Erratum: Witnessing entanglement in quantum magnets using neutron scattering [Phys. Rev. B 103, 224434 (2021)]

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Two mistakes were made in our paper concerning the definition of the fluctuation-dissipation theorem. The expression given in Sec. II C is $\chi''(\omega) = \frac{1}{\hbar}\tanh(\hbar\omega\beta/2)\tilde{S}(\omega)$ (suppressing wave-vector dependence). This expression should read $\chi''(\omega) = \frac{\pi}{\hbar}\tanh(\hbar\omega\beta/2)\tilde{S}(\omega)$ in order to be consistent with the Van Hove formulation of the dynamic structure factor conventionally used

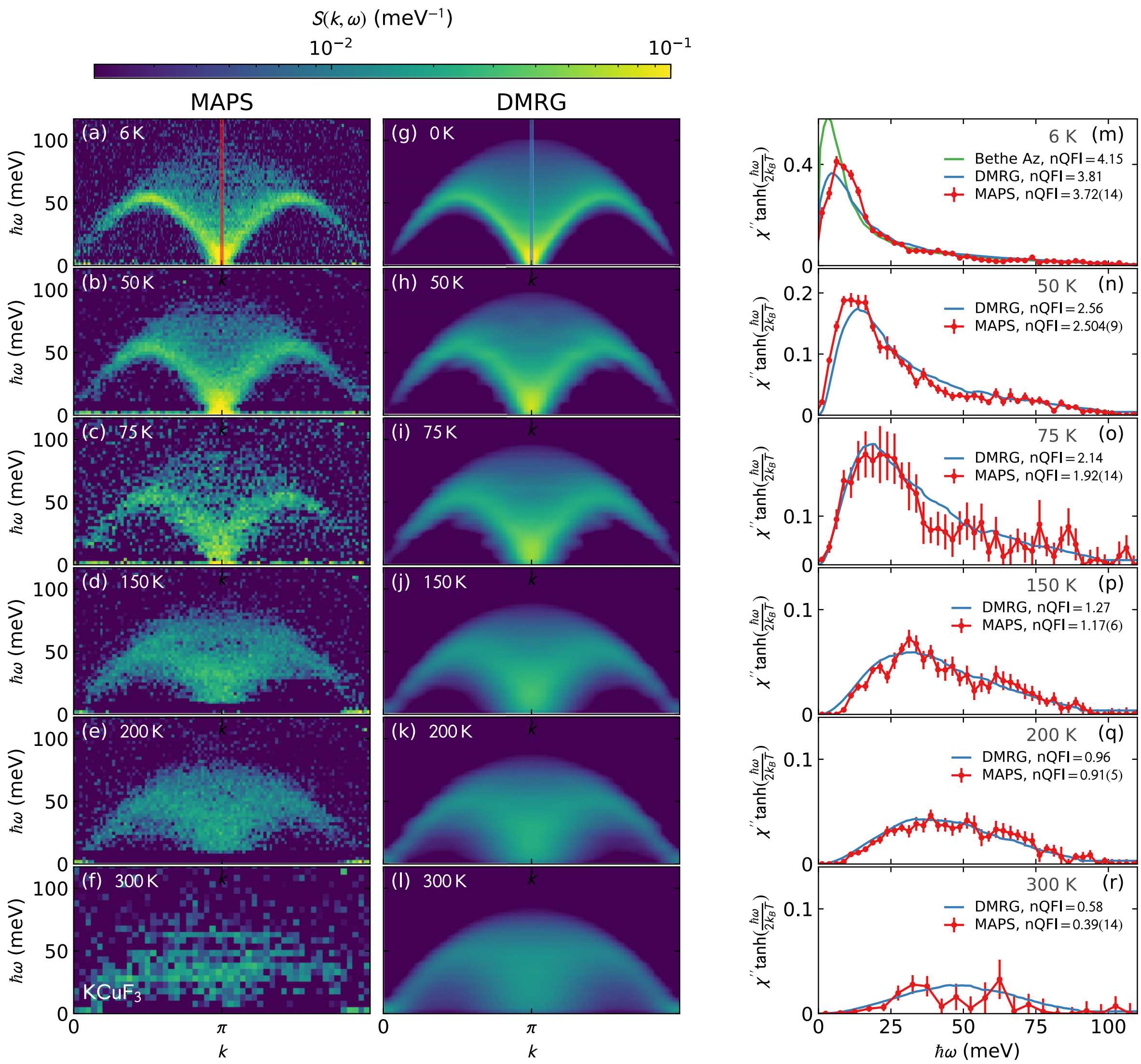

FIG. 1. Spectra and corrected quantum Fisher information. (a)–(f) Neutron scattering of $KCuF_3$ measured on MAPS at different temperatures. (g)–(l) DMRG simulated scattering from a 1D HAF with experimental resolution broadening applied. (m)–(r) QFI integrand at $k = \pi$, shown with normalized quantum Fisher information $(\frac{f_Q}{12 \cdot S^2})$ calculated at that point. At 6 K the data are also compared with the algebraic Bethe ansatz result (m).

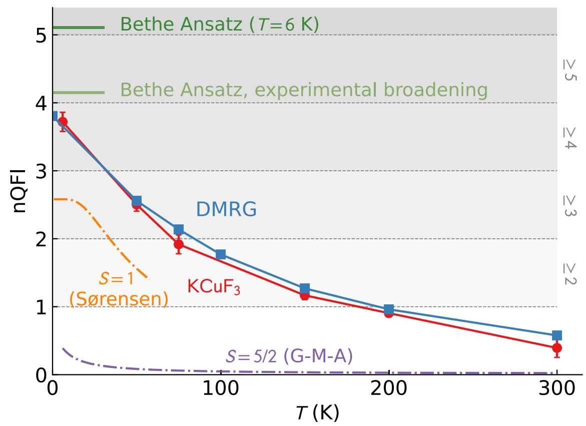

FIG. 2. Corrected normalized quantum Fisher information as a function of temperature. When nQFI $> m$, where $m > 0$ is an integer, the system is in a state with $\geqslant(m+1)$-partite entanglement. Note that at the lowest temperatures, $\geqslant$4-partite entanglement is witnessed in $KCuF_3$.

in neutron scattering, $S(\omega) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dt e^{-i\omega t}\langle\hat{S}\hat{S}(t)\rangle$ (suppressing position dependence), rather than the definition in Eq. (S13) of Hauke *et al.* [1].

However, the above fluctuation-dissipation theorem is only valid for $S(\omega)$ symmetrized over ω, $\tilde{S}(\omega) = S(\omega) + S(-\omega)$, which our data were not (no negative ω data were collected in the experiment). The correct fluctuation-dissipation theorem to use for nonsymmetrized data is [2]

$$\chi''(\omega) = \pi(1 - e^{-\hbar\omega\beta})S(\omega), \tag{1}$$

which applies to dynamic susceptibilities defined as

$$\chi''(\omega) = -i\int_0^\infty dt\, e^{i\omega t}\phi(t), \quad \phi(t) \equiv \frac{i}{\hbar}\langle[\hat{S}(t), \hat{S}]\rangle, \tag{2}$$

i.e., where the time integral runs from 0 to ∞, as is assumed by both Hauke *et al.* [1] and Lovesey [2].

Due to an error in the code used to calculate χ'' (and hence QFI) we inadvertently instead used $\chi''(\omega) = \frac{\pi}{2}(1 - e^{-\hbar\omega\beta})S(\omega)$. As a result, all reported χ'' and QFI values are smaller than their real values by a factor of two. This implies that QFI is a more powerful probe of solid state entanglement than previously realized. In particular, we now conclude that at least quadpartite entanglement is witnessed in $KCuF_3$ at the lowest temperatures, and that bipartite entanglement is present up to at least 150 K. Corrected versions of Figs. 2 and 5 of our original paper are provided in Figs. 1 and 2 of this Erratum, respectively.

Recently, we became aware of the need for this correction through work reported in Ref. [3], and thank Varun Menon for originally raising the issue. We note that this incorrect factor of 1/2 also affects the results of Refs. [4,5], the χ'' and QFI values of which will be corrected in the same fashion.

[1] P. Hauke, M. Heyl, L. Tagliacozzo, and P. Zoller, Measuring multipartite entanglement through dynamic susceptibilities, Nat. Phys. **12**, 778 (2016).

[2] S. Lovesey, *Theory of Neturon Scattering from Condensed Matter* (Clarendon Press, Oxford, UK, 1984).

[3] V. Menon, N. E. Sherman, M. Dupont, A. O. Scheie, D. A. Tennant, and J. E. Moore, Multipartite entanglement in the one-dimensional spin-$\frac{1}{2}$ Heisenberg antiferromagnet, Phys. Rev. B **107**, 054422 (2023).

[4] P. Laurell, A. Scheie, C. J. Mukherjee, M. M. Koza, M. Enderle, Z. Tylczynski, S. Okamoto, R. Coldea, D. A. Tennant, and G. Alvarez, Quantifying and Controlling Entanglement in the Quantum Magnet Cs_2CoCl_4, Phys. Rev. Lett. **127**, 037201 (2021).

[5] P. Laurell, A. Scheie, D. A. Tennant, S. Okamoto, G. Alvarez, and E. Dagotto, Magnetic excitations, nonclassicality, and quantum wake spin dynamics in the Hubbard chain, Phys. Rev. B **106**, 085110 (2022).