Fermi surface symmetric mass generation

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Symmetric mass generation (SMG) is a mechanism to give gapless fermions a mass gap by nonperturbative interactions without generating any fermion bilinear condensation. The previous studies of SMG have been limited to Dirac/Weyl/Majorana fermions with zero Fermi volume in the free fermion limit. In this paper, we generalize the concept of SMG to Fermi liquid (FL) with a finite Fermi volume and discuss how to gap out the Fermi surfaces (FSs) by interactions without breaking the U(1) loop group symmetry or developing topological orders. We provide examples of FS SMG in both (1+1)-dimensional [(1+1)D] and (2+1)-dimensional FL systems when several FSs together cancel the FS anomaly. However, the U(1) loop group symmetry in these cases is still restrictive enough to rule out all possible fermion bilinear gapping terms, such that a nonperturbative interaction mechanism is the only way to gap out the FSs. This symmetric FS reconstruction is in contrast to the conventional symmetry-breaking gapping mechanism in the FL. As a side product, our model provides a pristine one-dimensional lattice regularization for the (1+1)D U(1) symmetric chiral fermion model (e.g., the 3-4-5-0 model) by utilizing a lattice translation symmetry as an emergent U(1) symmetry at low energy. This opens up the opportunity for efficient numerical simulations of chiral fermions in their own dimensions without introducing mirror fermions under the domain wall fermion construction.

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I. INTRODUCTION

Fermi liquids (FLs) are gapless quantum many-body systems of fermions that possess Fermi surfaces (FSs) and well-defined quasiparticle excitations at low energy. They are the models for the most commonly seen metallic materials in nature. They are probably also some of the most studied quantum phases of matter in condensed matter physics since Landau [1,2]. However, there are still many aspects of FLs that might not have been well recognized. In this paper, we explore one such aspect: the phenomenon of *symmetric mass generation* (SMG, see a recent overview [3] and references therein) in FLs.

One intriguing property of the FL is the surprising stability of the FS under generic local interactions of fermions. Although the system is gapless with vastly degenerated ground states, local interactions often do not immediately lift the ground state degeneracy and destabilize the FL toward gapped phases. Early understanding of this property came from the perturbative renormalization group (RG) analysis, as the FL theory can emerge as a stable RG fixed point of interacting fermion systems [4–10].

Recently, a modern understanding arose under the name of a *FS anomaly* [11–13], which states that the stability of the FS can be viewed as protected by the quantum anomaly of an emergent LU(1) loop group symmetry at low energy, extending and unifying many related discussions [14–29] about Luttinger's theorem [30] and the Lieb-Schultz-Mattis (LSM) theorem [31] in fermionic systems. Loosely speaking, the LU(1) symmetry corresponds to the fermion number $n_{\mathbf{k}}$ conservation at each momentum point $\mathbf{k}$ on the FS, which is preserved by the Landau FL Hamiltonian $H_{\mathrm{FL}} = \sum_{\mathbf{k}\in\mathrm{FS}} \epsilon_{\mathbf{k}} n_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{k}'\in\mathrm{FS}} f_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}} n_{\mathbf{k}'} + \cdots$. In the presence of the FS anomaly, the FL can only be gapped by either (i) spontaneously breaking the LU(1) symmetry or (ii) spontaneously developing anomalous topological orders (or other non-FL exotic states) that saturate the FS anomaly. The anomaly matching is a kinematic constraint, which is nonperturbative and more robust than the perturbative RG analysis of the FL low-energy dynamics.

Over the past decade, the quantum anomaly [32–35] has been realized as an important theoretical tool in analyzing the protected gapless boundary states of interacting topological insulators/superconductors, which belong to symmetry-protected topological (SPT) phases in a grand scope (see overviews [36–38] and references therein). An interesting phenomenon, known as SMG [39–51], was discovered in the study of interacting fermionic SPT states. It was realized that certain SPT states might look nontrivial at the free-fermion (noninteracting) level but can be smoothly deformed into a trivial gapped phase with a unique ground state by fermion interactions. This implies some integer $\mathbb{Z}$ classification of noninteracting SPT states can be reduced to a finite Abelian elementary order-n group $\mathbb{Z}_n$ classification for some interacting SPT states, emphasized by Fidkowski and Kitaev [39,40]. Correspondingly, their gapless boundary states can be gapped out by (and only by) interaction without breaking the symmetry or developing the topological order (breaking emergent higher-form symmetry). This provides a

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mechanism to generate a mass for zero-density relativistic gapless fermions (e.g., Dirac/Weyl/Majorana fermions occupying only Fermi points with zero Fermi volumes at the Fermi level, colloquially known as Dirac/Weyl/Majorana cones) without symmetry breaking, which has been proposed to provide lattice regularization for the standard model and grand unified theories [45,52–56]. This mechanism is called SMG, or a mass-without-mass term [57,58], which is distinct from the conventional Higgs mechanism that relies on symmetry breaking for fermion mass generation.

However, the SMG mechanism has not yet been extended to fermion systems at a finite filling (with a finite density). The FL is the most notable examples of such, which possesses a FS enclosing a finite Fermi volume. It is natural to ask: Can SMG happen on the FS as well, gapping out the FS by interaction without breaking the loop group symmetry of interest? As we will demonstrate in this paper, the answer is yes.

Given the spacetime-internal symmetry G of a fermion system, the conditions [3] for SMG to happen are (i) the system must be free from G anomaly such that symmetric gapping (without topological order) becomes possible, and (ii) the symmetry G must be restricted enough to rule out any symmetric fermion bilinear gapping term such that the gapping can only be achieved by interaction. These defining conditions of SMG can be applied to the FL system by considering G as the emergent loop group symmetry on the FS. Based on this understanding, we will investigate the FS SMG in the presence of the LU(1) symmetry. The general feature is that, even though a single FS is anomalous, it is possible to cancel the FS anomaly among multiple FSs (or FSs with multiple fermion flavors), such that interactions can drive the transition from the FL phase to a symmetric gapped phase. We shall name this phenomenon as the FS SMG.

The FS SMG provides us a different possibility to create a gap to all excitations on the FS without condensing any fermion bilinear order parameter, which makes it distinct from the superconducting gap (i.e., condensing Cooper pairs) or the density wave gap (i.e., condensing excitons) that are more familiar in condensed matter physics. Nevertheless, it does involve condensing some multifermion bound states that transform trivially under the symmetry transformation. The simplest example is the charge-$4e$ superconductor [59–66], which condenses fermion quartets (four-fermion bound states) that preserve at least the $\mathbb{Z}_4$ subgroup of the charge U(1) symmetry. In this paper, we provide more carefully designed examples preserving the full U(1) symmetry (and other lattice symmetries), but the essential idea of condensing symmetric multifermion operators to generate a many-body excitation gap is the same. Therefore, the FS SMG is intrinsically a strong nonperturbative interaction effect of fermions. The interaction may look irrelevant at the free-fermion (or the FL) fixed point. However, strong enough interaction can still drive the gap-opening transition through nonperturbative effects.

This paper is organized as follows. In Sec. II, we present a lattice model of FS SMG in (1+1) dimensions [(1+1)D], as the pristine lattice regularization of the 3-4-5-0 chiral fermion model, whose phase diagram can be reliably analyzed by the RG approach. In Sec. III, we extend the discussion of FS SMG to (2+1) dimensions [(2+1)D] in a concrete lattice model, which can be exactly solved in both the weak and strong interaction limits. Through these examples, we establish the FS SMG as a general mechanism to gap out anomaly-free FSs in different dimensions. We summarize our result and discuss its connection to future directions in Sec. IV.

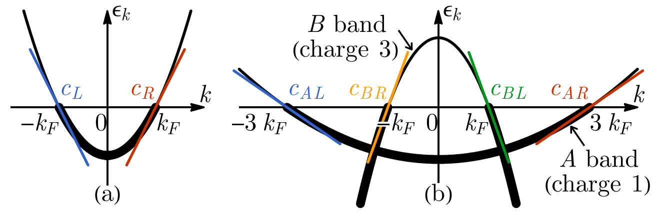

FIG. 1. (a) A typical single-band Fermi liquid (FL) with Fermi surface (FS) anomaly. (b) Two-band model of a FL with the FS anomaly canceled. Chiral fermions with linearized dispersions around different Fermi points emerge at low energy.

II. FS SMG IN (1+1)D

A. (1+1)D Fermi liquid and Fermi surface anomaly

In the free-fermion limit, the (1+1)D FL can be realized as a system of fermions occupying a segment of single-particle momentum eigenstates in the one-dimensional (1D) momentum space (or Brillouin zone), which can be described by a Hamiltonian $H = \sum_k c_k^\dagger \epsilon_k c_k$, where c_k (or $c_k^\dagger$) is the fermion annihilation (or creation) operator of the single-particle mode at momentum k. For now, we only consider spinless fermions, such that the c_k operator does not carry spin (or any other internal degrees of freedom). As an example, suppose the band structure is described by $\epsilon_k = (k^2 - k_F^2)/(2m)$ for nonrelativistic fermions with a finite chemical potential $\mu = k_F^2/(2m)$. The ground state of the Hamiltonian H will have fermions occupying the momentum segment $k \in [-k_F, k_F]$ bounded by the Fermi momentum k_F, as illustrated in Fig. 1(a).

The low-energy degrees of freedom in the (1+1)D FLs can be modeled by the chiral fermions near the zero-dimensional (0D) FSs (namely, Fermi points) at $\pm k_F$, which are described by the following Lagrangian density:

$$\mathcal{L} = c_L^\dagger (i\partial_t - v_F i\partial_x) c_L + c_R^\dagger (i\partial_t + v_F i\partial_x) c_R, \tag{1}$$

where $v_F = k_F/m$ is the Fermi velocity. The operator c_L (or c_R) annihilates the left (or right)-moving fermion modes, defined as

$$c_{R/L}(x) = \int_{-\Lambda}^{\Lambda} d\kappa\ c_{\pm k_F + \kappa} \exp[i(\pm k_F + \kappa)x] \tag{2}$$

around the Fermi points within a small momentum cutoff $\Lambda \ll k_F$. The low-energy effective theory $\mathcal{L}$ in Eq. (1) has an emergent $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ symmetry (more precisely as an emanant symmetry [67] since the translation and charge conservation symmetry are not the subgroup of $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ symmetry), corresponding to the separate charge conservation of the left- and right-moving chiral fermions. Under the symmetry transformation with the periodic ϕ_L and ϕ_R in $[0, 2\pi)$:

$$\begin{aligned} \mathrm{U}(1)_L &: c_L \to \exp(i\phi_L) c_L, \quad c_R \to c_R; \\ \mathrm{U}(1)_R &: c_L \to c_L, \quad c_R \to \exp(i\phi_R) c_R. \end{aligned} \tag{3}$$

They can be as well understood as a recombination of the vector $\mathrm{U}(1)_V$ and axial $\mathrm{U}(1)_A$ symmetries by rewriting $\phi_L = \phi - k_F\,\delta x$ and $\phi_R = \phi + k_F\,\delta x$:

$$\mathrm{U}(1)_V : c_k \to e^{i\phi} c_k \Rightarrow \begin{cases} c_L \to e^{i\phi} c_L, \\ c_R \to e^{i\phi} c_R; \end{cases}$$

$$\mathrm{U}(1)_A : c_k \to e^{ik\,\delta x} c_k \Rightarrow \begin{cases} c_L \to \exp(-ik_F\,\delta x) c_L, \\ c_R \to \exp(+ik_F\,\delta x) c_R. \end{cases} \quad (4)$$

More precisely, the combined symmetry group should be denoted as $\mathrm{U}(1)_V \times_{\mathbb{Z}_2^F} \mathrm{U}(1)_A \equiv \frac{\mathrm{U}(1)_V \times \mathrm{U}(1)_A}{\mathbb{Z}_2^F}$ because the $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_A$ symmetries share the fermion parity $\mathbb{Z}_2^F$ subgroup (under which $c_{L,R} \to -c_{L,R}$). The physical meaning of the vector $\mathrm{U}(1)_V$ symmetry is the total U(1) charge conservation of the fermions, and the axial $\mathrm{U}(1)_A$ symmetry can be considered an effective representation of the translation symmetry in the infrared (IR) limit (that translates all fermions by displacement δx along the 1D system). Although translation symmetry is described by a noncompact symmetry group $\mathbb{Z}$ at the lattice scale, its action on the low-energy chiral fermion fields c_L, c_R behaves as a compact $\mathrm{U}(1)_A$ emergent symmetry [26,68].

The stability of the FL is protected by the FS anomaly, which can be viewed as the mixed anomaly between the $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_A$ symmetries. The anomaly index is given by [24,30,31]

$$1 \times k_F - 1 \times (-k_F) = 2k_F = 2\pi\nu, \quad (5)$$

which can be related to the fermion filling fraction ν. The system is anomalous if the filling ν is not an integer. Without breaking the charge U(1) and translation symmetries, it is impossible to drive the FL to a trivial gap phase due to the nonvanishing FS anomaly. This can be viewed as a consequence of the LSM theorem [31]. The situation is also like the chiral fermion edge states on the (1+1)D boundary of a (2+1)D quantum Hall insulator.

B. Two-band model and anomaly cancellation

To generate a gap for these low-energy fermions in (1+1)D FLs, the FS anomaly must be canceled. Here, we present a two-band toy model that achieves anomaly cancellation and enables gapping out the FS without breaking the charge U(1) and translation symmetries and without generating any Fermi bilinear condensation. It will provide a concrete example of SMG in (1+1)D FLs.

Consider a 1D lattice (a chain of sites) with two types of fermions c_{iA} and c_{iB} per site. The A-type fermion c_{iA} carries charge q_A under a global U(1) symmetry, and the B-type fermion c_{iB} carries charge q_B under the same U(1) symmetry. The Hamiltonian takes the general form of

$$H = -\sum_{ij} \left(t_{ij}^A c_{iA}^\dagger c_{jA} + t_{ij}^B c_{iB}^\dagger c_{jB} + \mathrm{H.c.}\right) - \sum_i (\mu_A c_{iA}^\dagger c_{iA} + \mu_B c_{iB}^\dagger c_{iB}) + H_{\mathrm{int}}, \quad (6)$$

with H_{int} being some fermion interactions to be specified later in Eq. (16). The specific details of the hopping coefficients t_{ij}^A and t_{ij}^B are not important to our discussion if they produce a band structure that looks like Fig. 1(b) in the Brillouin zone. The A-type fermion forms an electronlike band, and the B-type fermion forms a holelike band. The two bands overlap in the energy spectrum. This will realize a two-band FL in general. The Hamiltonian H in Eq. (6) has a $\mathrm{U}(1) \times (\mathbb{Z} \rtimes \mathbb{Z}_2)$ symmetry (parameterized by a periodic angle $\phi \in [0, 2\pi)$ and an integer $n \in \mathbb{Z}$ as follows):

$$\begin{aligned} \mathrm{U}(1) &: c_{iA} \to \exp(iq_A\phi) c_{iA}, \quad c_{iB} \to \exp(iq_B\phi) c_{iB}; \\ \mathbb{Z} &: c_{iA} \to c_{(i+n)A}, \quad c_{iB} \to c_{(i+n)B}; \\ \mathbb{Z}_2 &: c_{iA} \to c_{(-i)A}, \quad c_{iB} \to c_{(-i)B}. \end{aligned} \quad (7)$$

They correspond to the total charge conservation symmetry U(1), the lattice translation symmetry $\mathbb{Z}$, and the lattice reflection symmetry $\mathbb{Z}_2$. The question is whether we can gap the FL without breaking all these symmetries in (1+1)D.

One significant obstruction toward gapping is the FS anomaly, which can also be interpreted as a mixed anomaly between the charge U(1) and (the IR correspondence of) the translation symmetry. To cancel the FS anomaly, we need to fine-tune the chemical potentials μ_A and μ_B such that the anomaly index vanishes:

$$q_A\nu_A + q_B\nu_B = 0 \mod 1, \quad (8)$$

where ν_A and ν_B are the filling fractions of the A and B bands (for the holelike B band, we may assign $\nu_B < 0$ such that $|\nu_B|$ corresponds to the hole-filling). This is also known as the *charge compensation* condition in semiconductor physics.

If the A- and B-type fermions carry the same charge as $q_A = q_B = 1$, the anomaly cancellation condition in Eq. (8) simply requires $\nu_A = -\nu_B$. In this case, the electronlike FS of the A-type fermion and the holelike FS of the B-type fermion are perfectly nested (with zero nesting momentum). A gap can be opened simply by tuning on a fermion bilinear term $\sum_i (c_{iA}^\dagger c_{iB} + \mathrm{H.c.})$ in the Hamiltonian, which preserves the full $\mathrm{U}(1) \times (\mathbb{Z} \rtimes \mathbb{Z}_2)$ symmetry. This is the familiar band hybridization mechanism to open a band gap in a charge-compensated FL, which drives a metal to a band insulator without breaking symmetry.

However, we are more interested in the nontrivial case when the fermions carry different charges $q_A \neq q_B$. For example, let us consider the case of $q_A = 1$ and $q_B = 3$. Then the anomaly cancellation condition in Eq. (8) requires $\nu_A = -3\nu_B$, i.e., the electronlike Fermi volume in the A band must be three times as large as the holelike Fermi volume in the B band to cancel the FS anomaly. Defining the fermion operators c_{kA}, c_{kB} in the momentum space by the Fourier transformation:

$$c_{kA} = \sum_i c_{iA} e^{-iki}, \quad c_{kB} = \sum_i c_{iB} e^{-iki}, \quad (9)$$

the desired band structure can be effectively described by the following band Hamiltonian (suppressing the interaction for now):

$$H = \sum_k (c_{kA}^\dagger \epsilon_{kA} c_{kA} + c_{kB}^\dagger \epsilon_{kB} c_{kB}), \quad (10)$$

with the band dispersions [see Fig. 1(b)]:

$$\epsilon_{kA} = \frac{k^2 - (3k_F)^2}{2m_A}, \quad \epsilon_{kB} = -\frac{k^2 - k_F^2}{2m_B}. \quad (11)$$

TABLE I. Charge assignments of low-energy fermions. See also the model in Ref. [69] on the same charge assignments.

Fermion c_a	Chirality $\operatorname{sgn} v_a$	$\mathrm{U}(1)_V$ q_a^V	$\mathrm{U}(1)_A$ q_a^A	$\mathrm{U}(1)_{\frac{3V+A}{2}}$ $\frac{1}{2}(3q_a^V+q_a^A)$	$\mathrm{U}(1)_{\frac{3V-A}{2}}$ $\frac{1}{2}(3q_a^V-q_a^A)$
c_{AR}	-1 (left)	1	3	3	0
c_{BR}	-1 (left)	3	-1	4	5
c_{BL}	$+1$ (right)	3	1	5	4
c_{AL}	$+1$ (right)	1	-3	0	3

Here, we assume $m_A, m_B > 0$. The Fermi momentum $k_F = |\nu_B|\pi$ is set by the filling $|\nu_B|$ which is typically an irrational number (without fine-tuning). The key feature is that the Fermi momenta of the A and B energy bands must have a 3 : 1 ratio that matches the inverse charge ratio $(q_A/q_B)^{-1}$ precisely. In this case, the energy band hybridization is forbidden by the charge U(1) symmetry as the two bands now carry different charges. Even if the band hybridization is spontaneously generated at the price of breaking the U(1) symmetry, it does not gap the FL because the FSs of the two bands are no longer nested at the Fermi level, such that the band hybridization will only create some avoided energy band crossing below the Fermi level. Then the system remains metallic because the (upper) hybridized band still crosses the Fermi level.

One can show that it is impossible to symmetrically gap the FL by any fermion bilinear terms in this charge-compensated two-band system with $q_A = 1$ and $q_B = 3$, even if the FS anomaly has already been canceled by the charge-compensated filling $\nu_A = -3\nu_B$. Although the anomaly vanishes (i.e., there is no obstruction toward gapping in principle), the symmetry is still restrictive enough to forbid any fermion bilinear gapping term, such that the only possible gapping mechanism rests on nonperturbative fermion interaction effects.

To see this, we can single out the low-energy chiral fermions near the four Fermi points:

$$\begin{aligned} c_{AR} &= c_{(3k_F)A}, \quad c_{BR} = c_{(-k_F)B}, \\ c_{BL} &= c_{(k_F)B}, \quad c_{AL} = c_{(-3k_F)A}, \end{aligned} \tag{12}$$

where A, B label the bands that they originated from and L, R label their chiralities (i.e., left- or right-moving), according to Fig. 1(b). Like Eq. (1), the low-energy effective Lagrangian density reads

$$\mathcal{L} = \sum_a c_a^\dagger (i\partial_t + v_a i\partial_x) c_a, \tag{13}$$

where the index a sums over the four Fermi point labels AR, BR, BL, and AL. Here, v_a denotes the Fermi velocity near the Fermi point a.

The original $\mathrm{U}(1)\times\mathbb{Z}$ symmetry at the lattice fermion level reduces to the emergent $\mathrm{U}(1)_V \times_{\mathbb{Z}_2^F} \mathrm{U}(1)_A$ symmetry for the low-energy chiral fermions c_a (see Appendix A for more explanations):

$$\begin{aligned} \mathrm{U}(1) &\Rightarrow \mathrm{U}(1)_V : c_a \to \exp\left(iq_a^V\phi_V\right)c_a, \\ \mathbb{Z} &\Rightarrow \mathrm{U}(1)_A : c_a \to \exp\left(iq_a^A\phi_A\right)c_a. \end{aligned} \tag{14}$$

Table I summarizes their charge assignment under $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_A$, where the vector $\mathrm{U}(1)_V$ symmetry is just the charge U(1) symmetry, and the axial $\mathrm{U}(1)_A$ symmetry is an emergent symmetry corresponding to the lattice translation symmetry $\mathbb{Z}$. Alternatively, they can be recombined into the $\mathrm{U}(1)_{\frac{3V+A}{2}} \times \mathrm{U}(1)_{\frac{3V-A}{2}}$ symmetry, such that it becomes obvious that all fermion bilinear back-scattering terms (either the Dirac mass $c_a^\dagger c_b$ or the Majorana mass $c_a c_b$ for $a \neq b$ and $a, b \in \{AR, BR, BL, AL\}$) are forbidden by the symmetry because they are all charged nontrivially under the $\mathrm{U}(1)_{\frac{3V+A}{2}} \times \mathrm{U}(1)_{\frac{3V-A}{2}}$ symmetry due to the distinct charge assignment to every chiral fermion. Given this situation, the only hope to gap the FL is to evoke the SMG mechanism that generates the mass for all chiral fermions by nonperturbative multifermion interactions.

C. SMG interaction and RG analysis

It is worth mentioning that the charge-compensated two-band model with $q_A = 1$ and $q_B = 3$ essentially regularizes the 3-4-5-0 chiral fermion model [70,71] on a pristine 1D lattice (without introducing any compact extra dimensions). The emergent $\mathrm{U}(1)_{\frac{3V\pm A}{2}}$ symmetries act as the lattice translations decorated by appropriate internal U(1) rotations, described by the following $\mathbb{Z}$ symmetry groups (parameterized by integer $n \in \mathbb{Z}$) at the lattice level (see Appendix A for derivation):

$$\mathbb{Z}\left(for\frac{3V \pm A}{2}\right) : \begin{cases} c_{iA} \to \exp(\pm i3k_F n)c_{(i+n)A}, \\ c_{iB} \to \exp(\pm i9k_F n)c_{(i+n)B}. \end{cases} \tag{15}$$

The 3-4-5-0 model is a toy model for studying the long-standing problem: the lattice regularization of the chiral fermion theory in high-energy physics [55,72–79]. Many variants of the model are studied in the lattice community (see references therein [80,81]). This model is anomaly-free—perturbative local gauge anomaly free within any linear combination of the $\mathrm{U}(1)_V \times_{\mathbb{Z}_2^F} \mathrm{U}(1)_A$ checked by the Adler-Bell-Jackiw method [82,83], perturbative local gravitational anomaly free because of the zero chiral central charge $c_L - c_R = 0$, also nonperturbative global anomaly free from any gauge or gravitational fields checked by the cobordism [84]. However, it was known much later that symmetric gapping can only be achieved by minimally six-fermion interactions among the four flavors of 3-4-5-0 fermions. The SMG interaction was proposed by Wang and Wen [46,51], which was later discussed by Tong [85] and only recently verified by the density matrix RG (DMRG) [86,87] numerical simulation in Ref. [88].

Given the existing knowledge about the SMG interaction in the 3-4-5-0 chiral fermion model, we can map the Wang-Wen interaction [46,51] back to our lattice model following the correspondence listed in Table I, which gives us the following SMG interaction (see Appendix B for more details):

$$H_{\text{int}} = g \sum_i c_{(i-1)B}^\dagger c_{(i-1)A} c_{iB} c_{iA} c_{(i+1)B}^\dagger c_{(i+1)A} + \text{H.c.} \tag{16}$$

This is a six-fermion interaction across three adjacent sites on the 1D lattice. It describes the process that first annihilates both A- and B-type fermions on the center site (which annihilates four units of charges on the site i) and then separately

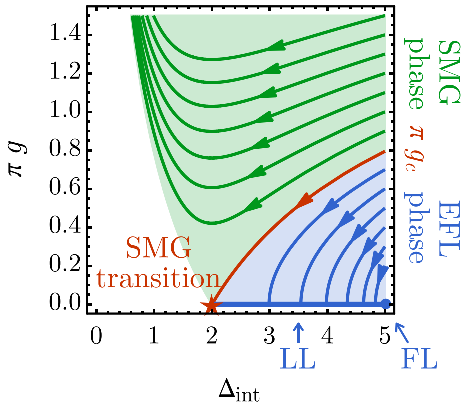

FIG. 2. The renormalization group (RG) flow of the coupling g and the scaling dimension $\Delta_{\rm int}$ of the SMG interaction. The abbreviations stand for the following terminology: SMG for symmetric mass generation, FL for Fermi liquid, EFL for ersatz FL, LL for Luttinger liquid.

converts A-type fermions to B-type fermions on the two adjacent sites (which creates two units of charges on each of the site $i-1$ and $i+1$), such that the $\mathrm{U}(1)_V$ charge is conserved. The interaction is also manifestly translation and reflection symmetric, so the full $\mathrm{U}(1)_V \times (\mathbb{Z} \rtimes \mathbb{Z}_2)$ symmetry is preserved by the interaction as expected. With this interaction, we claim that the lattice model in Eq. (6) will exhibit an (ersatz) FL to SMG insulator transition when the interaction strength g exceeds a finite critical value g_c.

To show that the proposed interaction in Eq. (16) indeed drives the FL to a gapped interacting insulator, we bosonize [89,90] the fermion operator $c_a \sim\, :\exp(i\varphi_a):$ (with $a \in \{AR, BR, BL, AL\}$) and cast the lattice model to an effective Luttinger liquid (LL) theory, described by the following Lagrangian density:

$$\mathcal{L} = \frac{1}{4\pi}(\partial_t \varphi^\intercal K \partial_x \varphi - \partial_x \varphi^\intercal V \partial_x \varphi) + \sum_{\alpha=1,2} g_\alpha \cos\left(l_\alpha^\intercal \varphi\right), \tag{17}$$

where $\varphi = (\varphi_{AR}, \varphi_{BR}, \varphi_{BL}, \varphi_{AL})^\intercal$ are compact scalar bosons. The K matrix and the l_α vectors are given by

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad l_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 2 \end{bmatrix}, \quad l_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \\ 1 \end{bmatrix}. \tag{18}$$

As shown in Appendix B, the six-fermion interaction $H_{\rm int}$ in Eq. (16) translates to the cosine terms g_1 and g_2 in the LL theory in Eq. (17), with $g_1 = g_2 = g$ enforced by the $\mathbb{Z}_2$ reflection symmetry (as the $\mathbb{Z}_2$ transformation exchanges the g_1 and g_2 terms). The RG flow in the log energy scale $\ell = -\ln\Lambda$ is given by [91,92]

$$\frac{dg}{d\ell} = (2-\Delta_{\rm int})g, \quad \frac{d\Delta_{\rm int}^{-1}}{d\ell} = \pi^2 g^2, \tag{19}$$

where $\Delta_{\rm int}$ is the scaling dimension of the SMG interaction. The RG flow diagram is shown in Fig. 2.

At the FL fixed point, we have $\Delta_{\rm int} = \frac{1}{2} l_\alpha^\intercal l_\alpha = 5 > 2$, meaning that the SMG interaction is perturbatively irrelevant. If the bare coupling g (the interaction strength at the lattice scale) is weak ($g < g_c$), it will just flow to zero and disappear in the IR theory. However, the scaling dimensions of all operators will be renormalized as the coupling g flows toward zero. Therefore, the FL fixed point will be deformed into the LL fixed-line, along which the fermion quasiparticle is no longer well defined, but the system remains gapless. Despite the different dynamical properties, the LL still preserves all the kinematic properties (e.g., emergent symmetries and anomalies) as the FL, which can be unified under the concept of *ersatz FL* (EFL) [11].

If the bare coupling g is strong enough ($g > g_c$), the scaling dimension $\Delta_{\rm int}$ can be reduced to $\Delta_{\rm int} < 2$ such that the SMG interaction becomes relevant and flows strong. As the cosine term in Eq. (17) gets strong, the corresponding vertex operators $\exp(i l_\alpha^\intercal \varphi)$ ($\alpha = 1, 2$) condense. Any other operators that braid nontrivially with the condensed operators will be gapped, which includes all the fermion operators. Therefore, the system enters the SMG insulating phase with all fermion excitations gapped without breaking the $\mathrm{U}(1) \times (\mathbb{Z} \rtimes \mathbb{Z}_2)$ symmetry. This has been confirmed by the DMRG simulation in Ref. [88] for a related model using the domain wall fermion construction, where it has been verified that the fermion two-point function indeed decays exponentially in the SMG phase—a direct piece of evidence for the gap generation. On the lattice level, this corresponds to condensing the six-fermion bound state by developing the ground state expectation value of $\langle c_{(i-1)B}^\dagger c_{(i-1)A} c_{iB} c_{iA} c_{(i+1)B}^\dagger c_{(i+1)A}\rangle \neq 0$. So the gapping is achieved by the multifermion condensation (involving more than two fermions), which is distinct from the fermion bilinear condensation in the conventional gapping mechanisms of FLs (such as the band hybridization or Cooper pairing mechanisms).

The RG analysis also indicates that the EFL-to-SMG insulator transition (at $g = g_c$) is of the Berezinskii-Kosterlitz-Thouless (BKT) [93–95] transition universality in (1+1)D.

The above analysis established the FS SMG phenomenon in the lattice model in Eq. (6) [equipped with the gapping interaction in Eq. (16)]. The significance of this lattice model is that it provides a pristine 1D lattice regularization of the 3-4-5-0 chiral fermion model by using lattice translation to realize the axial $\mathrm{U}(1)_A$ symmetry at low energy. In contrast to the domain wall fermion constructions [51,55,88], our construction does not require the introduction of a (2+1)D bulk to realize the chiral fermions as boundary modes. Such a pristine 1D lattice regularization is advantageous for the numerical simulation of chiral fermions, as the model contains no redundant bulk (or mirror) fermions, such that the computational resources can be used more efficiently. We will leave the numerical exploration of this model to future research.

III. FS SMG IN (2+1)D

A. (2+1)D Fermi liquid and Fermi surface anomaly

Given the example of FS SMG in (1+1)D, we would like to further explore similar physics in higher dimensions. The most important low-energy features of a (2+1)D FL are the gapless fermions on its 1D FS. Suppose we parametrize the 1D FS $\mathbf{k}_F(\theta) \in \partial\mathcal{V}_F$ by a continuous and periodic parameter θ, such that $\mathbf{k}_F(\theta + 2\pi) = \mathbf{k}_F(\theta)$ (where we do not require θ to literally represent the geometrical angle, as the FS may not be a perfect circle in general). The fermions c_θ on the FS have an emergent symmetry described by the loop group of U(1) [11,12], denoted as LU(1), under which

$$\mathrm{LU}(1) : c_\theta \to \exp[i\phi(\theta)]c_\theta, \tag{20}$$

where the U(1) phase factor $\exp[i\phi(\theta)]$ is a smooth function of θ with the periodicity $\exp[i\phi(\theta + 2\pi)] = \exp[i\phi(\theta)]$. Both the (global) charge U(1) and the translation symmetries $\mathbb{R}^2$ are subgroups of LU(1):

$$\mathrm{U}(1) : c_\theta \to e^{iq\phi}c_\theta, \quad \mathbb{R}^2 : c_\theta \to \exp[i\,\delta\mathbf{x}\cdot\mathbf{k}_F(\theta)]c_\theta, \tag{21}$$

assuming the fermions c_θ carry charge q under the global U(1) symmetry and are translated by the vector $\delta\mathbf{x} \in \mathbb{R}^2$.

The presence of the FS causes a mixed anomaly between the U(1) and translation symmetries [96], which is characterized by the anomaly index:

$$\frac{q}{2(2\pi)^2}\oint d\theta(\mathbf{k}_F \times \partial_\theta\mathbf{k}_F)_3 = \frac{q\mathcal{V}_F}{(2\pi)^2} = q\nu, \tag{22}$$

where $\mathcal{V}_F$ stands for the Fermi volume in the momentum space, and ν is the filling factor. If the FS anomaly is nonvanishing, it is impossible to trivially gap out the FL without breaking any symmetry or developing any topological order. The FS SMG is only possible if the FL system contains multiple FSs of opposite anomaly indices, such that their anomalies cancel as a whole.

B. Kagome-triangular lattice model

We present a concrete lattice model to demonstrate the FS SMG in (2+1)D. Consider two types of spinless fermions labeled by A and B that are charged under a global U(1) symmetry with charges $q_A = 1$ and $q_B = 3$, respectively. The A-type (or B-type) fermion is defined on a kagome (or triangular) lattice. As depicted in Fig. 3(a), the kagome and the triangular lattices lie on top of each other, with the site I of the triangular lattice aligned with the upper triangle $\triangle_I$ on the kagome lattice. We will use the lowercase letters i, j (or the uppercase letters I, J) to label the kagome (or the triangular) lattice sites.

The lattice model is described by the following Hamiltonian:

$$\begin{aligned} H &= H_A + H_B + H_{\text{int,CF}},\\ H_A &= -t_A\sum_{\langle ij\rangle}(c_i^\dagger c_j + \text{H.c.}) - \mu_A\sum_i c_i^\dagger c_i,\\ H_B &= -t_B\sum_{\langle IJ\rangle}(c_I^\dagger c_J + \text{H.c.}) - \mu_B\sum_I c_I^\dagger c_I,\\ H_{\text{int,CF}} &= -g\sum_I\sum_{ijk\in\triangle_I}(c_I^\dagger c_i c_j c_k + \text{H.c.}), \end{aligned} \tag{23}$$

where $\langle ij\rangle$ (or $\langle IJ\rangle$) denotes the nearest-neighboring link on the A (or B) lattices, and $ijk \in \triangle_I$ stands for the three A-sites i, j, k at the vertices of the upper triangle surrounding the B-site labeled by I. The model has a U(1) symmetry that acts as

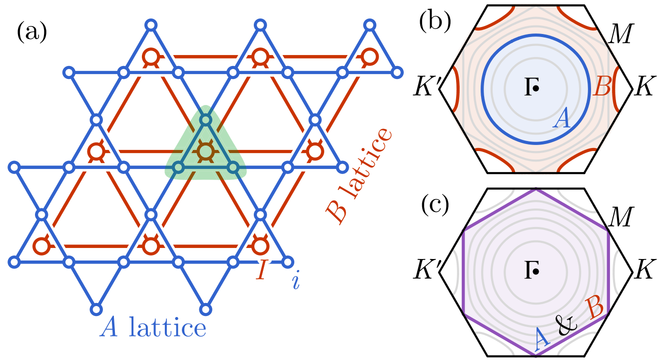

FIG. 3. (a) In the real space, we design the overlapping kagome (A) and triangular (B) lattices. The green triangle marks out the unit cell. In the momentum k space, we draw many contours to represent various equal energy curves of the energy band, at different filling levels (equally spaced by $\frac{1}{8}$ filling fraction). We illustrate the A-type (in blue) and B-type (in red) Fermi surfaces (FSs) (b) at a general filling such as $\nu_A = \frac{3}{8}$ and $\nu_B = \frac{7}{8}(= -\frac{1}{8})$, or (c) at a special filling $\nu_A = \nu_B = \frac{3}{4}(= -\frac{1}{4})$ where the FSs coincide.

$$\mathrm{U}(1) : c_i \to e^{i\phi}c_i, \quad c_I \to e^{i3\phi}c_I. \tag{24}$$

The Hamiltonian in Eq. (23) preserves the internal U(1) symmetry and all symmetries of the kagome-triangular lattice (most importantly, the lattice translation symmetry).

The model in Eq. (23) describes the two types of fermions hopping separately on their corresponding lattices. Because every unit cell contains four sites (three from the kagome lattice and one from the triangle lattice), the hopping model will give rise to four energy bands (three bands for A-type fermions and one band for B-type fermions). The chemical potentials μ_A and μ_B are adjusted to ensure the desired filling of these fermions. We will focus on a simple case when only the lowest A-type (kagome lattice) bands and the single B-type (triangular lattice) bands are filled by filling fractions ν_A and ν_B, respectively, such that the FS only involves two of the four bands.

The A- and B-type fermions are coupled together only through a four-fermion interaction $H_{\text{int,CF}}$ in Eq. (23) that fuses three A-type (charge-1) fermions to one B-type (charge-3) fermions (and vice versa) within each unit cell. We will call it a *charge fusion* (CF) interaction. The CF interaction breaks the separate U(1) charge conservation laws for A- and B-type fermions in the hopping model to a joint U(1) charge conservation, associated with the symmetry action in Eq. (24). Similar interactions also appear in a recent study [97] of quantum breakdown.

Without interaction ($g = 0$), the system is in a FL phase. According to Eq. (22), the FS anomaly cancellation condition requires

$$q_A\nu_A + q_B\nu_B = 0 \mod 1. \tag{25}$$

Given the charge assignment of $q_A = 1$ and $q_B = 3$, it requires $\nu_A = -3\nu_B$. There is no further requirement on the choice of ν_A itself. With a generic choice of filling (assuming $\nu_A < 3/4$) as in Fig. 3(b), the A-type fermions (on the kagome lattice) will form an electronlike FS, whose Fermi volume is three times as large as that of the holelike FSs formed by the B-type fermions (on the triangular lattice). Although the FL has a vanishing FS anomaly, the charge U(1) and the lattice translation symmetries are still restrictive enough to forbid any gap opening on the free-fermion level. For example, any pairing (charge-$2e$ superconducting) gap will break the U(1) symmetry. The only possibility to gap the FL relies on the multifermion interaction.

We claim that the CF interaction $H_{\text{int,CF}}$ in Eq. (23) is a valid SMG interaction that drives the FL into a trivially gaped insulator without breaking symmetry (or developing any topological order). To see this, we go to the strong coupling limit by taking $g \to \infty$. Of course, the chemical potentials μ_A, μ_B must increase correspondingly to keep the fermion fillings fixed. The model Hamiltonian decouples to each unit cell in the strong coupling limit:

$$H = \sum_{I|ijk\in\triangle_I} -\mu_A(n_i + n_j + n_k) - \mu_B n_I - g(c_I^\dagger c_i c_j c_k + \text{H.c.}), \tag{26}$$

where $n_i = c_i^\dagger c_i$ (and $n_I = c_I^\dagger c_I$) denotes the fermion number operator. Within each unit cell, there are only two relevant states $|1110\rangle$ and $|0001\rangle$ (in the Fock state basis $|n_i n_j n_k n_I\rangle$) acted upon by the Hamiltonian. Their hybridization will produce the ground state in each unit cell. The full-system ground state will be the following direct product state:

$$|\text{SMG}\rangle = \bigotimes_I (\sqrt{p}|1110\rangle + \sqrt{1-p}|0001\rangle)_I, \tag{27}$$

where $p = \frac{1}{2}[1 + \frac{-3\mu_A+\mu_B}{\sqrt{(-3\mu_A+\mu_B)^2+4g^2}}]$ is the probability to observe the $|1110\rangle$ state in the unit cell, which is tunable by adjusting μ_A, μ_B relative to g. The fermion fillings (per unit cell) in the ground state $|\text{SMG}\rangle$ will be

$$\nu_A = 3p, \quad \nu_B = 1 - p = -p \mod 1, \tag{28}$$

which automatically satisfies the anomaly cancellation condition $\nu_A = -3\nu_B$ (as it should be). The ground state $|\text{SMG}\rangle$ is nondegenerated and gapped from all excited states (with a gap of the order g). It also preserves the charge U(1) and all the lattice symmetries and does not have topological order. Therefore, we have explicitly shown that the system ends up in the SMG insulator phase as $g \to \infty$. As a gapped phase, we expect it to be stable against perturbations (such as the hopping terms t_A, t_B) over a finite region in the parameter space. The SMG phase is a strongly interacting insulating phase, which has no correspondence in the free-fermion picture.

Having established the FL (metallic) phase at $g = 0$ and the SMG insulator phase at $g \to \infty$, there must be an SMG transition (an interaction-driven metal-insulator transition) at some intermediate coupling strength g_c. However, the nature of the transition is still an open question, which we will leave for future numerical study. In the following, we will only analyze the SMG transition at a special filling: $\nu_A = \nu_B = \frac{3}{4}$, where the FSs coincide precisely and take the perfect hexagon shapes, as shown in Fig. 3(c). This allows us to gain some analytic control of the problem.

C. RG analysis of the SMG transition

In this subsection, we analyze the interaction effect in Eq. (23) when the filling is $\nu_A = \nu_B = \frac{3}{4}$. In this case, the FS of the system contains three Van Hove singularities (VHSs), also known as hot spots, located at three distinct M points, as shown in Fig. 3(c). This allows us to study the interaction effects using the hot-spot RG method at the one-loop level [98–105]. The hot-spot RG approach assumes that the low-energy physics emerges from the correlated effects of fermions near the VHSs, where the density of states diverges. This divergence leads to the a high instability toward gap opening.

Under RG, the CF interaction $H_{\text{int,CF}}$ will generate two types of density-density interactions at the one-loop level, namely, $H_{\text{int,AA}} = \sum_{i,j} n_i n_j$ and $H_{\text{int,AB}} = \sum_{i,I} n_i n_I$, as well as other (less important) exchange interactions. These density-density interactions are more important in the sense that they will in turn contribute to the correction of $H_{\text{int,CF}}$. Therefore, we should include $H_{\text{int,CF}}$, $H_{\text{int,AA}}$, $H_{\text{int,AB}}$ altogether in the RG analysis and study the RG flow jointly.

To proceed, we transform the interactions into the momentum space. The fermion operators are labeled by the flavor index $S = A, B$ and the hot-spot index $\alpha, \beta \in \{1, 2, 3\}$ (referring to the three different VHSs). We note that $H_{\text{int,CF}}$ would vanish if it is naively restricted to the hot spots because the momentum conservation requires multiple A-type fermion operators to appear on the same hot spot, which violates the Pauli exclusion principle of fermions. Thus, we need to introduce point splitting in the momentum space around each hot spot. Our strategy is to further split the A-type fermion into three modes A_s labeled by $s = 1, 2, 3$, and define the interaction:

$$\begin{aligned} H_{\text{int,CF}} = g_{\text{rs}} \sum_\alpha \epsilon^{ijk} c_{B\alpha}^\dagger c_{A_i\alpha} c_{A_j\alpha} c_{A_k\alpha} \\ + g_{\text{rt}} \sum_{\alpha\neq\beta} \epsilon^{ijk} c_{B\alpha}^\dagger c_{A_i\alpha} c_{A_j\beta} c_{A_k\beta} + \text{H.c.}, \end{aligned} \tag{29}$$

where g_{rs} and g_{rt} are the CF interaction decomposed into different momentum transfer channels: the intra-hot-spot scattering g_{rs} and the inter-hot-spot scattering g_{rt}.

These CF interactions receive corrections from the following density-density interactions at the one-loop level:

$$\begin{aligned} H_{\text{int,AA}} + H_{\text{int,AB}} = g_{\text{as}} \sum_{\alpha,st} n_{A_s\alpha} n_{A_t\alpha} + (A_s \leftrightarrow A_t) \\ + g_{\text{bt}} \sum_{\alpha\neq\beta,s} n_{B\alpha} n_{A_s\beta} + (A_s \leftrightarrow B) \\ + \text{H.c.} + \dots, \end{aligned} \tag{30}$$

where ... refers to the other interactions that are decoupled from $g_{\text{rs}}, g_{\text{rt}}, g_{\text{as}}, g_{\text{bt}}$ in the RG equations. The scattering processes of these four important interactions are illustrated in Fig. 4. The complete set of all possible interactions is presented in Appendix C.

We derive the RG equations based on the systematic approach developed in Ref. [106]. Since we are interested in the

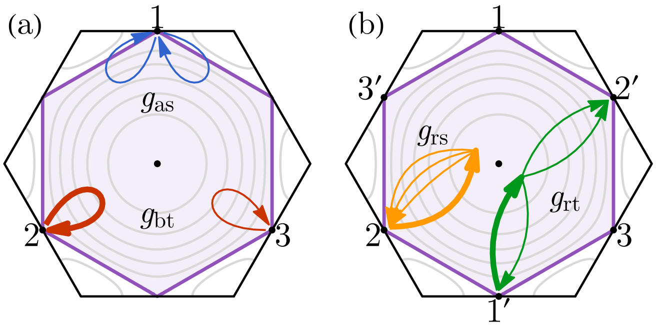

FIG. 4. Scattering of fermions between Van Hove singularities (VHSs) by (a) density-density interactions $g_{\rm bt}$ (red), $g_{\rm as}$ (blue) and (b) nonvanishing processes $g_{\rm rs}$ (yellow), $g_{\rm rt}$ (green) of $H_{\rm int,CF}$. Thin (or thick) arrows correspond to A-type (or B-type) fermions.

flow of $H_{\rm int,CF}$, the relevant part of the RG equations reads

$$\frac{dg_{\rm bt}}{d\ell} = 2d_0 d_{\rm AB} g_{\rm bt}^2, \quad \frac{dg_{\rm as}}{d\ell} = -2g_{\rm as}^2,$$
$$\frac{dg_{\rm rs}}{d\ell} = -6g_{\rm as}g_{\rm rs}, \quad \frac{dg_{\rm rt}}{d\ell} = 4d_0 d_{\rm AB} g_{\rm bt} g_{\rm rt} - 2g_{\rm as}g_{\rm rt}. \tag{31}$$

where the RG parameter is defined by the Cooper-pairing susceptibility of A-type fermions $\ell = \chi_{pp,\rm AA}(\mathbf{k}=0,E) \sim \nu_0 \ln^2(\Lambda/E)$, in which $\nu_0 \ln(\Lambda/E)$ is the diverging density of states at the VHS, E is the running energy scale, and Λ is the high-energy cutoff. Here, $d_0 = d\chi_{ph,\rm AA}(\mathbf{Q})/d\ell \leqslant 1$ is the nesting parameter of A-type fermions, which saturates to one in the perfectly nested limit ($d_0 \to 1$). In our case, different VHSs are half-nested (only one of the two crossing FSs is perfectly nested between every pair of different VHSs), so $d_0 = \frac{1}{2}$ is a suitable estimation. Similarly, we define $d_{\rm AB} = d\chi_{pp,\rm AB}(\mathbf{0})/d\ell$, which depends on the energies of A- and B-type fermions near the VHS. The full RG equations and details are listed in Appendix C.

According to the one-loop RG equations, if the density-density interactions $g_{\rm bt}, g_{\rm as}$ are initially zero, then the CF interactions $g_{\rm rs}, g_{\rm rt}$ remain marginal along the RG flow. However, if we turn on small density-density interactions $g_{\rm bt}, g_{\rm as}$ with correct signs ($g_{\rm bt} > 0$ or $g_{\rm as} < 0$), the CF interactions $g_{\rm rs}, g_{\rm rt}$ will be marginally relevant. The solutions of the RG equations in Eq. (31) are

$$g_{\rm bt}(\ell) = \frac{g_{\rm bt}(0)}{1 - 2d_0 d_{\rm AB} g_{\rm bt}(0)\ell}, \quad g_{\rm as}(\ell) = \frac{g_{\rm as}(0)}{1 + 2g_{\rm as}(0)\ell},$$
$$g_{\rm rs}(\ell) = \frac{g_{\rm rs}(0)}{[1 + 2g_{\rm as}(0)\ell]^3},$$
$$g_{\rm rt}(\ell) = \frac{g_{\rm rt}(0)}{[1 + 2g_{\rm as}(0)\ell][1 - 2d_0 d_{\rm AB} g_{\rm bt}(0)\ell]^2}. \tag{32}$$

As the RG parameter ℓ increases under the RG flow, the coupling strengths can diverge at some critical scale ℓ_c, when any of the denominators in Eq. (32) vanish. The critical scale is set by the bare density-density interaction strengths $g_{\rm bt}(0)$ and $g_{\rm as}(0)$, but the CF interaction strengths $g_{\rm rs}, g_{\rm rt}$ diverge faster than the density-density interactions as the critical scale is approached. Therefore, the RG fixed points are characterized by the behavior of $g_{\rm rs}, g_{\rm rt}$.

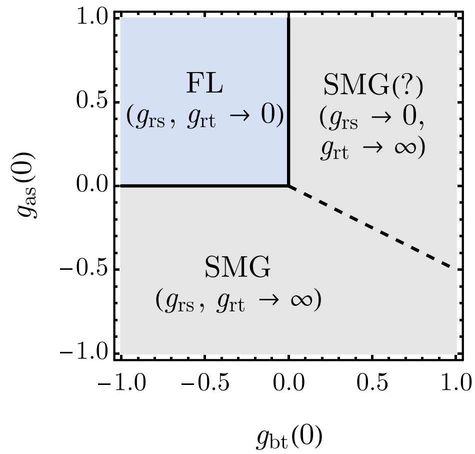

FIG. 5. The renormalization group (RG) phase diagram with respect to the density-density interactions $g_{\rm as}, g_{\rm bt}$. In the Fermi liquid (FL) phase, the gapping interaction flows to zero. In the symmetric mass generation (SMG) phase, the gapping interaction flows to infinity.

Depending on the bare density-density interaction strengths $g_{\rm as}(0)$ and $g_{\rm bt}(0)$, the system can flow toward different RG fixed points, as shown Fig. 5. When $g_{\rm as}(0) > 0$ and $g_{\rm bt}(0) < 0$, all interactions flow to zero, which corresponds to the FL fixed point. When $g_{\rm as}(0) < \min[0, -d_0 d_{\rm AB} g_{\rm bt}(0)]$, both CF interactions $g_{\rm rs}, g_{\rm rt}$ flow to infinity, which should correspond to the SMG phase according to the previous lattice model analysis. However, we also find a region in the phase diagram, described by $g_{\rm bt} > \max(0, -g_{\rm as}/d_0 d_{\rm AB})$, where $g_{\rm rs} \to 0$ and $g_{\rm rt} \to \infty$, i.e., flowing toward different limits. We are not sure how to interpret the physical meaning of this RG fixed point. It might still be in the SMG phase as one interaction still flows strong, but it could as well end up in a spontaneous symmetry breaking (SSB) phase that breaks the LU(1) symmetry since the A- and B-type FSs have pretty strong nesting instability. This might also be an artifact of the hot-spot RG method, as it does not fully capture all low-energy fermionic degrees of freedom of the FS.

Admittedly, it is not possible to determine whether the full FS is gapped using the hot-spot RG analysis alone. This is because the hot-spot RG approach only considers the fermions near the VHSs and not the FS freedom away from the VHSs. To determine whether the strong coupling fixed point is a fully gapped state, we have to rely on lattice model analysis in the strong coupling limit. The exact ground-state solution in Eq. (27) provides evidence to support the argument that the strong coupling fixed point is indeed a fully gapped state.

To improve, functional RG [107–110] might provide a better resolution of the FS and remove the uncertainty in the phase diagram in Fig. 5. A recent study [111] has demonstrated the functional RG method in a triangle lattice model with spinless fermions. The same technique might apply to our model as well. However, we will leave such a study for future research.

By tuning $g_{\rm as}(0)$ across zero on the $g_{\rm bt}(0) < 0$ side, one can drive a FL-to-SMG transition. The gapping interaction is marginally relevant at the transition point. According to

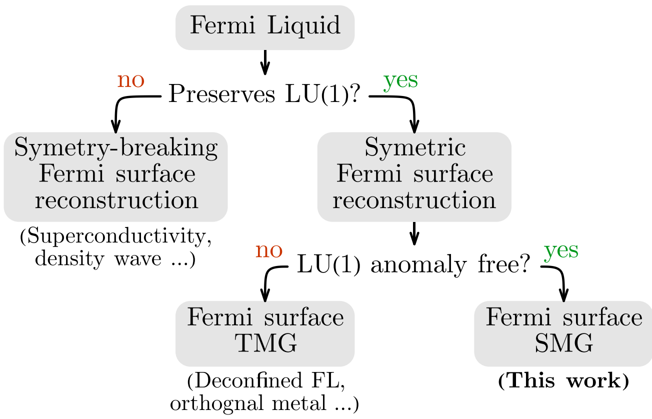

FIG. 6. Classification of Fermi surface (FS) reconstruction mechanisms, based on the LU(1) loop group symmetry. Symmetric FS reconstruction (SFSR) contains two broad classes: (1) FS symmetric mass generation (SMG) if the total FS anomaly is canceled and (2) FS topological mass generation (TMG) if the total FS anomaly is matched by topological order with low energy topological field theory.

the solution of the RG equations in Eq. (32), the coupling diverges at the critical scale $\ell_c \sim \nu_0 \ln^2(\Lambda/\Delta_{\text{SMG}})$ when the denominator $[1 + 2g_{\text{as}}(0)l_c]$ vanishes. This implies that the SMG gap Δ_{SMG} (the energy gap between the ground state and the first excited state) opens up as [112,113]

$$\Delta_{\text{SMG}} \sim \Lambda \exp\left[-\frac{c}{\sqrt{g_{\text{as}}(0)\nu_0}}\right], \tag{33}$$

where Λ is the ultraviolet (UV) cutoff energy scale, ν_0 is the coefficient in front of the diverging density of state at the VHS, and c is some nonuniversal constant.

IV. SUMMARY AND DISCUSSION

In this paper, we propose the concept of FS SMG: a mechanism to gap out FSs by nonperturbative interaction effects without breaking the LU(1) symmetry. This phenomenon can only happen when the FS anomaly is canceled out in the fermion system. We present (1+1)D and (2+1)D examples of FS SMG. We expect that the mechanism can generally occur in all dimensions.

FS SMG belongs to a broader class of phenomena, called the symmetric FS reconstruction (SFSR), as summarized in Fig. 6. The SFSR is in contrast to the more conventional symmetry-breaking FS reconstruction, where the FS is reconstructed (or gapped) by developing SSB orders. Depending on the cancellation of the FS anomaly, the SFSR further splits into two classes: the FS SMG if the anomaly vanishes or the FS topological mass generation (TMG) if the anomaly does not vanish. The former class, the FS SMG, is the focus of this paper. The latter class, the FS TMG, is also discussed in the literature, where the nonvanishing FS anomaly is absorbed by an anomalous topological quantum field theory, such that the SFSR is achieved by developing the corresponding topological order. This gives rise to deconfined/fractionalized FL (FL*) [114–116] or orthogonal metal [117–119]. Symmetry extension [120] has provided a unified framework to understand TMG and SMG for bosons or fermions of zero Fermi volume [121–126], where the symmetric gapping can be achieved by extending the symmetry group to lift any gapping obstruction that was otherwise imposed by the symmetry. Similar constructions can be applied to understand SFSR more generally.

FS SMG deforms an anomaly-free (charge-compensated) FL state to a fully gapped product state. Although the resulting SMG gapped state does not have nontrivial features like topological order, the SMG transition from the FL phase to the SMG phase can still be quite exotic. The SMG transition of relativistic fermions has been proposed as a deconfined quantum critical point [127,128], where the physical fermion fractionalizes to bosonic and fermionic partons with emergent gauge fluctuations at and only at the critical point. It is conceivable that similar scenarios might apply to the FS SMG transition as well, where deconfined FL (orthogonal metal) could emerge at the critical point. The lattice models presented in this paper lay the ground for future theoretical and numerical studies of the exotic SMG transition in these models.

It is also known that the fermion single-particle Green's function has symmetry-protected zeros at zero frequency in the SMG phase [129–132]. It will be interesting to investigate further the Green's function structure in the FS SMG phase. Whether the SMG interaction will replace the original FS (a loop of poles) with a loop of zeros in the Green's function is still an open question to explore.

Another potential experimental connection is to apply the FS SMG to understand the nature of pseudogap phases, which is an exotic state of electrons where the FS is partially gapped without obvious symmetry breaking. It has been observed in many correlated materials. The recent proposal of the ancilla qubit approach [133,134] for pseudogap physics draws a connection between the pseudogap metal-to-FL transition with the FS SMG transition in the ancilla layers, as both transitions are described by field theories of fermionic deconfined quantum critical points [127,128,135–137]. The FS anomaly constrains the dynamical behavior of such field theories and can potentially shed light on the open problem of pseudogap transition in correlated materials.

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APPENDIX A: EMERGENT U(1) SYMMETRIES IN THE (1+1)D TWO-BAND MODEL

Start from the definition of charge U(1) (parameterized by a periodic angle $\phi \in [0, 2\pi)$) and lattice translation $\mathbb{Z}$ (parameterized by an integer $n \in \mathbb{Z}$) symmetries as defined in Eq. (7):

$$\begin{aligned} \mathrm{U}(1) : c_{iA} &\to \exp(iq_A\phi)c_{iA}, \quad c_{iB} \to \exp(iq_B\phi)c_{iB}; \\ \mathbb{Z} : c_{iA} &\to c_{(i+n)A}, \quad c_{iB} \to c_{(i+n)B}. \end{aligned} \tag{A1}$$

Follow the definition in Eq. (9) of the fermion operators in the momentum space:

$$c_{kA} = \sum_i c_{iA}e^{-iki}, \quad c_{kB} = \sum_i c_{iB}e^{-iki}, \tag{A2}$$

where the wave number $k \in [-\pi, \pi)$ is a dimensionless periodic variable defined in the first Brillouin zone. (Note: the dimensionful momentum p should be related to the dimensionless wave number k by $p = \hbar k/a$, with a being the lattice constant, and the site coordinate $x \in \mathbb{R}$ is related to the site index $i \in \mathbb{Z}$ by $x = ai$, such that the Fourier factor $e^{-ipx/\hbar} = e^{-iki}$ is consistent with the quantum mechanics convention.) It is straightforward to show that the $\mathrm{U}(1) \times \mathbb{Z}$ symmetry acts in the momentum space as

$$\begin{aligned} \mathrm{U}(1) : c_{kA} &\to \exp(iq_A\phi)c_{kA}, \quad c_{kB} \to \exp(iq_B\phi)c_{kB}; \\ \mathbb{Z} : c_{kA} &\to e^{ikn}c_{kA}, \quad c_{kB} \to e^{ikn}c_{kB}. \end{aligned} \tag{A3}$$

Apply these transformations to the low-energy fermion near the four Fermi points. According to Eq. (12):

$$\begin{aligned} c_{AR} &= c_{(3k_F)A}, \quad c_{BR} = c_{(-k_F)B}, \\ c_{BL} &= c_{(k_F)B}, \quad c_{AL} = c_{(-3k_F)A}, \end{aligned} \tag{A4}$$

Eq. (A3) becomes

$$\begin{aligned} \mathrm{U}(1) : &\begin{cases} c_{AR} \to \exp(iq_A\phi)c_{AR}, \\ c_{BR} \to \exp(iq_B\phi)c_{BR}, \\ c_{BL} \to \exp(iq_B\phi)c_{BL}, \\ c_{AL} \to \exp(iq_A\phi)c_{AL}; \end{cases} \\ \mathbb{Z} : &\begin{cases} c_{AR} \to \exp(3ik_F n)c_{AR}, \\ c_{BR} \to \exp(-ik_F n)c_{BR}, \\ c_{BL} \to \exp(ik_F n)c_{BL}, \\ c_{AL} \to \exp(-3ik_F n)c_{AL}. \end{cases} \end{aligned} \tag{A5}$$

Because $k_F = |\nu_B|\pi$ is almost always (i.e., with probability 1) an irrational multiple of π (because $|\nu_B|$ is almost always an irrational number without fine tuning), $k_F n \mod 2\pi$ can approach any angle in $[0, 2\pi)$ (with 2π periodicity) as close as we want (given $n \in \mathbb{Z}$). This allows us to define two angular variables ϕ_V and ϕ_A, both are periodic in $[0, 2\pi)$:

$$\phi_V = \phi, \quad \phi_A = k_F n \mod 2\pi, \tag{A6}$$

then Eq. (A5) can be compactly written as

$$\begin{aligned} &\text{UV symmetry} \Rightarrow \text{IR symmetry} \\ &\mathrm{U}(1) \Rightarrow \mathrm{U}(1)_V : c_a \to \exp\left(iq_a^V\phi_V\right)c_a, \\ &\mathbb{Z} \Rightarrow \mathrm{U}(1)_A : c_a \to \exp\left(iq_a^A\phi_A\right)c_a, \end{aligned} \tag{A7}$$

for $a = AR, BR, BL, AL$, enumerating over the four Fermi point labels, together with the charge vectors (given that $q_A = 1$ and $q_B = 3$):

$$\mathbf{q}^V = \begin{bmatrix} q_A \\ q_B \\ q_B \\ q_A \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{q}^A = \begin{bmatrix} 3 \\ -1 \\ 1 \\ -3 \end{bmatrix}. \tag{A8}$$

Therefore, the global charge U(1) symmetry is simply reinterpreted as the $\mathrm{U}(1)_V$ vector symmetry, and the translation symmetry (described by a noncompact $\mathbb{Z}$ group) in the UV becomes an emergent $\mathrm{U}(1)_A$ axial symmetry (described by a compact U(1) group) in the IR. The symmetry transformation in Eq. (A7) precisely matches Eq. (14) with the correct charge assignment as listed in Table I.

Recombining the charge vectors of $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_A$, we can define two alternative emergent U(1) symmetries, denoted as $\mathrm{U}(1)_{\frac{3V\pm A}{2}}$ with the charge vectors:

$$\mathbf{q}^{\frac{3V\pm A}{2}} = \tfrac{1}{2}(3\mathbf{q}^V \pm \mathbf{q}^A), \tag{A9}$$

as their names implied. More explicitly, the charge vectors match the chiral charge assignments for the 3-4-5-0 fermions:

$$\mathbf{q}^{\frac{3V+A}{2}} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix}, \quad \mathbf{q}^{\frac{3V-A}{2}} = \begin{bmatrix} 0 \\ 5 \\ 4 \\ 3 \end{bmatrix}. \tag{A10}$$

The fermions are expected to transform under $\mathrm{U}(1)_{\frac{3V\pm A}{2}}$ as (parameterized by the periodic angles $\phi_\pm \in [0, 2\pi)$)

$$\mathrm{U}(1)_{\frac{3V\pm A}{2}} : c_a \to \exp\left[i\frac{1}{2}\left(3q_a^V \pm q_a^A\right)\phi_\pm\right]c_a. \tag{A11}$$

This can be viewed as the combined transformation of $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_A$ with the vector rotation angle ϕ_V and the axial rotation angle ϕ_A given by

$$\phi_V = \tfrac{3}{2}\phi_\pm, \quad \phi_A = \pm\tfrac{1}{2}\phi_\pm, \tag{A12}$$

as can be verified by comparing Eq. (A11) with Eq. (A7). Now we can connect these rotation angles back to the original $\mathrm{U}(1) \times \mathbb{Z}$ symmetry of the lattice fermions using the relation in Eq. (A6):

$$\phi = \tfrac{3}{2}\phi_\pm, \quad \pm\tfrac{1}{2}\phi_\pm = k_F n \mod 2\pi. \tag{A13}$$

Eliminate $\phi_\pm$ from the equations, and we obtain the relation:

$$\phi = \pm 3k_F n \mod 2\pi, \tag{A14}$$

for the $\mathrm{U}(1)_{\frac{3V\pm A}{2}}$ symmetries. Therefore, to reproduce the IR emergent $\mathrm{U}(1)_{\frac{3V\pm A}{2}}$ symmetries, the corresponding UV symmetries (at the lattice level) must be implemented such that every n-step translation should be followed by a charge U(1) rotation with the rotation angle $\phi = \pm 3k_F n$. Thus, we establish the following correspondence between the IR and UV symmetries:

$$\text{IR symmetry} \Rightarrow \text{UV symmetry}$$
$$\mathrm{U}(1)_{\frac{3V\pm A}{2}} \Rightarrow \mathbb{Z}\left(\frac{3V\pm A}{2}\right): \begin{cases} c_{iA} \to \exp(\pm 3iq_A k_F n)c_{(i+n)A}, \\ c_{iB} \to \exp(\pm 3iq_B k_F n)c_{(i+n)B}. \end{cases} \tag{A15}$$

Here, the compact U(1) symmetries in the IR get mapped to the noncompact symmetries $\mathbb{Z}$ in the UV because the UV symmetries are parameterized by the integer variable $n \in \mathbb{Z}$. Given that $q_A = 1$ and $q_B = 3$, Eq. (A15) becomes Eq. (15), as claimed in the main text. Therefore, the 3-4-5-0 chiral fermion model is indeed realized by the (1+1)D two-band lattice model at low energy.

APPENDIX B: WANG-WEN INTERACTION

In the bonsonization language, the Wang-Wen interaction is described by

$$\mathcal{L}_{\text{int}} = \sum_{\alpha=1,2} g_\alpha \cos\left(l_\alpha^\intercal \varphi\right), \tag{B1}$$

with $\varphi = (\varphi_{AR}, \varphi_{BR}, \varphi_{BL}, \varphi_{AL})^\intercal$ and the interaction vectors given by

$$l_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 2 \end{bmatrix}, \quad l_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \\ 1 \end{bmatrix}. \tag{B2}$$

Mapping back to the chiral fermions by the correspondence $c_a \sim : \exp(i\varphi_a):$, the interaction reads

$$\begin{aligned} H_{\text{int}} &= \frac{g_1}{2}(c_{AR}c_{BL})(c_{BR}^\dagger c_{AL})^2 + \text{H.c.} \\ &\quad + \frac{g_2}{2}(c_{BR}c_{AL})(c_{AR}c_{BL}^\dagger)^2 + \text{H.c.} \end{aligned} \tag{B3}$$

According to Eq. (12) and using the inverse Fourier transformation:

$$\begin{aligned} c_{AR} &= c_{(3k_F)A} = \sum_i c_{iA}\exp(3ik_F i), \\ c_{BR} &= c_{(-k_F)B} = \sum_i c_{iB}\exp(-ik_F i), \\ c_{BL} &= c_{(k_F)B} = \sum_i c_{iB}\exp(ik_F i), \\ c_{AL} &= c_{(-3k_F)A} = \sum_i c_{iA}\exp(-3ik_F i). \end{aligned} \tag{B4}$$

Plugging Eq. (B4) into Eq. (B3), the interaction becomes

$$H_{\text{int}} = \sum_{i_1,\cdots,i_6} g_{i_1\cdots i_6}\left(c_{i_1B}^\dagger c_{i_2A}\right)\left(c_{i_3B}c_{i_4A}\right)\left(c_{i_5B}^\dagger c_{i_6A}\right) + \text{H.c.}, \tag{B5}$$

with

$$\begin{aligned} g_{i_1\cdots i_6} &= \frac{g_1}{2}\exp[ik_F(i_1 - 3i_2 + i_3 + 3i_4 + i_5 - 3i_6)] \\ &\quad + \frac{g_2}{2}\exp[ik_F(-i_1 + 3i_2 - i_3 - 3i_4 - i_5 + 3i_6)]. \end{aligned} \tag{B6}$$

Notice that, under lattice reflection symmetry $\mathbb{Z}_2: c_{iA} \to c_{(-i)A}, c_{iB} \to c_{(-i)B}$, g_1 and g_2 map to each other. To simplify, we can impose the reflection symmetry which requires $g_1 = g_2 = g$. Then the coupling coefficient is

$$g_{i_1\cdots i_6} = g\cos\left[k_F(i_1 - 3i_2 + i_3 + 3i_4 + i_5 - 3i_6)\right]. \tag{B7}$$

The dominant s-wave interaction is given by

$$i_1 - 3i_2 + i_3 + 3i_4 + i_5 - 3i_6 = 0, \tag{B8}$$

such that $g_{i_1\cdots i_6} = g$ is uniform. We seek a local interaction that has minimal span on the lattice. The optimal solution of Eq. (B8) is given by

$$i_1 = i_2 = i - 1, \quad i_3 = i_4 = i, \quad i_5 = i_6 = i + 1, \tag{B9}$$

for any choice of i. With this solution in Eq. (B9), Eq. (B5) reduces to

$$H_{\text{int}} = g\sum_i c_{(i-1)B}^\dagger c_{(i-1)A}c_{iB}c_{iA}c_{(i+1)B}^\dagger c_{(i+1)A} + \text{H.c.}, \tag{B10}$$

which is the SMG interaction in Eq. (16) proposed in the main text.

APPENDIX C: FULL RG EQUATIONS

We start with the interaction $H_{\text{int,CF}}$:

$$\begin{aligned} H_{\text{int,CF}} &= g_{\text{rs}}\sum_\alpha \epsilon^{ijk}c_{B\alpha}^\dagger c_{A_i\alpha}c_{A_j\alpha}c_{A_k\alpha} \\ &\quad + g_{\text{rt}}\sum_{\alpha\neq\beta}\epsilon^{ijk}c_{B\alpha}^\dagger c_{A_i\alpha}c_{A_j\beta}c_{A_k\beta} + \text{H.c.} \end{aligned} \tag{C1}$$

Under RG, the following density-density and exchange interactions will be generated:

$$\begin{aligned} H_{\text{int,AA}} &= g_{\text{as}}\sum_{\alpha,st} n_{A_s\alpha}n_{A_t\alpha} + g_{\text{at}}\sum_{\alpha\neq\beta,st} n_{A_s\alpha}n_{A_t\beta} \\ &\quad + g_{\text{ae}}\sum_{\alpha\neq\beta,st} c_{A_s\alpha}^\dagger c_{A_s\beta}c_{A_t\beta}^\dagger c_{A_t\alpha} \\ &\quad + (A_s \leftrightarrow A_t) + \text{H.c.}, \end{aligned} \tag{C2}$$

and

$$\begin{aligned} H_{\text{int,AB}} &= g_{\text{bs}}\sum_{\alpha,s} n_{B\alpha}n_{A_s\alpha} + g_{\text{bt}}\sum_{\alpha\neq\beta,s} n_{B\alpha}n_{A_s\beta} \\ &\quad + g_{\text{be}}\sum_{\alpha\neq\beta,s} c_{B\alpha}^\dagger c_{B\beta}c_{A_s\beta}^\dagger c_{A_s\alpha} + (A_s \leftrightarrow B) + \text{H.c.} \end{aligned} \tag{C3}$$

There is an additional density-density interaction that will correct $H_{\text{int,AA}}$, $H_{\text{int,AB}}$:

$$H_{\text{int,BB}} = g_{\text{bb}} \sum_{\alpha\beta} n_{B\alpha} n_{B\beta} - c^\dagger_{B\alpha} c_{B\beta} c^\dagger_{B\beta} c_{B\alpha}. \tag{C4}$$

Putting all interactions together, the complete RG equations are

$$\frac{dg_{\text{bb}}}{d\ell} = 4d_0 d_{\text{BB}} g_{\text{bb}}^2 + 3d_0 g_{\text{be}}^2,$$

$$\frac{dg_{\text{bs}}}{d\ell} = -2d_{\text{AB}} g_{\text{bs}}^2 + \frac{9g_{\text{rs}}^2}{2} + g_{\text{rt}}^2,$$

$$\frac{dg_{\text{bt}}}{d\ell} = 2d_0 d_{\text{AB}} g_{\text{bt}}^2,$$

$$\frac{dg_{\text{be}}}{d\ell} = -6d_0 g_{\text{ae}} g_{\text{be}} + 2d_0 g_{\text{at}} g_{\text{be}} + 4d_0 d_{\text{BB}} g_{\text{bb}} g_{\text{be}} + 3g_{\text{rs}} g_{\text{rt}} + \frac{g_{\text{rt}}^2}{2},$$

$$\frac{dg_{\text{as}}}{d\ell} = -2g_{\text{as}}^2,$$

$$\frac{dg_{\text{at}}}{d\ell} = 2d_0 g_{\text{at}}^2 - d_0 d_{\text{AB}} g_{\text{rt}}^2,$$

$$\frac{dg_{\text{ae}}}{d\ell} = -d_0 d_{\text{AB}} g_{\text{rt}}^2 + 4d_0 g_{\text{ae}} g_{\text{at}} - 6d_0 g_{\text{ae}}^2 - 2d_0 d_{\text{BB}} g_{\text{be}}^2,$$

$$\frac{dg_{\text{rs}}}{d\ell} = -6g_{\text{as}} g_{\text{rs}},$$

$$\frac{dg_{\text{rt}}}{d\ell} = 4d_0 d_{\text{AB}} g_{\text{bt}} g_{\text{rt}} - 2g_{\text{as}} g_{\text{rt}},$$

where $d_{\text{AB}} = d\chi_{pp,\text{AB}}(\mathbf{0})/d\ell$, $d_{\text{BB}} = d\chi_{pp,\text{BB}}(\mathbf{0})/d\ell$. These ratios depend on the energies of A- and B-type fermions near the VHSs. The two types of fermions have similar band structures, which can be approximated as $E^{A,B}_{\mathbf{k}} = \epsilon^{A,B} f(\mathbf{k})$. The ratios are then given by $d_{\text{AB}} = \frac{2|\epsilon^A|}{|\epsilon^A|+|\epsilon^B|}$ and $d_{\text{BB}} = \frac{|\epsilon^A|}{|\epsilon^B|}$. If A- and B-type fermions have the same band structure, then $d_{\text{AB}} = d_{\text{BB}} = 1$.

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