Two copies of spin helices with stretching pitch and compensating helicity

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Persistent spin helix (PSH) manifests itself as an effective knob to tackle spin decay inevitably occurring in disordered two-dimensional electron gases. Here, for ordinary (110)-oriented two-subband GaInAs wells subjected to top and back gate voltages, we theoretically achieve adjusting the Dresselhaus terms of the two bands while meanwhile consistently *pinning* the system at symmetric configuration [i.e., locking the Rashba spin-orbit (SO) terms to zero], thus enabling *simultaneous* formation of two copies of PSHs of flexible control. Strikingly, we are able to stretch the pitch—spin density wave length—of PSH by far more than one period, enabling *helix-stretch functional* spin field-effect transistor (FET), with both on and off states protected by the PSH symmetry. Moreover, we attain a scenario in which the helicities of the two copies of PSHs are sufficiently compensated. This makes possible a new concept: "orbit (band) filter," which resembles spin FET while with novel functionality of orbit filtering, opening up a new route towards spintronic and orbitronic combined applications.

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I. INTRODUCTION

The spin-orbit (SO) interaction facilitates coherent spin manipulation $[1,2]$, and is of profound importance for diverse fields of condensed matter such as topological insulators [\[3\]](#page-4-0) and Majorana fermions [\[4,5\]](#page-4-0). However, the SO interaction inevitably causes spin decay and rotation-angle randomization [\[6–8\]](#page-4-0), which acts as fundamental challenges, not only limiting the functionality of SO based devices such as spin field-effect transistor (FET) but also setting great restrictions in quantum information science.

The persistent spin helix (PSH), which features invariance with respect to spin rotations [SU(2) symmetry] and is robust against any time-reversal conserving interaction [\[9–17\]](#page-4-0), provides a route to overcome spin decay occurring in disordered two-dimensional electron gases (2DEGs) [\[18](#page-4-0)[–22\]](#page-5-0). Since the experimental verification of PSH through transient spin grating spectroscopy [\[11\]](#page-4-0) and time-resolved Kerr rotation [\[12\]](#page-4-0), it has been exploited in various different forms including the drifting PSH driven by an in-plane electric field [\[23–27\]](#page-5-0) and the spin relaxation anisotropy mediated by an external magnetic field [\[28,29\]](#page-5-0), as well as its recent extension to Josephson junctions $[30,31]$ and even cavity photons $[32]$. Our recent proposals on the stretchable PSH [\[33\]](#page-5-0) and its symmetry breaking [\[34\]](#page-5-0) as well as the PSH-based persistent skyrmion lattice [\[35\]](#page-5-0), further manifest the importance of PSH.

Here, we theoretically demonstrate the emergence of two copies of PSHs, having not only *stretchy* pitch—spin density wave length—but also *compensating* helicity, in (110)-oriented GaInAs wells with two subbands. We utilize a technique, which relies on a combination of top (V_T) and back (V_B) gates [Fig. [1\(a\)\]](#page-1-0). With the help of self-consistent calculation of the Poisson and Schrödinger equations under the Hartree approximation, we obtain a full dual-gate multiband SO control [\[36\]](#page-5-0). In particular, we achieve adjusting the Dresselhaus SO couplings of the two bands while meanwhile *locking* the Rashba SO terms to zero, enabling *simultaneous* formation of two copies of PSHs (one for each band). Strikingly, we are able to stretch the pitch of PSH by far more than one period, enabling *helix-stretch functional* spin FET [Fig. $2(a)$], which works in two-dimensional (2D) diffusive regime with both on and off states protected by the PSH symmetry. Further, we attain a scenario that the helicities of the two copies of PSHs are sufficiently compensated [Fig. $1(e)$], resulting in persistent spin textures in both real and momentum spaces. This makes possible a new concept: "orbit (band) filter" [Fig. $2(b)$], which resembles spin FET while with novel functionality of *orbit filtering*, opening up a new route towards spintronic and orbitronic combined applications.

II. THEORETICAL FRAMEWORK: FROM A 3D TO AN EFFECTIVE 2D HAMILTONIAN

We start with the three-dimensional (3D) Rashba-Dresselhaus SO Hamiltonian for electrons hosted in

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FIG. 1. (a) Growth profile of a (110)-oriented $Al_{0.48}In_{0.52}As$ $Ga_{0.47}In_{0.53}As$ well subjected to top (V_T) and back (V_B) gates, and (b) its potential profile and wave functions of two subbands. (c) Three distinct regimes of dual-gate SO control: (i) $\alpha_{1,2} = 0$ (green); (ii) $\alpha_2 = 0$ (blue); and (iii) $\beta_1 + \beta_2 = 0$ (red). The contours of constant density (in unit of 10^{11} cm⁻²; gray) are also shown. (d) SO coefficients (upper panel: interband; lower panel: intraband) against V_T and *V*_B along the line of $\alpha_{1,2} = 0$ in (c). Black circle in (c) and (d) marks the "overlap" between regimes (i) and (iii). (e) Gate control of twoband PSHs *via* β_{ν} ($\nu = 1, 2$), the value of which complies with the lower panel in (d). The PSH for band 2 (green arrows) is greatly stretched, though only one period of stretching with β_2 ranging from 0.1 to 0.2 meV \cdot nm is shown, as guided by the vertical and curved (dashed) black lines; the PSH for band 1 (pink arrows) essentially maintains the same pitch, cf. β_1 and β_2 in the lower panel of (d). The shadowed regions at $\beta_1 = -\beta_2 = -0.18$ meV · nm indicate that the helicities of the copies of PSHs are sufficiently compensated. (f), (g) Schematic of opposite Dresselhaus fields for the first (g) and second (f) bands, with $\beta_1 < 0$ and $\beta_2 > 0$.

(110)-oriented wells in a reference frame of $x||[001]$, $y||[110]$, and *z*||[110] (for convenience):

$$
H = H_{\rm QW} + \beta(z)\sigma_z k_y + \alpha(z)(\sigma_y k_x - \sigma_x k_y), \qquad (1)
$$

where $H_{\text{QW}} = \frac{(k_x^2 + k_y^2)}{2m^* + k_z^2} / 2m^* + V_{\text{sc}}(z)$ is spin independent, with m^* the effective electron mass, $k_{x,y,z}$ the wave vector components, and V_{sc} the *self-consistent* potential comprising the structural V_w , the electron Hartree V_e , the doping V_d , and the gate $V_g(V_T, V_B)$ contributions. The second (third) term describes the Dresselhaus (Rashba) SO

interaction, in which $\beta(z) = -(1/2)[k_z\gamma(z)k_z + \gamma(z)(2k_x^2 - z)]$ (k_y^2)] and $\alpha(z) = \eta_w \partial_z V_w + \eta_H \partial_z (V_g + V_e + V_d)$ define the cor-responding SO strength [\[37\]](#page-5-0) and $\sigma_{x,y,z}$ are the spin Pauli matrices. Here, η_w and η_H contain bulk quantities of the well layer [\[33,35,38\]](#page-5-0), and $\gamma(z)$ the layer-dependent bulk Dresselhaus parameter. Note that, for $\beta(z)$, only dominant terms are kept, as widely done in literatures [\[8,21,](#page-4-0)[39\]](#page-5-0), and for the other terms, see the Supplemental Material (SM, Sec. I) [\[40\]](#page-5-0).

By projecting the 3D form $[Eq. (1)]$ onto the two spindegenerate eigensolutions of H_{QW} : $\langle \mathbf{r} | \mathbf{k}, v, \sigma \rangle = e^{i\mathbf{k} \cdot \mathbf{r}} \psi_{v}(z) | \sigma_{z} \rangle$, $\nu = 1, 2, \sigma_z = \uparrow, \downarrow, \text{ with energies } \varepsilon_{\nu,k} = \varepsilon_{\nu} + \hbar^2 k^2 / 2m^*$, where **k** is the in-plane electron wave vector and ε_v is the *v*th energy level, we obtain the 4×4 2D Hamiltonian

$$
\mathcal{H} = \left(\frac{\hbar^2 k^2}{2m^*} + \varepsilon_+\right) 1 \otimes 1 - \varepsilon_- \tau_z \otimes 1 + \mathcal{H}_{\text{RD}},\qquad(2)
$$

in which $\varepsilon_{\pm} = (\varepsilon_2 \pm \varepsilon_1)/2$, $\tau_{x,y,z}$ denote the "pseudospin" Pauli matrices in the orbital (band) subspace, and H_{RD} describes the 2D Rashba and Dresselhaus couplings

$$
\mathcal{H}_{\text{RD}} = \frac{1}{2} g \mu_B \sum_{\nu=1,2} \left[\tau_{\nu} \otimes \boldsymbol{\sigma} \cdot \mathbf{B}_{\text{SO}}^{\nu} + \tau_{\text{x}} \otimes \boldsymbol{\sigma} \cdot \mathbf{B}_{\text{SO}}^{12} \right], \quad (3)
$$

with *g* the electron *g* factor, μ_B the Bohr magneton, and $\tau_{1,2}$ = $(1 \pm \tau_z)/2$. The intraband SO field reads

$$
\mathbf{B}_{\rm SO}^{\nu} = -\frac{2}{g\mu_{\rm B}}\mathbf{k} \bigg[\alpha_{\nu}(\sin\theta\hat{\mathbf{x}} - \cos\theta\hat{\mathbf{y}}) + \frac{1}{2}\beta_{\nu}\sin\theta\hat{\mathbf{z}} \bigg]. \quad (4)
$$

Here we have defined $\tan \theta = k_y/k_x$, and the *intraband* SO couplings $\alpha_v = \langle v | \alpha(z) | v \rangle$ (Rashba) and $\beta_v = \beta_{1,v} - \beta_{3,v}$ (Dresselhaus), with $\beta_{1,v} = \langle v|k_z \gamma(z)k_z|v \rangle$ the linear term hav-ing interface contribution [\[2,](#page-4-0)[37\]](#page-5-0) and $\beta_{3,\nu} = \langle \nu | \gamma(z) | \nu \rangle k^2/4$ the cubic renormalization. The interband SO field is written as

$$
\mathbf{B}_{\rm SO}^{12} = -\frac{2}{g\mu_{\rm B}}k \bigg[\eta(\sin\theta \hat{\mathbf{x}} - \cos\theta \hat{\mathbf{y}}) + \frac{1}{2}\Gamma\sin\theta \hat{\mathbf{z}} \bigg], \quad (5)
$$

where $\eta = \langle v | \alpha(z) | v' \rangle$ (Rashba) and $\Gamma = \langle v | \beta(z) | v' \rangle$ (Dresselhaus) denote the *interband* SO couplings. For detailed derivations of 2D Hamiltonian including both the first and third harmonic terms, see the SM (Sec. I) [\[40\]](#page-5-0).

III. RESULTS AND DISCUSSION

A. Three distinct regimes for dual-gate SO control

We consider an $\text{Al}_{0.48}\text{In}_{0.52}\text{As/Ga}_{0.47}\text{In}_{0.53}\text{As/Al}_{0.48}\text{In}_{0.52}$ As quantum well grown along the *z*||[110] direction, of width 24 nm subjected to top (V_T) and back (V_B) gates [Fig. 1(a)], similar to experimental samples of Ref. [\[41\]](#page-5-0) while with two bands [Fig. 1(b)]. Our structure contains two *symmetrically* doped layers of width 6 nm sitting 18 nm away from either side of the well, with the donor concentration $\rho =$ 10×10^{18} cm⁻³. By adjusting *V*_T and *V*_B, which may compensate each other in varying the well symmetry, we achieve three distinct regimes for SO control: (i) Rashba terms α_1 and α_2 are simultaneously *locked* to zero, following the well being *pinned* at symmetric configuration [green line in Fig. 1(c)]; (ii) α_2 maintains zero but α_1 is largely finite [blue line in Fig. 1(c)]; and (iii) Dresselhaus terms β_1 and β_2 have equal

FIG. 2. Side view of dual-gate helix-stretch functional spin FET (a) and orbit filter (b). The labels S and D respectively denote ferromagnetic source and drain with the 2DEG channel sandwiched in between, and 1 (2) stands for the first (second) band. (c)–(f) Coherent superposition of two copies of PSHs, when $s_1(0) = s_2(0) = |\uparrow_x\rangle$ (c), (d); **s**₁(0) = **s**₂(0) = $|\uparrow_{xz}\rangle$ (e); and **s**₁(0) = $|\uparrow_{y}\rangle$, **s**₂(0) = $|\uparrow_{x}\rangle$ (f), where $|\uparrow_{xz}\rangle$ refers to the spin state pointing along the direction bisecting the angle between *x* and *z* axes. The size of circles (arrows) denote s_z ($s_{x,y}$), and red circles stand for spin up. In (c), $\beta_2 = -2\beta_1$; in (d)–(f), $\beta_2 = -\beta_1$, with $\beta_1 = -0.18$ meV nm [Fig. [1\(e\)\]](#page-1-0).

strength but opposite signs [red line in Fig. $1(c)$]. These three regimes underlie our PSH control, as we discuss next.

B. PSH pitch stretching and helix-stretch functional spin-FET in symmetric well

In regime (i), since the Rashba α_1 and α_2 both identically vanish, the overall SO field within the νth band is solely determined by the Dresselhaus field \mathbf{B}_{D}^{ν} , which is *intrinsically* perpendicular to the 2DEG plane [Figs. $1(f)$ and $1(g)$], ensuring *simultaneous* formation of two copies of PSHs (one for each band). In particular, for band 2, we are able to enhance the Dresselhaus strength β_2 by nearly a factor of four as V_B (\approx V_T) varying from −0.5 to 0.5 eV [Fig. [1\(d\)\]](#page-1-0), resulting in the PSH pitch, $P_2 = 2\pi/Q_2$, $Q_2 = m^* \beta_2/\hbar^2$, a stretch of far more than one period, see the *stretching* green arrows from $y = 5.05$ to 10.05 μ m in Fig. [1\(e\).](#page-1-0) This offers a unique platform for 2D diffusive spin-FET functioning for disordered electrons [Fig. $2(a)$], with both on and off states controlled by $V_g(V_T, V_B)$ and protected by the PSH symmetry, robust against any time-reversal conserving interactions (e.g., disorder). In contrast, regarding band 1, we reveal that β_1 exhibits inertia against $V_g(V_T, V_B)$, thus the corresponding PSH essentially maintains unstretched [pink arrows in Fig. $1(e)$].

C. Subband-selective PSH in asymmetric well

The Rashba terms are usually nonzero in structurally asymmetric wells [\[42\]](#page-5-0). Interestingly, for 2DEGs of double occupancy, due to delicate interplay of distinct SO contributions from several constituent potentials [i.e., V_w , V_e , V_d , and $V_g(V_T, V_B)$, we reveal that the electrons occupying the second band may see a *local* symmetry such that α_2 vanishes even in the well with an *overall* asymmetry (though α_1 is nonzero). This refers to our regime (ii), in which the PSH only survives for the second band, allowing *subband-selective* PSH control in asymmetric wells. And, here we achieve *continuous* selective control of the PSH, see in Fig. $1(c)$ the blue line, along which the SO manipulation is given in the SM (Sec. IV) [\[40\]](#page-5-0).

D. PSHs with compensating helicity: Orbit (pseudospin) filter

We attain a scenario which features an *overlap* between regimes (i) and (iii), namely the relations $\alpha_1 = \alpha_2 = 0$ and $\beta_1 = -\beta_2$ simultaneously hold, see the black circle in Figs. $1(c)$ and $1(d)$. The former relation, originating from the well being pinned at symmetric configuration, ensures that the two copies of PSHs—one for each band—form simultaneously. And, the latter one leads to the two PSHs being of not only equal pitch $P_1 = P_2$, $P_\nu = 2\pi / |Q_\nu|$, $Q_\nu = m^* \beta_\nu / \hbar^2$, but also opposite helicity, see in Fig. $1(e)$ the lower (band 1) and upper (band 2) shadowed regions. The compensating helicity of the two PSHs directly follows from that the first- and second-band electrons see the SO field of opposite directions [cf. Figs. $1(f)$ and $1(g)$].

These features underneath the two copies of PSHs make possible a new concept: "orbit (band) filter." It *resembles* spin FET while embraces novel functionality of *orbit filtering* [Fig. $2(b)$], which is controllable by $V_g(V_T, V_B)$, opening up a new route towards spin-orbitronic applications. And, the electrons occupying the first band in Fig. 2(b) are filtered out due to opposite spins between the 2DEG channel and the drain, facilitating band-selective spin manipulation.

In contrast to (110) wells considered here, note that in (001) wells, as we recently proposed persistent skyrmion lat-tice formed by the so-called two-band "crossed" PSHs [\[35\]](#page-5-0), the SO fields of the two bands align in the 2DEG plane and are orthogonal. Also, to stretch the PSH in (001) wells, one needs to tune the Rashba and Dresselhaus SO terms *independently* so that the two terms are not only adjusted *simultaneously* but also locked to equal strengths [\[33,34\]](#page-5-0). Thus, in practice, it is essentially not feasible to achieve the orbit filter by resorting to a well grown along the (001) direction.

E. Robust persistent spin textures in real space: Spatial superposition of two copies of PSHs

The unidirectional Dresselhaus field for (110) wells defines the persistent spin texture in momentum space [\[44–47\]](#page-5-0). Now, we move to spatial superposition of PSHs with compensating helicity. For realistic considerations, we take into account the presence of spin-independent potential $V(\mathbf{r})$, which may arise from a nonmagnetic disorder, to determine *robust* eigenspinors for 2DEGs hosted in (110) wells with two bands. Then, the electron Hamiltonian reads, $H_{\text{im}} = H + (1 \otimes 1)V(\mathbf{r})$, with H given in Eq. [\(2\)](#page-1-0). For GaAs and GaInAs based wells of typical electron densities such that the Fermi wave vector is far away from the crossings of energy

dispersions of the two bands, the interband terms can be treated as a perturbation [\[35,48\]](#page-5-0). Accordingly, we have $\mathcal{H}_{im} \to \sum_{v} \tau_{v} \otimes \mathcal{H}_{im}^{v}$, with $\mathcal{H}_{im}^{v} = [\varepsilon_{v} + \hbar^{2} k^{2}/2m + V(\mathbf{r})] \mathbb{1} + (1/2) g \mu_{B} B_{D}^{v} \sigma_{z}$, which admits eigenstates of the form $\psi_v^{\uparrow_z}(\mathbf{r}) = \phi(\mathbf{r})e^{i\frac{Q_v y}{2}}|\uparrow_z\rangle$ and $\psi_v^{\downarrow_z}(\mathbf{r}) = \phi(\mathbf{r})e^{-i\frac{Q_v y}{2}}|\downarrow_z\rangle$. Also, the function $\phi(\mathbf{r})$ fulfills the spin-independent equation $[-(\hbar^2/2m^*)\nabla^2 + V(\mathbf{r})]\phi(\mathbf{r}) =$ $(\varepsilon - \varepsilon_v + m^* \beta_v^2 / 8\hbar^2) \phi(\mathbf{r})$, where ε is the eigenvalue for either $\psi_{\nu}^{\uparrow z}(\mathbf{r})$ or $\psi_{\nu}^{\downarrow z}(\mathbf{r})$. Thus, the two spin states are doubly degenerate owing to time reversal symmetry and are robust against any nonmagnetic scatterings. The underlying physics is rooted in the commutation relation $[\mathcal{H}_{im}^{\nu}, \sigma_z] = 0$, valid for both bands. Next, we construct superposition of two copies of PSHs of compensating helicity.

Let $\psi_{\nu}(\mathbf{r}) = \phi(\mathbf{r})[\exp(iQ_{\nu}y/2)|\uparrow_z\rangle + \exp(-iQ_{\nu}y/2)|\downarrow_z\rangle)]$ $\sqrt{2}$ such that the stationary spin states at $\mathbf{r} = \mathbf{0}$ point along the *x* direction for both bands $v = 1, 2$. This results in the *v*th band spin density $\mathbf{s}_{\nu}(\mathbf{r}) = (1/2)\psi_{\nu}^{\dagger}(\mathbf{r})\sigma\psi_{\nu}(\mathbf{r})$. Then, for coherent superposition of stationary states $\psi(\mathbf{r}) =$ (1/ $\sqrt{2}$)[ψ_1 (**r**) \oplus ψ_2 (**r**)], the overall spin density **s**(**r**) = $(1/2)\psi^{\dagger}(\mathbf{r})(1 \otimes \sigma)\psi(\mathbf{r})$ reads

$$
\mathbf{s}(\mathbf{r}) = \frac{1}{4} |\phi(\mathbf{r})|^2 \sum_{\nu} [\cos(Q_{\nu} y)\hat{x} - \sin(Q_{\nu} y)\hat{y}]. \tag{6}
$$

Here the last term of $sin(Q_v y)$, which depends on the sign of β_{ν} , dominates the helicity of $s(r)$. Besides the quantummechanical approach, we also obtain Eq. (6) *via* the semiclassical approach based on 2D diffusive kinetic equation [\[26,27\]](#page-5-0) (see the SM, Sec. V) [\[40\]](#page-5-0). Accordingly, in the scenario of $\beta \equiv \beta_1$ (= - β_2), i.e., $Q \equiv Q_1$ (= - Q_2), the helicity of the overall spin density vanishes as a result of the helicities of **s**¹ and **s**² being sufficiently compensated (see the SM, Sec. III) [\[40\]](#page-5-0), leading to persistent (unidirectional) spin texture even in real space [Fig. $2(d)$]. Without lack of generality, to ob-tain Fig. [2\(d\),](#page-2-0) we have taken $|\phi(\mathbf{r})|^2 = 1$, referring to weak disorder, for which $\phi(\mathbf{r}) \rightarrow \exp(i\mathbf{k} \cdot \mathbf{r})$. In contrast, when β_1 and β_2 have different magnitudes, the overall spin density $s(r)$ still displays partial helicity, owing to distinct pitches of the two-band PSHs [Fig. $2(c)$]. In addition, when both s_1 and s_2 at $\mathbf{r} = 0$ are oriented in a direction bisecting the *x* and *z* axes, the s_x component exhibits similar spatial distribution to that in Fig. $2(d)$, while the s_z component maintains unchanged as it aligns with the Dresselhaus field [cf. Figs. $2(e)$ and $2(d)$]. Further, it is noteworthy that in the case of $\beta_1 = -\beta_2$, even when $s_1(0)$ and $s_2(0)$ point along distinct directions [\[49\]](#page-5-0), e.g., *y* and *x* directions for bands 1 and 2, respectively, the helicity of coherently superimposed **s**(**r**) also vanishes [Fig. [2\(f\)\]](#page-2-0).

F. The interband SO (Γ and η) contributions: **Band dispersion, spin texture, and spin relaxation**

For the well being pinned at symmetric configurations so that the two copies of PSHs form simultaneously, we reveal that the interband Dresselhaus Γ vanishes due to distinct parities of the wave functions of the two bands [upper panel of Fig. [1\(d\)\]](#page-1-0), similar to intraband Rashba α_{ν} . When β_1 and β_2 have opposite signs, referring to the scenario of two PSHs having compensating helicity, we observe that η maintains the feature of crossing of uncoupled ($\eta = 0$) band

FIG. 3. (a) Spin-resolved energy dispersion (scaled by a factor of 50 for visibility) for the (110) well in the scenario of $\beta_1 = -\beta_2$, $\alpha_{1,2} = 0$ (intraband), and $\Gamma = 0$, $\eta \neq 0$ (interband), as marked by black circle in Figs. $1(c)$ and $1(d)$. The size of circles denotes the magnitude of spin and the color stands for s_z , with the red (blue) referring to spin up (down), indicating that there is no spin hybridization occurring. (b) A counterpart to (a) while with β_1 and β_2 having the same sign. (c) Constant energy contours and spin textures at $E = E_F = -0.37$ eV [see (b)] [\[43\]](#page-5-0). The shadowed region indicates spin hybridization of distinct spin branches. (d) $\langle \sigma_z \rangle$ versus k_y , with the line type (and color) corresponding to respective energy branches in (c). The parameters in (c) and (d) are the same as those in (b).

dispersions, as indicated by the spin-polarized electronic structure in Fig. $3(a)$. Clearly, the spins for all the four energy branches (two for each band) remain invariant, which indicates that the spin states maintain unhybridized even in the presence of interband coupling, greatly quenching the detrimental effect of η on the two copies of PSHs. Also, the D'yakonov-Perel spin relaxation [\[50\]](#page-5-0) due to either third harmonic terms or interband corrections is suppressed, as both factors do not alter the fundamental symmetry of the SO field in (110)-oriented wells (see the SM, Secs. I and II) [\[40\]](#page-5-0). While the interband scattering may lead to the Elliott-Yafet type spin relaxation $[8,51,52]$ $[8,51,52]$ we obtain, the relaxation time is comparable to that of the cubic Dresselhaus (D'yakonov-Perel type) in (001) wells [\[10–12\]](#page-4-0) in limiting the PSH lifetime (see the SM, Sec. VI) [\[40\]](#page-5-0). All these justify our proposed two copies of PSHs as well as the related applications (e.g., diffusive spin-FET and orbit filter) in (110) wells being feasible for experimental realizations.

For a full picture, in Fig. $3(b)$ we show the spin-resolved electronic structure for the *usual* scenario of β_1 and β_2 having the same sign [e.g., see the SM, Fig. S2] [\[40\]](#page-5-0). Clearly, the original crossing for the uncoupled bands turns to be an avoided crossing, cf. black lines and colored circles. By setting the Fermi level $E_F = -0.37$ eV [\[43\]](#page-5-0), we also reveal anisotropic energy dispersions and the spin hybridization between the middle two branches, see the shadowed region in Fig. $3(c)$. And, the spin polarization may even vanish near the avoided crossing [Fig. $3(d)$]. These vertex corrections due to interband effect may lead to intriguing possibilities for spintronic applications, e.g., high-efficiency spin-charge conversion devices [\[53\]](#page-5-0).

IV. CONCLUDING REMARKS

Multiband SO phenomena are now attracting growing interest [\[35,](#page-5-0)[54–57\]](#page-6-0). Here, focusing on (110) GaInAs wells with two bands, we have achieved three distinct regimes of dual-gate SO manipulation, and further demonstrated the emergence of two copies of PSHs with not only stretchy pitch but also compensating helicity. Benefiting from them, we have not only proposed helix-stretch functional spin FET with both on and off states protected by the PSH symmetry, but also put forward a new concept: "orbit (band) filter," opening up a new avenue for spintronic and orbitronic combined applications. Our work would attract diverse interests in the communities of spintronics, orbitronics, and even quantum information science requiring long time and long-ranged coherent spin control. As a final remark, similar SO control can

also be achieved *via* optical means that we recently proposed in Ref. [\[58\]](#page-6-0), by resorting to intense high-frequency laser fields (see also the SM, Sec. VII) [\[40\]](#page-5-0), making our results more reliable for experimental realizations.

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