Letter

Disorder-induced spin-charge separation in the one-dimensional Hubbard model

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Many-body localization is believed to be generically unstable in quantum systems with continuous non-Abelian symmetries, even in the presence of strong disorder. Breaking these symmetries can stabilize the localized phase, leading to the emergence of an extensive number of quasilocally conserved quantities known as local integrals of motion, or l bits. Using a sophisticated nonperturbative technique based on continuous unitary transforms, we investigate the one-dimensional Hubbard model subject to both spin and charge disorder, compute the associated l bits and demonstrate that the disorder gives rise to a novel form of spin-charge separation. We examine the role of symmetries in delocalizing the spin and charge degrees of freedom, and show that while symmetries generally lead to delocalization through multiparticle resonant processes, certain subsets of states appear stable.

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I. INTRODUCTION

The study of localization in low-dimensional quantum systems due to disorder has a long history, from the original works of Anderson [1,2] through to modern ideas of many-body localization [3–10]. Conventionally, many-body localization (MBL) in systems with a random disordered potential is now understood in terms of the existence of an extensive number of local integrals of motions (LIOMs, or l bits), which decay exponentially in space and are related to the constituents of the microscopic model via a quasilocal unitary transform [11–15]. Several methods for constructing these l bits exist, including computing them directly via unitary transforms [16–20], extracting them from the long-time behavior of various observables [21,22], and from other hydrodynamic considerations [23].

Much of the focus has been on spinless fermionic systems or spin-1/2 chains, however, experiments [24–26] have also studied localization in systems of spinful fermions, which exhibit an SU(2) spin rotation symmetry. It has been argued that full many-body localization cannot exist unless all continuous non-Abelian symmetries are broken [27–37], and that even then, not all eigenstates may display typical MBL phenomena [38]. Models of spinful fermions present a major challenge to exact numerical methods due to the additional degree of freedom compared with spinless fermions, and this places strong limitations on the system sizes which can be studied numerically. In particular, little is known about the form of l bits of spinful fermions [34,39], despite their experimental relevance and the central role that l bits play in describing and understanding many-body localized quantum matter. The study of MBL in spinful fermionic systems is also relevant for the broader question of the stability of MBL, as the interaction between fermionic species can be viewed as a pseudo-system-bath coupling.

In this work, we make use of a technique designed to compute the local integrals of motion associated to a model with multiple fermionic species, and show that the existence of any continuous non-Abelian symmetry prevents the Hamiltonian from being written as a sum of mutually commuting local integrals of motion. When the symmetry is broken, we demonstrate that it is possible to write an effective Hamiltonian describing the system in terms of local integrals of motion associated to charge and spin degrees of freedom. This is reminiscent of the spin-charge separation seen in various regimes of the disorder-free Hubbard model, notably in the Luttinger liquid regime [40–42], the related strongly interacting t-J model [43–48] as well as in novel driven phases [49], but here it arises solely due to the presence of disorder [33].

II. MODEL

We will study the behavior of spinful fermions on a lattice in one dimension using the Fermi-Hubbard model, sketched in Fig. 1, which is given by:

$$\mathcal{H} = \sum_i \sum_{\sigma=\uparrow,\downarrow} h_i^\sigma n_{i,\sigma} - \sum_i \sum_{\sigma=\uparrow,\downarrow} t^\sigma (c_{i,\sigma}^\dagger c_{i+1,\sigma} + \text{H.c.}) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}, \tag{1}$$

where $h_i^\sigma \in [-d, d]$ represents the (spin-dependent) on-site disorder potential. Throughout, we will use $t^\sigma \equiv t = 1.0$ as the unit of energy, and set $U/t = 0.1$ unless otherwise stated. The disorder-free ($d = 0$) model has an SU(2) symmetry associated to global spin rotations, as well as an SU(2) pseudospin symmetry associated with charge degrees of freedom [38,50–54]. We will consider two types of disorder. The case of $h_i^\uparrow = h_i^\downarrow$ represents disorder in the charge degrees of freedom only, which preserves the SU(2) spin symmetry but breaks the SU(2) pseudospin symmetry, while the opposite case of $h_i^\uparrow = -h_i^\downarrow$ represents spin disorder, which breaks the SU(2) spin symmetry but preserves the pseudospin symmetry. In the noninteracting case of $U/t = 0$, the spin-up and spin-down fermions become completely decoupled and one can imagine the Hamiltonian Eq. (1) as describing two

independent chains of noninteracting particles. In the presence of disorder ($h_i^{\uparrow} \neq 0, h_i^{\downarrow} \neq 0$), these noninteracting chains will both be Anderson localized.

III. METHOD

We diagonalize the Hamiltonian up to terms of quartic order in the fermionic operators using the tensor flow equation (TFE) method [55,56], implemented using PYFLOW [57] and building on a variety of earlier works [20,58–60]. It is based on the Głazek-Wegner-Wilson flow, a method that has been widely used in condensed matter physics (and beyond) for a large variety of problems [17,19,61–73], including time-dependent [60,74–76] and open [77] systems. This technique is nonperturbative [70], but its success in fully diagonalizing the Hamiltonian depends on the microscopic model having a well-defined separation of energy scales. The method works by successively applying infinitesimal unitary transforms to the Hamiltonian until convergence is achieved, at each step removing some fraction of off-diagonal terms at the cost of inducing new higher-order terms proportional to powers of the interaction strength. In order to keep these terms under control, we will mainly work at weak interactions $U/t \ll 1$, such that any newly generated terms are small and can be neglected. A full error analysis is available in Ref. [56]. Here, we will focus on the qualitative form of the transformed Hamiltonian:

$$\begin{aligned}\tilde{\mathcal{H}} = &\sum_i (\tilde{h}_i^{\uparrow} : \tilde{n}_{i,\uparrow} : + \tilde{h}_i^{\downarrow} : \tilde{n}_{i,\downarrow} :) + \sum_{ij} + \tilde{\Delta}_{ij} : \tilde{n}_{i,\uparrow}\tilde{n}_{j,\downarrow} : \\ &+ \sum_{ij} (U_{ij}^* : \tilde{c}_{i,\uparrow}^{\dagger}\tilde{c}_{j,\uparrow}\tilde{c}_{i,\downarrow}^{\dagger}\tilde{c}_{j,\downarrow} : + \overline{U}_{ij} : \tilde{c}_{i,\uparrow}^{\dagger}\tilde{c}_{j,\uparrow}\tilde{c}_{j,\downarrow}^{\dagger}\tilde{c}_{i,\downarrow} :) \\ &+ \sum_{ij} (\tilde{\Delta}_{ij}^{\uparrow} : \tilde{n}_{i,\uparrow}\tilde{n}_{j,\uparrow} : + \tilde{\Delta}_{ij}^{\downarrow} : \tilde{n}_{i,\downarrow}\tilde{n}_{j,\downarrow} :) + \cdots , \end{aligned} \tag{2}$$

where the ... refers to higher-order terms which are neglected within the approximations made here, the $: O :$ notation signifies normal ordering, and the tilde notation indicates that all quantities are expressed in the transformed basis. The form of this Hamiltonian is independent of the dimensionality of the system.

A few comments are in order about the form of this effective Hamiltonian. First, and in contrast with previous implementations of the TFE method, this Hamiltonian is not always fully diagonalized. The presence of the spin SU(2) symmetry leads to a degeneracy between spin-up and spin-down sectors of the Hamiltonian, which in turn leads to a spin-flip term $\overline{U}_{ij}$. Similarly, the presence of the pseudospin SU(2) symmetry leads to a pair hopping term U_{ij}^*. If either symmetry is broken, and the resulting degeneracy lifted, the corresponding coupling constant is zero. These resonant terms couple degenerate states, and therefore cannot be removed by the TFE method. They do not commute with the rest of the Hamiltonian, ruling out the possibility of writing Eq. (1) solely in terms of mutually commuting quasilocal integrals of motion (as removing real-space degeneracies in this framework typically requires a nonlocal unitary transform). We will nonetheless refer to this as the diagonal basis, as all single-particle off-diagonal and multiparticle off-diagonal, off-resonant terms have been removed, and refer to number operators in this basis as l bits. Second, the action of the flow equation method leads to the emergence of new interaction terms $\tilde{\Delta}_{ij}^{\sigma} \propto tU^2$, which act within each spin species: these terms arise due to nonperturbative corrections computed with respect to an appropriate reference state (here taken to be an excited state of the noninteracting system) via a normal-ordering procedure [56], and are in some cases equivalent to one-loop renormalization group contributions [70,78]. The computational cost of each step of this method is $O(L^6)$, restricting us to small system sizes. For normal ordering with respect to the vacuum state, the corrections are zero and $\tilde{\Delta}_{ij}^{\sigma} = 0 \; \forall \; \sigma, i, j$. For normal ordering with respect to an arbitrary excited state, more appropriate for the study of many-body localization, these corrections are nonzero. This suggests that the low-energy and high-energy sectors of Eq. (1) may behave differently in the presence of disorder. Here, we choose the reference state to be a normalised eigenstate of the noninteracting system, which in the diagonal basis takes the form $|0, \uparrow\downarrow, 0, \uparrow\downarrow \ldots\rangle$ for charge disorder, and $|\uparrow, \downarrow, \uparrow, \downarrow \ldots\rangle$ for spin disorder [56]. We assume that superconductivity plays no role (as we consider a disordered system far from the ground state) and consequently no anomalous terms appear in Eq. (2).

Using the definition of charge ($\rho_i =: \tilde{n}_{i,\uparrow} : + : \tilde{n}_{i,\downarrow} :$) and spin ($\sigma_i =: \tilde{n}_{i,\uparrow} : - : \tilde{n}_{i,\downarrow} :$), we can (partially) rewrite this Hamiltonian in terms of mutually commuting l bits associated with spin and charge degrees of freedom:

$$\begin{aligned}\tilde{\mathcal{H}} = &\sum_i (h_i\rho_i + \overline{h}_i\sigma_i) + \sum_{ij} \left(\Delta_{ij}^{\rho}\rho_i\rho_j + \Delta_{ij}^{\sigma}\sigma_i\sigma_j\right) \\ &+ \sum_{ij} (\overline{U}_{ij} S_i^+ S_j^- + U_{ij}^* P_i^+ P_j^-) \\ &+ \sum_{ij} \Omega_{ij}(\rho_i\sigma_j + \sigma_i\rho_j) + \cdots \end{aligned} \tag{3}$$

with $h_i = \frac{1}{2}(\tilde{h}_i^{\uparrow} + \tilde{h}_i^{\downarrow})$, $\overline{h}_i = \frac{1}{2}(\tilde{h}_i^{\uparrow} - \tilde{h}_i^{\downarrow})$, $\Delta_{ij}^{\rho} = \frac{1}{4}(\tilde{\Delta}_{ij}^{\uparrow} + \tilde{\Delta}_{ij}^{\downarrow} + \tilde{\Delta}_{ij})$, $\Delta_{ij}^{\sigma} = \frac{1}{4}(\tilde{\Delta}_{ij}^{\uparrow} + \tilde{\Delta}_{ij}^{\downarrow} - \tilde{\Delta}_{ij})$, $\Omega_{ij} = \frac{1}{4}(\tilde{\Delta}_{ij}^{\uparrow} - \tilde{\Delta}_{ij}^{\downarrow})$ and we define the spin-flip term $S_i^+ S_j^- =: \tilde{c}_{i,\uparrow}^{\dagger}\tilde{c}_{j,\uparrow}\tilde{c}_{j,\downarrow}^{\dagger}\tilde{c}_{i,\downarrow} :$ and the pair hopping term $P_i^+ P_j^- =: \tilde{c}_{i,\uparrow}^{\dagger}\tilde{c}_{j,\uparrow}\tilde{c}_{i,\downarrow}^{\dagger}\tilde{c}_{j,\downarrow} :$ [79]. The first line of Eq. (3) represents two independent systems of charge and spin l bits, respectively, while the second line contains a coupling between them. In the case of charge disorder where only the spin SU(2) symmetry is preserved, we have $\overline{h}_i = 0 \; \forall i$ and $U_{ij}^* = \Omega_{ij} = 0 \; \forall i, j$, implying that in the absence of any other coupling all degrees of freedom would be localized. The spin-flip coefficient $\overline{U}_{ij}$ is not zero, however, and this term does not commute with the σ_i operators. This means that even though the charge degrees of freedom may be localized, the spin degrees of freedom are not. Similarly, in the case of spin disorder, the presence of an SU(2) pseudospin symmetry leads to the emergence of the pair hopping term $U_{ij}^* P_i^+ P_j^-$, which does not commute with the ρ_i operators, resulting in localization of the spin degrees of freedom but not the charges. In this case, however, both spin and charge degrees of freedom may eventually be able to equilibrate via the nonzero coupling Ω_{ij}, although in practice we find that this term is small and decays exponentially with distance, and as such delocalization via this term is likely to become extremely slow at strong disorder strengths. The spin-flip term

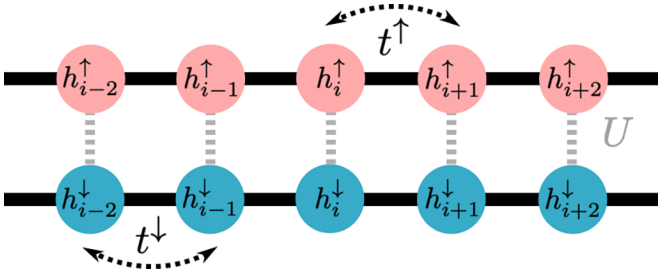

FIG. 1. A sketch of the Hubbard model described in Eq. (1) as two coupled chains of spinless fermions subject to a disordered on site potential h_i^{σ} and with hopping amplitude t^{σ}, interacting via an on-site repulsion U.

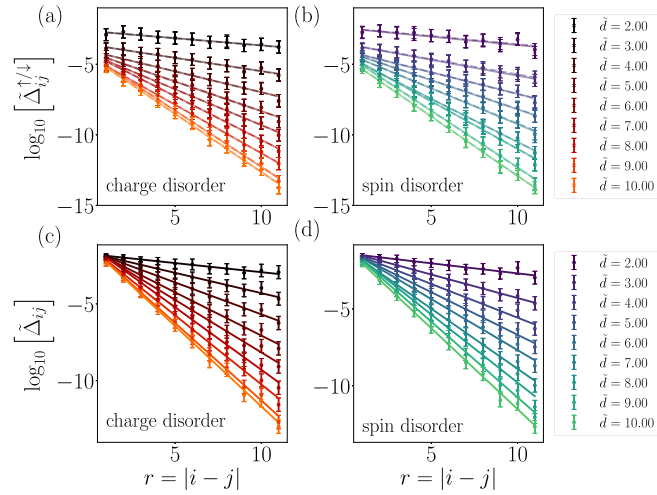

FIG. 2. Coupling constants extracted from the diagonal Hamiltonian [Eq. (2)] for a system of size $L = 12$ with $N_s = 128$ in the case of charge disorder (left column) and spin disorder (right column), with $\tilde{d} = d/t$. (a) Spin-up (crosses joined with solid lines) and spin-down (open circles joined with dashed lines) interactions. The solid and dashed lines are in most cases directly superimposed. b) The same, but here for spin disorder. (c) The coefficient of the mixed spin interaction term $\tilde{\Delta}_{ij}$ in the case of charge disorder. (d) The same again, but here for spin disorder. The decay of these interactions is essentially independent of the type of disorder used.

$S_i^+ S_j^-$, when rewritten in terms of the original operators in the $l = 0$ basis via the operator expansion $: \tilde{c}^{\dagger}_{i,\sigma} := \sum_{j,\sigma'} \tilde{\alpha}^{(i,\sigma)}_{j,\sigma'} : c^{\dagger}_{j,\sigma'} : + \sum_{jkq,\sigma',\sigma''} \tilde{\beta}^{(i,\sigma)}_{jkq,\sigma'\sigma''} : c^{\dagger}_{j,\sigma'} c^{\dagger}_{k,\sigma''} c_{q,\sigma''} : + \cdots$ [56], contains a spin-mediated hopping term of precisely the type obtained in Ref. [36], as well as additional higher-order terms, which combine to constitute a (slow) relaxation channel. The pair hopping term $P_i^+ P_j^-$ has a similar effect. As the microscopic interaction strength U is increased, both processes will become increasingly relevant for the dynamics, favoring thermalization. Curiously, there are also exceptional states where these terms can have no effect. Consider for example applying the spin-flip term to a CDW state in the diagonal basis, e.g., $|\tilde{\psi}\rangle = |0, \uparrow\downarrow, 0, \uparrow\downarrow \ldots\rangle$ This term can only exchange pairs of opposite spins on different sites, and cannot modify the CDW state as double-occupancy of the same spin species is forbidden by fermionic statistics. Similarly, if we consider applying the pair hopping term to a spin density wave (SDW) state in the diagonal basis, $|\tilde{\psi}\rangle = |\uparrow, \downarrow, \uparrow, \downarrow \ldots\rangle$, we find that it cannot change the state, as it can only move pairs of spins from one site to another. This suggests that for both types of disorder, although typical states will be delocalized, there may be rare states in which localization is stable to long times, although we cannot rule out the existence of weak higher-order processes involving three or more particles, which may destabilize these states on long timescales. This has clear parallels with weak ergodicity breaking, e.g., quantum many-body scars [80–82], and is consistent with similar observations in Refs. [38,83].

IV. NUMERICAL RESULTS

The results of the flow equation procedure are shown in Fig. 2 for both charge and spin disorder. We will use a system of size $L = 12$ and show the typical (median) magnitudes of various quantities $[\hat{O}]$ computed from $N_s \in [100, 128]$ disorder realizations. Error bars represent the statistical uncertainty (median absolute deviation) unless otherwise stated, and all joining lines are guides to the eye. A few general features are worth commenting on. First, the same-species interactions for spin-up (solid lines) and spin-down (dashed lines) l bits behave almost indistinguishably regardless of the type of disorder used, in all cases exhibiting an exponential decay with distance. The mixed spin interaction term $\tilde{\Delta}_{ij}$ behaves similarly. Interestingly, these coupling constants are all largely independent of the presence of either SU(2) symmetry, meaning those effects must be solely felt elsewhere. Second, for small values of d/t, all coupling constants become highly extended, at which point the neglected high-order terms in Eq. (3) become relevant and must be included. It is also interesting to see how the observed features change as the microscopic interaction strength U is increased. In Fig. 3(a), we show the coupling constants Δ_{ij}^{ρ} and Δ_{ij}^{σ} for a fixed (charge) disorder strength of $d/t = 3.0$ and for varying interaction strengths (with $N_s = 100$). Both interactions increase with U. For $U \gtrsim 1$, the neglected higher-order terms in Eq. (2) will become relevant and the current approximation will break down, but the qualitative behavior is nonetheless instructive and remains well controlled.

To unveil the role of the SU(2) symmetries, we now turn to the coupling between spin and charge l bits, namely the spin-charge coupling Ω_{ij}, the pair hopping term U_{ij}^*, and the spin-flip term $\overline{U}_{ij}$ in Eq. (3), shown in Figs. 3(b)–3(d), respectively ($N_s = 128$). The spin-flip term is only nonzero when the SU(2) spin symmetry is preserved, while the spin-charge coupling term is only nonzero when this symmetry is broken, and the pair hopping term is only nonzero in the presence of the SU(2) pseudospin symmetry. Figures 3(c) and 3(d) show that for weak disorder the couplings U_{ij}^* and $\tilde{U}_{ij}$ decay slowly and long-range resonances are possible, while at stronger disorder they are exponentially suppressed, suggesting delocalization will become parametrically slow for strong disorder.

We can also directly compute the integrals of motion associated with the spin-up and spin-down fermions, using the TFE method to transform the local operators in Eq. (2) back

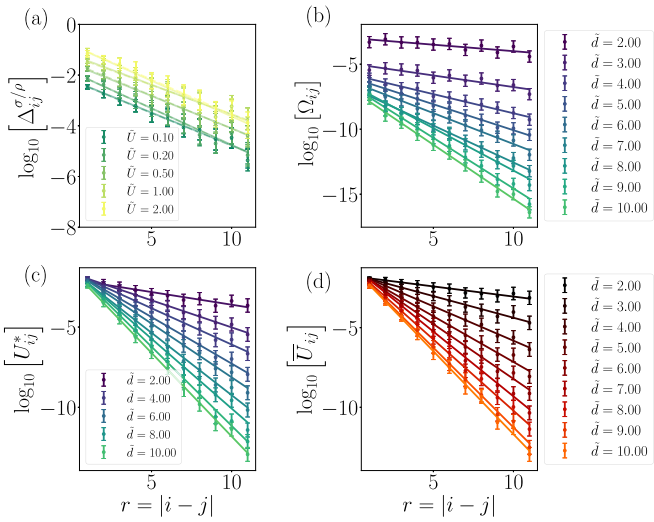

FIG. 3. (a) The coupling constants Δ_{ij}^{ρ} (solid lines) and Δ_{ij}^{σ} (dashed lines) extracted from Eq. (3) at fixed (charge) disorder $d/t = 3.0$ and varying interactions $\tilde{U} = U/t$. (b) The coefficient of the spin-charge coupling term in Eq. (3), for spin disorder and fixed interaction strength $U/t = 0.1$ and varying disorder strengths $\tilde{d} = d/t$. (c) The coefficient of the pair hopping term U_{ij}^{*} in Eq. (3), again for spin disorder and fixed interaction strength while varying the disorder. (d) The coefficient of the spin-flip term $\overline{U}_{ij}$ in Eq. (3), for charge disorder.

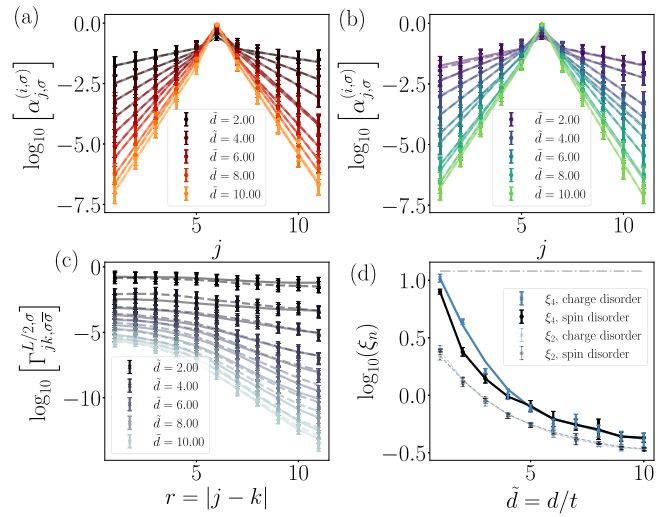

FIG. 4. (a) The quadratic terms $\alpha_{j,\sigma}^{(i,\sigma)}$ of Eq. (4) in the case of charge disorder, for $\sigma = \uparrow$ (solid lines) and $\sigma = \downarrow$ (dashed lines). (b) The same quantity in the case of spin disorder. (c) The quartic terms $\Gamma_{jk,\uparrow\downarrow}^{(i,\uparrow)} \equiv \zeta_{jjkk,\uparrow\downarrow}^{(i,\uparrow)}$ of Eq. (4) for charge disorder (solid) and spin disorder (dashed). (d) localization lengths ξ_2 (dashed) and ξ_4 (solid), for charge disorder (blue) and spin disorder (black). Error bars indicate the fitting uncertainty. The gray dashed line indicates the system size, $L = 12$.

into the original basis:

$$: \tilde{n}_{i,\sigma} := \sum_{j,\sigma'=\uparrow,\downarrow} \alpha_{j,\sigma'}^{(i,\sigma)} : n_{j,\sigma'} : + \sum_{jk,\sigma'} \beta_{jk,\sigma'}^{(i,\sigma)} : c_{j,\sigma'}^{\dagger} c_{k,\sigma'} : + \sum_{jkpq,\sigma'\sigma''} \zeta_{jkpq,\sigma'\sigma''}^{(i,\sigma)} : c_{j,\sigma'}^{\dagger} c_{k,\sigma'} c_{p,\sigma''}^{\dagger} c_{q,\sigma''} : + \cdots . \tag{4}$$

The quadratic terms $\alpha_{j,\sigma}^{(i,\sigma)}$ for both $\sigma = \uparrow, \downarrow$ are shown in Figs. 4(a)–4(b) as a function of disorder strength, for both spin and charge disorder ($N_s = 128$). They decay exponentially with distance $r = |i - j|$ and can be fitted with an exponential decay of the form $\sim e^{-r/\xi_2}$ to extract a localization length ξ_2. Similarly, we can plot the quartic coefficient $\zeta_{jkpq,\sigma'\sigma''}^{(i,\sigma)}$ [the $\Gamma_{jk,\uparrow\downarrow}^{(i,\uparrow)} \equiv \zeta_{jjkk,\uparrow\downarrow}^{(i,\uparrow)}$ component is shown in Fig. 4(c) other components behave similarly], which also decays approximately exponentially at large distance for all combinations of distances (e.g., $|i - j|$, $|i - k|$, $|j - k|$, and so on) for any choice of σ' and σ'', and we can fit the tails of these coefficients with a function $\sim e^{-r/\xi_4}$ to extract a second localization length ξ_4. Both localization lengths are shown in Fig. 4(d), and both exhibit similar monotonic decay with increasing disorder strength. At weak disorder, the localization length ξ_4 associated to the interacting part of the l bits becomes much larger then the noninteracting localization length ξ_2 and approaches the system size, suggestive of interaction-driven delocalization at sufficiently low values of d/t.

V. DISCUSSION

Starting from two decoupled, Anderson-localized chains of noninteracting spinful fermions, our results demonstrate that when a repulsive Hubbard contact interaction between both species is switched on, two different delocalization mechanisms emerge. One is the presence of the pair hopping and spin-flip terms, which lead to transport of the type identified in Refs. [36,38], while the other arises due to the slow decay of the coupling constants in the transformed Hamiltonian at weak disorder, resulting in a large l bit localization length. The former mechanism is activated by the presence of SU(2) symmetry (either spin or pseudospin), while the latter is unrelated to symmetry and results from the interplay of the microscopic interactions U/t and disorder strength d/t. While the existence of weak resonant terms in the presence of either SU(2) symmetry implies thermalization in the long-time limit, they do not rule out the emergence of long-lived prethermal or glassy states at large d where nonergodic features may persist for long times. Further investigating the form of this relaxation, and particularly whether it exhibits glassy effects such as aging, may be an interesting topic for future work.

VI. CONCLUSION

In this work, we have investigated the role of continuous non-Abelian symmetries on the formation of local integrals of motion in the one-dimensional Fermi-Hubbard model, and shown the origin of a form of disorder-induced spin-charge separation. While these numerical results specify to one dimension, the qualitative form of the Hamiltonian does not depend on the dimensionality, and it seems reasonable to expect that the presence of a continuous non-Abelian symmetry will generically forbid a multicomponent system from being written solely in terms of mutually commuting local integrals of motion in any dimension. This will prevent

full many-body localization, although there may exist atypical states, which the resonant terms cannot act upon [83]. These results obtained from the l bits complement prior studies, which examined the effects of SU(2) symmetry on the entanglement structure [30,38] and more general symmetry considerations [28]. Any perturbations, which break both the spin and pseudospin SU(2) symmetries, no matter how slight (e.g., uncontrolled impurities or stray magnetic fields) will act to stabilize localization. It would be very interesting to apply similar techniques in the limit of strongly interacting spinful fermions, e.g., to the disordered t-J model [43,84,85] or t-0 model recently studied in the context of MBL [86,87], and examine the effects of disorder in a sector where the clean system already exhibits strong spin-charge separation. In this regime, one might expect to see even more dramatic differences between spin and charge l bits.

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