




Unraveling the excitonic states in bulk 2H-MoS₂ via their giant Stark shift

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The interlayer exciton (IL) in bilayer 2H-MoS₂ has earlier been shown to undergo large Stark splitting under electric field $F_z \parallel c$ axis. We show that the excited state exciton A_{2s} in bulk 2H-MoS₂ undergoes nearly three times as large splitting, with a dipole moment magnitude 1.46 enm. The nature and evolution of different exciton species with F_z is verified by comparison with *ab initio* GW-Bethe-Salpeter equation (BSE) calculations that include the full electron-hole correlations. The excitonic wave functions reveal the individual character of the exciton states at high F_z as the ground state A_{1s} , split interlayer IL_- , IL_+ , and split excited state A_{2s-} . Extrapolation to low F_z indicates that IL and A_{2s} mix strongly. We try to rationalize the large dipole moment values by comparing GW-BSE results with the hydrogenic exciton model. Although the dominant A_{1s} shows an insignificant Stark shift, its large anticrossing with A_{2s-} provides a pathway for its modulation and control using an electric field.

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Introduction. Excitons in transition-metal dichalcogenide (TMD) semiconductors of the type MX_2 ($M=Mo$ and W , and $X=S$, Se , and Te), which can form stable mono/few-layer films, have large oscillator strengths and large binding energies that exceed thermal energy $k_B T \sim 26$ meV at room temperature. Consequently, excitons dominate the optical spectrum of TMDs. Many of the new electronic structure related properties discovered in TMDs in recent years, such as spin-valley coupling [1], spin-layer locking [2], non-radiative dark states [3], are manifest through excitons, which have been studied extensively [4]. Unlike typical group III-V semiconductors, the presence of excitons at room temperature in TMDs makes their study important for any optoelectronic device application [5]. In this context, there have been several studies, which investigated the response of excitons in few-layer TMDs to electric fields. In monolayer WS₂, films energy of the ground-state A_{1s} exciton and its charged counterpart the trion, could be tuned indirectly by ~ 20 meV through influencing their binding energy via a gate voltage-induced change in the background carrier concentration [6]. Large changes in reflectivity due to such shifts in the A_{1s} exciton in monolayer MoSe₂ has been demonstrated, suggesting the possibility of making electrically switchable mirrors [7]. More recently, in bi/trilayer 2H-MoS₂ films, a newly identified exciton state called the interlayer exciton [8–10], was found to show very large Stark splitting of its transition energy under an electric-field $F_z \parallel c$ axis [11–13]. This indicated a large electric dipole moment p associated with such states, that have a considerable oscillator strength. In the case of bilayer 2H-MoS₂ Lorchat *et al.* [11] reported $p = 0.48$ enm for interlayer excitons. Leisgang *et al.* [12] found $p = 0.47$ and $p = -0.39$ enm for the

two Stark split branches of the interlayer exciton and reported the anticrossing of one of the branches with the A_{1s} exciton. Peimyoo *et al.* [13] demonstrated tunability of the interlayer exciton transition energies over 70 meV with F_z ranging from -0.04 to 0.07 V/nm. Such field split interlayer exciton transitions also show up prominently in photoluminescence under the crossed circular excitation-detection mode [14]. Anomalous weaker Stark shifts of excitons have also been reported in monolayer WS₂ [15].

The above studies focused on few-layer films, however, thicker films have higher absolute reflectance (R) and are, therefore, of interest from a device application perspective especially since interlayer excitons have also been identified in bulk 2H-MoS₂ [16]. Here, we report a study of the influence of an electric-field $F_z \parallel c$ axis on excitons in bulk 2H-MoS₂ around its direct gap at the K point of the Brillouin zone, using reflectance spectroscopy. We compare the experimental results with first-principles electronic structure calculations that take into account electron-hole Coulomb interaction in the presence of F_z . The analysis of spatial correlation of the electron-hole wave function helped us to identify the field split exciton states, one of which is found to have a surprisingly large p value. We further investigate if the hydrogenic model of an exciton can help to understand such large p values. The Letter clarifies how excited states of excitons in bulk 2H-MoS₂ and many similar materials should be identified.

Experimental details. The 2H-MoS₂ crystals used in this Letter are of geological origin, obtained from SPI supplies. Thin flakes were prepared for measurement by mechanical exfoliation. The carrier concentration n was determined through Hall measurements in van der Pauw geometry on flakes ~ 60 nm thick with photolithographically [17] defined Al contacts. The estimated value of $n \sim 3 \times 10^{17}$ cm⁻³ at 25 K agrees well with other reports on material from this

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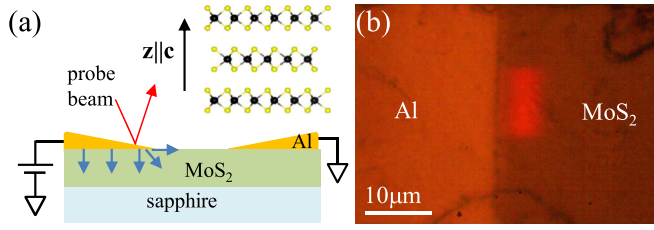


FIG. 1. (a) Arrangement for electric-field-dependent reflectance spectroscopy on bulk $2H$ - MoS_2 . Small arrows indicate the field direction at the Al- $2H$ - MoS_2 interface. The inset is a $2H$ - MoS_2 crystal structure schematic indicating the sample orientation. (b) Optical image showing the focused probe beam close to the thick edge of the Al electrode.

source [18]. For optical measurements under the electric field, Al electrodes were deposited on thick flakes by thermal evaporation in such a way that the electrodes have a gradually thinning edge see the Supplemental Material [19] as schematically shown in Fig. 1(a). R measurements were performed with the probe beam focused on the thin semitransparent part of the electrode below which the electric field is governed by the metal-semiconductor Schottky junction with field $F_z \parallel c$ axis of $2H$ - MoS_2 . An image of the focused probe beam next to the thick edge of the electrode is shown in Fig. 1(b). For a bias of 9 V, the maximum F_z at the metal-semiconductor interface is estimated to be $\sim 0.086 \text{ V nm}^{-1}$ see the Supplemental Material [19]. The probe beam was obtained by dispersing light from a laser driven broadband Xe light source using a 0.5-m focal length monochromator. It was focused on to the sample using a microscope arrangement with a $50\times$ long working distance objective, resulting in a spot size of $6 \times 12 \mu\text{m}^2$. A lock-in amplifier was used for phase-sensitive detection of the signal from a photomultiplier tube detector. The samples were cooled using a pulse-tube He refrigerator with a sample mount designed to minimize vibrations.

Results and Discussion. Figure 2 shows the R spectrum of bulk $2H$ - MoS_2 at 25 K for different bias voltages. For the lowest bias, a dominant resonant feature around 1.91 eV arising from the ground-state A_{1s} transition at the K point of the Brillouin zone of $2H$ - MoS_2 is found. The smaller feature around 1.95 eV labeled X1 was previously identified as an interlayer (IL) exciton transition in bulk $2H$ - MoS_2 [16]. There is also a weak bump around 2.0 eV labeled as X2. At 2.12 eV, we observe a feature due to the ground-state B_{1s} transition at the K point. With increasing voltage, X1 seems to redshift and gain strength, whereas X2 gains strength too and splits up into two features labeled IL_- and IL_+ . These labelings at high fields are based on comparison with theory results to be discussed later. A_{1s} and B_{1s} energy positions seem not affected by the voltage. However, between 4 and 5 V, the X1 and A_{1s} show anticrossing behavior wherein X1 comes closest to A_{1s} gains strength and then we see it at energies below A_{1s} with diminished strength, labeled as A_{2s-} . The energy positions of these transitions marked by arrows were inferred from reflectance line-shape simulations [19]. For the X1 feature, we observe a Stark shift as large as 60 meV with a moderate voltage of 9 V.

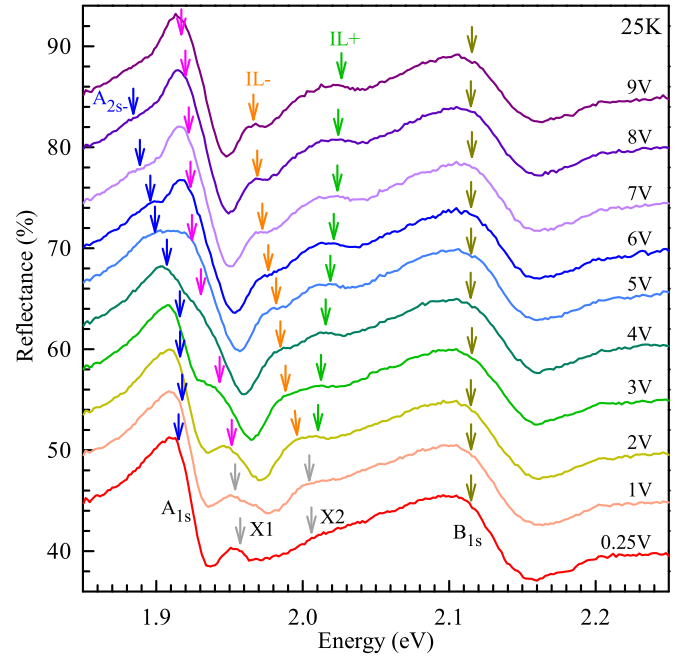


FIG. 2. Reflectance spectrum of $2H$ - MoS_2 as a function of applied voltage, which results in a field $F_z \parallel c$ axis. The plots are progressively shifted up by 5% for clarity.

Using the Schottky barrier formula [19] an effective electric field at the Al- MoS_2 interface can be obtained for an applied voltage. Figure 3 shows the variation of the transition energies of the different features as a function of the electric-field F_z . The continuous lines are either linear fits or, for the case marked A_{1s} and A_{2s-} , the fit is to an interacting two-level model, which reproduces the anticrossing behavior. The slope of a linear variation of the transition energy with a field reveals the dipole moment p . For features labeled IL_+ and IL_- , linear fits at high fields yield $p_{IL+} = 0.36 \pm 0.02 \text{ enm}$ and $p_{IL-} = -0.66 \pm 0.02 \text{ enm}$. These values are similar to dipole moments measured for the interlayer exciton in bilayer $2H$ - MoS_2 [11,12]. For B_{1s} , the dipole moment $p_{B_{1s}}$ is vanishingly small. For A_{2s-} and A_{1s} , using the interacting two-level model

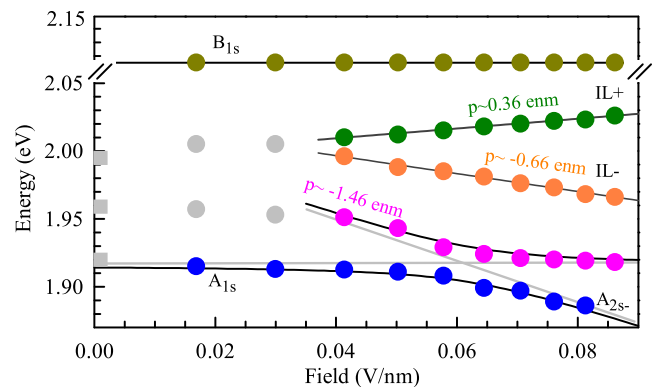


FIG. 3. Dependence of exciton transition energies on electric-field $F_z \parallel c$ axis. Their identification at high fields is based on comparison with theory results. The black lines are fits, and the gray lines are a guide to the eye. The squares are data from Ref. [16].

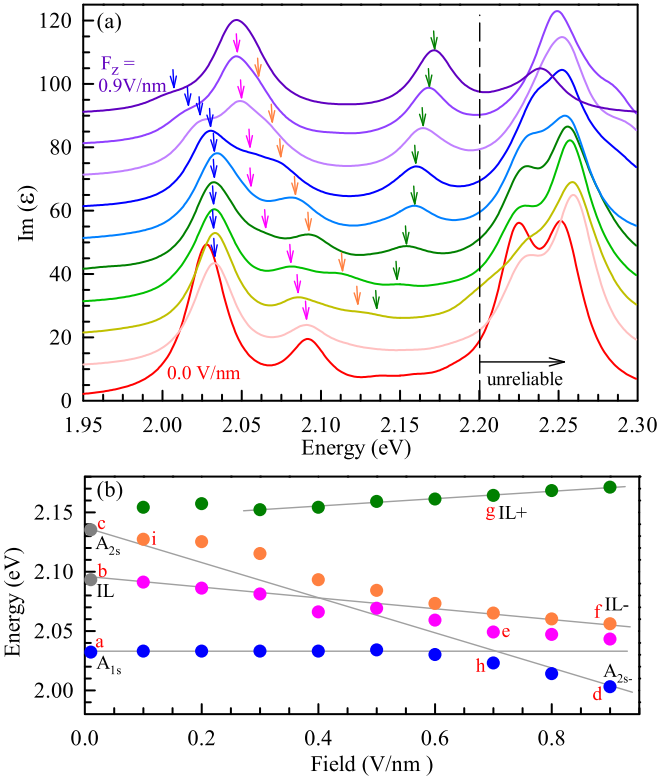


FIG. 4. Calculated (a) imaginary part of the dielectric function and (b) exciton transition energies in $2H$ -MoS $_2$ under electric-field $F_z \parallel c$ axis using GW-BSE. The lines in (b) are a guide to the eye. The points marked “a” to “i” in (b) represent exciton states for which $\bar{\chi}_S^2(z_h, z_e)$ calculated from the exciton wave function are plotted in Fig. 5.

fitting [19] with $p_{A_{1s}} = 0$ for A_{1s} , yielded $p_{A_{2s-}} = -1.46 \pm 0.03$ enm for A_{2s-} . This value of $p_{A_{2s-}}$ is more than twice as large as the interlayer exciton dipole moment reported in few-layer $2H$ -MoS $_2$ [11–13]. A very recent study reported dipole moment of 0.73 ± 0.01 enm for interlayer excitons in bilayer and trilayer $2H$ -MoSe $_2$ [20].

To understand these observations, we have performed density functional theory (DFT) based electronic band structure calculations, including the effect of the electric-field F_z . The electron-hole Coulomb interaction was included through solving the BSE [21]. Details of the calculations are given in Ref. [19]. In Fig. 4(a), we plot the theoretically calculated imaginary part of the dielectric function, which is proportional to the absorption spectrum as a function of energy for different F_z 's. The peaked features represent resonant exciton absorption. With increasing F_z , they split and shift in a manner very similar to the experimental data in Fig. 2. Although the relative energies for the low-energy excitons (below 2.2 eV) are converged to about 10 meV, the absolute energies are shifted with respect to experiment as discussed in literature [28]. Also the absolute value of the fields cannot be directly compared with measurements due to the different environment of the calculation [13]. For $F_z = 0$, we find that the ground-state A-exciton A_{1s} , a new state seen in such TMD semiconductors called the interlayer exciton IL and the first excited state of A-exciton A_{2s} [8,16]. Evolution of these calculated exciton

transition energies with F_z is shown in Fig. 4(b). Their trend is in good agreement with the data in Fig. 3. For finite F_z , the states IL and A_{2s} split into IL/IL_+ and A_{2s-}/A_{2s+} , respectively, and intermix as will be shown next. Note that our calculations are performed for pure $2H$ -MoS $_2$ bulk, whereas in experiment the asymmetric dielectric environment at the metal-semiconductor interface can affect the transition energies of a split exciton pair differently. This can partially explain the observed differences with experiment.

To verify the nature of these split exciton states, we calculated the exciton wave function $\chi_S(\mathbf{r}_h, \mathbf{r}_e) = \sum_v^{\text{occ}} \sum_c^{\text{emp}} A_{vc}^S \psi_v^*(\mathbf{r}_h) \psi_c(\mathbf{r}_e)$ where the occupied (empty) band indices v (c) includes the spin and the \mathbf{k} points. $\psi_{v,c}$'s are single-particle wave functions and \mathbf{r}_h , \mathbf{r}_e are the positions of the hole and the electron. As the wave function is a complicated six-dimensional object, we focus on the z_e - z_h spatial correlation defined as $\bar{\chi}_S^2(z_h, z_e) = \int |\chi_S(\mathbf{r}_h, \mathbf{r}_e)|^2 dx_h dy_h dx_e dy_e$ in which the in-plane coordinates of the hole and electron are integrated. $\bar{\chi}_S^2(z_h, z_e)$ represents the probability of finding the hole (electron) along the z direction at z_h (z_e).

Figures 5(a)–5(i) show $\bar{\chi}_S^2(z_h, z_e)$ plotted as a function of z_e and z_h for exciton states with a specific transition energy at certain values of F_z , which were previously marked as a to i in Fig. 4(b). In the case of the ground-state exciton A_{1s} with $F_z = 0$, the electron is predominantly [8,16] in the same layer as the hole, consequently $\bar{\chi}_S^2(z_h, z_e)$ is series of bright spots along the central diagonal in Fig. 5(a) representing $z_h = z_e$. Similarly, with $F_z = 0$, for IL and A_{2s} states where the electron is expected to be mostly one and two layers away from the hole, respectively, the $\bar{\chi}_S^2(z_h, z_e)$ plot should have two series of bright spots along diagonals one and two layers away from the central diagonal, respectively, as seen in Figs. 5(b) and 5(c). For fields $F_z \neq 0$, IL/IL_+ and A_{2s-}/A_{2s+} become nondegenerate, only one of the two diagonal spot series have significant weight. With this understanding, we can now identify the states in Fig. 5 as (a) A_{1s} , (b) IL , (c) A_{2s} , predominantly (d) A_{2s-} , predominantly A_{1s} with a mixture of IL_- and (e) A_{2s-} , (f) predominantly IL_- , (g) predominantly IL_+ , (h) a mixture A_{1s} and A_{2s-} , and (i) a mixture of A_{2s-} and IL_+ . By comparison with these results, we identified the experimentally observed split exciton features in Figs. 2 and 3.

What we find is that with increasing F_z , there is large Stark splitting of IL and A_{2s} , the latter being much larger, but no splitting of A_{1s} . Importantly, we observe intermixing of split IL , A_{2s} and A_{1s} exciton states. We do not see a clear signature of A_{2s+} in the experimental data possibly because, unlike A_{2s-} , which mixes with a sharp and strong A_{1s} and picks up oscillator strength from it, A_{2s+} can only interact with the broad B_{1s} and is, therefore, smeared out. We underline the difference compared to previous reports on few-layer $2H$ -MoS $_2$ where a pure IL exciton state splits under F_z and it is IL_- that blue shifts towards A_{1s} [11–13]. A very recent report on bilayer $2H$ -MoS $_2$ shows evidence of mixing and anticrossing of IL_- with A_{1s} [22], but no mixing of A_{1s} and A_{2s-} . However, in case of bilayer $2H$ -MoSe $_2$, mixing and anticrossing of A_{1s} and A_{2s} states has been reported recently [20].

In early literature, the feature X1 had been wrongly identified as a pure A_{2s} state [23], then as a transition at the H point

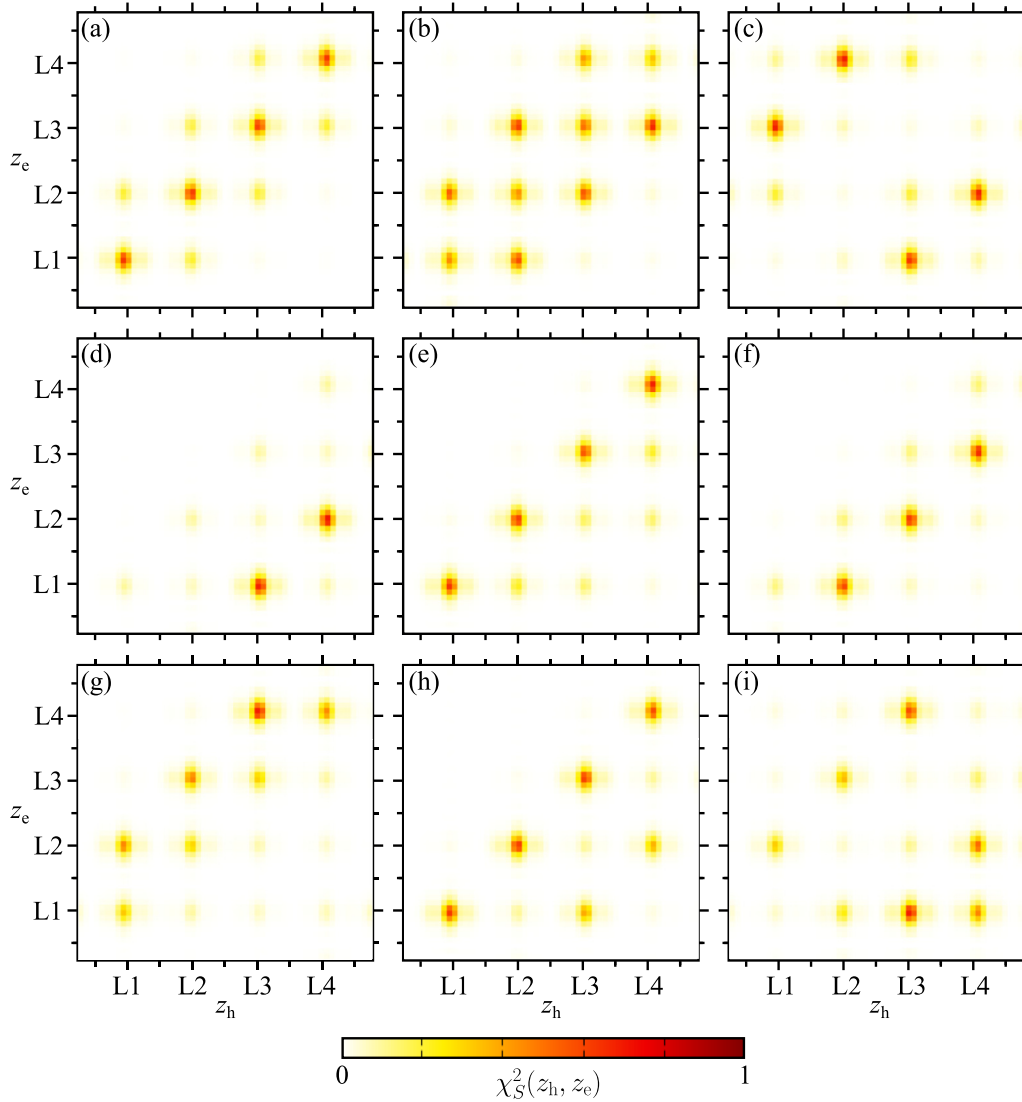


FIG. 5. (a)–(i) $\overline{\chi_S^2}(z_h, z_e)$ as a function of z_e and z_h calculated from the wave function of certain exciton states as they evolve under F_z . These states were marked in Fig. 4(b). z_e and z_h are in units of $L \sim 0.61$ nm, the distance between adjacent S-Mo-S layers in $2H$ -MoS₂ along the \mathbf{c} axis. For a detailed discussion of the Stark shift of pure A_{1s} , A_{2s} , IL excitons, and their mixtures, see the main text.

of the Brillouin zone [24,25] until, finally, it was understood to be a manifestation in bulk [16] of the IL exciton seen in few-layer $2H$ -MoS₂ [8,10]. Now, if we trace back the energies of the exciton features identified at high F_z in Fig. 3 to low F_z , we come to the following interesting conclusion: In bulk $2H$ -MoS₂, even under very weak F_z , the feature $X1$, which was most recently identified as an IL exciton, has contributions from A_{2s} , which gain strength with increasing fields, and similarly the $X2$ feature, which was thought to be a pure A_{2s} , also gains contribution from IL .

With the above identification, we can explain the measured dipole moment values. In a hydrogen atom, the $n = 1$ ground state does not possess a permanent dipole moment in first-order perturbation theory and is, therefore, not affected by an applied electric field [26]. This illustrates the insensitivity of the A_{1s} and B_{1s} ground-state excitons to F_z . However, the first excited $n = 2$ state of the hydrogen atom does possess a permanent dipole moment with magnitude $p = 3ea_0$, where e is the electronic charge and $3a_0$ is half the average distance of

the electron from the origin in the $2s$ state. In contrast to highly symmetric crystals, such as Cu₂O [27] no unique a_0 can be defined in $2H$ -MoS₂, which is highly anisotropic comparing in- and out-of-plane directions [8,16]. For estimating the dipole moments of IL and A_{2s} , we employ the average distance of the electron from the hole (mostly localized in the center of each layer) along the \mathbf{c} axis. The highest probability of finding the electron is in the adjacent layer (at a distance of ~ 0.61 nm from the hole) in case of the IL state (66%), and in the next layer at ~ 1.22 nm in the A_{2s} state (62%) [8,16]. Accounting for these probabilities [19], we estimated the dipole moment for IL and A_{2s} to be $\sim 0.52 \pm 0.10$ and $\sim 1.3 \pm 0.2$ enm, respectively. These numbers are in fair agreement with our measured average dipole moments for the IL and A_{2s} excitons and also explain why A_{2s} with its larger electron spread along the \mathbf{c} axis, has the highest dipole moment.

The A_{1s} exciton has the largest oscillator strength and shows up most prominently in reflectance. Its control with an electric field in bulk $2H$ -MoS₂ would be useful for device

applications, such as tunable mirrors [7]. Normally, it is insensitive to electric-fields F_z as explained earlier. However, for the anticrossing of A_{1s} and A_{2s-} , we find the maximum sensitivity of the A_{1s} feature to F_z . This provides an indirect way of controlling the A_{1s} exciton in thicker $2H$ -MoS₂ films using an electric field.

Conclusion. In conclusion, we showed that several low-energy exciton features in bulk $2H$ -MoS₂ undergo very large Stark splitting in electric-fields F_z applied $\parallel \mathbf{c}$ axis. We identified the different split exciton states at high fields by comparison with DFT-BSE-based calculations. We found that the A_{2s} exciton state has a dipole moment more than twice as large as the highest previously reported for an exciton species in TMDs. The results suggest that even for weak F_z , the A_{2s} and IL states are mixed in bulk $2H$ -MoS₂. This adds a new perspective to the debate on the identification of the feature adjacent to A_{1s} in the absorption/reflectance spectrum of bulk $2H$ -MoS₂, which had previously been attributed to

a pure A_{2s} or IL transition. Although the A_{1s} exciton with the largest oscillator strength has insignificant dipole moment, its anticrossing with the Stark split A_{2s-} state leads to a change in both its oscillator strength and energy position. This can be useful for electrical control of A exciton for device applications. Furthermore, we note that Stark shifts can be expected for several excitons in other layered materials as well. Thus, electric fields are a powerful tool to disentangle their nature.

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