Erratum: Spin-functional renormalization group for the $J_1J_2J_3$ quantum Heisenberg model [Phys. Rev. B 106, 174412 (2022)]

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We want to point out that the expressions for the mixed five- and six-point vertices as stated in Eqs. (27) and (28) of the original publication are not correct. Instead they should read

$$\Gamma_0^{zzz\eta^-\eta^+}(0,0,0,-\omega,\omega)=0, \tag{1}$$

$$\Gamma_0^{zzzz\eta^-\eta^+}(0,0,0,0,-\omega,\omega)=\frac{4b_3}{\beta b_1^4\omega^2}. \tag{2}$$

This change can be explained by the inclusion of additional higher-order diagrams in the tree expansion of both vertices, see Eqs. (C8) and (C13), which were omitted by mistake. The correct tree expansions of both vertices are

$$\begin{aligned}\Gamma_0^{zzz\eta^-\eta^+}(0,0,0,-\omega,\omega)=&-G_0^{-3}G_0^{zzz+-}(0,0,0,\omega,-\omega)+\Gamma_0^{zzzz}(0,0,0,0)G_0\Gamma_0^{z\eta^-\eta^+}(0,-\omega,\omega)\\&-6(-G_0)^{-2}\big[\Gamma_0^{z\eta^-\eta^+}(0,-\omega,\omega)\big]^3,\end{aligned} \tag{3}$$

$$\begin{aligned}\Gamma_0^{zzzz\eta^-\eta^+}(0,0,0,0,-\omega,\omega)=&-G_0^{-4}G_0^{zzzz+-}(0,0,0,0,\omega,-\omega)+8\Gamma_0^{zzzz}(0,0,0,0)\big[\Gamma_0^{z\eta^-\eta^+}(0,-\omega,\omega)\big]^2\\&+24\big[\Gamma_0^{z\eta^-\eta^+}(0,-\omega,\omega)\big]^4(-G_0)^{-3},\end{aligned} \tag{4}$$

where in the second equation we already took into account that the five-point vertex is zero. The same is true for contributions containing the mixed four-point vertex, whose vanishing was already acknowledged in the original publication. The appropriate replacement for Fig. 11, showing the tree expansion of the corresponding correlation functions, is displayed in Fig. 1 herein. As a consequence, the flow equation for the static four-point vertex U_Λ, given in Eq. (31b), has to be modified. The new version reads

$$\begin{aligned}\partial_\Lambda U_\Lambda=&\frac{T}{N}\sum_{\boldsymbol q}\dot G_\Lambda(\boldsymbol q)\left[\frac{7}{10}V_0-\frac{11}{3}U_\Lambda^2G_\Lambda(\boldsymbol q)\right]-\frac{4}{N}\sum_{\boldsymbol q}[\partial_\Lambda J_\Lambda(\boldsymbol q)]\left[\frac{b_3}{b_1^4}S_2(\tilde\Omega_\Lambda(\boldsymbol q))\right]\\&-\frac{24}{NT^3b_1^4}\sum_{\boldsymbol q}\frac{\partial_\Lambda J_\Lambda(\boldsymbol q)}{G_\Lambda(\boldsymbol q)G_\Lambda^2(\boldsymbol q+\boldsymbol Q)}S_5(\tilde\Omega_\Lambda(\boldsymbol q),\tilde\Omega_\Lambda(\boldsymbol q+\boldsymbol Q)),\end{aligned} \tag{5}$$

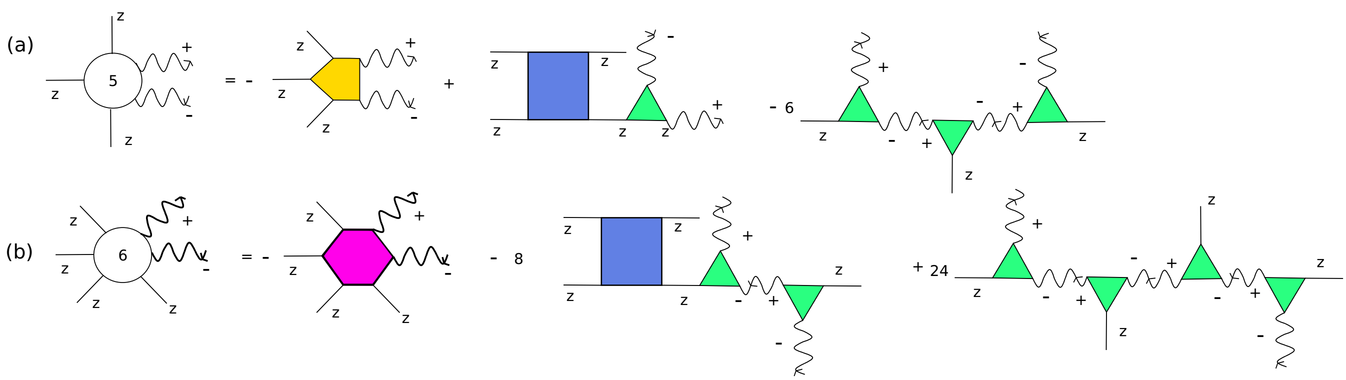

FIG. 1. Tree expansion of the mixed five-spin and six-spin correlation functions $G_0^{zzz+-}(0,0,0,\omega,-\omega)$ and $G_0^{zzzz+-}(0,0,0,0,\omega,-\omega)$ in terms of irreducible vertices. Note that for the second line we already took into account that the mixed five-point vertex (in yellow) vanishes.

TABLE I. Corrected values for T_c, using the high-temperature limit of $\tilde{\Pi}_\Lambda(K)$ during the integration of the flow equations. For comparison, see Table III in the original paper.

			T_c/T_c^{MF}		Rel. error/%
S	J_1	J_3/J_1	Switch	Benchmark	Switch
1/2	<0	0	0.545	0.559	2.5
1/2	>0	0	0.640	0.629	1.7
1	<0	0	0.651	0.650	0.2
1	>0	0	0.697	0.684	1.9
3/2	<0	0	0.688	0.685	0.4
3/2	>0	0	0.715	0.702	1.9
1/2	>0	0.2	0.752	0.722	4.2
1/2	>0	0.4	0.799	0.768	4.0
1/2	>0	0.6	0.823	0.794	3.7
1/2	>0	0.8	0.834	0.808	3.2

which is actually more compact than the previous one, due to the absence of two terms. Fortunately, the effect of that absence on the static self-energy Σ_Λ is modest. The inverse susceptibility curves of both cases are quite similar to each other. In particular, the numerical values of the transition temperatures T_c are only weakly affected, by a change of at most 1 or 2 in the last significant digit, i.e., they deviate by less than half a percent from the old results. Using again the high-temperature ansatz for $\tilde{\Pi}_\Lambda(K)$ during the flow, the new values for T_c are given in Table I. We refrain from showing new plots for $G^{-1}(\boldsymbol{Q})$ due to their similarity to the old results. Finally, we also checked the approximation, where we determine $\tilde{\Pi}_\Lambda(K)$ from the solution of a self-consistency equation, and we found the change to be of a similar magnitude. All in all, none of the conclusions regarding our approach have to be changed.