Bath-induced phase transition in a Luttinger liquid

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We study an XXZ spin chain, where each spin is coupled to an independent ohmic bath of harmonic oscillators at zero temperature. Using bosonization and numerical techniques, we show the existence of two phases separated by a Kosterlitz-Thouless transition. At low coupling with the bath, the chain remains in a Luttinger liquid (LL) phase with a reduced but finite spin stiffness, while above a critical coupling, the system is in a dissipative phase characterized by a vanishing spin stiffness. We argue that the transport properties are also inhibited: The LL is a perfect conductor, while the dissipative phase displays finite resistivity. Our results show that the effect of the bath can be interpreted as annealed disorder-inducing signatures of localization.

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I. INTRODUCTION

Localization is a spectacular quantum effect in which transport properties are totally suppressed. It is mainly due to the presence of quenched impurities, as predicted by Anderson [1] for free fermions and recently argued to persist in the presence of interactions via the so-called many-body localization (MBL) [2–7]. However, localization can also be induced by quantum measurements or by the presence of an external bath. The first case gives rise to the Zeno effect, originally introduced as the mechanism that froze the dynamics of a two-level system [8,9]. Today, it is generalized to many-body systems in which the frequency of quantum measurement is at the origin of phase transitions from a volume law to an area law for entanglement entropy [10–19]. Localization induced by external bath is much less studied. It is known that a subohmic bath at zero temperature can freeze the quantum dynamics of simple systems, such as a single spin or particle [20–23]. In this paper, we investigate if these mechanisms are also relevant in a many-body system. We show that a bath that produces local phonons is a source of annealed disorder and study how this disorder affects the transport properties. We focus on a one-dimensional (1D) system that can be mapped to a two-dimensional (2D) field theory, already studied by bosonization [24] and Monte Carlo techniques [25] in a different context. However, its phase diagram remains controversial, and it is not clear how many phases appear varying the strength of the coupling between the bath and the system. Here, we introduce an approach which directly simulates the bosonized action and allows us to reach large system sizes. Our results show a simple scenario of two phases with a Kosterlitz-Thouless (KT) transition between them. Increasing the coupling strength, a dissipative phase with suppressed transport takes over a perfectly conducting Luttinger liquid (LL) phase.

II. MODEL

We consider an XXZ spin chain with the Hamiltonian $H_S = \sum_{j=1}^{N} J_z S_j^z S_{j+1}^z + J_{xy}(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y)$ and $J_z/J_{xy} \in (-1, 1)$. This model displays a gapless low-energy spectrum, and it is in a perfectly conducting phase known as LL [26]. Each spin j of the chain is in contact with its own independent bath of harmonic oscillators with the Hamiltonian $H_B = \sum_{jk} \frac{P_{jk}^2}{2m_k} + \frac{m_k \Omega_k^2}{2} X_{jk}^2$ (see Fig. 1). A different choice for local baths was studied in Ref. [27]. The complete Hamiltonian is given by

$$H = H_S + H_B + H_{SB},$$
$$H_{SB} = \sum_{j=1}^{N} S_j^z \sum_k \lambda_k X_{jk}. \tag{1}$$

Note that the coupling term $h_j(t) = \sum_k \lambda_k X_{jk}$ is equivalent to a time-dependent magnetic field interacting with the spins. The time-independent limit $h_j(t) = h_j$ corresponds to a quenched disordered magnetic field. This case is well studied by bosonization [28] or powerful simulation techniques [29], and a zero temperature localization transition from LL toward a Bose glass phase takes place by varying the disorder strength. Here, we employ bosonization to study the time-dependent (annealed disorder) case. To fully characterize the bath, we need to specify the low-frequency behavior of the spectral function, defined as

$$J(\Omega) = \frac{\pi}{2} \sum_k \left(\frac{\lambda_k^2}{m_k \Omega_k} \right) \delta(\Omega - \Omega_k). \tag{2}$$

In general, one has $J(\Omega) = \pi \alpha \Omega^s$ for $\Omega \in (0, \Omega_D)$. Here, α denotes the effective coupling strength with the bath, the cutoff Ω_D is the Debye frequency, and s sets the nature of the bath. For our study, we take $s = 1$, which corresponds to an ohmic bath.

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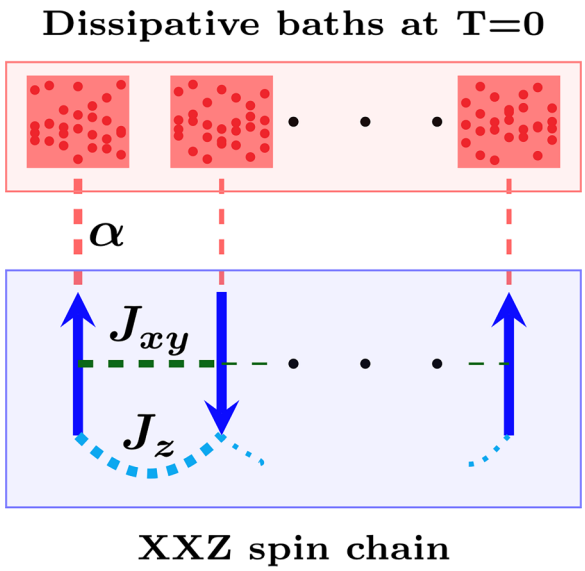

FIG. 1. Schematic representation of the microscopic system: A one-dimensional XXZ spin chain (blue color) with each spin coupled to its individual dissipative bath (red color). The baths are described by a collection of simple harmonic oscillators kept at zero temperature. The parameter α is a measure of the coupling strength between the bath and the associated spin.

III. BOSONIZED ACTION

The bosonization procedure of the XXZ spin chain is well known [26]. We map the chain with periodic condition into a 1D fermionic system using Jordan-Wigner transformation. For $J_z = 0$, we recover the free-fermion problem that can be diagonalized in the momentum space $q = 2\pi l/(Na)$ with a as lattice spacing and $l \in (-N/2, N/2)$. The Fermi momentum depends on the total magnetization M of the spin chain, namely, $q_F = \pi(N - M)/(2Na)$, where q_F is the Fermi momentum of the spin chain and a is the lattice spacing kept for dimensional matching reasons. Two cases should be distinguished: In the zero sector of magnetization, q_F is commensurate with the lattice space, while it is incommensurate for the nonzero magnetization sector. Here, we focus on the incommensurate case. Away from the sector of zero magnetization, by linearizing the spectrum around q_F, one recovers the action for the well-known LL model:

$$S_{\rm LL} = \frac{1}{2\pi}\int dx d\tau \left\{\frac{1}{uK}[\partial_\tau \phi(x,\tau)]^2 + \frac{u}{K}[\partial_x \phi(x,\tau)]^2\right\}. \tag{3}$$

Here, $\phi(x,\tau)$ is a 2D field living in the physical space $x \in (0, L)$ and in imaginary time $\tau \in (0, \beta)$, β being the inverse temperature of the system. At zero magnetization, there is an extra term in the action $S_{\cos} = -\frac{J_z}{2\pi^2}\int dx d\tau \cos[4\phi(x,\tau)]$, which is irrelevant for $K > \frac{1}{2}$. The constants u and K are called LL parameters, and they depend on J_{xy} and J_z. These parameters can be exactly calculated from the Bethe ansatz (e.g., $K^{-1}_{\text{Bethe ansatz}} = (2/\pi)\arccos[-J_z/J_{xy}]$), and they match with the bosonization prediction in the regime $J_z \ll J_{xy}$ ($K^{-1}_{\text{bosonization}} = \sqrt{1 + 4J_z/\pi J_{xy}}$). However, away from half-filling (nonzero magnetization sector), the bosonization prediction between the LL parameters and the spin chain are slightly more complicated and given by $uK = aJ_{xy}\sin(q_F a)$ and $u/K = uK\{1 + \frac{2aJ_z}{\pi v_F}[1 - \cos(2q_F a)]\}$.

To tackle the dissipative problem, there are two different approaches. Here, we map to an equivalent fermionic system via Jordan-Wigner transformation and apply bosonization to arrive at a 2D field theory. Alternatively, the quantum Hamiltonian can be mapped onto a hard-core bosonic system via Holstein-Primakoff transformation and then numerically simulated via quantum Monte Carlo methods (indeed, bosons do not suffer from the sign problem). In both cases, to integrate the bath degrees of freedom, one must introduce the path integral description of the system. The action associated with the bath and the interaction between the bath and the system are identical for bosons and fermions and are given by

$$S_{\rm B} + S_{\rm SB} = \int_0^\beta d\tau \sum_{j=1}^N \left[n_j(\tau) - \frac{1}{2}\right]\sum_k \lambda_k X_{kj} + \sum_{j=1}^N \sum_k \left(m_k \dot{X}_{kj}^2 + \frac{m_k \Omega_k^2}{2} X_{kj}^2\right), \tag{4}$$

where $n_j = S_j^z + \frac{1}{2}$ is the density operator. Now, we can integrate out the bath degrees of freedom and arrive at an effective action for the system degrees of freedom only, where the effect of the bath is encoded in the interacting part $S_{\rm int}$:

$$S_{\rm int} = -\iint_0^\beta d\tau d\tau' \sum_{j=1}^N \left[n_j(\tau) - \frac{1}{2}\right] \times D(\tau - \tau')\left[n_j(\tau') - \frac{1}{2}\right]. \tag{5}$$

Here, $D(\tau - \tau')$ is the dissipative kernel which is produced from integrating over the bath modes. Its Fourier transform can be expressed in terms of the bath spectral function $J(\Omega)$:

$$D(\omega_n) = \frac{2}{\pi}\int_0^\infty J(\Omega)\frac{\Omega}{\omega_n^2 + \Omega^2}. \tag{6}$$

Using the form $J(\Omega) = \pi\alpha\Omega$ ($s = 1$), we get $D(\tau - \tau') \sim \alpha|\tau - \tau'|^{-2}$.

To bosonize Eq. (5), we recall that the bosonized version of S_j^z is given by

$$\hat{S}^z = -\frac{1}{\pi}\nabla\phi + \frac{1}{\pi a}\cos(2\phi - 2q_F x). \tag{7}$$

Using Eq. (5), the dissipative part of the action is given by

$$S_{\rm int} = -\frac{1}{2\pi^2}\int_0^L dx \iint_0^\beta d\tau d\tau' \times \left\{-\nabla\phi(x,\tau) + \frac{1}{a}\cos[2\phi(x,\tau) - 2q_F x]\right\} \times D(\tau - \tau')\left\{-\nabla\phi(x,\tau') + \frac{1}{a}\cos[2\phi(x,\tau') - 2q_F x]\right\}. \tag{8}$$

After multiplying all the terms, one can put $a = 1$, which was there for dimensional purposes. For the expansion, we will be making a few observations here:

(1) At equal time ($\tau = \tau'$), the dissipative action is $(S_j^z)^2$, which is identity. Hence, this term does not contribute anything to the physics, and we can neglect the constant term in $D(\tau - \tau')$.
(2) The terms of the form $\nabla\phi(\tau)\cos[2\phi(\tau') - 2q_F x]$ will oscillate rapidly for nonzero magnetization due to the $2q_F x$ term, and hence, it will integrate to zero. Similarly, the $\cos[2\phi(\tau) - 2q_F x]\cos[2\phi(\tau') - 2q_F x]$ term can be broken up into two terms; one of these terms will be of the form $\cos\{2[\phi(\tau) + \phi(\tau')] - 4q_F x\}$. This term can also be integrated to zero due to the rapidly oscillating term $4q_F x$.
(3) The $\nabla\phi(\tau)\nabla\phi(\tau')$ term is the forward scattering term and is irrelevant by power counting.

Hence, the action of the full system turns out to be

$$S_{\text{tot}} = S_{\text{LL}} + S_{\text{int}}, \tag{9}$$

$$S_{\text{int}} = -\frac{\alpha}{4\pi^2}\int dx d\tau d\tau' \frac{\cos\{2[\phi(x,\tau) - \phi(x,\tau')]\}}{|\tau - \tau'|^2}. \tag{10}$$

IV. OBSERVABLES AND BOSONIZATION

Thermodynamic quantities of the spin chain can be expressed in terms of the correlation functions of the field ϕ. The propagator $G(q,\omega_n) = \langle\phi(q,\omega_n)\phi(-q,-\omega_n)\rangle$ can be related to the susceptibility χ and spin stiffness ρ_s by the two equations:

$$\chi = \lim_{q\to 0}\lim_{\omega_n\to 0}\frac{q^2}{\pi^2}G(q,\omega_n), \tag{11}$$

$$\rho_s = \lim_{\omega_n\to 0}\lim_{q\to 0}\frac{\omega_n^2}{\pi^2}G(q,\omega_n). \tag{12}$$

Here, $\omega_n = 2\pi n/\beta, n \in (-\infty,\infty)$ are the Matsubara frequencies. In the LL phase, $G_{\text{LL}}(q,\omega_n) = \pi K/(\omega_n^2/u + uq^2)$, and hence, $\chi = K/(u\pi)$ and $\rho_s = uK/\pi = K^2/(\pi^2\chi)$. The bath introduces a long-range cosine interaction in the τ direction only, and the strength of this potential is controlled by the parameter α. A perturbative renormalization group study [24] shows that, for $K < K_c = 0.5$, the cosine term is relevant, and the LL phase is destroyed, whereas for $K > K_c$ and small α, the system stays in the LL phase but with renormalized LL parameters K_r and u_r. For $K \gtrsim K_c$, the transition is of the KT type: The critical point $\alpha_c(K)$ is still LL with $K_r = K_c = 0.5$. The nature of the dissipative phase is not clear: For moderate K and very large α, the action should be gapless and harmonic, obtained by the quadratic expansion of the cosine term. For $K \gg K_c$, a large-N argument suggests the existence of a gapped disordered phase. Monte Carlo simulations [25] were performed on the 1D hard-core bosonic chain, which can be mapped to free fermions ($K = 1$). Increasing α, they found that χ increases, and at α_c, the system undergoes a continuous second-order phase transition with vanishing ρ_s. Below, we propose a simple scenario able to conciliate the puzzle of contradictory results.

V. METHODS

To make progress, on one side, we compute the correlation function $G(q,\omega_n)$ numerically by generating equilibrated configurations $\phi(x,\tau)$ from the action in Eq. (9) with the help of Langevin dynamics (see Appendix C). The long-distance, low-energy behavior of this correlation function allows us to classify the system in two possible phases. One possibility is that the system remains in the LL phase with renormalized values of u and K. The second possibility is the appearance of a new dissipative phase, where α becomes relevant. The analytical behavior of $G(q,\omega_n)$ in this new phase was proposed in Ref. [24] using a harmonic expansion around the cosine potential. Here, we use a variational approach and propose an improved expression of the correlation function in the dissipative phase:

$$G_{\text{var}}^{-1}(q,\omega_n) = \frac{u_r q^2}{2\pi K_r} + \frac{\alpha_r}{\pi^2}|\omega_n| + a_1|\omega_n|^{3/2} + a_2\omega_n^2. \tag{13}$$

The macroscopic behavior of this phase depends only on the two parameters u_r/K_r and α_r. The parameters a_1 and a_2 are introduced to account for finite-sized effects. From the analysis of our result, we will show that, by varying α, the long-distance properties are always captured either by the LL or by the variational propagator [Eq. (13)] with renormalized parameters u_r, K_r, and α_r.

A. Variational ansatz

To derive the correlation function of Eq. (13), we need to find an effective quadratic action of the form $S_{\text{var}} = \frac{1}{2\beta L}\sum_{q,\omega_n}\phi^*(q,\omega_n)G_{\text{var}}^{-1}(q,\omega_n)\phi(q,\omega_n)$. We use the variational method: We minimize the free energy $F_{\text{var}} = -\frac{1}{\beta}\sum_{q,\omega_n}\log G_{\text{var}} + \frac{1}{\beta}\langle S - S_{\text{var}}\rangle_{S_{\text{var}}}$ [with $(S - S_{\text{var}})$ averaged over S_{var}] with respect to the variational Green's function:

$$G_{\text{var}}^{-1} = \frac{1}{2\pi K}\left(uq^2 + \frac{\omega_n^2}{u}\right) + \frac{\alpha}{\pi^2}\int d\tau D(\tau)(1 - \cos\omega\tau) \times \exp\left\{\left[-\frac{1}{\pi^2}\int_{-\infty}^{\infty} dq d\omega\, G_{\text{var}}(q,\omega)(1 - \cos\omega\tau)\right]\right\}. \tag{14}$$

We try to solve this self-consistent equation by making the following ansatz: $G_{\text{var}}^{-1}(q,\omega) = \frac{1}{2\pi K}(uq^2 + \frac{\omega_n^2}{u}) + \frac{\alpha}{\pi^2}F(\omega)$, where $F(\omega) = a(\alpha)|\omega|^{\psi} + b(\alpha)|\omega|$. With this assumption, for large τ, the behavior of $G(\omega)$ is governed by the $|\omega|$ term. It can be easily shown that $\int_{\infty}^{\infty} dq d\omega\, G_{\text{var}}(q,\omega)(1 - \cos\omega\tau) \approx C(\alpha) - [\frac{\tau_c(\alpha)}{\tau}]^{1/2}$, where $C(\alpha)$ and $\tau_c(\alpha)$ are α-dependent constants. For a more systematic expansion in powers of $1/\tau$, one can use the results in Ref. [30]. Putting this back into the self-consistent equation for $F(\omega)$, we obtain

$$\begin{aligned} a(\alpha)|\omega|^{\psi} + b(\alpha)|\omega| &= \int d\tau D(\tau)(1 - \cos\omega\tau) \times \exp\left(-\left\{C(\alpha) - \left[\frac{\tau_c(\alpha)}{\tau}\right]^{1/2}\right\}\right) \\ &\overset{\text{large }\tau}{\approx} \int d\tau D(\tau)(1 - \cos\omega\tau) \times e^{-C(\alpha)}\left\{1 + \left[\frac{\tau_c(\alpha)}{\tau}\right]^{1/2}\right\}. \end{aligned} \tag{15}$$

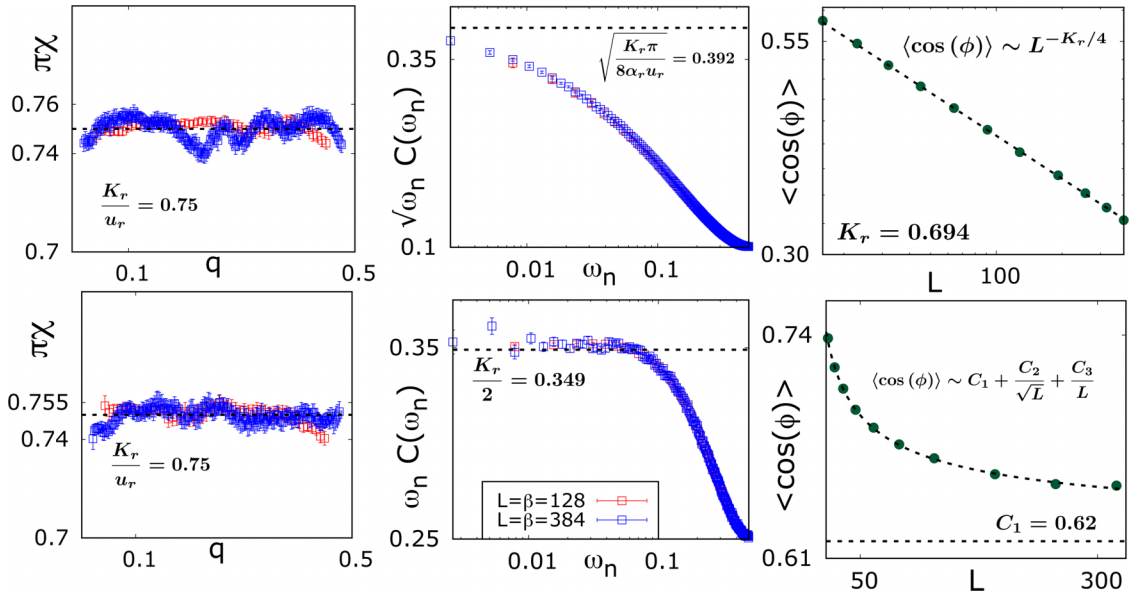

FIG. 2. Calculation of different quantities for $K = 0.75$ that characterizes Luttinger liquid (LL; $\alpha = 1$, top row) and dissipative phase ($\alpha = 8$, bottom row). Blue and red points correspond to $L = \beta = 384$ and 128, respectively. The average is performed over 6000 to 12 000 configurations. (left) Due to symmetry, $\pi\chi = K_r/u_r$ is equal to $K/u = 0.75$ for all values of α and all length scales. (middle) For $\alpha = 1$, $\omega_n C(\omega_n)$ saturates to $K_r/2 = 0.349$ as $\omega_n \to 0$; whereas for $\alpha = 8$, $\sqrt{\omega_n}C(\omega_n)$ saturates to $[K_r\pi/(8\alpha_r u_r)]^{1/2} = 0.392$. The other fitting constants are $a_1 = 0.2493$ and $a_2 = 3.572$. (right) For $\alpha = 1$, $\langle\cos(\phi)\rangle$ decays as a power law, which allows us to extract $K_r = 0.694$, consistent with the fit of $\omega_n C(\omega_n)$. For $\alpha = 8$, it saturates to a constant, as predicted by the variational ansatz (the fit gives $c_1 = 0.62$, $c_2 = 0.603$, and $c_3 = -0.531$).

The ω dependence can be easily extracted from these equations, which turns out to be $|\omega|$ and $|\omega|^{3/2}$. The coefficient of $|\omega|$ should be determined self-consistently, and in our analysis, we take it as a fitting parameter α_r/π^2. The coefficient in front of ω^2 will be renormalized by higher-order terms from variational analysis. Hence, the variational propagator is given at low order in ω by Eq. (13).

VI. RESULTS

A. Phase diagram

In the following, we present our results for the correlation functions of the action $S = S_{\rm int} + S_{\rm LL}$, with $u = 1$, $K = 0.75$, and different α. For our simulations, we set $\beta = L$. The first observation is that the action of Eq. (10) is invariant under tilt transformation (see Appendix B). As a consequence, χ is not affected by the presence of $S_{\rm int}$. We measure K_r/u_r both at low and high α, as shown in Fig. 2, left. Note that the susceptibility corresponds to the $q \to 0$ limit, but due to the symmetry, K_r/u_r is invariant at all length scales and all values of α. We conclude that $K_r/u_r = K/u$ for all values of α. In Fig. 2, middle, we present our results for $C(\omega_n) = (1/\pi L)\sum_q \langle|\phi(q,\omega_n)|^2\rangle = \frac{1}{\pi L}\sum_q G(q,\omega)$. Using the LL and the variational propagator, we find that

$$C(\omega_n \to 0) = \begin{cases} \frac{K_r}{2\omega_n} & \text{LL} \\ \sqrt{\frac{K_r\pi}{8\alpha_r u_r}}\frac{1}{\sqrt{\omega_n}} & \text{variational.} \end{cases} \tag{16}$$

We see that, indeed, for small α, $C(\omega_n)$ behaves as expected for the LL phase, while for large α, $C(\omega_n)$ shows an agreement with the variational approach. To confirm our prediction, we compute an independent quantity, namely, $\langle\cos(\phi)\rangle$. This quantity decreases with a characteristic finite-sized behavior: It goes to zero as $\langle\cos(\phi)\rangle_{\rm LL} \sim L^{-K_r/4}$ in the LL phase and saturates to a constant as $\langle\cos(\phi)\rangle_{\rm var} \sim c_1 + c_2/\sqrt{L} + c_3/L$ within the variational ansatz (here, c_1, c_2, and c_3 are fitting cutoff-dependent parameters, see Appendix A). Figure 2, right, confirms the scenario of a transition between a LL to a dissipative phase described by the variational ansatz. Moreover, the value of K_r extracted from $\langle\cos(\phi)\rangle$ matches nicely with the prediction of $C(\omega_n)$. In Fig. 3, we rationalize our results of the renormalized parameters, obtained by varying α. For $K = 0.75$, we observe that the stiffness decreases with α in the LL phase and vanishes in the critical region $\alpha \in (3, 4)$. Moreover, just before the transition, K_r approaches $K_c = 0.5$ and $\rho_{s_c} = 1/(4\pi^2\chi)$, as predicted by the KT transition. In the Appendix, we provide further results for $u = 1$, $K = 0.55$ (see Fig. 4), which are also in agreement with this picture.

B. Transport properties

With our approach, one can compute thermodynamic quantities without direct access to transport properties. However, via Wick rotation, the conductivity can be related to the propagator:

$$\sigma(\omega) = \frac{e^2}{\pi^2\hbar}[\omega_n G(q=0,\omega_n)]_{i\omega_n\to\omega+i\epsilon}.$$

For LL, the DC conductivity $\sigma_{\rm DC} \equiv {\rm Re}[\sigma(\omega \to 0)] = (e^2 uK/\hbar)\delta(\omega)$, which shows the system is perfectly conducting. For the dissipative phase, we use Eq. (13) for Wick rotation and get $\sigma_{\rm DC} = e^2/\hbar\alpha_r$, proving that the system has finite conductivity. For a generic bath, $G(q=0,\omega_n) \sim 1/(\alpha_r|\omega_n|^s)$,

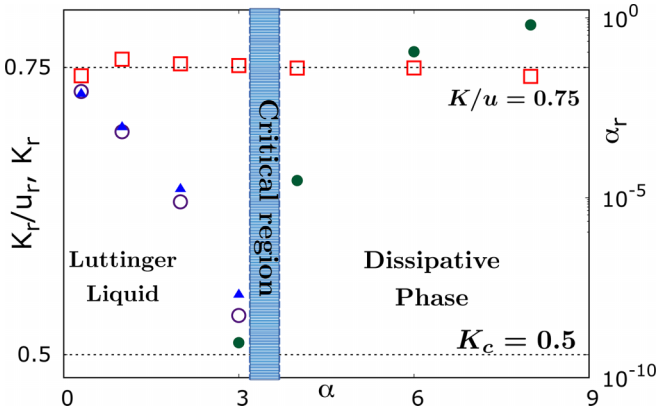

FIG. 3. Behavior of different renormalized parameters as a function of dissipative coupling α. K_r/u_r (red square points) remains constant and equal to $K/u = 0.75$ for all values of α. K_r, extracted from $\langle\cos(\phi)\rangle$ analysis (purple circular points) agrees with the one from the $C(\omega_n)$ analysis (blue triangular points). It approaches $K_c = 0.5$ as α reaches the critical point. The parameter α_r (green circular points) starts to be defined in the dissipative phase and increases rapidly with increase in α. This behavior of the parameter allows us to locate the different phases: For $\alpha < 3$, the system is in Luttinger liquid (LL) phase, whereas for $\alpha > 4$, the system is in dissipative phase. The phase transition takes place for $\alpha \in (3, 4)$.

and hence, $\mathrm{Re}[\sigma(\omega)] = (e^2/\hbar\alpha_r)(\epsilon/(\omega^2+\epsilon^2)^{s/2})$. Especially when the bath is subohmic ($s < 1$), the DC conductivity of the system goes to zero, which is a signature of bath-induced localization in the system.

VII. DISCUSSION AND CONCLUSIONS

It remains important to clarify how to conciliate our observations of a KT transition with $K_c = 0.5$ and the hard-core bosonic Monte Carlo simulations (at $K = 1$ instead of $K = 0.75$) that show a vanishing stiffness at the transition [25]. A possibility is that it is an artifact of the commensurate-incommensurate crossover of the system as the system size, as well as the incommensurate parameter, used in the Monte Carlo study is small. Another possibility remains that our action misses some term that is relevant for the microscopic lattice model.

On a more general framework, many efforts are currently being made to observe localization transition in open quantum systems. The most popular approach is to consider the bath as a perturbative source of quantum measurements. In this Markovian limit, one can rely either on the Lindblad formalism [31], which is microscopically more accurate but is limited to very small system sizes, or introduce models with quantum circuits which display localization transitions but are very simplistic. In both cases, localization appears as a many-body Zeno effect. Here, the bath is non-perturbative and equivalent to annealed disorder. Hence, the localization observed here is a non-Markovian effect, more like the localization due to quenched impurities. It remains an open question to compare the differences of these two kinds of bath-induced localization.

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APPENDIX A: SYSTEM SIZE DEPENDENCE OF ORDER PARAMETER

In this section, we find an analytical expression for the quantity $\langle\cos[\phi(x,\tau)]\rangle$, which is equal to $\exp\{-\frac{1}{2}\langle[\phi(\vec{r})]^2\rangle\}$ for Gaussian theories.

For our variational ansatz, we need to calculate

$$\langle\cos[\phi(\vec{r})]\rangle = \exp(S_1) = \exp\left(-\frac{1}{2\beta L}\sum_{q,\omega_n}\frac{1}{\frac{u}{\pi K}q^2 + \frac{2\alpha}{\pi^2}|\omega_n| + 2a_1|\omega_n|^{3/2} + 2a_2\omega_n^2}\right).$$

To calculate the sum inside the exponential, we send the limit of integration over q from zero to infinity and the integral over ω from $1/\beta$ to $1/l_0$, where l_0 is the microscopic cutoff. By doing so, one can find the small ω behavior of the sum as below:

$$S_1 = \frac{1}{4}\sqrt{\frac{K}{\pi u}}\left[\frac{a_1\pi^3}{(2\alpha)^{3/2}l_0} - \frac{2\pi}{\sqrt{2\alpha l_0}}\right] + \sqrt{\frac{\pi K}{8u\alpha}}\frac{1}{\sqrt{\beta}} - \frac{a_1\pi^3}{4}\sqrt{\frac{K}{8u\alpha^3}}\frac{1}{\beta}. \tag{A1}$$

We put this back into the expression of $\langle\cos[\phi(\vec{r})]\rangle$, and from a large β (zero temperature limit) expansion, we obtain the finite-sized dependence of the order parameter:

$$\begin{aligned}\langle\cos[\phi(\vec{r})]\rangle_{\mathrm{var}} &= c_1 + \frac{c_2}{\sqrt{\beta}} + \frac{c_3}{\beta},\\ c_1 &= \exp\left\{\frac{1}{4}\sqrt{\frac{K}{\pi u}}\left[\frac{a_1\pi^3}{(2a)^{3/2}l_0} - \frac{2\pi}{\sqrt{2\alpha l_0}}\right]\right\},\\ c_2 &= c_1\sqrt{\frac{\pi K}{8u\alpha}},\\ c_3 &= c_1\left(\frac{\pi K}{16u\alpha} - \frac{a_1\pi^3}{4}\frac{K}{8u\alpha^3}\right).\end{aligned} \tag{A2}$$

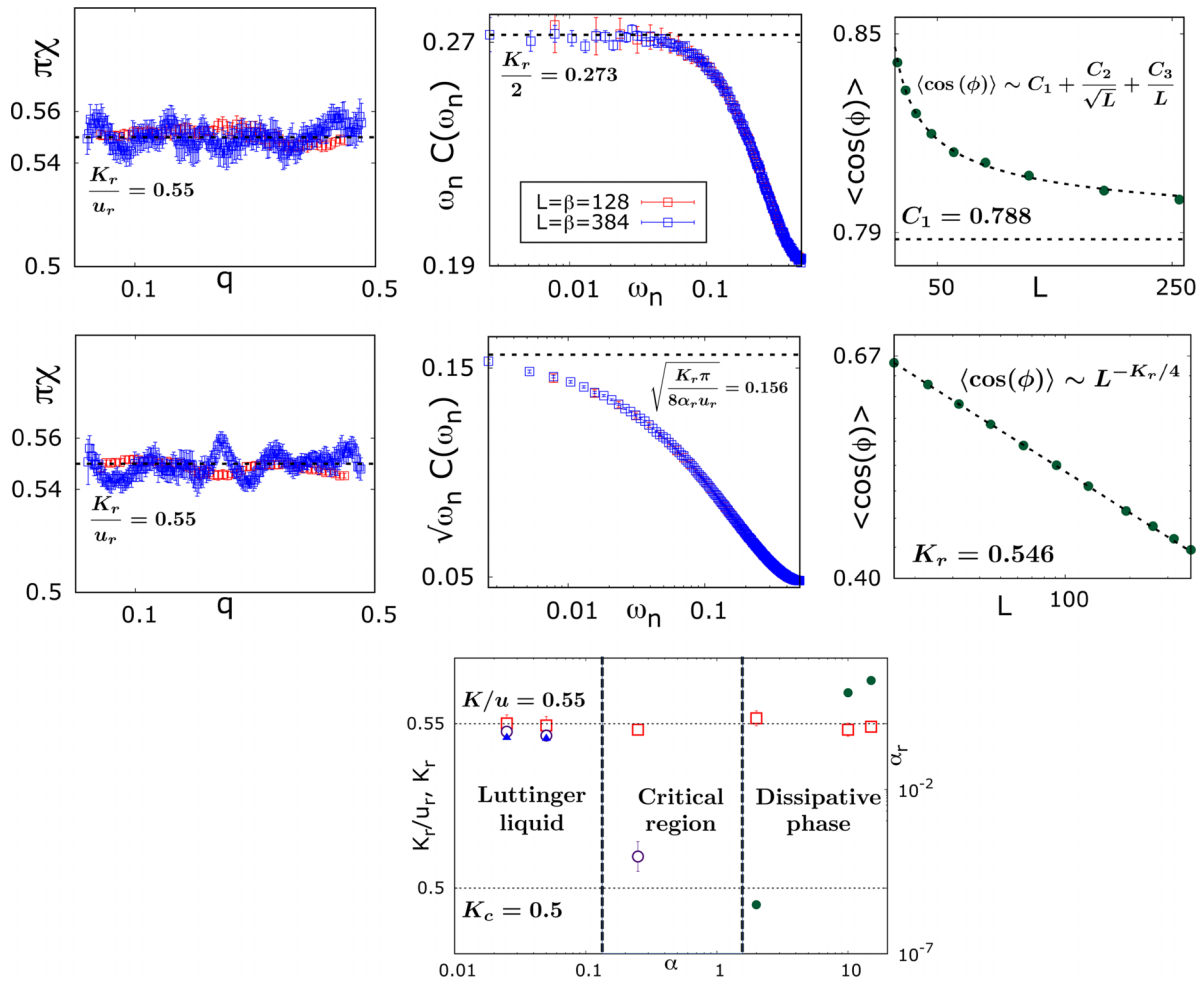

FIG. 4. Calculation of different quantities for $K = 0.55$ that characterizes Luttinger liquid (LL; $\alpha = 0.05$, top row) and dissipative phase ($\alpha = 10$, bottom row). Blue and red points correspond to $L = \beta = 384$ and 128, respectively. (left) Due to symmetry, $\pi\chi = K_r/u_r$ is equal to $K/u = 0.55$ for all values of α and all length scales. (middle) For $\alpha = 0.05$, $\omega_n C(\omega_n)$ saturates to $K_r/2 = 0.273$ as $\omega_n \to 0$; whereas for $\alpha = 10$, $\sqrt{\omega_n}C(\omega_n)$ saturates to $[K_r\pi/(8\alpha_r u_r)]^{1/2} = 0.156$. The other fitting constants are $a_1 = 16.61$ and $a_2 = 571.4$. (right) For $\alpha = 0.05$, $\langle\cos(\phi)\rangle$ decays as a power law, which allows us to extract $K_r = 0.546$, consistent with the fit of $\omega_n C(\omega_n)$. For $\alpha = 10$, it saturates to a constant, as predicted by the variational ansatz (the fit gives $c_1 = 0.788$, $c_2 = 0.215$, and $c_3 = 0.012$). (bottom row) Behavior of different renormalized parameters as a function of dissipative coupling α. K_r/u_r (red square points) remains constant and equal to $K/u = 0.55$ for all values of α. K_r, extracted from $\langle\cos(\phi)\rangle$ analysis (purple circular points) agrees with the one from the $C(\omega)$ analysis (blue triangular points). It approaches $K_c = 0.5$ as α reaches the critical point. The parameter α_r (green circular points) starts to be defined in the dissipative phase and increases rapidly with increase in α. This behavior of the parameter allows us to locate the different phases: For $\alpha < 0.25$, the system is in LL phase, whereas for $\alpha > 2$, the system is in dissipative phase. The phase transition takes place for $\alpha \in (0.25, 2)$. We believe $\alpha = 0.25$ to be in the critical region as K_r extracted from $\langle\cos(\phi)\rangle$ is very close to $K_c = 0.5$, α_r is very small, and $\omega_n C(\omega_n)$ saturates for a long range of ω_n but then starts decreasing.

APPENDIX B: TILT SYMMETRY OF THE ACTION

In this section, we explain why the parameter K/u, which identifies with the susceptibility χ, remains constant for all dissipative coupling α [32,33]. To compute the susceptibility, we introduce a finite magnetic field h in the z direction. Then the susceptibility can be written as $\chi = \frac{\partial^2}{\partial(h\beta)^2}(\ln Z[h])$, where Z is the partition function and β is the inverse temperature of the system. In the bosonized language, the term $-h\sum_j S_j^z$ in the Hamiltonian gives rise to the term $-\frac{h}{\pi}\int[\nabla\phi(x,\tau)]dxd\tau$ in the action. Hence, the partition function of the system can be written as

$$Z[h] = \int \mathcal{D}[\phi]\exp\left\{\left[-S_{\rm LL} - S_{\rm int} + \frac{h}{\pi}\int\nabla\phi(x,\tau)dxd\tau\right]\right\}. \tag{B1}$$

One can rewrite the terms $\frac{u}{2\pi K}(\nabla\phi)^2 - \frac{h}{\pi}\nabla\phi$ as $\frac{u}{2\pi K}(\nabla\phi - \frac{hK}{u})^2 - \frac{h^2K}{2\pi u}$. Introducing the tilt $\tilde{\phi} \to \phi - \frac{hKx}{u}$, the partition function can be rewritten:

$$Z[h] = \int \mathcal{D}[\tilde{\phi}]\exp\left\{\left(-S_{\rm LL}[\tilde{\phi}] - S_{\rm int}[\tilde{\phi}] + \frac{\beta^2h^2K}{2u\pi}\right)\right\}. \tag{B2}$$

The key point is that the interacting action $S_{\rm int}$ is invariant under the tilt transformation $S_{\rm int}(\tilde{\phi}+\frac{hkx}{u})=S_{\rm int}(\tilde{\phi})$. From the previous equation, it can be easily seen that

$$\ln Z[h]=\frac{\beta^2h^2K}{2u\pi}+\ln Z[h=0]. \tag{B3}$$

From this expression, the susceptibility can be easily computed, which is finally given by

$$\chi=\frac{\partial^2}{\partial(\beta h)^2}\frac{\beta^2h^2K}{2u\pi}=\frac{K}{u\pi}. \tag{B4}$$

APPENDIX C: NUMERICAL DETAILS

In this section, we describe the numerical procedure for this paper. We denote the discretized two-dimensional field as ϕ_{ij}, where $i\in[1,L]$ and $j\in[1,\beta]$ with periodic boundary conditions in both directions. Our strategy is to start from a flat interface $\phi_{ij}=0$ at $t=0$ and then let it evolve according to the Langevin equation [34]:

$$\frac{d\phi_{ij}(t)}{dt}=-\frac{\delta S[\phi_{ij}(t)]}{\delta\phi_{ij}}+\eta_{ij}(t), \tag{C1}$$

where $\eta_{ij}(t)$ is a white noise, specified by the correlations $\langle\eta_{ij}(t)\rangle=0$ and $\langle\eta_{ij}(t)\eta_{i'j'}(t')\rangle=2\delta_{i,i'}\delta_{j,j'}\delta(t-t')$. Note that the time t that appears in Eq. (C1) should not be confused with the imaginary time τ. When $t\to\infty$, the surface $\phi_{i,j}(t)$ obtained by direct integration of Eq. (C1) is equilibrated with the action $S[\phi]$. Hence, the Langevin equation, which we numerically simulate, is given by

$$\begin{aligned}\frac{d\phi_{ij}(t)}{dt}=&\frac{\alpha}{\pi^2}\sum_{j'}D(|j-j'|)\sin[2(\phi_{ij'}-\phi_{ij})]\\&+\frac{1}{uK\pi}[\phi_{i,j+1}+\phi_{i,j-1}-2\phi_{i,j}]\\&+\frac{u}{K\pi}[\phi_{i+1,j}+\phi_{i-1,j}-2\phi_{i,j}]+\eta_{ij}(t).\end{aligned} \tag{C2}$$

To obtain a correct discretization of the long-range kernel $D(j-j')$, we use the same protocol as in Ref. [35]. For $\beta\to\infty$, we set

$$\begin{aligned}D(j-j')&=\int_0^{2\pi}\frac{d\omega}{2\pi}\exp[i\omega(j-j')]\{2[1-\cos(\omega)]\}^{1/2}\\&=\frac{1}{(j-j')^2-\frac{1}{4}}.\end{aligned}$$

At finite β, the periodic boundary conditions are implemented as

$$D(j-j')=\sum_{k=-\beta/2}^{\beta/2}\frac{1}{(|j-j'|+k\beta)^2-\frac{1}{4}}. \tag{C3}$$

To conclude, we remark that, in the numerical integration, the term $\frac{\delta S[\phi_{ij}(t)]}{\delta(\phi)}$ is multiplied by Δt, whereas $\eta_{ij}(t)$ is multiplied by $\sqrt{\Delta t}$. Here, we use the stochastic second-order Runge-Kutta algorithm for white noise [34]. Using this is preferable, as this is much faster than the standard Euler's algorithm. We choose the value of the Langevin time step $\Delta t=0.05$. To benchmark the equilibration time of the surface, we used the harmonic approximation $\sin[2(\phi_{ij'}-\phi_{ij})]\to 2(\phi_{ij'}-\phi_{ij})$ that can be analytically solved.

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