Letter

Topological symplectic Kondo effect

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Multiple conduction channels interacting with a quantum impurity—a spin in the conventional "multichannel Kondo effect" or a topological mesoscopic device ("topological Kondo effect")—has been proposed as a platform to realize anyonic quasiparticles. However, the above implementations require either perfect channel symmetry or the use of Majorana fermions. Here we propose a Majorana-free mesoscopic setup which implements the Kondo effect of the symplectic Lie group and can harbor emergent anyons (including Majorana fermions, Fibonacci anyons, and $\mathbb{Z}_3$ parafermions) even in the absence of perfect channel symmetry. In addition to the detailed prescription of the implementation, we present the strong coupling solution by mapping the model to the multichannel Kondo effect associated to an internal SU(2) symmetry and exploit conformal field theory to predict the nontrivial scaling of a variety of observables, including conductance, as a function of temperature. This work does not only open the door for robust Kondo-based anyon platforms, but also sheds light on the physics of strongly correlated materials with competing order parameters.

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Introduction. The realization of fault tolerant quantum computation is a major goal of present day quantum research. Amongst the various hardware platforms suitable for this application, topologically ordered states with anyonic excitations are particularly appealing [1], as the robustness against noise and errors is a fundamental, intrinsic property of these quantum many-body phases. A classic platform for realizing anyons which has gained renewed interest in mesoscopic systems are frustrated and overscreened Kondo impurity models [2,3].

The SU(2) Kondo effect is a paradigmatic model of quantum many-body physics [2–7] which merges the physics of strong electronic correlations and entanglement, while its strong coupling physics is still amenable to nonperturbative analytical methods such as Bethe *ansatz* [8–10], conformal field theory (CFT) [11,12], and Abelian bosonization [13]. Even though the impurity spin in the conventional Kondo effect is perfectly screened at strong coupling, the overscreened multichannel Kondo (MCK) effect, in which $k > 2S$ electronic baths compete for screening a single spin S, is one of the earliest examples of quantum criticality and local non-Fermi liquid (FL) behavior and harbors a remnant zero temperature impurity entropy [14–16] $S_{\text{imp}} = \ln(g_k)$, with $g_k = \sqrt{2}, (1+\sqrt{5})/2, \sqrt{3}, \ldots$ for $S = 1/2$ and $k = 2, 3, 4, \ldots$ consistent with the quantum dimensions of Ising, Fibonacci, and $\mathbb{Z}_3$ parafermionic anyons. It has thus recently been proposed to exploit these anyons for quantum information theoretical applications [17–21], but a major technical difficulty is that multichannel Kondo physics, even with SU(N) and $N > 2$, is unstable with respect to unequal coupling to different electronic baths.

Stable overscreened fixed points may be achieved by using strongly interacting [22,23] or higher spin [24] conduction

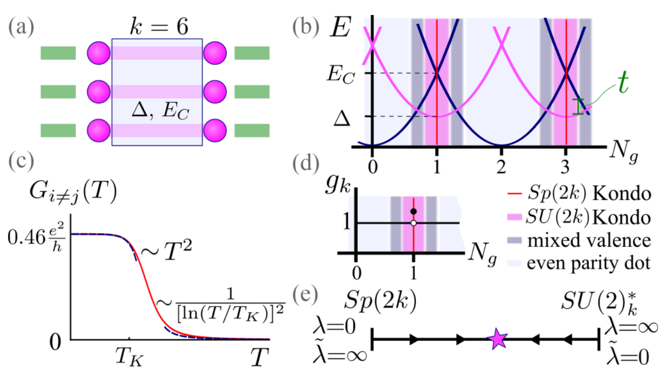

FIG. 1. (a) Schematics of proposed implementation for $k = 6$ with Δ the proximity-induced gap and E_C the charging energy. Light green squares are the leads and light purple dots are the ends of one-dimensional topological systems. The light gray square is a superconducting island. (b) Energy levels as a function of gate voltage (as imposed by charge N_g; we take $\Delta = 0.4E_C$). The tunneling strength t is between the dots and the leads (nearest sites). Dark blue (light purple) curves correspond to states with even (odd) fermion parity. The background shading corresponds to different effective low-energy theories (see legend). (c) Transconductance. The solid red line interpolates between the low- and high-temperature asymptotic behavior. The conductance quantization at $T = 0$ is universal, cf. Eq. (9), and equal to $(4/3)\sin^2(\pi/5) \approx 0.46e^2/h$ for $k = 3$. In the weak coupling regime, the conductance has a logarithmic temperature dependence. (d) Ground state degeneracy. (e) Schematic RG flow illustrating the duality between Sp($2k$) Kondo effect and k-channel SU(2) Kondo effect at spin $S = (k-1)/2$.

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electrons, or by going beyond the conventional SU(2) group. A recent example of the latter is the orthogonal Kondo effect in which spin-polarized conduction electrons couple to an impurity spin transforming under the group SO(M). The orthogonal Kondo effect for arbitrary M can be realized with the use of Majorana Cooper pair boxes [25–28], in which case it is called the topological Kondo effect. While fascinating, this implementation is temporarily elusive as the control over Majorana devices is still developing. Another, Majorana-free implementation for the special case $M = 5$ was recently proposed [29,30] and it was argued that Ising anyons (Majorana) are emergent at the infrared.

In this paper, we propose a mesoscopic setup, see Fig. 1, realizing the symplectic Kondo effect as a platform for anyons and potentially for measurement-only topological quantum computation [31]. Following Cartan's classification of Lie groups [32,33], we explore here the third remaining type of Lie group Sp($2k$), i.e., a symplectic Kondo Hamiltonian

$$H_K = \lambda \sum_{A=1}^{k(2k+1)} S^A J^A, \tag{1}$$

in which the symplectic impurity "spin" operators S_A transform in the fundamental $2k$-dimensional representation and $J^A = c_0^\dagger T_A c_0$ is the symplectic spin of conduction electrons; the spinor $c_a = (c_{a,1,\uparrow}, \dots, c_{a,k,\uparrow}, c_{a,1,\downarrow}, \dots, c_{a,k,\downarrow})^T$ for site a has $2k$ components with $i = 1, \dots, k$ denoting the lead index and $\sigma = \uparrow, \downarrow$ the physical spin. Despite the $2k$ components of the spinor, Eq. (1) is still a one-channel Kondo model and will therefore not suffer from channel anisotropy. The $2k \times 2k$ matrices $T_A = -\sigma_y T_A^T \sigma_y$ denote Sp($2k$) generators in the fundamental representation [34]. We present a mesoscopic implementation of this effect for arbitrary k, the phase diagram for this nanodevice, and characteristic signatures in transport measurements, as well as a solution of the symplectic Kondo effect in the strong coupling limit.

From the perspective of materials science, symplectic Kondo models are theoretically appealing as they allow for a proper definition of time reversal symmetry and thus for large-N descriptions of heavy fermion superconductors [35]. At the same time SO(5) $\sim$ Sp(4) theories of cuprates are popular approaches to account for competing orders [36]. From the viewpoint of quantum information theory, the symplectic Kondo effect allows for the arguably most robust way of realizing anyons in impurity models: in addition to the aforementioned stable implementation of nontrivial anyons, earlier work using CFT [29,30,33] demonstrates that—contrary to standard multichannel Kondo phenomenology—the leads behave FL-like (suggesting relatively strong decoupling of anyons and conduction electrons) and that Fibonacci anyons (which are the simplest anyons allowing for universal quantum computation) cannot be realized in the simplest realization of the topological Kondo effect, but are accessible in the present Sp(6) setup.

Implementation of the Sp($2k$) Kondo model. We consider k spinful fermionic zero-energy states coupled to a floating s-wave superconductor; see Fig. 1(a). These states may stem from a time-reversal symmetric higher-order topological insulator, resonant levels of quantum dots, or a set of Su-Schrieffer-Heeger chains. The low-energy Hamiltonian of our

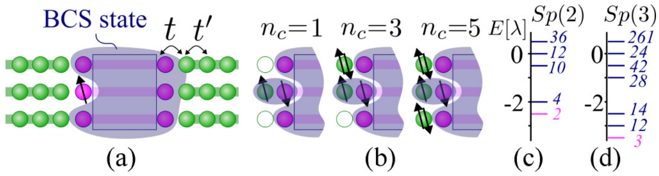

FIG. 2. (a) $2k$-degenerate ground state in the odd parity sector is given by a BCS state supplemented by one unpaired electron. (b) Illustration of k charge degenerate ground states in the extreme strong coupling limit $t' = 0$ and (c),(d) the corresponding energy spectrum.

topological quantum dot is

$$H_\mathrm{d} = E_C(2\hat{N}_C + \hat{n}_d - N_g)^2 - \frac{1}{2}\Delta \sum_{i=1}^{k} \sum_{\sigma\sigma'} e^{-i\phi} d_{i,\sigma}^\dagger (\sigma_y)_{\sigma\sigma'} d_{i,\sigma'}^\dagger + \text{H.c.}, \tag{2}$$

where $\hat{n}_d = \sum_{i,\sigma} d_{i,\sigma}^\dagger d_{i\sigma}$ is the total charge in the edge states and $\hat{N}_C = -i\partial_\phi$ is the number operator of the Cooper pairs of the s-wave superconductor. The Hamiltonian Eq. (2) conserves the total number of electrons $\hat{N}_\mathrm{tot} = 2\hat{N}_C + \hat{n}_d$, controllable by the gate charge N_g. We assume that the island size exceeds the superconducting coherence length, so that crossed-Andreev reflection as well as hybridization of zero modes can be neglected and that the proximity-induced gap Δ on the boundary states of the topological wires is less than the bulk gap, allowing us to ignore quasiparticle states of the parent superconductor in Eq. (2). We also take the gap to be smaller than the charging energy, $\Delta < E_C$, enabling a ground state with an odd number of electrons. We ignore additional mutual charging energies between the zero modes, which is a good assumption when the central superconducting island has a large normal-state conductivity [37,38].

In the absence of Δ, each state with even N_tot is degenerate, with allowed values $n_d = 0, 2, 4, \dots, 2k$ and all possibilities to distribute these electrons over the topological edge states. Similarly, the states with odd N_tot are also degenerate with $n_d = 1, 3, 5, \dots, 2k - 1$ allowed. The presence of Δ lifts the degeneracy as it allows one to connect different states and favors a single BCS-like ground state $|\mathrm{BCS}\rangle_d$ in the even sector (see Supplemental Material [39] for details). In the odd sector, there are $2k$ ground states given in which one of the k spin-degenerate boundary states is singly occupied, while the remaining $k - 1$ are occupied by a BCS-like state; see Fig. 2(a). The ground state energy of the even sector is

$$E_\mathrm{even}(N_\mathrm{tot}) = E_C(N_\mathrm{tot} - N_g)^2 - \Delta k, \tag{3}$$

while $E_\mathrm{odd} = E_\mathrm{even} + \Delta$. These energies are plotted in panel (b) of Fig. 1 (there, all energies E are measured with respect to $-\Delta k$).

In the $2k$-fold degenerate odd sector the quantum dot acts as an effective Sp($2k$) impurity. We will therefore consider N_g close to 1, where the $2k$ odd parity states with $N_\mathrm{tot} = 1$ are lowest in energy, while the lowest excited states (with $N_\mathrm{tot} = 0, 2$) are separated by an energy gap $\Delta E_\pm = E_\mathrm{even}(N_\mathrm{tot} = 1 \pm 1) - E_\mathrm{odd}(N_\mathrm{tot} = 1)$. To derive the effective Kondo interaction, we next consider tunneling between the electrons on the dot and the first site ($a = 0$) of the lead, $H_t = -\sum_{i=1}^{k}\sum_{\sigma=\uparrow,\downarrow} t_i c^\dagger_{0,i,\sigma} d_{i\sigma} + \text{H.c.}$ At low temperatures and bias voltages in the weak tunneling limit, $k_BT, eV, t_i \ll \Delta E_\pm$, the dot occupation cannot change and H_t induces an effective Kondo interaction in second order perturbation theory. When we fine-tune all $t_i = t$ ($\forall i = 1,\dots,k$), we get [39]

$$H_{\text{eff}} = -\lambda_1 (d^\dagger c_0)(c_0^\dagger d) - \lambda_2 (d^\dagger \sigma_y c_0^*)(c_0^T \sigma_y d), \tag{4}$$

where $c_a^* = (c_a^\dagger)^T$ and Gutzwiller projection to the $2k$ $N_{\text{tot}} = 1$ states is understood. The coupling constants are $\lambda_1 = 2t^2/(\Delta E_-) > 0$ and $\lambda_2 = 2t^2/(\Delta E_+) > 0$. Exactly at $N_g = 1$, and after using the completeness relation of symplectic generators, $\sum_A T_A^{ij} T_A^{kl} = [\delta_{il}\delta_{jk} - (\sigma_y)_{ki}(\sigma_y)_{jl}]/2$, Eq. (4) becomes a Kondo-type interaction, Eq. (1), with (bare) coupling constant $\lambda = 2\lambda_1 = 2\lambda_2 = 4t^2/(E_C - \Delta)$. As we will see below, the anisotropy of tunneling strength t_i is irrelevant.

Weak and strong coupling. Perturbation theory in the Kondo term H_K leads to the usual logarithmic divergence at second order [40]. We therefore use the renormalization group (RG) technique to analyze Eq. (1) upon lowering the bare bandwidth/cutoff $D_0 \sim E_C - \Delta$ to a running cutoff $D = D_0 e^{-l}$ [41,42]. We find the RG equation,

$$\frac{d\lambda}{dl} = (k+1)\rho_0\lambda^2, \tag{5}$$

where $\rho_0 = (\pi\hbar v_F)^{-1}$ denotes the lead density of states per spin per length and v_F is the Fermi velocity. Equation (5) implies that λ flows towards stronger coupling upon reducing the energy cutoff (set by, e.g., the temperature). We estimate the strong coupling scale to be $T_K \sim (E_C - \Delta)e^{-1/[\rho_0\lambda(D_0)2(k+1)]}$ in terms of the bare coupling.

Given that the isotropic weak-coupling fixed point ($\lambda = 0$) is unstable, with RG flow towards strong coupling, we will next investigate the stability of the strong-coupling fixed point, where the local Kondo interaction (1) is the dominant term in the Hamiltonian and we can treat kinetic energy $t' \sim 1/\rho_0$ of the leads perturbatively. On the bare level, this corresponds to the limit $t' \ll t^2/\Delta E_\pm$ of the mesoscopic device introduced above; see Fig. 2(a).

We start by finding the unperturbed ground state of Eq. (1), without kinetic terms. Similarly to Nozières' [43] description of the conventional SU(2) Kondo problem, the strong coupling ground state is given by singlets formed by the impurity and the conduction electrons. We systematically derived the spectrum [39] of this problem using representation theory, and additionally explicitly constructed the singlet ground state wave functions for all k and the excited states for $k = 2, 3$; see Figs. 2(c) and 2(d). We find that the Sp($2k$) Kondo Hamiltonian is overscreened, with k degenerate ground states at $t' = 0$; e.g., for $k = 2$ these are the Sp($2k$) singlets,

$$|N=1\rangle_{\text{singlet}} = -i(d^\dagger\sigma_y c_0^*)\,|0\rangle_c \otimes |\text{BCS}\rangle_d, \tag{6a}$$

$$|N=3\rangle_{\text{singlet}} = (d^\dagger c_0)\,|4\rangle_c \otimes |\text{BCS}\rangle_d, \tag{6b}$$

where $|4\rangle_c$ is the state in which all electronic states on the first site of the lead are filled, while $|0\rangle_c$ is the empty state. For generic k, the degeneracy is a consequence of the symplectic symmetry associated to superconductivity [44]: given that $|N=1\rangle_{\text{singlet}}$ is a singlet, the states $(c_0^\dagger\sigma_y c_0^*)|N=1\rangle_{\text{singlet}},\dots,(c_0^\dagger\sigma_y c_0^*)^{k-1}|N=1\rangle_{\text{singlet}}$ transform trivially under Sp($2k$), as well [45]; see Fig. 2(b). The above states, Eqs. (6a) and (6b), are related by particle-hole symmetry (PHS). More generally, PHS implies an inherent SU(2) symmetry in Nambu space for the symplectic Kondo Hamiltonian [35], which can be made apparent by writing the Sp($2k$) currents as symmetric form,

$$J^A = \frac{1}{2}\begin{bmatrix} c_0^\dagger & c_0^T(i\sigma_y)\end{bmatrix}\begin{pmatrix} T_A & \mathbf{0} \\ \mathbf{0} & T_A\end{pmatrix}\begin{pmatrix} c_0 \\ (-i\sigma_y)c_0^*\end{pmatrix}, \tag{7}$$

which is invariant under SU(2) rotations in particle-hole space. We used here the property $T_A^T = -\sigma_y T_A \sigma_y$ of Sp($2k$) generators. After having established the $t' = 0$ ground states, we now incorporate the nearest-neighbor hopping $H_{\text{NN}} = -t'\sum_{i=1}^{k}\sum_{\sigma=\uparrow,\downarrow}(c^\dagger_{0,i,\sigma}c_{1,i,\sigma} + c^\dagger_{1,i,\sigma}c_{0,i,\sigma})$ as a perturbation to study the stability of the strong coupling fixed point. H_{NN} will couple the degenerate strong coupling ground states, Eqs. (6a) and (6b), in second order perturbation theory, while preserving the SU(2) symmetry. Inspired by the SU(2) symmetry in the particle-hole space [see Eq. (7)] and the k singlets distributing in all odd-number particle sectors [39], we thus conjecture that the strong coupling Hamiltonian takes the form of the channel-isotropic k channel Kondo model,

$$H_s = \tilde{\lambda}\mathbf{S}\cdot\sum_{i=1}^{k}\mathbf{s}_i, \tag{8}$$

where the impurity SU(2) spin-$(k-1)/2$ operator $\mathbf{S}$ acts in the k-dimensional subspace [spanned by Eqs. (6a) and (6b) for $k = 2$], $\mathbf{s}_i = f_i^\dagger(\boldsymbol{\sigma}/2)f_i$ and $f_i = (f_{i\uparrow}, f_{i\downarrow})^T \equiv (c^\dagger_{1,i,\uparrow}, c_{1,i,\downarrow})^T$, with $i = 1,\dots,k$ labeling the effective channel of conduction electrons. Since $S = (k-1)/2 < k/2$, the MCK Hamiltonian (8) is overscreened [46]. We have explicitly proven the conjecture for $k = 2$ ($k = 3$) by second-order perturbation theory (Schrieffer-Wolff transformation), for which virtual fluctuations into the 62 (381) excited states lead to $\tilde{\lambda} = 24t'^2/(5\lambda)$ [$\tilde{\lambda} = 128t'^2/(21\lambda)$], respectively [39]. In this context it is also worthwhile to point out a hidden (larger) Sp($2k$) symmetry in the k-channel SU(2) Kondo effect [47].

Since the weak-coupling limit of the overscreened multichannel SU(2) model is unstable [46,48], the above map relating it to the strong-coupling limit of the symplectic Kondo model implies also the instability of the latter fixed point; see Fig. 1(e). Together with the instability of the weak-coupling fixed point of the Sp($2k$) Kondo problem, see Eq. (5), these findings indicate a single stable fixed point between the two, i.e., at an intermediate coupling. Our conjecture of a single fixed point is supported by the low-temperature impurity entropy (below) which is found to have the same value, when approaching from the weak [Sp($2k$)] and strong [k-channel SU(2)] coupling sides. Since Sp(2) is isomorphic to SU(2), our model provides an example of the level-rank duality [33] relating the weak and strong coupling theories.

Observables: Thermodynamics. Above, we argued that, near strong coupling, the model can be mapped to an overscreened k-channel spin-$(k-1)/2$ Kondo model, which has a stable intermediate coupling fixed point. We can use the impurity entropy [49] to characterize the effective residual ground state degeneracy g_k of this fixed point. The ground state degeneracy associated to screening a spin $(k-1)/2$ with k spin-1/2 channels is well known, $g_k = 2\cos[\pi/(k+2)]$

[14–16,49]. This result agrees with the impurity entropy of the Sp($2k$) Kondo problem, calculated using CFT [33] and Bethe ansatz [50].

In particular, we note that the case $k = 3$ has $g_3 = (1 + \sqrt{5})/2 = \varphi$, the golden ratio, indicating an emergent Fibonacci anyon. Crucially, in our symplectic Kondo model this Fibonacci anyon occurs even in the single-channel case [in the sense that our model, Eq. (1), is of level 1] and is therefore not subject to instability due to channel anisotropy, unlike previous examples in the three-channel Kondo [47,51] and two-channel topological Kondo [52] models.

Despite this appearance of the same anyon-like ground-state degeneracies and an unstable strong coupling fixed point which is equivalent to the k-channel spin-$(k-1)/2$ SU(2) Kondo model, we emphasize that in our model due to PHS, not all operators of the SU(2) Kondo model are effective. For example, the symplectic susceptibility involves the excitation of states outside the low-energy manifold Eqs. (6a) and (6b), leading to less singular behavior than for the SU$(2)_k$ susceptibility [39]. More generally, we expect that the irrelevant operator of scaling dimension $1 + 2/(2 + k)$ is forbidden for the dual Kondo problem, Eq. (8). This implies Fermi-liquid-like temperature and field dependence of thermodynamic quantities, consistent with results [29,30,33,50] based on the weak coupling Hamiltonian, Eq. (1).

We note that CFTs in which certain operators are symmetry disallowed are well known in the theory of (e.g., confinement-deconfinement) phase transitions in gauge theories and usually denoted by an asterisk [53–55]. In view of the relationship between deconfining gauge theories and overscreened Kondo impurities [56], we borrow the notation employed for the latter phenomena and denote the boundary CFT describing the dual Kondo problem, Eq. (8), as SU$(2)^*_k$.

Observables: Transport. We propose to test the nontrivial nature of the symplectic Kondo effect in a charge transport experiment across the mesoscopic island. As we explicitly demonstrate [39] using the CFT method [12,49,57–63], the fixed point off-diagonal conductance, Eq. (9), of Sp($2k$) Kondo model and the spin-1/2, k-channel SU(2) charge Kondo model [64,65] are identical up to normalization [66]. Nevertheless, we emphasize that our result is valid far from the charge degeneracy points, in the regime of elastic cotunneling akin to spin Kondo effect [67]. At low temperatures, $T \ll T_K$, near the intermediate coupling fixed point, the off-diagonal Sp($2k$) charge conductance is

$$G_{i\neq j}(T) = \frac{4e^2}{hk}\sin^2\left(\frac{\pi}{k+2}\right)\left[1 + c_{ij}\left(\frac{T}{T_K}\right)^2\right], \qquad (9)$$

where the $T = 0$ value is obtained in Ref. [39]. For $k = 2$ we have exactly half of the maximum conductance, analogous to halving of the conductance in the spin two-channel Kondo effect [68] and also similar to the conductance in the quarter-filling SU(4) Kondo model [69]. The finite-temperature correction with its dimensionless coefficient c_{ij} and the Kondo temperature T_K are determined from the microscopic physics; see below Eq. (5) for the latter. The temperature dependent transconductance (including larger temperature regimes) is plotted in Fig. 1(c). The exponent in the finite-temperature correction to $G_{ij}(T = 0)$ is determined by the scaling dimension $\Delta_{\rm LIO}$ of the leading irrelevant operator. Importantly, in the one-channel Sp($2k$) model the leading irrelevant operator [29,30,33] is local density-density interaction with $\Delta_{\rm LIO} = 2$, giving a FL-like temperature dependence while it is non-FL like for SU$(2)_k$. As explained above, also for SU$(2)^*_k$ the operator responsible for non-FL power laws is absent and we expect the exponent in Eq. (9) to be the same regardless of whether we approach the stable intermediate ($T = 0$) fixed point from weak or strong coupling. The exotic zero-temperature conductance value $G_{i\neq j}(0)$ reminiscent of the multichannel charge Kondo effect [3,4,64,65,70] together with FL corrections to it are unique signatures of the Sp($2k$) intermediate fixed point.

Effect of anisotropy and PHS breaking. When deriving the Sp($2k$) Kondo interaction, we required fine-tuning of the tunneling strengths $t_i = t$ ($\forall\, i = 1, \dots, k$) and particle-hole symmetry $N_g = 1$. Although these parameters can be controlled in experiments, we will discuss next what happens when we deviate from the requirements. We show that the former requirement can be relaxed but the deviation from PHS will drive the system towards an SU($2k$) Kondo fixed point. Let us first discuss the anisotropy of the tunneling amplitudes, while keeping the system PHS [39]. When we consider the anisotropic version of the effective Hamiltonian Eq. (4), the anisotropic tunneling strength $t_i > 0$ can be absorbed into the operators $\tilde{d} = \eta d$ and $\tilde{c}_0 = \eta c_0$, where $\eta = \mathbb{I} \otimes \mathrm{diag}(\sqrt{t_1}, \dots, \sqrt{t_k})/\sqrt{t}$, where t now denotes the geometric mean of t_i. The anisotropic Hamiltonian then takes the same form as Eq. (4), with the replacement $d, c_0 \to \tilde{d}, \tilde{c}_0$. Upon using the completeness relation and restoring the physical operators d, c_0, we obtain transformed generators $\eta T_A \eta$ in the operators S^A and J^A. The transformed generators are still Sp($2k$) generators because the matrix η commutes with $(\sigma_y \otimes \mathbb{I})$; thus $(\sigma_y \otimes \mathbb{I})(\eta T_A \eta)^T(\sigma_y \otimes \mathbb{I}) = -\eta T_A \eta$ according to the properties of Sp($2k$) generators [34]. Then, we can expand the transformed generators by the original generators: $\eta T_A \eta = \sum_B \kappa_{AB} T^B$. From this we see that the anisotropy of tunneling amplitudes is equivalent to the "exchange" anisotropy of the Sp($2k$) Kondo model, $H_K = \lambda \sum_{A,B} \kappa_{AB} S^A J^B$. Using the generalized version [39], weak anisotropies $|\kappa_{AB} - \delta_{AB}| \ll 1$ can be shown to be irrelevant on general grounds. The same situation occurs with SO(M), in the topological Kondo model [25,26,71–73], where the isotropic direction dominates the RG flow. We note however that in the effective strong coupling multichannel SU(2) model, Eq. (8), time-reversal symmetric tunneling anisotropy (unequal t'_i) corresponds to channel anisotropy which is a relevant perturbation. Thus the strong coupling multichannel Kondo physics requires fine-tuning of the Sp($2k$) symmetry.

While at weak coupling anisotropy in the tunnelcouplings is harmless, the Sp($2k$) is more sensitive to breaking of PHS. We first consider $N_g \neq 1$ ($\lambda_1 \neq \lambda_2$) in Eq. (4), while still requiring $t \ll \Delta E_\pm$ [regime of pink shading of Fig. 1(b)]. Then, we can rewrite Eq. (4) as a potential scattering term for conduction electrons and an anisotropic SU($2k$) Kondo interaction. This SU($2k$) Kondo model is exactly screened and has a FL fixed point and thus the non-FL fixed point of the Sp($2k$) Kondo model will be unstable. An example with $k = 2$ has been discussed in Ref. [30]. Also, the term arising from $\lambda_1 \neq \lambda_2$ maps to an effective magnetic field in the SU(2)

Kondo model in the strong coupling regime, similar to the case in charge Kondo [70]. Near the intermediate coupling fixed point such a perturbation is relevant, with a scaling dimension $\Delta_H = 2/(2+k)$, and drives the system to a FL fixed point [47]. Hence we conclude that the PHS breaking anisotropy ($N_g \neq 1$) is relevant. As N_g is further detuned from unity to a regime $t \sim \Delta E_\pm$ and further, we first enter an SU($2k$) mixed valence regime [dark gray in Fig. 1(b)], in which odd and even parity states are of comparable energy, and ultimately reach the regime in which the impurity ground state is nondegenerate. In the infrared, FL behavior persists; see Fig. 1(d).

Summary and conclusions. In summary, we proposed a mesoscopic implementation of the symplectic Kondo effect, in which the group Sp($2k$) naturally describes spin-1/2 fermions in k orbitals in a Coulomb blockaded island hosting k spinful topological Andreev states. We couple each Andreev state to a spinful fermion lead and found the symplectic Kondo Hamiltonian Eq. (1) for an odd-parity charge state of the Coulomb blockaded island.

Interesting open questions about the symplectic Cooper pair box setup include the Coulomb blockaded transport beyond $N_g = 1$ and complementary analytical, numerical, and experimental studies which should help shed light on the anyonic signatures and their quantum-information theoretic potential.

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