Universal photonic quantum gate by Cooper-pair-based optical nonlinearity

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We propose a compact and highly efficient scheme for solid-state photonic quantum gates. At the core of our scheme is a $SWAP^{\phi}$ gate based on giant Cooper-pair-based optical nonlinearity we predict in a semiconductor-superconductor structure, selectively introducing a phase to the $|\Psi_+\rangle$ Bell state. We theoretically demonstrate this scheme on a practical device based on a superconducting contact coupled to a GaAs/AlGaAs waveguide structure. We model the Cooper-pair-induced nonlinear change of the refractive index showing strong nonlinearities at energies close to the superconducting gap inside a semiconductor band. We calculate the fidelity of the proposed $SWAP^{\phi}$ gate, as well as the sensitivity of the gate to device parameters. As short photon wave packets are crucial for efficient higher-order interactions, we investigate the integrated fidelity for short wave packets with different central wavelengths and bandwidths, providing limiting factors as well as possible optimizations. This theoretically demonstrated concept can pave the way toward practical realizations of scalable photonic quantum circuits.

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Quantum information processing (QIP) [1] can provide exponential speed-up of various computational algorithms and in physically proven data security [2,3]. Alongside widely used matter-based realizations such as superconducting circuits and trapped ions [4–9], realizations employing photons are also used in applications such as cryptography [10], computing [11–15], teleportation [16,17], Bell inequality tests [18,19], and metrology [20]. Despite the wide use of photon qubits in QIP, implementation of quantum gates based on photons is extremely challenging due to the very weak photon-photon interaction in practical materials [21,22]. Giant nonlinearities in dilute atomic media have been shown to enable quantum nondemolition measurements [23] and quantum gate realization [24]. However, dilute atomic systems are not compatible with compact integrated photonic circuits, and thus prevent scaling of such realizations. Probabilistic quantum gates based on integrated photonic circuits have been demonstrated [25,26]. However, deterministic integrated quantum gates between photons have not been realized yet. Lately, a field of superconducting optoelectronics has emerged investigating light-matter interaction in semiconductor-superconductor structures [27–33]. Superconducting light-matter interaction was shown to result in novel processes based on strongly enhanced optical nonlinearities, including spontaneous two-photon emission [34] and two-photon gain [35], enabling various aspects of quantum information processing such as full Bell-state analysis [36] and entangled-photon generation [31,40]. Nevertheless, a complete infrastructure for superconductor-based photonic quantum information processing requires quantum gate schemes, which have not been studied so far.

Here we propose a new concept of an efficient universal quantum gate based on a giant Cooper-pair-based optical nonlinearity we predict in a semiconductor-superconductor structure. This giant nonlinearity selectively introduces a nonlinear phase to one specific Bell state, $|\Psi_+\rangle$, whereas the other three Bell states remain unaffected (Fig. 1), enabling the realization of an efficient photonic universal $SWAP^{\phi}$ gate in a solid-state system. We study the fidelity of the proposed gate for a practical realization based on a semiconductor-superconductor waveguide.

At the core of our proposal is the selective Bell-state nonresonant optical nonlinearity, related to two-photon absorption of a specific Bell state into a Cooper pair [36]. The presence of a superconducting gap inside the conduction band of the semiconductor inhibits single-photon absorption in a narrow spectral range due to the inability to raise single electrons from the valence band into the gapped region in the conductance band. Two-photon absorption, however, is possible as the superconducting gap supports the presence of Cooper pairs. Typically, for low-critical temperature (T_c) superconductors, Cooper pairs are found in the spin singlet state [37]. The ability to support only a spin singlet state, coupled with fermionic exchange minus sign, results in the demand for a specific polarization state of two absorbed photons, which is the $|\Psi_+\rangle$ Bell state. The one-photon absorption coefficient α is related to the refractive index n through the linear Kramers-Kronig relations. For higher order processes such as two-photon absorption, it has been established that such absorption at resonant energies is linked to nonresonant optical Kerr nonlinearity via a nonlinear version of the Kramers-Kronig relations [38]. These relations are a special case of the general quantum field theory optical theorem [39]. As the nonlinear Kramers-Kronig relations predict, the change in absorption $\Delta\alpha$ is related to the change in the refractive index Δn of the material. The change in the refractive index

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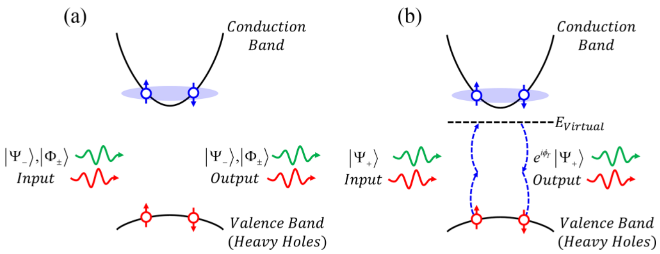

FIG. 1. (a) Due to both the singlet Cooper-pair state and the requirement of total angular momentum conservation, the $|\Psi_-\rangle$, $|\Phi_\pm\rangle$ Bell states will not interact with the superconducting condensate and will not aquire additional phase. (b) The $|\Psi_+\rangle$ Bell state will interact with the superconducting condensate, aquiring additional phase.

results in a different phase imprinted on the $|\Psi_+\rangle$ Bell state, singling it out of the rest of the Bell states. For two-photon absorption, the change in refraction index Δn is related to the absorption coefficient change $\Delta\alpha$ through the following form:

$$\Delta n\big(E;E_{q_\nu}\big)=\frac{c\hbar}{2\pi}P\left\{\int_{-\infty}^{\infty}\frac{\Delta\alpha\big(E_{q_\mu};E_{q_\nu}\big)}{E_{q_\mu}\big(E_{q_\mu}-E\big)}dE_{q_\mu}\right\} \tag{1}$$

where E_{q_μ} and E_{q_ν} are the energies of the photons in the absorbed pair and P denotes the Cauchi principle integral. The photon energy E is defined as the energy of a photon experiencing the change in the refractive index Δn related to the two-photon absorption $\Delta\alpha$. For the derivation in Eq. (1), E was chosen arbitrarily to represent E_{q_μ}, implying that E_{q_ν} applies a nonlinear phase change on E_{q_μ} (which is expressed through Δn). The photon energy E is expressed in normalized units $(2E-E_{\text{total}})/2\Delta_0$, where $\Delta_0=\Delta(T=0\,\text{K})$ is the superconducting order parameter. Due to the Bell-state selectivity of the absorption coefficient, $\Delta\alpha\neq 0$ only for the $|\Psi_+\rangle$ Bell state, resulting in a refractive index change Δn and thus a phase change only for the $|\Psi_+\rangle$ Bell state. In addition, the second-order correction to the refractive index is mediated via a virtual energy level. As the process occurs through a virtual energy state, no significant absorption of the two-photon state takes place inside the device, even though $\Delta\alpha$ is used to derive Δn. Furthermore, while there are significant absorption and refraction processes that are related via the linear Kramers-Kronig relations (first-order processes), intrinsic loss should appear only for photons with energies larger than the semiconductor bandgap, as they have a chance of getting absorbed in the semiconductor layers. However, the nonlinear optical effect described here can occur virtually without losses as each photon energy in the input state can be smaller than the bandgap, allowing it to undergo a nonlinear phase change while avoiding one-photon absorption in the semiconductor. The required photonic states may be preprepared and coupled into our proposed system [40,41] or prepared inside waveguides, for example, by using parametric downconversion [42]. In addition, nonlinear processes occurring at the single photon level inside waveguide structures have been demonstrated previously [43,44], indicating the feasibility of performing nonlinear interactions with a few photons, even without the strong superconducting enhancement described in our proposed scheme.

A simple scheme based on gratings and phase plates can convert $|\Psi_+\rangle$ and $|\Psi_-\rangle$ Bell states as well as $|\Phi_-\rangle$ and $|\Phi_+\rangle$ into each other [36]. The resulting Bell state with the induced phase can be set to be $|\Psi_-\rangle$. The universal quantum gate that can be implemented based on this selective photon Bell-state nonlinearity is the $SWAP^{\phi}$ gate:

$$SWAP^{\phi}=\begin{pmatrix}1&0&0&0\\0&\frac{1+e^{i\pi\phi(E;E_{q_\nu})}}{2}&\frac{1-e^{i\pi\phi(E;E_{q_\nu})}}{2}&0\\0&\frac{1-e^{i\pi\phi(E;E_{q_\nu})}}{2}&\frac{1+e^{i\pi\phi(E;E_{q_\nu})}}{2}&0\\0&0&0&1\end{pmatrix} \tag{2}$$

where $\phi(E;E_{q_\nu})=\Delta n(E;E_{q_\nu})\frac{E}{\pi\hbar c}L$ is given by the nonlinear phase obtained by a specific photon Bell sate in a waveguide of length L, and the Bell-states in polarization basis are defined as

$$|\Phi^+\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix},\quad |\Phi^-\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix},$$
$$|\Psi^+\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix},\quad |\Psi^-\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}. \tag{3}$$

The Bell-state basis is implemented as polarization-entangled photon pairs, with frequency serving as an index for the distinction between the two polarization Hilbert spaces (one per photon). The two-photon basis is $(L_{\omega_1}L_{\omega_2}, L_{\omega_1}R_{\omega_2}, R_{\omega_1}L_{\omega_2}, R_{\omega_1}R_{\omega_2})$. Various implementations of universal gates have been proposed based on the $SWAP^{\phi}$ gate, including a controlled-not (CNOT) gate scheme with six $SWAP^{1/2}$ gates, which have been shown to be sufficient to implement any two-qubit operation [45], as well as a scheme based on three $SWAP^{\phi}$ gates [46]. The resulting implementation of the $SWAP^{\phi}$ gate in general depends on the spectral dispersion of the nonlinear induced index change $\Delta n(E,E_{q_\nu})$ affecting the fidelity of the gate. The fidelity of the implemented $SWAP^{\phi}$ gate versus an ideal target gate $SWAP^{\phi_T}$ for four-dimensional Hilbert space is given by [47] and as noted in Supplemental Material [48]:

$$F\big(SWAP^{\phi(E;E_{q_\nu})\dagger},SWAP^{\phi_T}\big)=\frac{4+|3+e^{i\pi(E;E_{q_\nu})}|^2}{20}. \tag{4}$$

For maximum fidelity, the required phase difference $\phi_T-\phi(E_{q_\mu};E_{q_\nu})$ has to be in multiples of 2π. As the phase change depends strongly on the nonlinear refractive index change Δn (which in turn strongly depends on the wavelength), as well as the overall length of the proposed gate, the fidelity, and thus performance, of our gate scheme is sensitive to these parameters. Because our proposed quantum gate scheme is universal in nature, we can choose various target states depending on the desired quantum circuit. Such circuits include, for example, the CNOT gate scheme [45], and the more general three-$SWAP^{\phi}$ gates scheme for any two-qubit operations [46] that were mentioned earlier. Moreover, the proposed gate scheme is passive, not requiring external controls in order to affect the input photon pairs.

In order to assess the fidelity of the proposed gate scheme, we first estimate the magnitude of the nonlinear refractive index change induced by the single control photon on a target photon due to Cooper pair two-photon absorption related nonlinearity. It has been previously shown that in a semiconductor waveguide in proximity with a superconductor, the two-photon absorption coefficient (in units of m^{-1}) is given by [36]

$$\Delta\alpha\left(E_{q_\mu};E_{q_\nu}\right)=\frac{Sm_p^{HH}\xi^2q^4\hbar\Delta^2n}{cm_0^2\varepsilon_0^2V^2E_{q_\mu}\left(2E_{\mathrm{gap}}+E_{\mathrm{offset}}-E_{q_\mu}\right)\left(\left(2E_{q_\mu}-2E_{\mathrm{gap}}-E_{\mathrm{offset}}\right)^2-4\Delta^2\right)^2}, \tag{5}$$

where m_p^{HH} is the heavy hole effective mass, m_e is the free electron mass, q is the electron charge, $\Delta(T)$ is the temperature-dependent superconducting order parameter, n is the refractive index around E_{gap}, c is the speed of light, S is the total area of the superconducting contact, ε_0 is the vacuum permittivity, V is the photon mode volume, and ξ is the semiconductor light-matter coupling matrix element. It is assumed that $E_{q_\mu}+E_{q_\nu}=2E_{\mathrm{gap}}+E_{\mathrm{offset}}=E_{\mathrm{total}}$, with E_{offset} marking the detuning of the total two-photon energy from $2E_{\mathrm{gap}}$. The total two-photon energy E_{total} is assumed to be a constant. Using the Kramers-Kronig relations, we can obtain $\Delta n(E;E_{q_\nu})$:

$$\Delta n\left(E;E_{q_\nu}\right)=\frac{nSm_p^{HH}\xi^2q^4\hbar^2\Delta^2}{2\pi m_e^2V^2\varepsilon_0^2}P\left\{\int_{-\infty}^{\infty}\frac{1}{\left(2E_{\mathrm{gap}}+E_{\mathrm{offset}}-E_{q_\mu}\right)\left(E_{q_\mu}\right)^2\left(E_{q_\mu}-E\right)\left[\left(2E_{q_\mu}-2E_{\mathrm{gap}}-E_{\mathrm{offset}}\right)^2-4\Delta^2\right]^2}dE_{q_\mu}\right\}. \tag{6}$$

To demonstrate the efficiency of the nonlinear phase imprinting on the target photon by the control photon, we calculate for a typical semiconductor-superconductor optoelectronic waveguide based on Nb/AlGaAs [49,50] with $\xi=28.8$ eV; $m_p^{HH}=0.45m_e$, with m_e being free electron mass; $E_{\mathrm{gap}}=1.42$ eV; and $n=3.6$. Because of the narrow photon bandwidth, the refractive index is assumed to be constant. The cross-sectional area was chosen to be 1 $\mu\mathrm{m}^2$. The resulting imprinted nonlinear index change $\Delta n(E;E_{q_\nu})$ spectrum shows a strong peak at the resonances corresponding to the superconducting gap inside the semiconductor band, with the corresponding induced phase reaching values of π for waveguide lengths on the order of 100 μm. For our proposed device, disorder may be present in the dimensions of the device and thicknesses of the various semiconductor layers, which vary in each device and possibly between devices, but more importantly, in the spatial distribution of the superconducting order parameter Δ throughout a given device. While the former induces losses for all photons entering our device, the latter is more crucial for the $|\Psi_+\rangle$ Bell-state photons that undergo the nonlinear phase change, as disorder in the nonlinear phase change process will directly affect the performance of the device. The superconducting order parameter Δ can be classified as a microscopic parameter for a single device. In order to account for this disorder, a Gaussian distribution with a standard deviation Γ [Fig. 2(a)] was used as it is the most common type of naturally occurring distribution, also typically assumed for quantum wells with a long-range disorder [51,52]. The expression for $\Delta n(E;E_{q_\nu})$ has been convoluted with the Gaussian disorder distribution to obtain the disorder-induced broadening. Added disorder causes spectral broadening of the peaks and reduces the change in the refractive index. While disorder reduces $\Delta n(E;E_{q_\nu})$, it causes it to be more uniform over a broader spectral range. This result has the advantage of creating a more stable system that allows for high fidelity over a broad range of wavelengths. Therefore, the proposed effect of Cooper-pair-induced photon phase imprinting can be implemented in practical waveguide-based devices allowing potential realization of compact deterministic photonic $SWAP^{\phi}$ gates on a chip.

Because the strongest change in $\Delta n(E;E_{q_\nu})$ occurs close to the resonances, it is expected that the two-photon absorption $\Delta\alpha$ will be large as well. In order to assess this tradeoff, a comparison was made between $\Delta n(E;E_{q_\nu})$ and $\Delta\kappa(E;E_{q_\nu})$ ($\Delta\kappa(E;E_{q_\nu})=\Delta\alpha(E;E_{q_\nu})\times hc/4\pi E$) [Figs. 2(b)–2(e)]. It was observed that the attenuation coefficient $\Delta\kappa(E;E_{q_\nu})$ decays rapidly off-resonance, resulting in $\Delta n(E;E_{q_\nu})$ becoming dominant even for small detuning.

Based on the obtained spectrum of $\Delta n(E;E_{q_\nu})$, we calculated the fidelity of our proposed $SWAP^{\phi}$ gate (Fig. 3). The obtained fidelity has strong dependence on the photon energy, rapidly changing between maximum and minimum values. The change becomes slower as the disorder increases. The oscillations originate from the rapid phase changes, which are in turn caused by the sharp changes in $\Delta n(E;E_{q_\nu})$ around $\frac{E_{\mathrm{total}}}{2}\pm\Delta(T)$. A higher level of disorder reduces the magnitude of the changes in $\Delta n(E;E_{q_\nu})$ and broadens them, resulting in a slower phase change and a slower oscillation in fidelity. This indicates the importance of disorder for our proposed gate. The disorder, however, does not reduce fidelity by itself. This is because while the disorder reduces the nonlinear contribution to the refractive index Δn, it only slows the oscillations in fidelity as a fidelity of unity can still be achieved with a smaller Δn, although it does require a longer path to realize the phase change of the $|\Psi_+\rangle$ state fully. Practically, a broad range of photon energies, for which the fidelity is high, is desired. For low disorder levels, the rapid oscillations in fidelity versus photon energy result in multiple narrow spectral ranges of high fidelity, requiring high precision of photon energy, thus preventing the use of ultrafast broadband pulses. Moreover, high sensitivity to phase requires high accuracy in device length on the scale of the photon wavelength. Higher levels of disorder, however, result in broader spectral ranges for photon energies with high fidelity, allowing robust gate

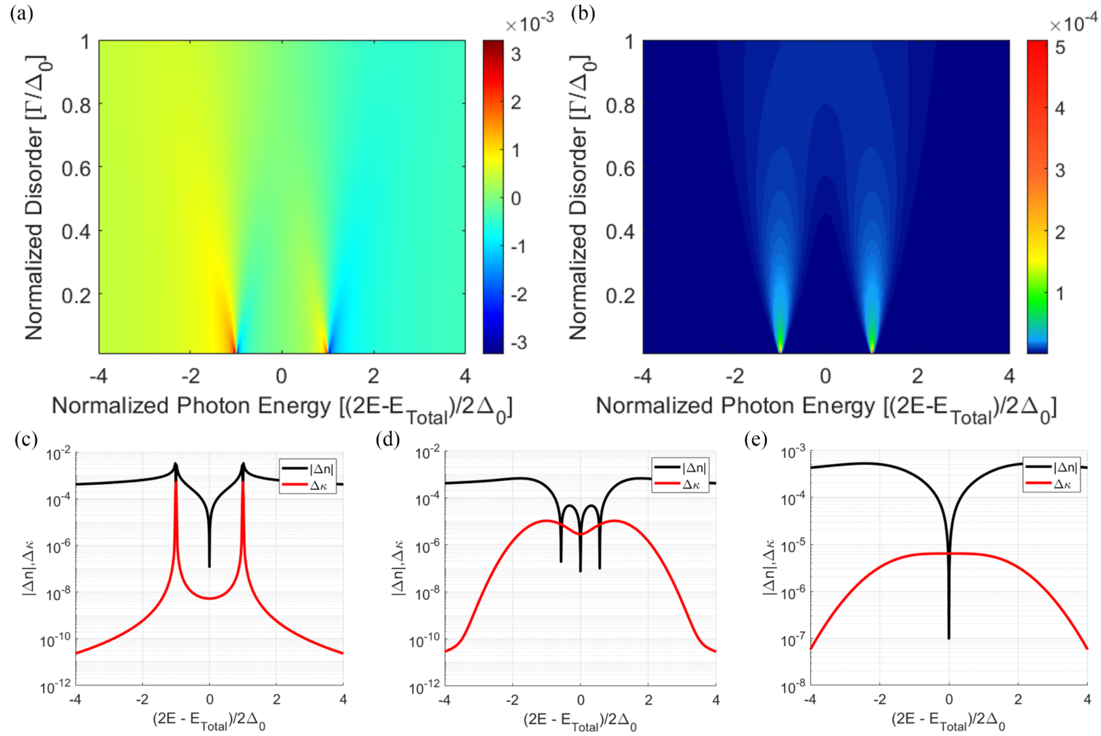

FIG. 2. (a) and (b) The value of induced change in refractive index Δn (a) and the imaginary coefficient $\Delta\kappa$ (b) at 0 K versus normalized photon energy and disorder for zero detuning. Two sharp features in Δn and $\Delta\kappa$ are evident for two-photon energies of $\sim E_{\text{total}} \pm 2\Delta(T)$, whereas disorder causes the changes to broaden and reduce in magnitude. (c)–(e) Absolute-valued $|\Delta n|$ (black) and $\Delta\kappa$ (red) for low ($\Gamma = 0.01\Delta_0$) (c), medium ($\Gamma = 0.5\Delta_0$) (d), and large ($\Gamma = 0.99\Delta_0$) (e) disorder in log scale. $\Delta\kappa$ is smaller than Δn by several orders of magnitude except at the resonances.

operation to both photon energy and device length inaccuracy. The inherent disorder can thus play an important role in obtaining a stable gate since it mitigates the effect that parameters, such the length of the waveguide, its cross section, and the superconducting order parameter, have on the fidelity spectra, allowing for slower phase changes and a more stable fidelity. Furthermore, another effect that results in broader ranges is operating the gate close to T_c of the superconductor. This is because the magnitude of the nonlinear change to $\Delta n(E; E_{q_v})$ depends on $\Delta(T)^2$. Thus, reducing $\Delta(T)$ results in a smaller change, which causes a slower phase change and hence slower oscillations in fidelity.

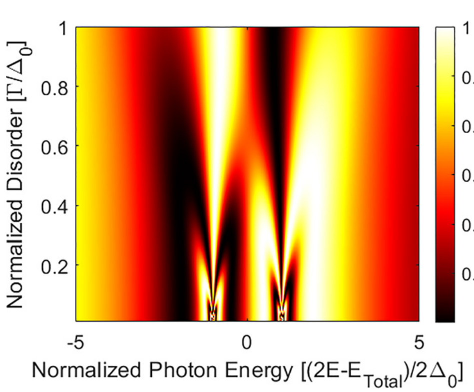

FIG. 3. Fidelity versus photon energy and disorder. The rapid oscillations in fidelity are the result of the large changes in the refractive index Δn, leading to rapid changes in the resulting phase change of the Bell state and thus the fidelity of the gate, with disorder mitigating the oscillations.

As short pulses are capable of concentrating high field strengths suitable for higher order processes, an analysis of the integrated fidelity resulting from short photon wave packets with a broad spectrum in our proposed scheme is important. We calculated the integrated fidelity resulting from a short wave packet with varying central wavelength and bandwidth (Fig. 4). Any linear dispersion effects during the propagation are equal for all four Bell states and do not modify the spectral shapes, and thus do not affect the gate operation. Furthermore, a nonlinear effect such as self-phase modulation [53], which could modify the spectral shape of a multiphoton state or a classical field, does not occur for the two-photon pulses in Bell states.

An interesting result is that the largest integrated fidelity is not obtained at $\frac{E_{\text{total}}}{2} \pm \Delta(T)$, but farther away in either higher or lower energies. This is due to the slower refractive index change experienced farther away from $\frac{E_{\text{total}}}{2} \pm \Delta(T)$. The slower fidelity change results in broader ranges of photon energies where the fidelity remains high. Thus, a wave packet containing photon energies inside such ranges will naturally result in a much higher integrated fidelity. In contrast, a wave packet containing photon energies in rapidly changing ranges close to $\frac{E_{\text{total}}}{2} \pm \Delta(T)$ will result in a much lower integrated fidelity.

Other than disorder-induced phase differences, several additional mechanisms can, in principle, reduce the overall fidelity compared to that derived in Eq. (4). First, for

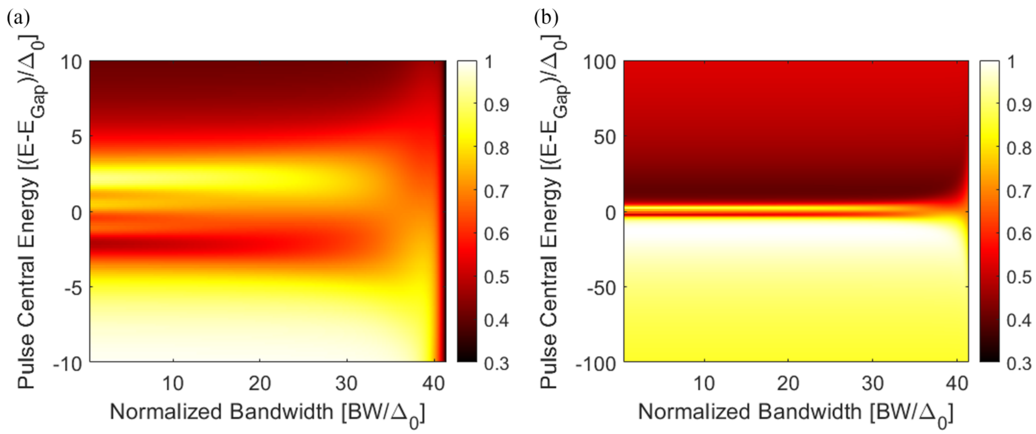

FIG. 4. (a) and (b) Integrated pulse fidelity as a function of the pulse central wavelength energy, and bandwidth (BW). (a) A narrower range of pulse central wavelengths. Multiple peaks in fidelity are evident. The highest integrated fidelity is obtained not at $\sim\frac{E_{\rm total}}{2}\pm\Delta(T)$, but rather at higher or lower energies. (b) For the wider range of pulse central wavelengths, the oscillations in fidelity become slower due to the smaller change in the refractive index.

materials such as GaAs, degeneracy of the light and heavy hole bands can readily result in loss of fidelity, as band degeneracy allows for the emission [40] and absorption of partially mixed photonic states, thus reducing the overall fidelity. However, the use of a quantum well as the active emitting layer lifts the degeneracy, separating the light and heavy hole bands, thus providing a solution to the light/heavy hole degeneracy problem. Another key mechanism that can reduce the overall fidelity is various losses in the waveguide system. Such losses include loss through absorption in the waveguide and loss due to fabrication-induced imperfections in the desired mode shape throughout the waveguide, causing unwanted scattering and thus loss. While such losses are detrimental to the success of any quantum photonic circuit, careful choice of materials and proper fabrication can result in devices with fidelity in excess of 99%, comparable to the highest reported fidelities [25,54–57].

In conclusion, we proposed a highly efficient scheme for scalable photonic quantum gates based on a $SWAP^{\phi}$ gate by employing giant Cooper-pair-based optical nonlinearity. We studied the nonlinear modification of the refractive index, showing the change is sufficiently large to induce a considerable phase change over short propagation distances, making our proposed gate feasible to implement in existing photonic structures. We calculated the resulting quantum gate fidelity, showing that disorder plays a major role in determining the photon energy range for which high fidelity is obtained. Disorder is also important in mitigating the effect of phase sensitivity of the devices to photon central wavelength and bandwidth, as well as device dimensions. Our integrated photonic quantum gate scheme can pave the way toward scalable realizations of advanced quantum information processing making use of hybrid superconductor-photonic quantum devices.

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