


# Degeneracy between even- and odd-parity superconductivity in the quasi-one-dimensional Hubbard model and implications for Sr<sub>2</sub>RuO<sub>4</sub>

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Based on a weak coupling calculation, we show that an accidental degeneracy appears between even- and odd-parity superconductivity in the quasi-one-dimensional (1D) limit of the repulsive Hubbard model on the square lattice. We propose that this effect could be at play on the quasi-1D orbitals Ru  $d_{zx}$  and  $d_{zy}$  of Sr<sub>2</sub>RuO<sub>4</sub>, leading to a gap of the form  $\Delta_{\text{even}} + i\Delta_{\text{odd}}$  which could help reconcile several experimental results.

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## I. INTRODUCTION

The presence of multiple components in the superconducting (SC) order parameter (OP) can lead to a flurry of interesting phenomena, like the spontaneous breaking of time-reversal symmetry (TRS) and the appearance of topological edge states [1–3]. Multicomponent superconductivity can either be symmetry imposed, corresponding to a multidimensional irreducible representation (irrep) of the point group, or it can be accidental, when two SC orders are accidentally close to degenerate. The latter scenario, although somewhat undesirable since it often requires fine tuning, has been invoked for a variety of superconductors [4–6] for which a multidimensional irrep is in apparent contradiction with certain experiments or when such an irrep does not exist altogether.

This paper is motivated by Sr<sub>2</sub>RuO<sub>4</sub>, for which the nature of the SC order remains an open question even 25 years after its discovery [7–14]. This material sounds like a perfect testbed to study unconventional superconductivity since its phase above  $T_c$  is a well-behaved, albeit renormalized, Fermi liquid, for which Fermi surfaces (FSs) have been measured with extreme accuracy [15–18]. However, the theoretical study of this material has been hampered by several complications, including the presence of multiple orbitals [the quasi-one-dimensional (1D) orbitals  $d_{xz}$  and  $d_{zy}$  and the quasi-two-dimensional (2D) orbital  $d_{xy}$ ] and their coupling via spin-orbit interaction. Despite the challenges, achieving a consistent match between theory and experiments for this material would be an important milestone and could shed light on a flurry of other unconventional superconductors.

The evidence for TRS breaking [19–21] and multicomponent superconductivity [22–24] in Sr<sub>2</sub>RuO<sub>4</sub> would naturally point toward a  $\vec{d} = (p_x + ip_y)\hat{z}$  state. However, such a state is in contradiction with the drop of spin susceptibility observed recently in nuclear magnetic resonance (NMR) [25,26]. Several other candidates have thus been proposed [6,27–32]. Accidental degeneracies between nonsymmetry-related

orders have been considered, like  $d + ig$  [6] or  $s' + id$  [29]. Nevertheless, there is at least one experimental fact which seems difficult to explain for any candidate OP: the absence of a specific heat anomaly [33] at the putative second transition under [1, 0, 0] strain revealed by muon spin resonance ( $\mu$ SR) [21].

In this paper, we propose another candidate for a combination of accidentally degenerate states with the potential to resolve several of these issues: states of the form  $\Delta_e + i\Delta_o$ , where  $\Delta_e$  is even parity and  $\Delta_o$  is odd parity. This proposal is based on our solution of the small- $U$  Hubbard model on a square lattice in the quasi-1D limit. We provide an analytical proof that this model exhibits an accidental degeneracy between even- and odd-parity representations (as previously pointed out in Ref. [34]). Since the Ru  $d_{zx}$  and  $d_{zy}$  orbitals in Sr<sub>2</sub>RuO<sub>4</sub> have a strongly 1D character, our hypothesis is that this mechanism could be at play on these orbitals, leading to a mixed-parity OP on them. Remarkably,  $\Delta_e$  and  $\Delta_o$  have the same magnitude everywhere on the FS, leading to a parametrically small specific heat jump. This mechanism therefore provides a microscopic justification for an accidental degeneracy, along with a justification for a parametrically small second specific heat jump.

In Sec. II, we provide an exact analytical solution for weak coupling superconductivity in the repulsive Hubbard model for a quasi-1D band on the square lattice. We show that there is an accidental degeneracy between even- and odd-parity SC orders across the entire spectrum and that this degeneracy is robust to changes in the dispersion relation. In Sec. III, we use a Ginzburg-Landau (GL) analysis to study the possible combinations of even- and odd-parity SC orders. We find that states of the type  $\Delta_e + i\Delta_o$  are favored. We then study two thermodynamic properties of these states: specific heat and spin susceptibility. In Sec. IV, we assume this mechanism is at play on the quasi-1D bands of Sr<sub>2</sub>RuO<sub>4</sub> and discuss the consequences for experiments.

## II. WEAK COUPLING CALCULATION

We study a single-orbital repulsive Hubbard model on a square lattice, with nearest neighbor hoppings  $t_x$  along the  $x$

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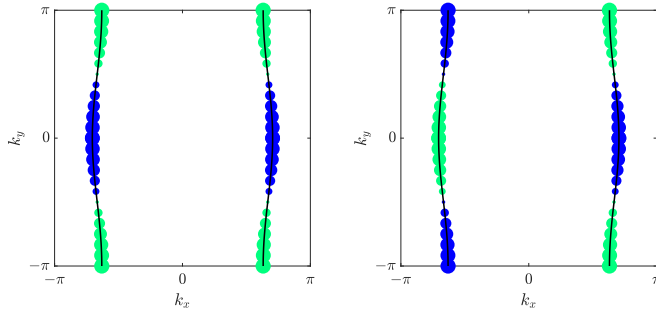


FIG. 1. Nearly degenerate dominant gap functions in the even-parity (left) and odd-parity (right) sector, evaluated at the Fermi surface, for  $t_x = 1$ ,  $t_y = 0.1$ , and  $\mu = 1$ . The size of the dots gives the gap magnitude, and the color gives the sign. The gap functions are well approximated by the analytical form of Eq. (8) ( $\Delta_{m=1,e}$  and  $\Delta_{m=1,o}$ ) obtained in the limit of  $t_y/t_x \rightarrow 0$ .

direction and  $t_y$  along the  $y$  direction. The Hamiltonian reads

$$H = \sum_{\mathbf{k}} \xi(\mathbf{k})(n_{\mathbf{k},\uparrow} + n_{\mathbf{k},\downarrow}) + \sum_{\mathbf{x}} U n_{\mathbf{x},\uparrow} n_{\mathbf{x},\downarrow}, \quad (1)$$

with a dispersion relation given by

$$\xi(k) = -2t_x \cos(k_x) - 2t_y \cos(k_y) - \mu. \quad (2)$$

We are interested in the quasi-1D limit:  $t_y \ll t_x$ . In that limit, the FSs are given by slightly corrugated vertical lines (see Fig. 1):

$$k_F(k_y) = k_F + \frac{2t_y \cos(k_y)}{v_F} + O(t_y^2), \quad (3)$$

with  $k_F = \arccos(-\mu/2t_x)$  and  $v_F = 2t_x \sin(k_F)$ .

Following the standard weak coupling approach [34–48], valid in the limit  $U/t \rightarrow 0$ , we must solve the following eigenvalue problem:

$$\frac{1}{(2\pi)^2} \int_{\text{FS}} \frac{d\hat{k}_2}{v(\hat{k}_2)} V(\hat{k}_1 - \hat{k}_2) \Delta(\hat{k}_2) = \lambda \Delta(\hat{k}_1), \quad (4)$$

where the integral is over the FS,  $V$  is the effective interaction in the Cooper channel, and  $v(\hat{k})$  is the norm of the Fermi velocity at momentum  $\hat{k}$ . Each solution with negative eigenvalue  $\lambda$  corresponds to a SC order with gap function  $\Delta(\mathbf{k})$  and critical temperature  $T_c \propto W e^{1/\lambda}$ , with  $W$  the bandwidth. The dominant OP has the most negative eigenvalue.

Since we are taking two limits ( $U/t \rightarrow 0$  and  $t_y/t_x \rightarrow 0$ ), it is important to specify the order in which they are taken. We first take the weak coupling limit before taking the quasi-1D limit, which means that the system above  $T_c$  behaves as a 2D Fermi liquid (as opposed to a Luttinger liquid if the other order of limits had been chosen). This order of limits therefore allows us to use a weak coupling approach in a quasi-1D system, even though this approach is not valid in a strictly 1D system [49]. Note that the present model also differs from the case of small- $U$  multileg Hubbard ladders [50] since we work directly in the thermodynamic limit in both the  $x$  and  $y$  directions.

In a single-orbital model,  $V$  takes a simple form [34,51]:

$$\begin{aligned} V_e(\hat{k}_1 - \hat{k}_2) &= U + U^2 \chi(\hat{k}_1 - \hat{k}_2), \\ V_o(\hat{k}_1 - \hat{k}_2) &= -U^2 \chi(\hat{k}_1 - \hat{k}_2), \end{aligned} \quad (5)$$

in the even- and odd-parity channel, respectively, and where  $\chi(\mathbf{q})$  is the Lindhard susceptibility:

$$\chi(\mathbf{q}) = \frac{-1}{(2\pi)^2} \int d\mathbf{k} \frac{n[\xi(\mathbf{k})] - n[\xi(\mathbf{k} + \mathbf{q})]}{\xi(\mathbf{k}) - \xi(\mathbf{k} + \mathbf{q})}, \quad (6)$$

with  $n[\xi]$  the Fermi-Dirac distribution.

As explained in the Appendix, Eq. (4) is analytically solvable in the limit of  $t_y/t_x \rightarrow 0$ , leading to the following negative eigenvalues:

$$\lambda_m = -\frac{U^2}{2(2\pi)^2 v_F^2} \frac{1}{m}, \quad (7)$$

for  $m = 1, 2, 3, \dots$ . The most negative eigenvalue thus corresponds to  $m = 1$ . Each eigenvalue is doubly degenerate, with an even-parity eigenvector and an odd-parity eigenvector. For odd  $m$ , these eigenvectors are given by

$$\begin{aligned} \Delta_{m,e} &= \cos(mk_y), \\ \Delta_{m,o} &= \cos(mk_y) \text{sign}(k_x). \end{aligned} \quad (8)$$

For even  $m$ , we find

$$\begin{aligned} \Delta_{m,e} &= \sin(mk_y) \text{sign}(k_x), \\ \Delta_{m,o} &= \sin(mk_y). \end{aligned} \quad (9)$$

For each  $m$ , we therefore have two degenerate eigenvectors which are simply related by a sign change between the left and right branches of the FS. The source of this degeneracy can be understood easily [34]. In the quasi-1D limit, the almost perfect nesting of the FSs leads to a strong peak in  $\chi(\mathbf{q})$  for  $q_x = \pm 2k_F$ . This means that the dominant type of scattering occurs between the two branches of the FS. By flipping the relative sign of the gap on the two branches, one can therefore effectively flip the sign of the effective interaction. This sign change exactly cancels out the sign difference for the  $U^2$  term in the effective interaction between even and odd parity [see Eq. (5)] [52].

Whereas the analytic results provided so far were obtained in the limit of  $t_y/t_x \rightarrow 0$ , we also numerically studied the case of small but finite  $t_y/t_x$ . As shown in Fig. 2, the dependence on  $t_y$  is extremely weak, and our analytic solution is therefore a good approximation for a broad range of  $t_y/t_x$ . The main effect of a finite  $t_y$  is to generate a small splitting between even- and odd-parity states which, for  $m = 1$ , favors the even-parity state. However, as shown in the right panel of Fig. 2, the splitting remains extremely small even for  $t_y/t_x \simeq 0.1$ , which is the range relevant for  $\text{Sr}_2\text{RuO}_4$ . The effect of finite  $t_y/t_x$  on eigenvectors is also small: They are still very well approximated by the simple cosine form given above even at  $t_y/t_x = 0.1$ . We also checked that changing the chemical potential does not produce any qualitative changes to these results.

### III. GL ANALYSIS AND THERMODYNAMIC PROPERTIES

In the previous section, we learned that the dominant SC orders in the quasi-1D Hubbard model are given by the two nearly degenerate  $m = 1$  states:

$$\begin{aligned} \Delta_e &\equiv \Delta_{m=1,e} = \cos(k_y), \\ \Delta_o &\equiv \Delta_{m=1,o} = \cos(k_y) \text{sign}(k_x). \end{aligned} \quad (10)$$

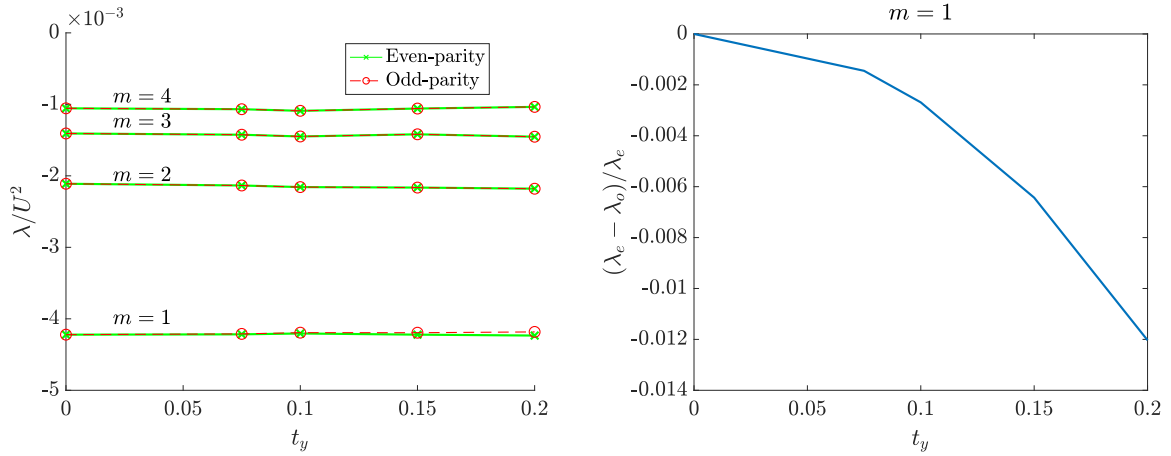


FIG. 2. Left: Four dominant eigenvalues in the even- and odd-parity sectors, for  $t_x = 1$  and  $\mu = 1$ . The points at  $t_y = 0$  were obtained analytically from Eq. (7), whereas the points at  $t_y > 0$  were obtained by numerically solving Eq. (4). The splitting between even- and odd-parity eigenvalues is so small that it is barely visible. Right: Splitting between the dominant (i.e.,  $m = 1$ ) even- and odd-parity eigenvalues, normalized by  $\lambda_e$ . The splitting increases quadratically with  $t_y$ , but remains extremely small for a wide range of  $t_y$ .

In this section, we use a GL analysis to study the possible combinations of these two OPs.

A combination of a singlet and triplet OP is nonunitary unless the relative phase between them is  $\pm i$ . Complex combinations of singlets and triplets are therefore generically favored [53]. We will thus consider the following OP:

$$\mathbf{\Delta}(\mathbf{k}) \equiv \Delta_{\uparrow\downarrow}(\mathbf{k}) = \psi_e \Delta_e(\mathbf{k}) + i\psi_o \Delta_o(\mathbf{k}), \quad (11)$$

with  $\psi_e$  and  $\psi_o$  real parameters, leading to  $|\Delta(\mathbf{k})|^2 = |\psi_e \Delta_e(\mathbf{k})|^2 + |\psi_o \Delta_o(\mathbf{k})|^2$  [54]. A typical GL free energy functional reads [55]

$$F = -a_e \psi_e^2 - a_o \psi_o^2 + b(\psi_e^2 + \psi_o^2)^2 + b'(\psi_e^2 - \psi_o^2)^2, \quad (12)$$

with  $a_e(T) \propto (T_{c,e} - T)$ ,  $a_o(T) \propto (T_{c,o} - T)$ , where  $T_{c,e}$  and  $T_{c,o}$  are the critical temperatures for each component when considered in isolation.

For  $b' < 0$ , the system favors having only one component at a time, whereas for  $b' > 0$ , the system favors a combination of the two. We will see below that  $b'$  is positive for the OPs obtained in the previous section, so we will focus on that case. The small splitting between eigenvalues which slightly favors the even-parity order (see Fig. 2) translates into a small difference between the critical temperatures:  $T_{c,e} = T_{c,o} + \delta$ , with  $\delta > 0$  small. In this scenario, the  $\psi_e$  component arises at the first transition  $T_{c,e}$ , and the  $\psi_o$  component arises at a second transition  $T^*$  given by  $T^* = T_{c,o} - \delta \frac{b-b'}{2b'}$ . We give a derivation of these results based on a GL analysis in Appendix B. An important approximation which was used in that analysis is that we assume the linear coefficients for  $a_e(T) \propto (T_{c,e} - T)$  and  $a_o(T) \propto (T_{c,o} - T)$  are equal. This assumption is justified by the fact that the two OPs have essentially the same magnitude everywhere on the FS, as we now discuss.

Whereas the GL analysis presented so far is standard, what is unusual about  $\Delta_e$  and  $\Delta_o$  is that they have the same magnitude everywhere on the FS (see Fig. 1):

$$|\Delta_e(\mathbf{k})|^2 \simeq |\Delta_o(\mathbf{k})|^2 \forall \mathbf{k} \in \text{FS}. \quad (13)$$

This property is really unique since one usually considers combinations of OPs that gap out different parts of the FS [like

$p_x + ip_y$  or  $d_{x^2-y^2} + ig_{(x^2-y^2)xy}$ ]. The main consequence is that the parameter  $b'$  is parametrically small (in  $t_y/t_x$ ), in contrast to standard two-component OPs for which it is of order one. This can be deduced from the following microscopic formula [56] for  $b'$ :

$$\frac{b'}{B} = \frac{1}{2} \left( \frac{1}{2} \langle |\Delta_e|^4 \rangle + \frac{1}{2} \langle |\Delta_o|^4 \rangle \right) - \frac{1}{2} \langle |\Delta_e|^2 |\Delta_o|^2 \rangle, \quad (14)$$

where  $B = \frac{7\zeta(3)}{16\pi^2(k_B T_c)^2} \rho$  and where  $\langle \dots \rangle$  is a FS average defined by

$$\langle f \rangle = \frac{1}{\rho} \frac{1}{(2\pi)^D} \int_{\text{FS}} d\hat{k} \frac{1}{v(\hat{k})} f(\hat{k}), \quad (15)$$

with  $\rho$  the density of states (DOS) at the Fermi level. The difference in Eq. (14) is usually of order one [e.g., for  $p_x + ip_y$  or  $d_{x^2-y^2} + ig_{(x^2-y^2)xy}$ ], but in our case, it is parametrically small. In other words, a unique feature of the current scenario is that the small parameter  $t_y/t_x$  leading to the near degeneracy of critical temperatures also leads to a small  $b'$  parameter.

### A. Specific heat

An important consequence of a small  $b'$  is that the jump in specific heat at the second transition  $T^*$  is parametrically small. As shown in Appendix B 1, the ratio of specific heat jumps is given by

$$\frac{\Delta C_{T^*}}{\Delta C_{T_c}} = \frac{\langle |\Delta(\mathbf{k})|^2 Y_{T^*}(\mathbf{k}) \rangle}{\langle |\Delta(\mathbf{k})|^2 \rangle} \frac{b'}{b}, \quad (16)$$

with

$$Y_T(\mathbf{k}) = \frac{1}{4} \int_{-\infty}^{\infty} dx \frac{1}{\cosh \left[ \frac{1}{2} \sqrt{x^2 + \beta^2 |\Delta(\mathbf{k})|^2} \right]^2} \quad (17)$$

the  $k$ -dependent Yosida function and  $\beta = 1/k_B T$ . When deriving Eq. (16), we made the approximation that  $|\Delta_e(\mathbf{k})|^2 = |\Delta_o(\mathbf{k})|^2 \equiv |\Delta(\mathbf{k})|^2$ .

From Eq. (16), we learn that two separate effects can lead to a reduction of the second specific heat jump: the effect of the Yosida function and the effect of a small  $b'/b$  ratio.

The first effect is always present for any two-component OP and would act in the same way in this case [6]. However, this effect can only give a substantial reduction of the second specific heat jump if  $T^*$  is much smaller than  $T_{c,e}$ . On the other hand, the effect of  $b' \ll b$  is unique to the current scenario and naturally leads to a parametric difference between the two specific heat jumps. As an illustration, for the numerical solution at  $t_y/t_x = 0.1$  obtained in the previous section, we find that  $b'/b \sim 10^{-5}$ .

### B. Spin susceptibility

The even- and odd-parity components have of course different effects on the spin susceptibility since the former is a spin singlet and the latter is a spin triplet (we neglect spin-orbit coupling for the time being). Taking advantage of the  $SU(2)$  symmetry of the Hubbard model, we did not have to specify the orientation of  $\vec{d}$  for the odd-parity, spin-triplet component in the previous discussion (see, e.g., Ref. [57] for a definition of  $\vec{d}$ ). It is however now necessary to specify it to discuss the spin susceptibility  $\chi$ . Whereas the susceptibility of a spin singlet goes to zero for any orientation of the magnetic field  $\vec{H}$ , the situation is more complex when a spin triplet component is present. When  $\vec{H}$  is parallel to  $\vec{d}$ , the singlet and triplet components lead to the same decay of  $\chi$ , with zero residual spin susceptibility. When  $\vec{H}$  is perpendicular to  $\vec{d}$ , the spin susceptibility is given by (see Appendix C for a derivation):

$$\frac{\chi(T)}{\chi_N} = \langle Y_T(\mathbf{k}) \rangle + \left\langle \frac{|\psi_o \Delta_o(\mathbf{k})|^2}{|\mathbf{\Delta}(\mathbf{k})|^2} [1 - Y_T(\mathbf{k})] \right\rangle, \quad (18)$$

with  $\chi_N$  the normal state Pauli susceptibility. At  $T = 0$ , one finds  $Y_T(\mathbf{k}) = 0$ , leading to

$$\frac{\chi(T=0)}{\chi_N} = \left\langle \frac{|\psi_o \Delta_o(\mathbf{k})|^2}{|\mathbf{\Delta}(\mathbf{k})|^2} \right\rangle \simeq \frac{|\psi_o|^2}{|\psi_e|^2 + |\psi_o|^2}, \quad (19)$$

where we made the approximation that  $|\Delta_e(\mathbf{k})|^2 = |\Delta_o(\mathbf{k})|^2$  in the last step. Assuming  $|\psi_o|^2 \simeq |\psi_e|^2$  at  $T = 0$  (which is expected if the two critical temperatures are close to each other), this leads to a residual susceptibility of  $\frac{1}{2}$  for  $\vec{d} \perp \vec{H}$ .

## IV. APPLICATION TO STRONTIUM RUTHENATE

As mentioned in the introduction, the main motivation behind this paper is the study of superconductivity in  $\text{Sr}_2\text{RuO}_4$ . The Hamiltonian studied above provides a good model for the quasi-1D Ru orbital  $d_{zx}$  (and of course for  $d_{zy}$  after a  $\pi/2$  rotation) of  $\text{Sr}_2\text{RuO}_4$ , if it could be considered in isolation. In this section, we will make the assumption that the above mechanism for accidental mixed-parity superconductivity is at play on each of these two orbitals, and we will analyze the consequences for experiments. We should emphasize that this assumption is purely empirical: We do not claim to have a microscopic justification for neglecting the coupling between the two quasi-1D orbitals, and between the quasi-1D orbitals and the  $d_{xy}$  orbital.

As thermodynamic measurements give evidence for a SC order of similar size on the three orbitals, we also need to make an assumption about the OP on the  $d_{xy}$  orbital (which contributes mostly to the  $\gamma$  band). Since there is no reason

to expect a degeneracy between even- and odd-parity components for  $d_{xy}$  (because it is not quasi-1D), we assume that only one component, the even one, is present on that orbital. To sum up, the proposed scenario is the following: An even-parity  $\Delta_e$  component appears at the first transition on all three orbitals, and an odd-parity component  $\Delta_o$  appears at a second transition only on the quasi-1D orbitals.

Before discussing in more detail the form  $\Delta_e$  and  $\Delta_o$  could take within a three-orbital model, we can already discuss the general properties of a state of the type  $\Delta_e + i\Delta_o$ . Such a state has several desirable features as a candidate for multicomponent superconductivity in  $\text{Sr}_2\text{RuO}_4$ . First, the accidental degeneracy between the two components has a microscopic justification based on the small parameter  $t_y/t_x$ . Second, the OP is still nodal even though it forms a complex linear combination since both components have cosine nodes at  $k_y = \pm\pi/2$  (resp.  $k_x = \pm\pi/2$ ) for  $d_{zx}$  (resp. for  $d_{zy}$ ). (The presence of nodes in the SC gap is well established [58–62], although their location remains controversial.) Third, the fact that  $|\Delta_e(\mathbf{k})|^2 = |\Delta_o(\mathbf{k})|^2$  everywhere on the FS leads to a parametrically small second specific heat jump, as required by recent measurements [33].

Another problem facing most proposals of TRS-breaking OPs is that it contradicts the absence of measurable edge currents revealed by magnetometry measurements [63]. Even though several effects have been predicted to reduce these currents [64–67], this remains a challenge for most OPs with TRS breaking, like  $p + ip$  or  $d + id$ . By contrast, a state of the type  $\Delta_e + i\Delta_o$  provides a natural way of breaking TRS without having edge currents (in a centrosymmetric crystal). Indeed, the gradient terms which usually lead to spontaneous edge currents are not allowed in this case since they do not respect parity [68]:

$$F \not\supset \int d\mathbf{x} (\partial_x \psi_e^*)(\partial_y \psi_o) + \text{c.c.}, \quad (20)$$

where  $\psi_o$  and  $\psi_e$  are the components as defined in Eq. (11).

If no edge currents are expected, what is the manifestation of TRS breaking for mixed parity states? It actually manifests itself through the spin degree of freedom rather than the orbital one. Indeed, mixed even-odd parity superconductors experience a spontaneous magnetization at any nonhomogeneities, like domain walls, edges, and defects [69–71]. The intuition is that the relative  $i$  phase is between two different spin (or rather helicity) components rather than two different orbital components (e.g.,  $p_x$  and  $p_y$ ). The orientation of the spontaneous magnetization depends on the orientation of  $\vec{d}$  and of the inhomogeneity. For example, for a state of the type  $d_{x^2-y^2} + i(p_x - p_y)\hat{z}$  (as proposed below), the following term would be allowed by symmetry [70]:

$$F \supset \int d\mathbf{x} m_z \psi_e^* (\partial_x + \partial_y) \psi_o, \quad (21)$$

where  $m_z$  is the  $z$  component of the magnetization. This term would create a spontaneous  $\hat{z}$  magnetization localized around inhomogeneities of the OP.

More generally, a magnetization  $\vec{m}$  localized around extended defects like domain walls or dislocations could explain the presence of a signal in  $\mu\text{SR}$  [19,21] (regardless of the orientation of  $\vec{m}$ ) and in the Kerr effect [20] (if  $\vec{m}$  has an

out-of-plane component). It could also explain the absence of a signal in scanning superconducting quantum interference device magnetometry measurements [63] since a surface magnetization does not produce stray fields. Note also that the scale of the magnetization would depend on microscopic details and is probably directly related to the strength of spin-orbit coupling. An additional phenomenon to consider when studying  $\mu$ SR is that the muon itself could create a local magnetization in a mixed-parity superconductor since it can be seen as a charged defect.

Additionally, the behavior of superconductivity in  $\text{Sr}_2\text{RuO}_4$  under  $[1, 0, 0]$  strain could also be explained by the current scenario. First, no cusp of  $T_c$  at zero strain is expected for an accidental degeneracy [72]. Second, it is natural to expect the even-parity component to undergo a large increase of  $T_c$  as the  $\gamma$  band approaches the van Hove singularity since the even-parity component is by assumption nonzero on that band and is antinodal at the van Hove point [73]. By contrast, one would only expect a small variation of the onset temperature for the odd-parity component since it only resides on the quasi-1D bands, which are comparatively little affected by strain. This would be consistent with the small variation of the onset temperature of the  $\mu$ SR signal observed in Ref. [21].

Further, the presence of an odd-parity, pseudospin triplet component would help explain a number of experiments which have been interpreted that way, like Josephson junction tunneling [74–76], the observation of half-quantum vortices [77], and  $\text{Sr}_2\text{RuO}_4$ -ferromagnet heterostructures [78].

In the next two subsections, we will discuss in more detail the different ways in which the two components  $\Delta_e$  and  $\Delta_o$  obtained in the simple model of Sec. II could be incorporated into a three-orbital model of  $\text{Sr}_2\text{RuO}_4$ . We will also examine the implications for other experiments, namely, the measurement of the Knight shift [25,26] and the jump in elastic moduli [22–24].

### A. Nature of the even-parity component

Assuming that a gap of the form  $\cos(k_y)$  [resp.  $\cos(k_x)$ ] is favored on  $d_{zx}$  (resp.  $d_{zy}$ ), there remains the question of the relative phase between the gaps in the two orbitals. If this phase is  $+1$  [resp.  $-1$ ], the resulting gap is in the  $A_{1g}$  (resp.  $B_{1g}$ ) representation:

$$\begin{aligned} A_{1g} : [\Delta_{e,d_{zx}}, \Delta_{e,d_{zy}}] &= [\cos(k_y), \cos(k_x)], \\ B_{1g} : [\Delta_{e,d_{zx}}, \Delta_{e,d_{zy}}] &= [\cos(k_y), -\cos(k_x)], \end{aligned} \quad (22)$$

where  $\Delta_{e,d_{zx}}$  (resp.  $\Delta_{e,d_{zy}}$ ) is the even-parity component on the  $d_{zx}$  (resp.  $d_{zy}$ ) orbital. The difference between  $A_{1g}$  ( $s'$ ) and  $B_{1g}$  ( $d_{x^2-y^2}$ ) only becomes important along the diagonals ( $[1, 1, 0]$  and  $[1, -1, 0]$  directions) since the  $B_{1g}$  gap has symmetry-imposed nodes along the diagonals, while the  $A_{1g}$  gap does not. By contrast, the cosine nodes at  $k_x = \pm\pi/2$  and  $k_y = \pm\pi/2$  are present for both  $A_{1g}$  and  $B_{1g}$ .

Within a two-orbital model, the splitting between  $A_{1g}$  and  $B_{1g}$  is a second-order effect since it only depends on the hybridization between the two orbitals, which is mostly localized in a small region along the diagonals. In fact, a close competition between these states has been reported in previous work, even in three-orbital models [29,79]. Both  $s'$  and

$d_{x^2-y^2}$  should therefore be considered as candidates for the even-parity component.

### B. Nature of the odd-parity component

We expect the odd-parity order to only arise on the  $d_{zx}$  and  $d_{zy}$  orbitals since the degeneracy between odd- and even-parity states relies on the quasi-1D limit. Starting from the  $\text{sign}(k_x) \cos(k_y)$  form found in the single-orbital model, two choices have to be made: the spin orientation of Cooper pairs (parameterized by  $\vec{d}$ ) on each orbital and the relative phase of the OPs between the two orbitals. Each choice corresponds to a different  $D_{4h}$  representation:

$$\begin{aligned} E_u : [\vec{d}_{d_{zx}}, \vec{d}_{d_{zy}}] &= \hat{z}[\eta_x \text{sign}(k_x) \cos(k_y), \eta_y \text{sign}(k_y) \cos(k_x)], \\ A_{1u} : [\vec{d}_{d_{zx}}, \vec{d}_{d_{zy}}] &= [\hat{x} \text{sign}(k_x) \cos(k_y), \hat{y} \text{sign}(k_y) \cos(k_x)], \\ A_{2u} : [\vec{d}_{d_{zx}}, \vec{d}_{d_{zy}}] &= [\hat{y} \text{sign}(k_x) \cos(k_y), -\hat{x} \text{sign}(k_y) \cos(k_x)], \\ B_{1u} : [\vec{d}_{d_{zx}}, \vec{d}_{d_{zy}}] &= [\hat{x} \text{sign}(k_x) \cos(k_y), -\hat{y} \text{sign}(k_y) \cos(k_x)], \\ B_{2u} : [\vec{d}_{d_{zx}}, \vec{d}_{d_{zy}}] &= [\hat{y} \text{sign}(k_x) \cos(k_y), \hat{x} \text{sign}(k_y) \cos(k_x)], \end{aligned} \quad (23)$$

where  $\vec{d}_{d_{zx}}$  (resp.  $\vec{d}_{d_{zy}}$ ) is the  $\vec{d}$  vector on the  $d_{zx}$  (resp.  $d_{zy}$  orbital) and  $\eta_x$  and  $\eta_y$  are free parameters. All these representations are degenerate for the  $SU(2)$ -symmetric single-orbital toy model considered in Sec. II. They would, however, be split by spin-orbit coupling in a realistic model, as studied in previous work (see Ref. [79] and references therein). Here, we will take a phenomenological approach and discuss the different representations in light of available experimental results.

#### 1. $E_u$ state

The favored  $E_u$  state can either be  $[1, 0, 0]$ -nematic  $\{p_x, p_y\}$ ,  $[1, 1, 0]$ -nematic  $p_x \pm p_y$ , or chiral  $p_x \pm ip_y$ . Whereas a chiral state is usually favored since it does not have any symmetry-imposed nodes, the situation is different here due to the presence of the even-parity component. It is indeed favorable for both  $p_x$  and  $p_y$  components to have a relative  $\pm i$  phase with respect to the even-parity component (to form a unitary state), which is of course incompatible with having a relative  $i$  phase between  $p_x$  and  $p_y$ . A nematic state could therefore be favored due to the presence of the even-parity component. Since a 100-nematic state seems unlikely due to the fact that it would only gap out one of the two quasi-1D orbitals, the most likely scenario would be a  $[1, 1, 0]$ -nematic state:  $p_x \pm p_y$ . Combining this with the above candidates for the even component, the OP would be of the form  $d_{x^2-y^2} + i(p_x \pm p_y)\hat{z}$  or  $s' + i(p_x \pm p_y)\hat{z}$ .

Neglecting spin-orbit coupling and assuming an equal amplitude of singlet and triplet components on the  $\alpha$  and  $\beta$  bands at  $T = 0$ , we can obtain an estimate of the residual spin susceptibilities based on Sec. III B:

$$\begin{aligned} \frac{\chi_{\parallel}(T=0)}{\chi_N} &= \frac{1}{2} \frac{\rho_{\alpha,\beta}}{\rho} \simeq 0.2, \\ \frac{\chi_{\perp}(T=0)}{\chi_N} &= 0, \end{aligned} \quad (24)$$

for in-plane and out-of-plane magnetic fields, respectively, where  $\rho_{\alpha,\beta}$  is the DOS at the Fermi level for the  $\alpha$  and  $\beta$  bands, and  $\rho = \rho_{\alpha,\beta} + \rho_{\gamma}$  is the total DOS. Quantum oscillation measurements give  $\frac{\rho_{\alpha,\beta}}{\rho} \simeq 0.4$  [10]. A residual susceptibility of 0.2 was consistent with earlier Knight shift measurements [25,26] but is inconsistent with the upper bound of 0.1 recently reported by Chronister *et al.* [80]. Based on our current estimate for the residual susceptibility, a mixed  $\Delta_e + iE_u$  state is therefore inconsistent with the latest Knight shift experiments. A more accurate estimate of  $\chi/\chi_N$  based on a microscopic calculation with spin-orbit coupling and multi-band effects is, however, warranted before the possibility of such a state is discarded altogether.

Regarding ultrasound experiments, an  $E_u$  component would explain the presence of a jump in the  $B_{2g}$  elastic modulus [22–24] but could also potentially have a jump in the  $B_{1g}$  channel, which was not observed (although there could be some microscopic reasons why the  $B_{1g}$  jump has a smaller prefactor).

## 2. Helical states ( $A_{1u}, A_{2u}, B_{1u}, B_{2u}$ )

Helical states have a  $\vec{d}$  vector that rotates in plane as one moves around the FS. An accurate calculation of the spin susceptibility is beyond the scope of this paper, but we can already obtain an estimate as follows. Assuming an approximately isotropic orientation of  $\vec{d}$  within the plane, helical states would have the following residual spin susceptibilities:

$$\begin{aligned} \frac{\chi_{\parallel}(T=0)}{\chi_N} &= \frac{1}{4} \frac{\rho_{\alpha,\beta}}{\rho} \simeq 0.1, \\ \frac{\chi_{\perp}(T=0)}{\chi_N} &= \frac{1}{2} \frac{\rho_{\alpha,\beta}}{\rho} \simeq 0.2, \end{aligned} \quad (25)$$

for in-plane and out-of-plane magnetic fields, respectively. To the best of our knowledge, these values are compatible with current NMR experiments but could potentially be disproved by further measurements [25,26].

It does not seem possible at this point to explain a jump in the  $B_{2g}$  elastic modulus without invoking an accidental combination of two different helical states, like  $B_{1u}$  and  $A_{2u}$ . However, a thorough analysis of possible couplings between elasticity and mixed even-odd OPs might reveal other possibilities, especially if inhomogeneities of the OP are considered.

A necessary (though not sufficient [81]) criterion to see a Kerr signal is to break TRS and all vertical mirror planes [82]. If inhomogeneities (e.g., domain walls) can be invoked to break certain mirror symmetries, the Kerr signal cannot discriminate between different helical states. However, if one requires all vertical mirror symmetries to be broken by the bulk OP, the presence of a Kerr signal imposes restrictions on the possible helical states: assuming that the even component is in  $A_{1g}$  or  $B_{1g}$ , only combinations of the type  $A_{1g} + iA_{1u}$  or  $B_{1g} + iB_{1u}$  would break all vertical mirrors.

## V. DISCUSSION

We have established an accidental degeneracy between even-parity [ $\Delta_e = \cos(k_y)$ ] and odd-parity [ $\Delta_o = \cos(k_y)\text{sign}(k_x)$ ] SC orders in the quasi-1D limit ( $t_y/t_x \rightarrow 0$ )

of the Hubbard model, in the weak  $U$  limit. Moving away from the purely 1D limit creates a small splitting between these orders by favoring the even-parity one. A GL analysis then revealed that a linear combination of the type  $\Delta_e + i\Delta_o$  can become favorable at a second transition. Remarkably, the degenerate orders have essentially the same gap magnitude over the entire FS, leading to a parametrically small  $b'$  coefficient in the GL free energy. This leads to a parametrically small specific heat jump at the second transition.

In Sec. IV, we assumed that this mechanism is at play on the quasi-1D orbitals of  $\text{Sr}_2\text{RuO}_4$ , and we analyzed the consequences for experiments. A state of the type  $\Delta_e + i\Delta_o$  has several desirable features. It explains the presence of nodes [58–62] in a TRS-breaking state, and it predicts a parametrically small specific heat jump [33]. It also reconciles the breaking of TRS [19,20] with the absence of edge currents [63]. Further, the presence of an odd-parity, pseudospin triplet component would help explain a number of measurements which have been interpreted as such [74,75,77,78].

Whereas our solution of the single-orbital Hubbard model is exact, its application to  $\text{Sr}_2\text{RuO}_4$  was purely empirical since we do not have a microscopic justification for neglecting interorbital effects. These effects have been studied extensively in the literature [34,45,79,83–87] and can often impact crucially the predictions of theoretical models. Our ambition with this paper was much smaller: We wanted to find a toy model which exhibits a second transition to a TRS-breaking state with a parametrically small specific heat jump, which we have found. A more realistic calculation which includes multiple orbitals and spin-orbit coupling would be necessary to go beyond this proof of principle. The main effect which could create substantial splitting between even- and odd-parity SC orders is interorbital interaction, as already observed in Ref. [34]. A thorough study of the fate of this degeneracy as a function of  $J/U$  is therefore warranted (where  $J$  is Hund's coupling, and  $U$  is the intra-orbital Hubbard interaction).

Additionally, the quasi-1D regime of the square lattice Hubbard model is relevant to a variety of materials, including Bechgaard salts [44,88,89] and  $\text{Li}_{0.9}\text{Mo}_6\text{O}_{17}$  [90]. This model can be generalized to the case of longer-range interaction and finite  $U/t$ , which leads to a variety of interesting SC phases, including odd-frequency superconductivity and Fulde-Ferrell-Larkin-Ovchinnikov phases [91–94]. Moving beyond the quasi-1D regime, an accidental degeneracy between even- and odd-parity superconductivity is an interesting possibility to consider [95], in the context of  $\text{Sr}_2\text{RuO}_4$  and other systems. In fact, the proximity to a quantum critical point was shown to provide another mechanism for a nearly degenerate pairing in even and odd channels [96–102].

One defining feature of a mixed-parity state is of course the breaking of inversion symmetry, which could be probed by nonlinear optical effects like second-harmonic generation [103–105]. Another way to measure a breaking of inversion symmetry is provided by phase-sensitive measurements which probe opposite sides of the sample [74]. A study of the nature of edge modes in a mixed-parity state could also reveal interesting properties and could be compared with existing experimental data [106]. Finally, the most direct way to put the present proposal to the test is probably the Knight shift [25,26]: The presence of a spin-triplet component could be

disproved if a residual susceptibility smaller than the ones predicted in Eq. (24) or (25) was measured.

*Note added.* As we were completing this paper, we received a manuscript by Chronister *et al.* [80] reporting Knight shift measurements in  $\text{Sr}_2\text{RuO}_4$ . These measurements provide a more constraining upper bound on the spin susceptibility of the condensate than previous work. Based on our estimates for the residual susceptibility, the results of Chronister *et al.* [80] do not rule out the possibility of a mixed-parity OP.

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### APPENDIX A: ANALYTIC SOLUTION OF THE WEAK COUPLING EQUATION

Within a weak coupling analysis of the SC instability, we must solve the following equation:

$$\frac{1}{(2\pi)^2} \int \frac{d\hat{k}_2}{|v(\hat{k}_2)|} V(\hat{k}_1 - \hat{k}_2) \Delta(\hat{k}_2) = \lambda \Delta(\hat{k}_1), \quad (\text{A1})$$

where

$$\begin{aligned} V_e(\hat{k}_1 - \hat{k}_2) &= U + U^2 \chi(\hat{k}_1 - \hat{k}_2), \\ V_o(\hat{k}_1 - \hat{k}_2) &= -U^2 \chi(\hat{k}_1 - \hat{k}_2) \end{aligned} \quad (\text{A2})$$

are the effective interactions in the even and odd sectors,  $\chi$  is the Lindhard susceptibility, and  $\hat{k}_1, \hat{k}_2$  live on the FS. In this Appendix, we will provide an analytic solution that is valid in the limit of  $t_y/t_x \rightarrow 0$ .

In this limit, the FSs are given by two sheets at  $k_x = \pm k_F(k_y)$ , with

$$k_F(k_y) = k_F + \frac{2t_y \cos(k_y)}{v_F} + O(t_y^2), \quad (\text{A3})$$

where  $v_F = 2t_x \sin(k_F)$  and  $k_F = \arccos(-\mu/2t_x)$ .

#### 1. Lindhard susceptibility

Since the FS is given by two separate sheets at  $k_x \simeq k_F$ , we only need the value of  $\chi(q_x, q_y)$  in two regimes:  $q_x \simeq 0$  (for intrasheet scattering) and  $q_x \simeq 2k_F$  (for intersheet scattering). For intrasheet scattering, one easily finds that

$$\chi(q_x \simeq 0, q_y) = \rho + O(t_y), \quad (\text{A4})$$

with  $\rho$  the DOS at the Fermi level in the vanishing  $t_y$  limit. We can therefore forget about intrasheet scattering since this constant term will only give a contribution in the trivial  $s$ -wave channel.

The intersheet case is more interesting: We will find that

$$\chi(q_x \simeq 2k_F, q_y) = C(t_y) + f(q_y) + O(t_y), \quad (\text{A5})$$

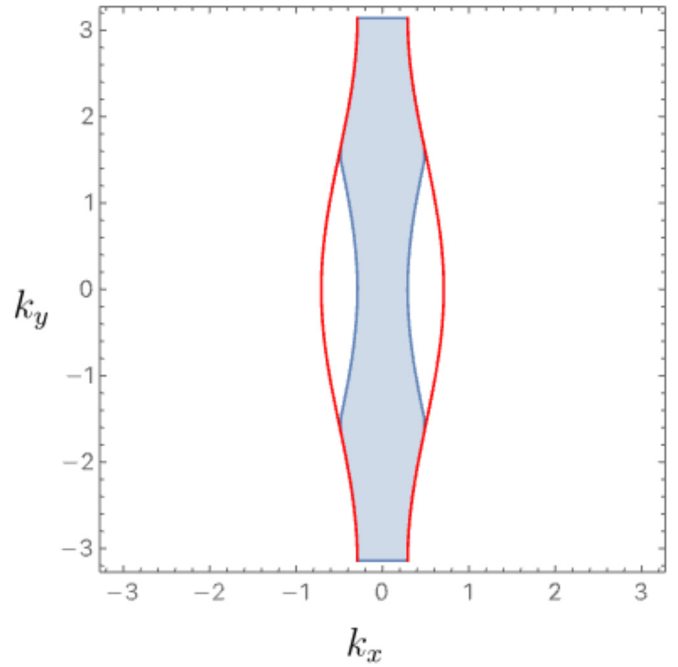


FIG. 3. The blue region shows a typical example of zone 2 defined in Eq. (A7). The Fermi surface is shown in red. Parameters are  $k_F = 0.5$ ,  $t_y/t_x = 0.1$ ,  $q_y = 0$ , and  $q_x = 1$ .

where  $C(t_y)$  is an unimportant constant since it will only give a contribution in the trivial  $s$ -wave channel, and  $f(q_y)$  is a nontrivial function that is independent of  $t_y$  and will need to be diagonalized to solve the problem at hand.

As a reminder, the susceptibility is defined as

$$\chi(\mathbf{q}) = -\frac{1}{(2\pi)^2} \int d\mathbf{p} \frac{n[\xi(\mathbf{p})] - n[\xi(\mathbf{p} + \mathbf{q})]}{\xi(\mathbf{p}) - \xi(\mathbf{p} + \mathbf{q})}. \quad (\text{A6})$$

The numerator is nonzero in two disjoint regions, one for which  $\xi(\mathbf{p}) > 0$  and  $\xi(\mathbf{p} + \mathbf{q}) < 0$  (zone 1) and one for which  $\xi(\mathbf{p}) < 0$  and  $\xi(\mathbf{p} + \mathbf{q}) > 0$  (zone 2). Since these two zones give the same contribution to the integral, we will only focus on zone 2. For a given  $k_y$ , the zone limits for zone 2 are  $k_{x,\text{start}} \leq k_x \leq k_{x,\text{end}}$  with

$$\begin{aligned} k_{x,\text{start}}(k_y) &= \max[k_F(k_y + q_y) - q_x, -k_F(k_y)], \\ k_{x,\text{end}}(k_y) &= \min[-k_F(k_y + q_y) + q_x, k_F(k_y)]. \end{aligned} \quad (\text{A7})$$

A typical example of zone 2 is shown in Fig. 3.

We are now interested in the locus of points  $\tilde{k}_x$  where the denominator vanishes [i.e., where  $\xi(\mathbf{p}) - \xi(\mathbf{p} + \mathbf{q}) = 0$ ] since the integrand will be peaked there. It is given, to leading order in  $t_y$ , by

$$\tilde{k}_x(k_y) = -k_F + \frac{1}{2}[k_F(k_y + q_y) - k_F(k_y) - (q_x - 2k_F)]. \quad (\text{A8})$$

It will be useful to define  $k_y^*$  as

$$k_F(k_y^* + q_y) = -k_F(k_y^*) + q_x. \quad (\text{A9})$$

In other words,  $k_y^*(q_x, q_y)$  is the value of  $k_y$  on the left branch such that  $k + q$  sits exactly on the right branch at  $k_y + q_y$ . With this parameterization, we find  $q_x = k_F(k_y^* + q_y) + k_F(k_y^*)$ .

Now we can expand the denominator  $D(p_x, p_y) \equiv \xi(\mathbf{p}) - \xi(\mathbf{p} + \mathbf{q})$  linearly along the  $k_x$  direction:

$$D(p_x, p_y) \simeq (p_x - \tilde{k}_x) (\partial_{p_x} D)|_{\tilde{k}_x} + O[(p_x - \tilde{k}_x)^2]. \quad (\text{A10})$$

To leading order in  $t_y$ , we find  $(\partial_{p_x} D)|_{\tilde{k}_x} = -2v_F$ ; thus,  $D(p_x, p_y) \simeq (p_x - \tilde{k}_x)2v_F$ .

Now the integral becomes

$$\begin{aligned} \chi(\mathbf{q}) &= -\frac{2}{(2\pi)^2} \int dp_y \int_{k_{x,\text{start}}(p_y)}^{k_{x,\text{end}}(p_y)} dp_x \frac{1}{-2v_F(p_x - \tilde{k}_x)} \\ &= \frac{2}{2v_F(2\pi)^2} \int dp_y \{ \ln[k_{x,\text{end}}(p_y) - \tilde{k}_x(p_y)] \\ &\quad - \ln[k_{x,\text{start}}(p_y) - \tilde{k}_x(p_y)] \} \\ &\simeq -\frac{2}{2v_F(2\pi)^2} \int dp_y \ln[k_{x,\text{start}}(p_y) - \tilde{k}_x(p_y)], \quad (\text{A11}) \end{aligned}$$

where in the last line, we used the fact that, in the small  $t_y$  limit,  $k_{x,\text{start}}(p_y) - \tilde{k}_x(p_y)$  goes to zero, while  $k_{x,\text{end}}(p_y) - \tilde{k}_x(p_y)$  is finite. We also find that

$$\begin{aligned} k_{x,\text{start}}(p_y) - \tilde{k}_x(p_y) &= |\tilde{k}_x - [-k_F(p_y)]| \\ &= \frac{1}{2} |k_F(p_y) + k_F(p_y + q_y) - k_F(k_y^* + q_y) - k_F(k_y^*)| \\ &= \frac{2t_y}{v_F} \frac{1}{2} |\cos(p_y) + \cos(p_y + q_y) \\ &\quad - \cos(k_y^* + q_y) - \cos(k_y^*)| + O(t_y^2), \quad (\text{A12}) \end{aligned}$$

which finally leads to

$$\begin{aligned} \chi(q) &= -2 \frac{1}{2v_F(2\pi)^2} \int dp_y \ln \left[ \frac{2t_y}{v_F} \frac{1}{2} |\cos(p_y) \right. \\ &\quad \left. + \cos(p_y + q_y) - \cos(k_y^* + q_y) - \cos(k_y^*)| \right]. \quad (\text{A13}) \end{aligned}$$

After some algebra, we find the simple relation:

$$\chi(\hat{k}_2 - \hat{k}_1) = \chi'_0 - \frac{1}{2v_F(2\pi)} \ln [\cos(k_{y,2} - k_{y,1}) + 1], \quad (\text{A14})$$

with  $\chi'_0 = \frac{1}{2v_F(2\pi)} \ln(\sqrt{2}v_F/t_y)$  an inconsequential constant since it will only give a repulsive contribution in the  $m = 0$  channel (see below).

## 2. Diagonalization

Starting from the initial eigenproblem [Eq. (A1)], we can make a further set of approximations which are valid to leading order in  $t_y$ . We can omit constant terms in the effective interaction since they will only contribute to the  $m = 0$  sector, which is always repulsive. This includes the  $U$  term in Eq. (A2), the intrasheet scattering (i.e., when  $\hat{k}_1$  and  $\hat{k}_2$  are on the same FS sheet), and the  $\chi'_0$  term in Eq. (A14). Finally, to leading order, we can take the Fermi velocity to be constant:  $v(k) = v_F$ . After all these approximations, the even- and odd-parity sector eigenproblems both simplify to the same

equation:

$$\frac{1}{(2\pi)^2 v_F} s_y \int_{-\pi}^{\pi} dk_{y,2} \chi(k_{y,1} - k_{y,2}) \Delta(k_{y,2}) = \lambda \Delta(k_{y,1}), \quad (\text{A15})$$

where  $\chi(k_{y,1} - k_{y,2})$  is given in Eq. (A14), and  $s_y$  is the sign change of  $\Delta$  under the  $y \rightarrow -y$  mirror symmetry.

Since  $\chi$  only depends on  $k_{y,2} - k_{y,1}$ , we can always diagonalize Eq. (A15) with Fourier series, leading to four sets of eigenvectors:

$$\begin{aligned} \Delta_{m,1,1}(k_x, k_y) &= \cos(mk_y), \\ \Delta_{m,-1,1}(k_x, k_y) &= \cos(mk_y) \text{sign}(k_x), \\ \Delta_{m,1,-1}(k_x, k_y) &= \sin(mk_y), \\ \Delta_{m,-1,-1}(k_x, k_y) &= \sin(mk_y) \text{sign}(k_x), \quad (\text{A16}) \end{aligned}$$

for  $m \geq 1$  (it is easy to check that the  $m = 0$  states are repulsive). Using the relation:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} dk_y \ln [\cos(k_y) + 1] \cos(mk_y) = \frac{(-1)^{m+1}}{m}, \quad (\text{A17})$$

valid for  $m \geq 1$ , one finds all the negative eigenvalues:

$$\lambda_m = -\frac{U^2}{2(2\pi)^2 v_F^2} \frac{1}{m}, \quad (\text{A18})$$

for  $m \geq 1$ . Each of these eigenvalues is doubly degenerate. For odd  $m$ , the eigenvectors are given by

$$\begin{aligned} \Delta_{m,e} &= \Delta_{m,1,1}(k_x, k_y) = \cos(mk_y), \\ \Delta_{m,o} &= \Delta_{m,-1,1}(k_x, k_y) = \cos(mk_y) \text{sign}(k_x). \quad (\text{A19}) \end{aligned}$$

For even  $m$ , we find

$$\begin{aligned} \Delta_{m,e} &= \Delta_{m,-1,-1}(k_x, k_y) = \sin(mk_y) \text{sign}(k_x), \\ \Delta_{m,o} &= \Delta_{m,1,-1}(k_x, k_y) = \sin(mk_y). \quad (\text{A20}) \end{aligned}$$

## APPENDIX B: GL ANALYSIS

We consider the following mixed-parity OP:

$$\mathbf{\Delta}(\mathbf{k}) \equiv \Delta_{\uparrow\downarrow}(\mathbf{k}) = \psi_e \Delta_e(\mathbf{k}) + i\psi_o \Delta_o(\mathbf{k}), \quad (\text{B1})$$

with  $\psi_e$  and  $\psi_o$  real parameters, leading to  $|\mathbf{\Delta}(\mathbf{k})|^2 = |\psi_e \Delta_e(\mathbf{k})|^2 + |\psi_o \Delta_o(\mathbf{k})|^2$ . The free energy is given by

$$F = -a_e \psi_e^2 - a_o \psi_o^2 + b(\psi_e^2 + \psi_o^2)^2 + b'(\psi_e^2 - \psi_o^2)^2, \quad (\text{B2})$$

with  $a_e(T) = a_0(T_{c,e} - T)$ ,  $a_o(T) = a_0(T_{c,o} - T)$ , where  $T_{c,e}$  and  $T_{c,o}$  are the critical temperatures for each component when considered in isolation.

The first transition occurs at  $T_{c,e}$ , and the second transition occurs at  $T^*$  given by

$$T^* = T_{c,o} - (T_{c,e} - T_{c,o}) \frac{b - b'}{2b'}. \quad (\text{B3})$$



The solution for the OP is

$$\left[ \psi_{e,0}^2 = \frac{a_e}{2(b+b')}, \psi_{i,0}^2 = 0 \right], \quad (\text{B4})$$

for  $T^* < T < T_{c,e}$  and

$$\left[ \psi_{e,1}^2 = \frac{1}{2} \left( \frac{a_e + a_o}{4b} + \frac{a_e - a_o}{4b'} \right), \psi_{o,1}^2 = \frac{1}{2} \left( \frac{a_e + a_o}{4b} - \frac{a_e - a_o}{4b'} \right) \right], \quad (\text{B5})$$

for  $T < T^*$ .

### 1. Specific heat discontinuity

The specific heat discontinuity at a temperature  $T$  is given by [57]

$$\Delta C = N_0 \frac{\beta}{4} \int d\xi \left\langle \frac{1}{\cosh(\beta E/2)^2} \left[ - \left( \frac{\partial |\Delta(\mathbf{k})|^2}{\partial T} \right)_{T_-} + \left( \frac{\partial |\Delta(\mathbf{k})|^2}{\partial T} \right)_{T_+} \right] \right\rangle_{\text{FS}}, \quad (\text{B6})$$

with

$$\begin{aligned} \langle f(k) \rangle_{\text{FS}} &= \frac{1}{N_0} \frac{1}{(2\pi)^D} \int_{\text{FS}} d\hat{k} \frac{1}{v(k)} f(\hat{k}), \\ N_0 &= \frac{1}{(2\pi)^D} \int_{\text{FS}} d\hat{k} \frac{1}{v(k)}, \end{aligned} \quad (\text{B7})$$

and where  $T_+$  and  $T_-$  are approaching  $T$  from above and below, respectively.

At the first transition, this leads to

$$\begin{aligned} \Delta C_{T_{c,e}} &= N_0 \frac{\beta}{4} \int d\xi \frac{1}{\cosh(\beta E/2)^2} \left\langle - \left( \frac{\partial |\Delta(\mathbf{k})|^2}{\partial T} \right)_{T_-} \right\rangle_{\text{FS}} \\ &= N_0 \frac{\beta}{4} \int d\xi \frac{1}{\cosh(\beta \xi/2)^2} \langle |\Delta_e(\mathbf{k})|^2 \rangle_{\text{FS}} \frac{a_0}{2(b+b')} \\ &= N_0 \langle |\Delta_e(\mathbf{k})|^2 \rangle_{\text{FS}} \frac{a_0}{2(b+b')}, \end{aligned} \quad (\text{B8})$$

where we used

$$\int d\xi \frac{1}{\cosh(\beta \xi/2)^2} = \frac{4}{\beta}. \quad (\text{B9})$$

At the second transition, we find

$$\begin{aligned} \Delta C_{T^*} &= N_0 \frac{\beta}{4} \int d\xi \left\langle \frac{1}{\cosh(\beta E/2)^2} \left[ - \left( \frac{\partial |\Delta(\mathbf{k})|^2}{\partial T} \right)_{T_-} + \left( \frac{\partial |\Delta(\mathbf{k})|^2}{\partial T} \right)_{T_+} \right] \right\rangle_{\text{FS}} \\ &= N_0 \frac{\beta}{4} \int d\xi \left\langle \frac{1}{\cosh(\beta E/2)^2} |\Delta(\mathbf{k})|^2 \right\rangle_{\text{FS}} \left[ \frac{a_0}{2b} - \frac{a_0}{2(b+b')} \right] \\ &= N_0 \langle Y_{T^*}(\mathbf{k}) |\Delta(\mathbf{k})|^2 \rangle_{\text{FS}} \left[ \frac{a_0}{2b} - \frac{a_0}{2(b+b')} \right], \end{aligned} \quad (\text{B10})$$

where we made the approximation that  $|\Delta_e(\mathbf{k})|^2 = |\Delta_o(\mathbf{k})|^2 \equiv |\Delta(\mathbf{k})|^2$ .

Equation (16) of the main text is obtained by taking the ratio of the two specific heat discontinuities, combined with the approximation  $|\Delta_e(\mathbf{k})|^2 = |\Delta_o(\mathbf{k})|^2 \equiv |\Delta(\mathbf{k})|^2$ .

### APPENDIX C: SPIN SUSCEPTIBILITY

In this Appendix, we calculate the spin susceptibility of a mixed singlet-triplet superconductor when the  $\vec{H}$  field is perpendicular to  $\vec{d}$ . Without loss of generality, we choose  $\vec{H} \parallel \hat{z}$  and  $\vec{d} = -i\psi_o \Delta_o(\mathbf{k}) \hat{y}$ . For a given  $\mathbf{k}$  value, the BCS Hamiltonian reads

[57]

$$H = \begin{bmatrix} c_{\uparrow}^{\dagger}(\mathbf{k}) & c_{\downarrow}^{\dagger}(\mathbf{k}) & c_{\uparrow}(\mathbf{k}) & c_{\downarrow}(\mathbf{k}) \end{bmatrix} \begin{bmatrix} \xi(\mathbf{k}) + h & 0 & \psi_o \Delta_o(\mathbf{k}) & \psi_e \Delta_e(\mathbf{k}) \\ 0 & \xi(\mathbf{k}) - h & -\psi_e \Delta_e(\mathbf{k}) & \psi_o \Delta_o(\mathbf{k}) \\ \psi_o \Delta_o(\mathbf{k}) & -\psi_e \Delta_e(\mathbf{k}) & -[\xi(\mathbf{k}) + h] & 0 \\ \psi_e \Delta_e(\mathbf{k}) & \psi_o \Delta_o(\mathbf{k}) & 0 & -[\xi(\mathbf{k}) - h] \end{bmatrix} \begin{bmatrix} c_{\uparrow}(\mathbf{k}) \\ c_{\downarrow}(\mathbf{k}) \\ c_{\uparrow}^{\dagger}(\mathbf{k}) \\ c_{\downarrow}^{\dagger}(\mathbf{k}) \end{bmatrix}, \quad (\text{C1})$$

where  $h$  is the Zeeman splitting. The resulting magnetization can be calculated analytically by performing a Bogolyubov transformation, and the magnetic susceptibility is read off from the linear-in- $h$  term.

Up to an overall multiplicative constant, the magnetic susceptibility is found to be

$$\begin{aligned} \chi &= \sum_{\mathbf{k}} \frac{\beta}{4} \frac{1}{\cosh(\frac{1}{2}\beta E)^2} \left\{ 1 - \frac{|\psi_o \Delta_o(\mathbf{k})|^2}{E^2} \left[ 1 - \frac{\sinh(\beta E)}{\beta E} \right] \right\} \\ &= \frac{1}{(2\pi)^D} \int d\xi \int d\hat{k} \frac{1}{v(k)} \frac{\beta}{4} \frac{1}{\cosh(\frac{1}{2}\beta E)^2} \left\{ 1 - \frac{|\psi_o \Delta_o(\mathbf{k})|^2}{E^2} \left[ 1 - \frac{\sinh(\beta E)}{\beta E} \right] \right\} \\ &= N_0 \langle Y_T(k) \rangle - N_0 \left\langle \frac{\beta}{4} \int d\xi \frac{1}{\cosh(\frac{1}{2}\beta E)^2} \frac{|\psi_o \Delta_o(\mathbf{k})|^2}{E^2} \left[ 1 - \frac{\sinh(\beta E)}{\beta E} \right] \right\rangle \\ &= N_0 \langle Y_T(k) \rangle + N_0 \left\langle \frac{|\psi_o \Delta_o(\mathbf{k})|^2}{|\Delta(\mathbf{k})|^2} [1 - Y_T(k)] \right\rangle, \end{aligned} \quad (\text{C2})$$

with  $|\Delta(\mathbf{k})|^2 = |\psi_o \Delta_o(\mathbf{k})|^2 + |\psi_e \Delta_e(\mathbf{k})|^2$ .

Since the normal state susceptibility is given by  $N_0$  in our units, one recovers Eq. (18) from the main text.

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- [1] A. J. Leggett, *Rev. Mod. Phys.* **47**, 331 (1975).
- [2] G. E. Volovik, *The Universe in a Helium Droplet*, Vol. 117 (Oxford University Press, Oxford, 2003).
- [3] X.-L. Qi and S.-C. Zhang, *Rev. Mod. Phys.* **83**, 1057 (2011).
- [4] S. Maiti and A. V. Chubukov, *Phys. Rev. B* **87**, 144511 (2013).
- [5] Z. F. Weng, J. L. Zhang, M. Smidman, T. Shang, J. Quintanilla, J. F. Annett, M. Nicklas, G. M. Pang, L. Jiao, W. B. Jiang *et al.*, *Phys. Rev. Lett.* **117**, 027001 (2016).
- [6] S. A. Kivelson, A. C. Yuan, B. J. Ramshaw, and R. Thomale, *npj Quantum Mater.* **5**, 43 (2020).
- [7] Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J. G. Bednorz, and F. Lichtenberg, *Nature (London)* **372**, 532 (1994).
- [8] T. M. Rice and M. Sigrist, *J. Phys.: Condens. Matter* **7**, L643 (1995).
- [9] G. Baskaran, *Phys. B: Condens. Matter* **223–224**, 490 (1996).
- [10] A. P. Mackenzie and Y. Maeno, *Rev. Mod. Phys.* **75**, 657 (2003).
- [11] Y. Maeno, S. Kittaka, T. Nomura, S. Yonezawa, and K. Ishida, *J. Phys. Soc. Jpn.* **81**, 011009 (2012).
- [12] C. Kallin and A. J. Berlinsky, *J. Phys.: Condens. Matter* **21**, 164210 (2009).
- [13] C. Kallin, *Rep. Prog. Phys.* **75**, 042501 (2012).
- [14] A. P. Mackenzie, T. Scaffidi, C. W. Hicks, and Y. Maeno, *npj Quantum Mater.* **2**, 40 (2017).
- [15] A. Damascelli, D. H. Lu, K. M. Shen, N. P. Armitage, F. Ronning, D. L. Feng, C. Kim, Z.-X. Shen, T. Kimura, Y. Tokura *et al.*, *Phys. Rev. Lett.* **85**, 5194 (2000).
- [16] C. Bergemann, S. R. Julian, A. P. Mackenzie, S. NishiZaki, and Y. Maeno, *Phys. Rev. Lett.* **84**, 2662 (2000).
- [17] C. Bergemann, A. P. Mackenzie, S. R. Julian, D. Forsythe, and E. Ohmichi, *Adv. Phys.* **52**, 639 (2003).
- [18] A. Tamai, M. Zingl, E. Rozbicki, E. Cappelli, S. Riccò, A. de la Torre, S. McKeown Walker, F. Y. Bruno, P. D. C. King, W. Meevasana *et al.*, *Phys. Rev. X* **9**, 021048 (2019).
- [19] G. M. Luke, Y. Fudamoto, K. M. Kojima, M. I. Larkin, J. Merrin, B. Nachumi, Y. J. Uemura, Y. Maeno, Z. Q. Mao, Y. Mori, H. Nakamura, and M. Sigrist, *Nature (London)* **394**, 558 (1998).
- [20] J. Xia, Y. Maeno, P. T. Beyersdorf, M. M. Fejer, and A. Kapitulnik, *Phys. Rev. Lett.* **97**, 167002 (2006).
- [21] V. Grinenko, S. Ghosh, R. Sarkar, J.-C. Orain, A. Nikitin, M. Elender, D. Das, Z. Guguchia, F. Brückner, M. E. Barber *et al.*, *Nat. Phys.* **17**, 748 (2021).
- [22] C. Lupien, Ultrasound attenuation in the unconventional superconductor  $\text{Sr}_2\text{RuO}_4$ , Ph.D. thesis, University of Toronto, 2002.
- [23] S. Ghosh, A. Shekhter, F. Jerzembeck, N. Kikugawa, D. A. Sokolov, M. Brando, A. P. Mackenzie, C. W. Hicks, and B. J. Ramshaw, *Nat. Phys.* **17**, 199 (2021).
- [24] S. Benhabib, C. Lupien, I. Paul, L. Berges, M. Dion, M. Nardone, A. Zitouni, Z. Q. Mao, Y. Maeno, A. Georges *et al.*, *Nat. Phys.* **17**, 194 (2021).
- [25] A. Pustogow, Y. Luo, A. Chronister, Y.-S. Su, D. A. Sokolov, F. Jerzembeck, A. P. Mackenzie, C. W. Hicks, N. Kikugawa, S. Raghu *et al.*, *Nature (London)* **574**, 72 (2019).
- [26] K. Ishida, M. Manago, and Y. Maeno, *J. Phys. Soc. Jpn.* **89**, 034712 (2020).
- [27] W. Huang and H. Yao, *Phys. Rev. Lett.* **121**, 157002 (2018).
- [28] A. Ramirez and M. Sigrist, *Phys. Rev. B* **100**, 104501 (2019).
- [29] A. T. Rømer, D. D. Scherer, I. M. Eremin, P. J. Hirschfeld, and B. M. Andersen, *Phys. Rev. Lett.* **123**, 247001 (2019).

- [30] H. Gyeol Suh, H. Menke, P. M. R. Brydon, C. Timm, A. Ramires, and D. F. Agterberg, *Phys. Rev. Res.* **2**, 032023 (2020).
- [31] W. Huang, Y. Zhou, and H. Yao, *Phys. Rev. B* **100**, 134506 (2019).
- [32] W. Huang, Y. Zhou, and H. Yao, [arXiv:1901.07041](https://arxiv.org/abs/1901.07041).
- [33] Y. S. Li, N. Kikugawa, D. A. Sokolov, F. Jerzembeck, A. S. Gibbs, Y. Maeno, C. W. Hicks, M. Nicklas, and A. P. Mackenzie, [arXiv:1906.07597](https://arxiv.org/abs/1906.07597).
- [34] S. Raghu, A. Kapitulnik, and S. A. Kivelson, *Phys. Rev. Lett.* **105**, 136401 (2010).
- [35] W. Kohn and J. M. Luttinger, *Phys. Rev. Lett.* **15**, 524 (1965).
- [36] M. A. Baranov and M. Y. Kagan, *Z. Phys. B* **86**, 237 (1992).
- [37] M. Y. Kagan and A. Chubukov, *JETP Lett.* **50**, 517 (1989).
- [38] A. V. Chubukov and J. P. Lu, *Phys. Rev. B* **46**, 11163 (1992).
- [39] M. A. Baranov, A. V. Chubukov, and M. Yu. Kagan, *Int. J. Mod. Phys. B* **06**, 2471 (1992).
- [40] A. V. Chubukov, *Phys. Rev. B* **48**, 1097 (1993).
- [41] H. Fukazawa and K. Yamada, *J. Phys. Soc. Jpn.* **71**, 1541 (2002).
- [42] R. Hlubina, *Phys. Rev. B* **59**, 9600 (1999).
- [43] S. Raghu, S. A. Kivelson, and D. J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010).
- [44] W. Cho, R. Thomale, S. Raghu, and S. A. Kivelson, *Phys. Rev. B* **88**, 064505 (2013).
- [45] T. Scaffidi, J. C. Romers, and S. H. Simon, *Phys. Rev. B* **89**, 220510(R) (2014).
- [46] F. Šimkovic, X.-W. Liu, Y. Deng, and E. Kozik, *Phys. Rev. B* **94**, 085106 (2016).
- [47] T. Scaffidi, *Weak-Coupling Theory of Topological Superconductivity: The Case of Strontium Ruthenate*, Springer Theses (Springer International Publishing AG, Cham, 2017).
- [48] H. S. Røising, F. Flicker, T. Scaffidi, and S. H. Simon, *Phys. Rev. B* **98**, 224515 (2018).
- [49] T. Giamarchi, *Quantum Physics in One Dimension*, International Series of Monographs on Physics, Vol. 121 (Clarendon Press, Oxford, 2004).
- [50] L. Balents and M. P. A. Fisher, *Phys. Rev. B* **53**, 12133 (1996).
- [51] Note that Ref. [34] uses  $V_e = U + U^2\chi(\hat{k}_1 + \hat{k}_2)$  instead of  $V_e = U + U^2\chi(\hat{k}_1 - \hat{k}_2)$ , but these two choices are equivalent since we only care about integrals of the type  $\int d\hat{k}_2 V_e(\hat{k}_1, \hat{k}_2)\Delta(\hat{k}_2)$  with even gap functions [ $\Delta(\hat{k}_2) = \Delta(-\hat{k}_2)$ ]. It is easy to see that these integrals are equal for either choice of  $V_e$ .
- [52] The  $U$  term in the singlet channel gives a zero contribution for any  $m > 0$ .
- [53] Y. Wang and L. Fu, *Phys. Rev. Lett.* **119**, 187003 (2017).
- [54] Here,  $\Delta_o$  should be described by a vector OP  $\vec{d}$  describing the spin component of the Cooper pair [57], but we are free to choose the orientation of  $\vec{d}$  without loss of generality since the model is  $SU(2)$  symmetric. We chose  $\vec{d} \parallel \hat{z}$  in this section to simplify notations.
- [55] We have omitted a term proportional to  $\psi_e^4 - \psi_o^4$  for simplicity because it does not change the physics qualitatively.
- [56] R. L. Frank and M. Lemm, *Ann. Henri Poincaré* **17**, 2285 (2016).
- [57] M. Sigrist, *AIP Conf. Proc.* **789**, 165 (2005).
- [58] S. NishiZaki, Y. Maeno, and Z. Mao, *J. Phys. Soc. Jpn.* **69**, 572 (2000).
- [59] I. Bonalde, B. D. Yanoff, M. B. Salamon, D. J. Van Harlingen, E. M. E. Chia, Z. Q. Mao, and Y. Maeno, *Phys. Rev. Lett.* **85**, 4775 (2000).
- [60] C. Lupien, W. A. MacFarlane, C. Proust, L. Taillefer, Z. Q. Mao, and Y. Maeno, *Phys. Rev. Lett.* **86**, 5986 (2001).
- [61] E. Hassinger, P. Bourgeois-Hope, H. Taniguchi, S. René de Cotret, G. Grissonnanche, M. S. Anwar, Y. Maeno, N. Doiron-Leyraud, and L. Taillefer, *Phys. Rev. X* **7**, 011032 (2017).
- [62] R. Sharma, S. D. Edkins, Z. Wang, A. Kostin, C. Sow, Y. Maeno, A. P. Mackenzie, J. C. S. Davis, and V. Madhavan, *Proc. Natl. Acad. Sci. USA* **117**, 5222 (2020).
- [63] C. W. Hicks, J. R. Kirtley, T. M. Lippman, N. C. Koshnick, M. E. Huber, Y. Maeno, W. M. Yuhasz, M. B. Maple, and K. A. Moler, *Phys. Rev. B* **81**, 214501 (2010).
- [64] W. Huang, S. Lederer, E. Taylor, and C. Kallin, *Phys. Rev. B* **91**, 094507 (2015).
- [65] W. Huang, E. Taylor, and C. Kallin, *Phys. Rev. B* **90**, 224519 (2014).
- [66] S. Lederer, W. Huang, E. Taylor, S. Raghu, and C. Kallin, *Phys. Rev. B* **90**, 134521 (2014).
- [67] T. Scaffidi and S. H. Simon, *Phys. Rev. Lett.* **115**, 087003 (2015).
- [68] These gradient terms become important close to sample edges, domain walls, or defects, in the vicinity of which the OP is not spatially homogeneous.
- [69] M. Achermann, T. Neupert, E. Arahata, and M. Sigrist, *J. Phys. Soc. Jpn.* **83**, 044712 (2014).
- [70] W. Yang, C. Xu, and C. Wu, *Phys. Rev. Res.* **2**, 042047 (2020).
- [71] A. Robins and P. Brydon, *J. Phys.: Condens. Matter* **30**, 405602 (2018).
- [72] C. W. Hicks, D. O. Brodsky, E. A. Yelland, A. S. Gibbs, J. A. N. Bruin, M. E. Barber, S. D. Edkins, K. Nishimura, S. Yonezawa, Y. Maeno *et al.*, *Science* **344**, 283 (2014).
- [73] A. Steppke, L. Zhao, M. E. Barber, T. Scaffidi, F. Jerzembeck, H. Rosner, A. S. Gibbs, Y. Maeno, S. H. Simon, A. P. Mackenzie, and C. W. Hicks, *Science* **355**, eaaf9398 (2017).
- [74] K. D. Nelson, Z. Q. Mao, Y. Maeno, and Y. Liu, *Science* **306**, 1151 (2004).
- [75] F. Kidwingira, J. D. Strand, D. J. Van Harlingen, and Y. Maeno, *Science* **314**, 1267 (2006).
- [76] M. S. Anwar, R. Ishiguro, T. Nakamura, M. Yakabe, S. Yonezawa, H. Takayanagi, and Y. Maeno, *Phys. Rev. B* **95**, 224509 (2017).
- [77] J. Jang, D. G. Ferguson, V. Vakaryuk, R. Budakian, S. B. Chung, P. M. Goldbart, and Y. Maeno, *Science* **331**, 186 (2011).
- [78] M. S. Anwar, M. Kunieda, R. Ishiguro, S. R. Lee, L. A. B. Olde Olthof, J. W. A. Robinson, S. Yonezawa, T. W. Noh, and Y. Maeno, *Phys. Rev. B* **100**, 024516 (2019).
- [79] Z. Wang, X. Wang, and C. Kallin, *Phys. Rev. B* **101**, 064507 (2020).
- [80] A. Chronister, A. Pustogow, N. Kikugawa, D. A. Sokolov, F. Jerzembeck, C. W. Hicks, A. P. Mackenzie, E. D. Bauer, and S. E. Brown, *Proc. Natl. Acad. Sci. USA* **118**, e2025313118 (2021).
- [81] E. Taylor and C. Kallin, *Phys. Rev. Lett.* **108**, 157001 (2012).
- [82] A. Kapitulnik, J. Xia, E. Schemm, and A. Palevski, *New J. Phys.* **11**, 055060 (2009).
- [83] M. W. Haverkort, I. S. Elfimov, L. H. Tjeng, G. A. Sawatzky, and A. Damascelli, *Phys. Rev. Lett.* **101**, 026406 (2008).

- [84] Q. H. Wang, C. Platt, Y. Yang, C. Honerkamp, F. C. Zhang, W. Hanke, T. M. Rice, and R. Thomale, *Europhys. Lett.* **104**, 17013 (2013).
- [85] J.-W. Huo, T. M. Rice, and F.-C. Zhang, *Phys. Rev. Lett.* **110**, 167003 (2013).
- [86] C. N. Veenstra, Z.-H. Zhu, M. Raichle, B. M. Ludbrook, A. Nicolaou, B. Slomski, G. Landolt, S. Kittaka, Y. Maeno, J. H. Dil *et al.*, *Phys. Rev. Lett.* **112**, 127002 (2014).
- [87] H. S. Røising, T. Scaffidi, F. Flicker, G. F. Lange, and S. H. Simon, *Phys. Rev. Res.* **1**, 033108 (2019).
- [88] C. Bourbonnais and D. Jérôme, in *The Physics of Organic Superconductors and Conductors*, edited by A. Lebed, Springer Series in Materials Science, Vol. 110, (Springer, Berlin, Heidelberg, 2008).
- [89] N. Doiron-Leyraud, P. Auban-Senzier, S. René de Cotret, C. Bourbonnais, D. Jérôme, K. Bechgaard, and L. Taillefer, *Phys. Rev. B* **80**, 214531 (2009).
- [90] W. Cho, C. Platt, R. H. McKenzie, and S. Raghu, *Phys. Rev. B* **92**, 134514 (2015).
- [91] K. Shigeta, Y. Tanaka, K. Kuroki, S. Onari, and H. Aizawa, *Phys. Rev. B* **83**, 140509(R) (2011).
- [92] H. Aizawa, K. Kuroki, T. Yokoyama, and Y. Tanaka, *Phys. Rev. Lett.* **102**, 016403 (2009).
- [93] Y. Tanaka and K. Kuroki, *Phys. Rev. B* **70**, 060502(R) (2004).
- [94] K. Shigeta, S. Onari, K. Yada, and Y. Tanaka, *Phys. Rev. B* **79**, 174507 (2009).
- [95] This possibility was mentioned by Leggett [1], but no microscopic model exhibiting this behavior was known at the time. We thank Catherine Kallin for bringing this point to our attention.
- [96] S. Lederer, Y. Schattner, E. Berg, and S. A. Kivelson, *Phys. Rev. Lett.* **114**, 097001 (2015).
- [97] V. Kozii and L. Fu, *Phys. Rev. Lett.* **115**, 207002 (2015).
- [98] J. Kang and R. M. Fernandes, *Phys. Rev. Lett.* **117**, 217003 (2016).
- [99] Y. Wang and A. V. Chubukov, *Phys. Rev. B* **92**, 125108 (2015).
- [100] Y. Wang, G. Y. Cho, T. L. Hughes, and E. Fradkin, *Phys. Rev. B* **93**, 134512 (2016).
- [101] J. Ruhman, V. Kozii, and L. Fu, *Phys. Rev. Lett.* **118**, 227001 (2017).
- [102] V. Kozii, H. Isobe, J. W. F. Venderbos, and L. Fu, *Phys. Rev. B* **99**, 144507 (2019).
- [103] L. Zhao, D. H. Torchinsky, H. Chu, V. Ivanov, R. Lifshitz, R. Flint, T. Qi, G. Cao, and D. Hsieh, *Nat. Phys.* **12**, 32 (2016).
- [104] L. Zhao, C. A. Belvin, R. Liang, D. A. Bonn, W. N. Hardy, N. P. Armitage, and D. Hsieh, *Nat. Phys.* **13**, 250 (2017).
- [105] T. Xu, T. Morimoto, and J. E. Moore, *Phys. Rev. B* **100**, 220501(R) (2019).
- [106] S. Kashiwaya, H. Kashiwaya, H. Kambara, T. Furuta, H. Yaguchi, Y. Tanaka, and Y. Maeno, *Phys. Rev. Lett.* **107**, 077003 (2011).