

Anisotropic optics and gravitational lensing of tilted Weyl fermions

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We show that tilted Weyl semimetals with a spatially varying tilt of the Weyl cones provide a platform for studying analogs to problems in anisotropic optics as well as curved spacetime. Considering particular tilting profiles, we numerically evaluate the time evolution of electronic wave packets and their current densities. We demonstrate that electron trajectories in such systems can be obtained from Fermat's principle in the presence of an inhomogeneous, anisotropic effective refractive index. On the other hand, we show how the electron dynamics reveal gravitational features and use them to simulate gravitational lensing around a synthetic black hole. These results bridge optical and gravitational analogies in Weyl semimetals, suggesting different pathways for experimental solid-state electron optics.

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Inspiration drawn from light optics has driven important fundamental and applied breakthroughs in electronic condensed matter systems. It led to the manipulation of electrons using inhomogeneous potentials [1], negative refractive indices, and Veselago lensing [2,3], tailoring the effective mass of electrons [4], a geodesic description for random variations in surface height [5], and strain-induced pseudomagnetic fields [6]. Even more exotic phenomena such as electron beam collimation [7] and cloaking [8–11] have been proposed. In the past decade, some of these advanced theoretical ideas have been realized in experiments which brought about a new era in the field of *solid-state electron optics* [12,13]. In this context, we consider systems with a specific spatially varying hopping profile rather than the more traditional inhomogeneous potential landscape [1], which allows us to devise an electronic version of anisotropic optics [14–16], thus opening the way to applications using anisotropic optics effects, such as birefringence, in electronic systems. In the long-wavelength limit this inhomogeneous hopping profile maps to a system with Weyl cones whose tilting depends on position.

This is of particular interest as close analogies can be drawn between electronic Weyl cones and the light cones defined in general relativity [17–20]. Recent developments showed behaviors analogous to gravitational horizons for a range of phenomena in inhomogeneous systems related to Weyl semimetals, including eternal slowdown and Unruh radiation [21–30]. There are already several proposals for experimentally realizing the modulations in tilting as a function of space, including the use of structural distortions, spin textures, and external position-dependent driving [31–37]. These novel possibilities enrich the field of *analog gravity* that have been proposed in a range of other types of systems [38–63].

Here, we consider in particular lattice systems of tilted Weyl cones whose tilt varies slowly compared to the lattice spacing, and we numerically evaluate the quantum dynamics of wave packets in such systems. Intriguingly, we show that the propagation in these systems can be understood from two seemingly disconnected perspectives: (a) in analogy to anisotropic continuum optics through Fermat's principle; and (b) mimicking analog black holes and gravitational lensing for electrons. These features are illustrated in Fig. 1 where the current density is depicted both in the absence [Figs. 1(a) and 1(c)] and presence [Figs. 1(b) and 1(d)] of tilting modulations mimicking a curved spacetime with a black hole horizon (black circle). Electrons injected from a lead attached to the upper left side are seen to bend around the synthetic black hole.

Systems with position-dependent tilting of Weyl cones provide a unique platform to study phenomena which brings anisotropic optical analogies and connections to gravitational physics together. To showcase the different facets of the approach, we address it at different levels of theory, including semiclassical trajectories, quantum wave packet dynamics, and experimentally accessible electronic current densities resulting from the presence of an event horizon.

Anisotropic optics of tilted Weyl cones. We consider the motion of wave packets in inhomogeneous systems with tilted Weyl cones. At every real space position the Hamiltonian can be described as a tilted Weyl cone in the local momentum space, and we allow this tilt to change as a function of position. This system is described by the real space Hamiltonian

$$H = \sum_j -i[\sigma_j - t_j(\mathbf{r})\sigma_0]\partial_j + i\frac{1}{2}\partial_j t_j(\mathbf{r})\sigma_0, \quad (1)$$

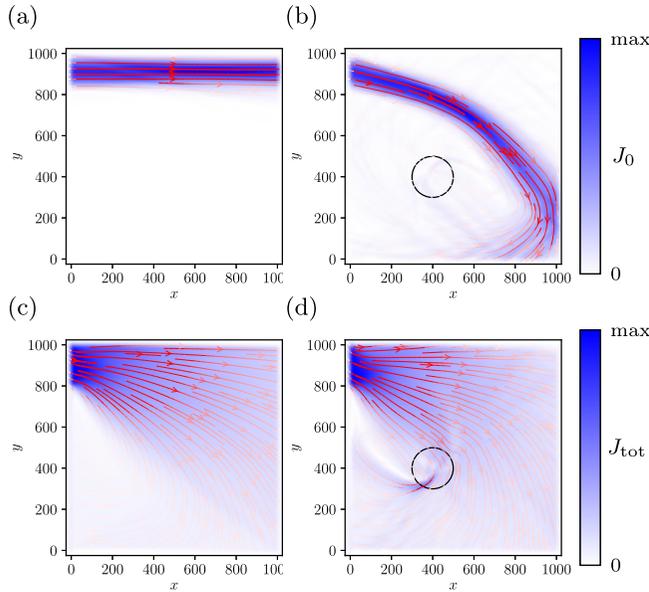


FIG. 1. Current density emanating from a lead in the upper left corner, flowing through a scattering region without [(a) and (c)] and with [(b) and (d)] a horizon (black circle). In (a) and (b) only a single mode is shown, while in (c) and (d) all modes are present. Color indicates the absolute value of the current density, red arrows its direction (see SM for details).

where the tilting \mathbf{t} depends on position, and σ_j are the Pauli matrices. The last term enforces the Hermiticity of the Hamiltonian.

In homogeneous systems, with constant \mathbf{t} , the Schrödinger equation can be easily solved using plane waves, but here the Hamiltonian is spatially varying. We assume that the tilting varies slowly with position, which allows us to use the eikonal approximation [14]. Details of the following derivation are available in the Supplemental Material (SM) [64]. Starting from a plane wave ansatz with position-dependent amplitude and phase, $\Phi_\alpha = A_\alpha(\mathbf{r}, t)e^{i\phi(\mathbf{r}, t)}$, and assuming slow variations of the amplitude, constant energy, and no interband mixing, we obtain the eikonal equation

$$E = \pm k - \sum_j t_j k_j, \quad (2)$$

where E is the fixed energy, and the wave vector is given by $\mathbf{k} = \nabla\phi$.

This problem is analogous to solving the Schrödinger equation of free electrons in an inhomogeneous potential [1], or solving the wave equation of light in inhomogeneous refractive media [14]. Fermat's principle stipulates that along the trajectories for which the variation of the optical path length is vanishing, we get constructive interference, and these will form the classical optical rays or trajectories. They can be obtained from a variational problem as

$$\delta \int \mathbf{k} \cdot d\mathbf{r} = \delta \int k \cos(\alpha - \vartheta) dl \equiv \delta \int n_{\text{eff}} dl = 0, \quad (3)$$

where n_{eff} is the effective refraction index and the angles are defined in Fig. 2. Notice that the effective refraction index depends on the direction of the group velocity (ϑ) and that the

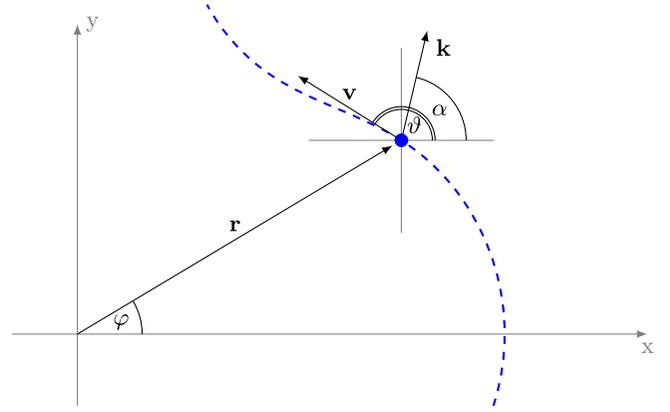


FIG. 2. Definition of the vectors of importance for the variational problem based on Fermat's principle. \mathbf{v} is the group velocity tangential to the trajectory (blue dashed line) with angle ϑ . \mathbf{k} is the wave vector with angle α . r and φ are the polar coordinates.

wave vector and the group velocity are not necessarily parallel ($\alpha \neq \vartheta$). These are major differences between the current problem and optics or electrons subjected to an inhomogeneous potential. They are classic signatures of anisotropic optics, seen also in for example crystal optics [14] or seismic wave propagation [16].

A striking consequence of its effect in classical anisotropic optics concerns refraction at perpendicular incidence to an interface, where the outgoing ray may leave at a finite angle. We calculated that our system does display an analog of this phenomenon, where an electron does not refract at a perpendicular angle upon impinging perpendicularly at an interface (see SM for a video).

Such anisotropic effects could be used to create electronic devices analogous to optical polarizers, which rely on birefringence. Indeed, one can consider a material with two Weyl cones, each forming a valley, with opposite tilting along the propagation direction. A wave packet formed of a superposition of the two valleys propagating in this system would split into two wave packets, each corresponding to a single valley. This *valley birefringence* could be used to create valley polarizers, with potential applications in the field of valleytronics [65].

To illustrate the rich anisotropic physics we can access in this inhomogeneously tilted Weyl system, we turn to two-dimensional systems with circular symmetry where the tilt can be expressed as

$$t_i(\mathbf{r}) = \frac{x_i}{r} t(r). \quad (4)$$

Transforming to polar coordinates (see Fig. 2), we can express the effective refraction index as [64]

$$n_{\text{eff}} = \frac{E}{v}, \quad v = \sqrt{1 - t^2 \sin^2(\vartheta - \varphi) - t \cos(\vartheta - \varphi)}. \quad (5)$$

with v the magnitude of the group velocity. Here, the energy E only appears as a multiplicative factor and does not affect the variational problem. Therefore, particles with different energies will trace out the same trajectories in space. This is a unique feature of the linear dispersion of the Weyl cones, and is not valid if the dispersion includes nonlinear terms.

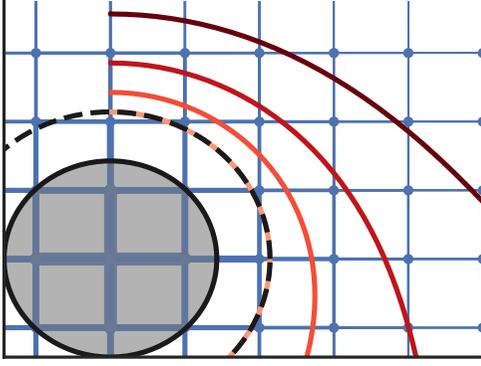


FIG. 3. Geodesics of light around a black hole (gray sphere) in the Painlevé-Gullstrand coordinates. The red lines represent light with a different initial position and zero radial velocity. The light gets deflected inward by the gravitational potential. The dashed black line is the photon sphere, the stable orbit of light. The blue dots and lines give a schematic representation of the lattice in a corresponding Weyl semimetal implementation, with thickness indicating the strength of hopping.

To solve the variational problem, we can take the commonly used arc-length parametrization of the trajectory [15,16] and find the Euler-Lagrange equations [64]. We then get a system of differential equations that, given an initial position (r_0, φ_0) and an initial direction (ϑ_0) , uniquely defines a classical trajectory for the electron. Importantly, it does not give any information regarding dynamics, such as the time it takes to travel along a trajectory.

Gravitational lensing. We now turn to a specific system with rotational symmetry, in which the anisotropic physics is directly inherited from relativistic light propagation in curved spacetime. Indeed, it has been shown recently that systems featuring tilted Weyl cones can be understood as analogs to propagation in spacetime in the proximity of a black hole horizon [17,18,21–29,35].

The connection to general relativity can be understood by choosing the following circularly symmetric metric in polar coordinates [66,67],

$$ds^2 = [t^2(r) - 1]dT^2 - 2t(r)drdT + dr^2 + r^2d\varphi^2, \quad (6)$$

where T denotes the temporal coordinate and $t(r)$ is the radial tilt defined in Eq. (4). This metric has a horizon when $t(r) = 1$. As shown in Ref. [28] the massless Dirac equation in a curved background defined by this metric leads to Eq. (1) in terms of tilted Weyl cones.

By choosing the specific tilting dependence

$$t^2(r) = \frac{2M}{r}, \quad (7)$$

the metric of Eq. (6) describes a Schwarzschild black hole with radius $r_s = 2M$ written in the Painlevé-Gullstrand coordinate system [64,68].

We are interested in the deflection of lightlike trajectories evidencing gravitational lensing, and its analogs in Weyl semimetals with inhomogeneous tilt profiles. The lightlike trajectories given by geodesics of the metric in Eq. (6) can be numerically calculated and are shown in Fig. 3 (for more details, see the SM). The different lines represent light rays ap-

proaching the black hole horizon with different impact factors. The black hole, whose horizon is represented by the black circle, can be clearly seen to curve the light inwards. At a distance $3r_s/2$ from the black hole center, we find the photon sphere (dashed circle) defined by light having a circular orbit around the black hole [69].

Note that the Painlevé-Gullstrand metric, which we use here, is related to the Schwarzschild metric by a transformation of the time coordinate while leaving the position coordinates unchanged. The spatial trajectories traced out by the light, and in particular the lensing due to the gravitational potential, are the same in both metrics, although the dynamics along those trajectories may differ (see SM for details).

Schwarzschild black hole with tilted Weyl nodes. In the following we consider a two-dimensional lattice model that hosts tilted Weyl cones at low energies, introduced in Ref. [28],

$$H = (\sigma_x - t_x \sigma_0) \sin k_x + (\sigma_y - t_y \sigma_0) \sin k_y + \sigma_z (2 - \cos k_x - \cos k_y), \quad (8)$$

where H is the local Bloch Hamiltonian. We focus on the specific case that maps to the Schwarzschild black hole where the radial tilting defined in Eq. (4) is $t(r) = \sqrt{r_s/r}$. This Hamiltonian is schematically depicted in Fig. 3: The lattice in blue has thicker edges where the hopping proportional to σ_0 is larger. The tilting therefore increases when approaching $r = 0$. The black line corresponds, in the electronic system, to the transition between type-I and type-II Weyl cones. In the gravitational system, this line corresponds to the black hole horizon.

To establish the extent of the connection between this tight-binding model and gravitational physics, we investigate the relation between the propagation of the wave packets in the tight-binding model and the geodesics of light defined by the metric. The wave packet trajectories in the tight-binding model for different impact parameters are shown in Fig. 4(a). These were obtained using two different methods: first, using Fermat's principle [Eq. (3)] and solving the Euler-Lagrange equation numerically [red curves in Fig. 4(a)], and second, by simulating the full quantum wave packet dynamics using the Chebyshev expansion method [70,71] on the tight-binding Hamiltonian given by Eq. (8) [blue lines in Fig. 4(a)]. The trajectories calculated using the two methods are in close agreement with each other and with the geodesics in Fig. 3. All trajectories are deflected, and decreasing the impact factor increases the curvature of the trajectory. If the starting position is on the photon sphere, the trajectory becomes circular, while smaller impact factors imply collapse towards the center of the analog black hole.

To understand the origin of the lensing effect in the context of tilted Weyl semimetals, we show a representation of the Fermi surface defined by the local Bloch Hamiltonian, at the energy of the wave packet, along five specific trajectories in Fig. 4(b), and on the whole lattice in Fig. 4(c). This energy is conserved and determines the dynamics of the wave packet. Far away from $r = 0$, the Fermi surfaces are almost circular, with the Γ point at the center. Getting closer to the horizon, the Fermi surfaces become elliptical, with the Γ point heavily shifted towards the horizon. In particular, this causes the wave vector and group velocity to not be parallel, similar to Eq. (3),

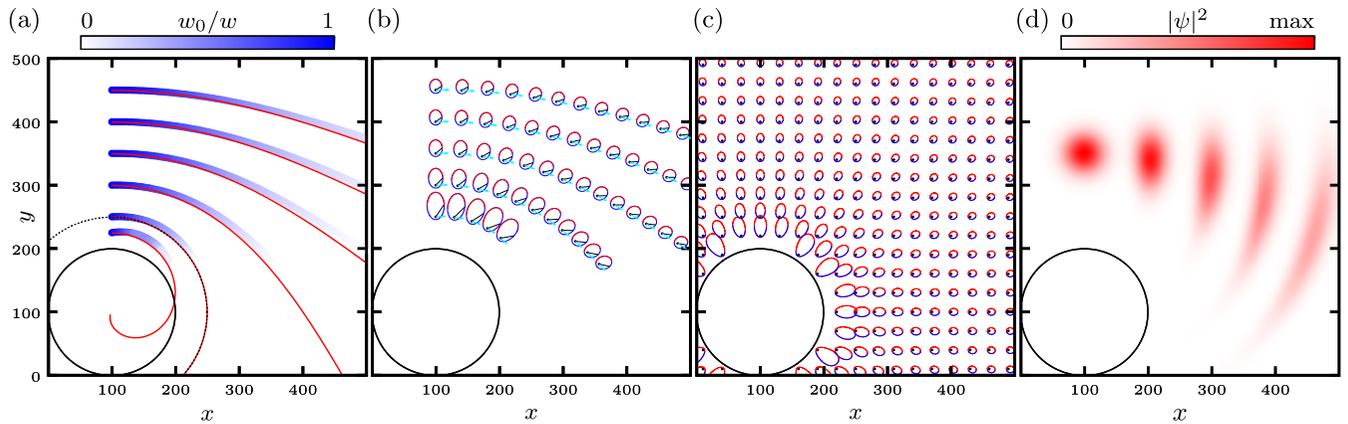


FIG. 4. (a) Calculated trajectories of wave packets using the Chebyshev expansion method (blue), and trajectories obtained using Fermat's principle (red), in an inhomogeneously tilted Weyl system. The black solid line delineates the transition between type-I and type-II Weyl cones, which corresponds to the black hole horizon in the analogous gravitational system. The dotted black line corresponds to the photon sphere. The shade of blue denotes the inverse of the width w of the wave packet relative to the initial width w_0 . (b) Constant energy lines in the local Bloch Hamiltonians along five different trajectories. The black and cyan lines indicate the wave vector and group velocity, respectively. (c) Constant energy lines for local Bloch Hamiltonians throughout the whole system. In (b) and (c) the Γ point of the local Brillouin zone is represented by the black point and the color of the energy lines denotes the sign of v_y , red (blue) is positive (negative). (d) Probability density of a propagating wave packet shown at five different time steps.

as depicted in Fig. 4(b). Along the trajectory, the wave vector must change continuously while conserving the energy. This, together with the rotation of the ellipses along the trajectory, leads to a rotation of the group velocity and to a bending of the trajectories around the black hole.

Moving beyond the semiclassical description, we can consider the actual wave packets and their broadening inside the system.

To quantify this, we calculate the maximal full width at half maximum (w) of the wave packet, represented in Fig. 4(a) by the color scale. The ratio of the initial width to the actual width (w_0/w) goes to zero for a delocalized wave function and is equal to one for the initial state. The w_0/w ratio decreases for all trajectories during their propagation through the system, but the decrease is much faster for trajectories that come close to the horizon. Snapshots of the propagation of one particular wave packet are depicted in Fig. 4(d).

The initially well-localized wave packet gets compressed in the direction of propagation and becomes wider in the perpendicular direction. The semiclassical description using Fermat's principle breaks down once the wave packet becomes too delocalized in real space, since the pointlike particle approximation is no longer valid. Furthermore, it also breaks down once the wave packet gets too squeezed, making it less localized in momentum space, and mixing in components for which the dispersion is no longer linear.

Current density. Having characterized the wave packet trajectories and their connection to geodesics and Fermat's principle, we turn to the effect of lensing on electronic currents, which can be accessed experimentally. We consider a multiterminal transport setup, where we attach leads to all sides of a square scattering region and use the KWANT package [72] to calculate the current density associated to the scattering states (see SM for details of the setup and calculations). The results are summarized in Fig. 1.

When inputting a single propagating mode with large momentum, shown in Figs. 1(a) and 1(b), the current densities resemble the semiclassical trajectories. Indeed, without any Weyl cone tilting, the current density is rectilinear and only sizable in a horizontal region, while including inhomogeneous Weyl cone tilting causes the current density to display a clear lensing effect.

Next, to consider the total transport through the scattering region, we include all propagating modes in the lead. Summing over these, we obtain the current density profile corresponding to what would be measured in a typical transport measurement, as shown in Figs. 1(c) and 1(d). The presence of the inhomogeneous Weyl cone tilting then bends the current density, which results in a significantly larger current reaching the bottom lead, although the effect is less prominent than when inputting a single mode. In terms of multiterminal conductances, increasing the radius of the overtilted region would make the conductance to the bottom lead grow while reducing the one to the right lead.

Conclusion. We showed that tilted Weyl semimetals with inhomogeneous tilting can be used to connect phenomena in distant fields including condensed matter physics, optics, and general relativity. We extended the well-known connection between general relativity and condensed matter physics to anisotropic optics.

Electronic analogies with anisotropic optics provide a promising platform to use phenomena related to exotic anisotropic effects, but with different ranges of wave numbers and energies. The trajectories of electrons can be obtained using an anisotropic effective refractive index.

In connection with general relativity, we related models of inhomogeneously tilted Weyl cones to massless particles propagating around a Schwarzschild black hole and suggested a setup to measure this phenomenon experimentally. We also showed how the lensing around a black hole is related to anisotropic optics.

With the rapidly growing number of realizations of tilted Weyl semimetals both in real condensed matter systems [22,73,74] and metamaterials [75,76], and different methods to engineer inhomogeneity [31,32,34–37] in the tilting, the effects discussed in this Letter can be experimentally accessed. Using the analogy to anisotropic optics one can devise effects and corresponding experimental realizations that would otherwise be impossible in systems of electrons in an inhomogeneous potential.

Note added. Recently, we became aware of a related work [77] that considers an alternative approach to gravitational lensing in a related system using semiclassical dynamics discussing Berry curvature effects. Where our studies overlap, we get consistent results.

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