi.org/10.1038/s41427-019-0158-2) NPG Asia Mater. **11**, 60 (2019).
- [16] J. Zhang, L. Duan, Z. Wang, X. Wang, J. Zhao, M. Jin, W. Li, C. Zhang, L. Cao, Z. Deng *et al.*, The synthesis of a quasi-onedimensional iron-based telluride with antiferromagnetic chains and a spin glass state, Inorg. Chem. **59**[, 5377 \(2020\).](https://doi.org/10.1021/acs.inorgchem.9b03592)
- [17] J. Zhang, M.-L. Jin, X. Li, X.-C. Wang, J.-F. Zhao, Y. Liu, L. Duan, W.-M. Li, L.-P. Cao, B.-J. Chen *et al.*, Structure-spintransport anomaly in quasi-one-dimensional  $Ba<sub>9</sub>Fe<sub>3</sub>Te<sub>15</sub>$  under high pressure, [Chin. Phys. Lett.](https://doi.org/10.1088/0256-307X/37/8/087106) **37**, 087106 (2020).
- <span id="page-7-0"></span>[18] J. Zhang, X. Wang, L. Zhou, G. Liu, D. T. Adroja, I. da Silva, F. Demmel, D. Khalyavin, J. Sannigrahi, H. S. Nair *et al.*, A [ferrotoroidic candidate with well-separated spin chains,](https://doi.org/10.1002/adma.202106728) Adv. Mater. **34**, 2106728 (2022).
- [19] L. Duan, X. C. Wang, J. Zhang, Z. Hu, J. F. Zhao, Y. G. Feng, H. L. Zhang, H.-J. Lin, C. T. Chen, W. Wu *et al.*, Synthesis, structure, and magnetism in the ferromagnet  $La<sub>3</sub>MnAs<sub>5</sub>$ : Well[separated spin chains coupled via itinerant electrons,](https://doi.org/10.1103/PhysRevB.106.184405) Phys. Rev. B **106**, 184405 (2022).
- [20] C. Ritter, A. K. Pathak, R. Filippone, A. Provino, S. K. Dhar, and P. Manfrinetti, Magnetic ground states of  $Ce<sub>3</sub>TiSb<sub>5</sub>$ ,  $Pr_3TiSb_5$  and  $Nd_3TiSb_5$  determined by neutron powder diffrac[tion and magnetic measurements,](https://doi.org/10.1088/1361-648X/abe9db) J. Phys.: Condens. Matter **33**, 245801 (2021).
- [21] J. F. Khoury, B. Han, M. Jovanovic, R. Singha, X. Song, R. Queiroz, N.-P. Ong, and L. M. Schoop, A class of Magnetic Topological Material Candidates with Hypervalent Bi Chains, [J. Am. Chem. Soc.](https://doi.org/10.1021/jacs.2c02281) **144**, 9785 (2022).
- [22] D.-C. Pan, Z.-M. Sun, and J.-G. Mao, Synthesis and crystal structures of La<sub>3</sub>MgBi<sub>5</sub> and LaLiBi<sub>2</sub>, J. Solid State Chem. 179, 1016 (2006).
- [23] See Supplemental Material at [http://link.aps.org/supplemental/](http://link.aps.org/supplemental/10.1103/PhysRevB.108.075157) 10.1103/PhysRevB.108.075157 for details of crystal structure and transport properties.
- [24] G. Kresse and J. Furthmüller, Efficient iterative schemes for ab initio total-energy calculations using a plane-wave basis set, Phys. Rev. B **54**[, 11169 \(1996\).](https://doi.org/10.1103/PhysRevB.54.11169)
- [25] G. Kresse and D. Joubert, From ultrasoft pseudopotentials to [the projector augmented-wave method](https://doi.org/10.1103/PhysRevB.59.1758) Phys. Rev. B **59**, 1758 (1999).
- [26] D. Shoenberg, *Magnetic Oscillations in Metals* (Cambridge University Press, Cambridge, UK, 1984).
- [27] J. Hu, Z. Tang, J. Liu, X. Liu, Y. Zhu, D. Graf, K. Myhro, S. Tran, C. N. Lau, J. Wei *et al.*, Evidence of Topological [Nodal-Line Fermions in ZrSiSe and ZrSiTe,](https://doi.org/10.1103/PhysRevLett.117.016602) Phys. Rev. Lett. **117**, 016602 (2016).
- [28] G. P. Mikitik and Yu. V. Sharlai, Manifestation of Berry's Phase in Metal Physics, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.82.2147) **82**, 2147 (1999).
- [29] W. Gao, X. Zhu, J. Hu, S. Li, F. Zheng, H. Zhang, M. Wu, G. Zheng, N. Hao, P. Zhang *et al.*, De Haas–van alphen study on three-dimensional topological semimetal pyrite PtBi<sub>2</sub>, Sci. Bull. **64**, 1496 (2019).
- [30] F. Fei, X. Bo, R. Wang, B. Wu, J. Jiang, D. Fu, M. Gao, H. Zheng, Y. Chen, X. Wang *et al.*, Nontrivial berry phase and type-II dirac transport in the layered material PdTe<sub>2</sub>, Phys. Rev. B **96**, 041201 (2017).
- [31] L. P. He, X. C. Hong, J. K. Dong, J. Pan, Z. Zhang, J. Zhang, and S. Y. Li, Quantum Transport Evidence for the Three-Dimensional Dirac Semimetal Phase in  $Cd<sub>3</sub>As<sub>2</sub>$ , Phys. Rev. Lett. **113**, 246402 (2014).
- [32] A. A. Taskin and Y. Ando, Berry phase of nonideal dirac [fermions in topological insulators,](https://doi.org/10.1103/PhysRevB.84.035301) Phys. Rev. B **84**, 035301 (2011).
- [33] J. Hu, J. Y. Liu, D. Graf, S. M. A. Radmanesh, D. J. Adams, A. Chuang, Y. Wang, I. Chiorescu, J. Wei, L. Spinu *et al.*, π Berry

[phase and zeeman splitting of weyl semimetal tap,](https://doi.org/10.1038/srep18674) Sci. Rep. **6**, 18674 (2016).

- [34] C. Shekhar, A. K. Nayak, Y. Sun, M. Schmidt, M. Nicklas, I. Leermakers, U. Zeitler, Y. Skourski, J. Wosnitza, Z. Liu *et al.*, Extremely large magnetoresistance and ultrahigh mobility [in the topological weyl semimetal candidate NbP,](https://doi.org/10.1038/nphys3372) Nat. Phys. **11**, 645 (2015).
- [35] X. Huang, L. Zhao, Y. Long, P. Wang, D. Chen, Z. Yang, H. Liang, M. Xue, H. Weng, Z. Fang *et al.*, Observation of the Chiral-Anomaly-Induced Negative Magnetoresistance in 3d Weyl Semimetal TaAs, Phys. Rev. X **5**[, 031023 \(2015\).](https://doi.org/10.1103/PhysRevX.5.031023)
- [36] Y. L. Wang, L. R. Thoutam, Z. L. Xiao, J. Hu, S. Das, Z. Q. Mao, J. Wei, R. Divan, A. Luican-Mayer, G. W. Crabtree *et al.*, Origin of the turn-on temperature behavior in WTe<sub>2</sub>, Phys. Rev. B **92**, 180402 (2015).
- [37] Q. Wu, S. Zhang, H.-F. Song, M. Troyer, and A. A. Soluyanov, WannierTools: An open-source software package for novel topological materials, [Comput. Phys. Commun.](https://doi.org/10.1016/j.cpc.2017.09.033) **224**, 405 (2018).
- [38] S. Zhang, Q. Wu, Y. Liu, and O. V. Yazyev, Magnetoresis[tance from fermi surface topology,](https://doi.org/10.1103/PhysRevB.99.035142) Phys. Rev. B **99**, 035142 (2019).
- [39] Y. Liu, H.-J. Zhang, and Y. Yao, *Ab initio* investigation of [magnetic transport properties by Wannier interpolation,](https://doi.org/10.1103/PhysRevB.79.245123) Phys. Rev. B **79**, 245123 (2009).
- [40] A. A. Mostofi, J. R. Yates, Y.-S. Lee, I. Souza, D. Vanderbilt, and N. Marzari, Wannier90: A tool for obtaining maximally[localised Wannier functions,](https://doi.org/10.1016/j.cpc.2007.11.016) Comput. Phys. Commun. **178**, 685 (2008).
- [41] K. Zhao, E. Golias, Q. H. Zhang, M. Krivenkov, A. Jesche, L. Gu, O. Rader, I. I. Mazin, and P. Gegenwart, Quantum oscillations and dirac dispersion in the  $BaZnBi<sub>2</sub>$  semimetal guaranteed by local Zn vacancy order, Phys. Rev. B **97**[, 115166 \(2018\).](https://doi.org/10.1103/PhysRevB.97.115166)
- [42] J. Y. Liu, J. Hu, Q. Zhang, D. Graf, H. B. Cao, S. M. A. Radmanesh, D. J. Adams, Y. L. Zhu, G. F. Cheng, X. Liu *et al.*, A Magnetic Topological Semimetal Sr<sub>1−y</sub>Mn<sub>1−*z*</sub>Sb<sub>2</sub>(y, *z* < 0.1), Nat. Mater. **16**[, 905 \(2017\).](https://doi.org/10.1038/nmat4953)
- [43] L. An, X. Zhu, Y. Qian, C. Xi, W. Ning, H. Weng, and M. Tian, Signature of Dirac semimetal states in gray arsenic studied by de Haas–van Alphen and Shubnikov–de Haas quantum oscillations, Phys. Rev. B **101**[, 205109 \(2020\).](https://doi.org/10.1103/PhysRevB.101.205109)
- [44] F.-X. Xiang, X.-L. Wang, M. Veldhorst, S.-X. Dou, and M. S. Fuhrer, Observation of topological transition of fermi surface from a spindle torus to a torus in bulk rashba spin-split BiTeCl, Phys. Rev. B **92**[, 035123 \(2015\).](https://doi.org/10.1103/PhysRevB.92.035123)
- [45] K. Akiba, A. Miyake, Y. Akahama, K. Matsubayashi, Y. Uwatoko, and M. Tokunaga, Two-carrier analyses of the transport [properties of black phosphorus under pressure,](https://doi.org/10.1103/PhysRevB.95.115126) Phys. Rev. B **95**, 115126 (2017).
- [46] D. E. Soule, Magnetic field dependence of the hall effect and [magnetoresistance in graphite single crystals,](https://doi.org/10.1103/PhysRev.112.698) Phys. Rev. **112**, 698 (1958).
- [47] P. M. C. Rourke and S. R. Julian, Numerical extraction of de Haas–van Alphen frequencies from calculated band energies, [Comput. Phys. Commun.](https://doi.org/10.1016/j.cpc.2011.10.015) **183**, 324 (2012).