

## Locality of spontaneous symmetry breaking and universal spacing distribution of topological defects formed across a phase transition

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The crossing of a continuous phase transition results in the formation of topological defects with a density predicted by the Kibble-Zurek mechanism (KZM). We characterize the spatial distribution of pointlike topological defects in the resulting nonequilibrium state and model it using a Poisson point process in arbitrary spatial dimensions with KZM density. Numerical simulations in a one-dimensional  $\phi^4$  theory unveil short-distance defect-defect corrections stemming from the kink excluded volume, while in two spatial dimensions, our model accurately describes the vortex spacing distribution in a strongly coupled superconductor indicating the suppression of defect-defect spatial correlations.

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*Introduction.* Spontaneous symmetry breaking in finite time leads to the formation of topological defects. The dynamics of a continuous phase transition, classical or quantum, is characterized by the breakdown of adiabaticity resulting from critical slowing down. Facing a degenerate vacuum manifold, spatially separated regions of the driven system make independent choices of the broken symmetry. Topological defects, such as domain walls in a ferromagnet or vortices in a superconductor, form at the interface between adjacent domains characterized by a homogeneous order parameter. Exploiting equilibrium scaling theory, the Kibble-Zurek mechanism (KZM) predicts the average number of spontaneously formed topological defects as a function of the quench time  $\tau_Q$  in which the phase transition is crossed, e.g., by modulating a control parameter  $\lambda$ , such as the temperature or an external magnetic field, across its critical value  $\lambda_c$  [1–5].

The KZM scaling has been verified in a variety of platforms, studying soliton formation in cigar-shaped Bose-Einstein condensates [6] and kinks in trapped ions [7–9], with moderate system sizes and inhomogeneous samples [10]. The scaling of the vortex density has been established in hexagonal manganites [11,12], Bose gases [13,14], and unitary Fermi gases [15]. Across quantum phase transitions, which can also be described by the KZM [16–20], the universal scaling has been probed using two-level systems [21–25], D-Wave machines [26–28], and Rydberg gases [29].

A crucial ingredient of the KZM is the local character of spontaneous symmetry breaking. Kibble proposed that defects formed according to the geodesic rule, in a probabilistic fashion [1], a feature that has been explored by merging experimentally independent Bose-Einstein condensates [30]. This observation has motivated the unraveling of universal signatures in critical dynamics that lie beyond the scope of

the KZM, e.g., by focusing on the full counting statistics of topological defects [25,27,28,31–36].

What kind of spatial correlations between topological defects are consistent with the locality of symmetry breaking? Pioneering works by Halperin [37] and Liu and Mazenko [38] focused on growth dynamics and phase-ordering kinetics. In these studies, the characterization of correlations between topological defects was pursued using two-point correlation functions, leaving room for more stringent tests involving higher-order correlations. An important precedent of our work is the observation by Zurek that spatial defect statistics can be used to characterize spontaneously formed supercurrents [39].

In this Research Letter, we explore the extent to which spontaneous formation of topological defects is correlated at different locations. Specifically, we focus on the spacing distribution of topological defects generated during the crossing of a continuous phase transition in finite time. We combine elements of stochastic geometry and the KZM to describe the formation of topological defects by a Poisson point process and put forward a universal prediction for the defect spacing distribution which varies with the spatial dimension. This theory is corroborated by numerical simulations of kink statistics in a one-dimensional  $\phi^4$  theory and vortex formation in a two-dimensional holographic superconductor.

*Universal defect spacing distribution.* At equilibrium, the correlation length  $\xi$  and the relaxation time  $\tau$  diverge according to the power laws  $\xi = \xi_0/|\epsilon|^{-\nu}$  and  $\tau = \tau_0/|\epsilon|^{-z\nu}$  as a function of the distance  $\epsilon = (\lambda - \lambda_c)/\lambda_c$  to the critical point  $\lambda_c$ . Here,  $\nu$  denotes the correlation-length critical exponent, and  $z$  is the dynamic critical exponent. For a linear quench of the form  $\epsilon = t/\tau_Q$ , the KZM [1–4] sets the average distance between topological defects equal to the KZM

correlation length

$$\hat{\xi} = \xi_0 \left( \frac{\tau_Q}{\tau_0} \right)^{\frac{\nu}{1+\nu}}. \quad (1)$$

Thus the typical density of topological defects is  $\rho \propto \hat{\xi}^{-d} = \xi_0^{-d} (\tau_0/\tau_Q)^{\frac{d\nu}{1+\nu}}$  in  $d$  spatial dimensions.

We consider a Poisson point process associated with the random distribution of  $N$  pointlike topological defects in a volume  $V$  with homogeneous density  $\rho$  fixed by the KZM. We use the well-known fact that a  $(d-1)$ -dimensional sphere of radius  $R$  enclosing a  $d$ -dimensional ball  $B_d(R) = \{x \in \mathbb{R}^d : \|x\| \leq R\}$  has surface  $S_{d-1}(R)$  and volume  $V_d(R)$  given by [40]

$$S_{d-1}(R) = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})} R^{d-1}, \quad V_d(R) = \frac{\pi^{d/2}}{\Gamma(\frac{d}{2} + 1)} R^d, \quad (2)$$

in terms of the gamma function  $\Gamma(x)$ . The distribution of  $N$  topological defects in volume  $V = V_d(R)$  leads to an effective volume occupied by each topological defect

$$v_d = \frac{V_d(R)}{N} = \frac{1}{\rho} \propto \hat{\xi}^d. \quad (3)$$

We choose the proportionality constant such that

$$V_d(R) = N v_d = N V_d(\hat{\xi}), \quad (4)$$

e.g., by considering each defect at the center of a  $d$ -dimensional ball of radius  $\hat{\xi}$ , which implies  $R = N^{\frac{1}{d}} \hat{\xi}$ .

The defect spacing distribution can be estimated taking as a reference a given defect and determining the probability of finding any of the other  $(N-1)$  defects at a distance between  $s$  and  $s+ds$ , with the remaining  $(N-2)$  defects being located farther away,

$$P(s)ds = (N-1) \frac{S_{d-1}(s)ds}{V_d(R)} \left[ 1 - \frac{V_d(s)}{V_d(R)} \right]^{N-2}. \quad (5)$$

We note that the above expression for  $P(s)$  fulfills the normalization condition  $\int_0^R P(s)ds = 1$ . This distribution generalizes the familiar distribution for a Poissonian random process on a plane, studied, e.g., in the context of random matrix theory. Making use of (4) and introducing the dimensionless scaled spacing  $X = s/R = s/(N^{\frac{1}{d}} \hat{\xi})$ , one finds the spacing distribution of topological defects  $P(s)ds = d(N-1)X^{d-1} [1 - X^d]^{N-2} dX$ . We shall focus on the large- $N$  limit, in which the normalized distribution reads

$$P(s)ds = dNX^{d-1} e^{-NX^d} dX. \quad (6)$$

The mean spacing,  $\langle s \rangle = \int_0^R P(s)sds$ , is given by

$$\langle s \rangle = dN^{1+\frac{1}{d}} \hat{\xi} \int_0^1 X^d e^{-NX^d} dX \approx \frac{\hat{\xi}}{d} \Gamma\left(\frac{1}{d}\right),$$

with the correlation length  $\hat{\xi}$  in Eq. (1) set by the KZM. To bring out the universal character of  $P(s)$  with the quench time, it proves convenient to introduce the dimensionless defect spacing in units of the mean  $S = s/\langle s \rangle$ . Using it, the normalized defect spacing distribution in the large- $N$  limit takes the form

$$P(S) = dr^d S^{d-1} e^{-r^d S^d}, \quad r = \frac{\langle s \rangle}{\hat{\xi}} = \frac{1}{d} \Gamma\left(\frac{1}{d}\right), \quad (7)$$

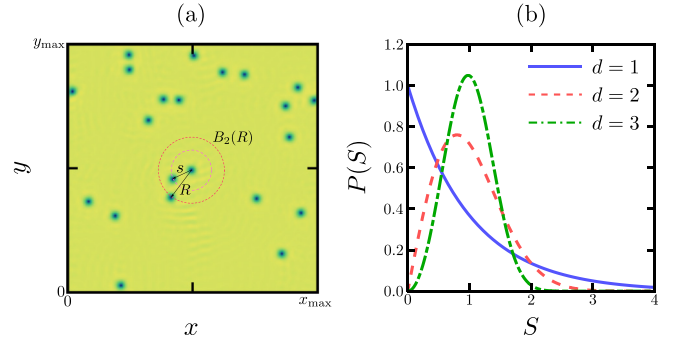


FIG. 1. Universal defect spacing distribution. (a) Schematic representation of  $N$  pointlike topological defects enclosed in a two-dimensional ball  $B_2(R)$  with radius  $R$ . (b) Distribution  $P(S)$  of the spacing between topological defects as a function of the dimensionless spacing normalized to the mean,  $S = sd/[\hat{\xi}\Gamma(\frac{1}{d})]$ , where  $s$  is the shortest distance between defects,  $d$  denotes the spatial dimension, and  $\hat{\xi}$  is the nonequilibrium correlation length predicted by the KZM, which exhibits the universal power-law scaling with the quench time in Eq. (1). In one dimension the distribution is exponential, while  $P(S)$  takes the form of a Wigner-Dyson distribution in  $d = 2$ .

which is thus independent of the quench time  $\tau_Q$ . This distribution is normalized in the domain  $S \in [0, \infty)$ , has unit mean  $\langle S \rangle = \int_0^\infty SP(S)dS = 1$ , and has a maximum at  $S_{\max} = (1 - \frac{1}{d})^d / \Gamma(1 + \frac{1}{d})$  with value  $P(S_{\max}) = (d-1)^{\frac{d-1}{d}} d^{\frac{1}{d}} e^{\frac{1}{d}-1} \Gamma(1 + \frac{1}{d})$ . In addition, the fluctuations in the defect spacing as quantified by the variance read  $\Delta S^2 = \Gamma(\frac{2+d}{d}) / \Gamma(1 + \frac{1}{d})^2 - 1$ , diminishing with increasing spatial dimension  $d$  and vanishing as  $d \rightarrow \infty$ . Specifically, we note that for spatial dimensions  $d = 1, 2, 3$ ,  $r = 1, \sqrt{\pi}/2, \frac{1}{3}\Gamma(\frac{1}{3})$ , and

$$d = 1, \quad P(S) = e^{-S}, \quad (8)$$

$$d = 2, \quad P(S) = \frac{\pi}{2} S e^{-\frac{\pi}{4} S^2}, \quad (9)$$

$$d = 3, \quad P(S) = \frac{1}{9} \Gamma\left(\frac{1}{3}\right)^3 S^2 e^{-\frac{1}{27} \Gamma\left(\frac{1}{3}\right)^3 S^3}, \quad (10)$$

with fluctuations  $\Delta S^2 \approx 1, 0.27$ , and  $0.13$ , respectively. These distributions are plotted in Fig. 1. For  $d = 1$  the distribution is exponential, familiar in the context of the energy-level spacing of integrable systems. For  $d = 2$ , the defect spacing distribution is given by the well-known Wigner-Dyson distribution, familiar from random matrix theory. For arbitrary  $d$ , it is known as the Brody distribution in the context of quantum chaos [41,42].

Point random processes assume no correlations between defects beyond the fact that the density is homogeneous throughout the system. As a result, they provide a structureless model, a natural reference one in the context of the KZM. We next explore the extent to which the spontaneous formation of topological defects can be described by point random processes with KZM density by considering some paradigmatic models.

*Example 1: Spontaneous  $\mathbb{Z}_2$  breaking in a one-dimensional system.* Let us consider a paradigmatic scenario of spontaneous symmetry breaking to explore the defect-defect spacing distribution following the crossing of a phase transition in

finite time: a Ginzburg-Landau theory with real field  $\phi$  undergoing Langevin dynamics. The validity of the KZM in this setting has been reported in Refs. [43–45]. In addition, the universality of the number distribution of topological defects, beyond the KZM, has also been established in Ref. [33]. We consider a generic Ginzburg-Landau potential given by  $V(\phi) = \frac{1}{8}(\phi^4 - 2\epsilon\phi^2 + 1)$ . The parameter  $\epsilon$  measures the distance from the phase transition. Minimizing  $V(\phi)$  with respect to  $\phi$  gives two possible minima given by  $\phi_{\min}(t) = \{0, \pm\sqrt{\epsilon(t)}\}$ . A second-order phase transition occurs if  $\epsilon$  changes its sign. When  $\epsilon < 0$ , the field  $\phi$  fluctuates around its expectation value  $\langle\phi\rangle = 0$ . However, when  $\epsilon > 0$ , the field  $\phi$  settles locally around one of two minima  $\langle\phi\rangle = \pm\sqrt{\epsilon}$ . We consider a quench described by

$$\epsilon(t) = \lambda_i + \frac{t}{\tau_Q}|\lambda_f - \lambda_i|. \quad (11)$$

The system is in contact with a thermal reservoir and obeys the Langevin equation

$$\ddot{\phi} + \eta\dot{\phi} - \partial_{xx}\phi + \partial_{\phi}V(\phi) = \xi(x, t), \quad (12)$$

where  $\eta > 0$  is a global damping constant. The noise  $\xi(x, t)$  is a real Gaussian process with zero mean and autocorrelation function  $\langle\xi(x, t)\xi(x', t')\rangle = 2\eta T\delta(x - x')\delta(t - t')$ . Details of the numerical integration to determine trajectories of the Ginzburg-Landau real field  $\phi$  can be found in the Supplemental Material [46]. We simulate the dynamics in the overdamped regime and fix the temperature  $T = 0.01$  and  $\eta = 1$  (the same regime of parameters used in Ref. [43]). The stochastic evolution is described using a computational numerical grid from  $x_{\min} = 0$  to  $x_{\max} = 500$  given by 5000 partitions. The correlation-length critical exponent takes the mean-field value  $\nu = 1/2$ . In the overdamped regime, the dynamic critical exponent is  $z = 2$ . The KZM prediction for the typical size of the domains is thus  $\hat{\xi} = \xi_0(\tau_Q/\tau_0)^{\frac{1}{4}}$ , numerically confirmed in Refs. [33,43].

The distribution  $P(S)$  of the spacing between adjacent topological defects, normalized to the mean, is shown in Fig. 2. The tail of distribution for  $S > 1.5$  is well described by the exponential function  $\exp(-S)$ . However, important deviations are manifested for smaller values of  $S$ . In particular,  $P(S)$  is highly suppressed for small values of  $S$ , while an enhancement of the probability for  $S \simeq 1$  is observed. This is to be expected as one dimension enhances correlations between topological defects. Furthermore, in one spatial dimension, a kink can only be surrounded by antikinks. Coarsening by the annihilation of kink and antikink pairs is thus unavoidable even in the KZM scaling regime of the dynamics for small lattice spacing. In addition, topological defects such as kinks are actually not pointlike and have a finite healing length.

One may wonder whether the distribution can be reproduced by considering the excluded volume of each defect. To this end, we consider the spacing between randomly distributed disks in one spatial dimension, characterized by Torquato, Lu, and Rubinstein (TLR). In terms of the packing fraction  $\phi = N\sigma/L \in [0, 1]$  for  $N$  topological defects of radius  $\sigma$ , the TLR spacing distribution reads [47,48]

$$P_{\text{TLR}}(S) = \frac{1 + \phi}{1 - \phi} \exp\left[-\frac{1 + \phi}{1 - \phi}S + \frac{2\phi}{1 - \phi}\right] \quad (13)$$

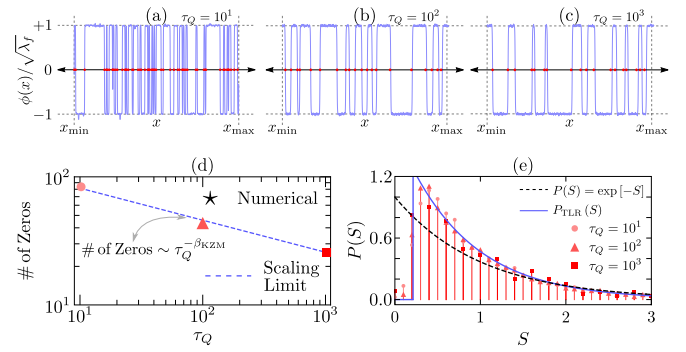


FIG. 2. Statistics of kinks spontaneously formed in a one-dimensional  $\phi^4$  theory. (a)–(c) A typical nonequilibrium field configuration  $\phi(x)$  supporting kinks is shown, when the transition is crossed at different quench times  $\tau_Q = 10^1, 10^2$ , and  $10^3$ . The position of every kink is represented by a red dot over  $\phi(x) = 0$ . (d) Scaling of the number of zeros as a function of the quench time. A fitting to the scaling of the average number of zeros with the quench time of the form (number of zeros  $\propto \tau_Q^{-\beta_{\text{KZM}}}$ ) yields a value of the power-law exponent  $\beta_{\text{KZM}} = 0.248 \pm 0.004$  in agreement with the KZM prediction  $d\nu/(1 + z\nu) = 1/4$  for  $d = 1$ ,  $\nu = 1/2$ , and  $z = 2$ . (e) Distribution  $P(S)$  of the spacing between nearest-neighbor topological defects. While the details of the distribution are well reproduced by an exponential decay, there are strong short-distance suppression of  $P(S)$  and bunching at a spacing comparable to the mean,  $S \sim 1$ . Deviations are captured by the TLR distribution with  $\phi = 0.11$ . For each value of  $\tau_Q$ , data are collected from 10 000 independent trajectories.

and vanishes identically for small spacing values when  $S < 2\phi/(1 + \phi)$ . The comparison with this distribution is shown as well in Fig. 2. The TLR distribution captures accurately the corrections to the ideal distribution that arise at all distances from the excluded volume, with a weak dependence of the optimal packing fraction  $\phi$  on the quench time; see Supplemental Material [46] for additional details.

*Example 2: Spontaneous U(1) breaking in a two-dimensional strongly coupled superconductor.* We next turn our attention to an example in one higher spatial dimension and characterize the defect spacing distribution of spontaneously formed vortices in a newborn strongly coupled superconductor. The latter can be described using holographic duality, which makes use of a gravitational theory with one additional spacelike dimension [49]. Holographic quantum matter provides a natural setting to explore the role of strong coupling in topological defect formation and has been used to establish the validity of the KZM in this regime, up to subleading corrections in the quench time [34,50–55].

The setup and numerical scheme we adopt are described in detail in the Supplemental Material [46], which includes Refs. [56–60]. We consider the paradigmatic Lagrangian of the Maxwell-complex scalar model for a holographic superconductor [58],

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\Psi|^2 - m^2|\Psi|^2, \quad (14)$$

where  $D = \nabla - iA$  with  $A$  being the U(1) gauge field and  $\Psi$  being the complex scalar field (using units with  $e = c = \hbar = k_B = 1$ ). The equations of motion read  $D_\mu D^\mu \Psi - m^2 \Psi = 0$ ,  $\nabla_\mu F^{\mu\nu} = i(\Psi^* D^\nu \Psi - \Psi (D^\nu \Psi)^*)$ . The ansatz we take is

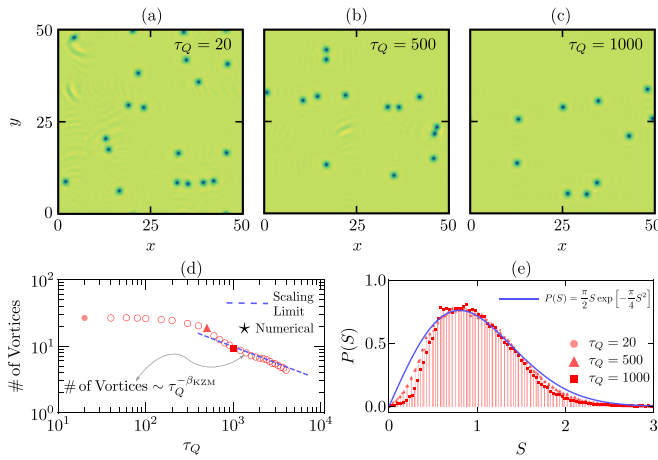


FIG. 3. Statistics of vortices in a newborn strongly coupled superconductor. (a)–(c) Spontaneously formed vortices in a newborn holographic superconductor are shown in density plots of the order parameter for three different quench rates. The average vortex number decreases with the quench rates. (d) The dependence of the average vortex number on the quench time is consistent with a power-law scaling, with fitted value  $\beta_{\text{KZM}} = 0.512 \pm 0.005$  in agreement with the prediction by the KZM with  $d\nu/(1+z\nu) = 1/2$  for  $d = 2$ ,  $\nu = 1/2$ , and  $z = 2$ . (e) Spacing distribution  $P(S)$  for three different quenches. The numerical data are consistent with the theoretical fitting line  $P(S) = \frac{\pi}{2} S \exp[-\frac{\pi}{4} S^2]$ . For each value of  $\tau_Q$ , data are collected from 10 000 independent trajectories.

$\Psi = \Psi(t, z, x, y)$ ,  $A_t = A_t(t, z, x, y)$ ,  $A_x = A_x(t, z, x, y)$ ,  $A_y = A_y(t, z, x, y)$ , and  $A_z = 0$ . In the simulation, we set the size of the system as  $(x, y) = (50, 50)$  and the mass square of the scalar field as  $m^2 = -2$ . Numerical simulations and theoretical analysis indicate that in this system the correlation-length critical exponent  $\nu = 1/2$  while the dynamic critical exponent  $z = 2$  [34,53], so that the KZM correlation length is consistent with  $\xi = \xi_0(\tau_Q/\tau_0)^{1/4}$ .

Before quenching, the system is thermalized by adding the noise  $\xi(x_i, t)$  into the bulk and satisfying the conditions  $\langle \xi(x_i, t) \rangle = 0$  and  $\langle \xi(x_i, t) \xi(x_j, t') \rangle = h \delta(t - t') \delta(x_i - x_j)$ , with a small amplitude  $h = 10^{-3}$ . Subsequently, a linear quench of the temperature, from  $T_i = 1.4T_c$  to  $T_f = 0.8T_c$ , prompts the system to evolve from a normal metal state to a superconductor state. As a result of the U(1) symmetry breaking, vortices form spontaneously as predicted by the KZM.

We evaluate the spatial distribution of vortices until the system enters the equilibrium state. Typical spatial distributions of the vortices are shown in Figs. 3(a)–3(c) for three different

kinds of quench rates,  $\tau_Q = 20, 500$ , and  $1000$ . The number of vortices decreases with the quench time. The scaling of the average number of vortices as a function of the quench time is shown in Fig. 3(d). For slow quenches, the scaling law satisfies the KZM with the power  $\beta_{\text{KZM}} \approx 1/2$  in two dimensions. The corresponding vortex spacing distributions are shown in Fig. 3(e). In obtaining Fig. 3, we first choose a vortex as a reference and then determine the closest vortex to it, regardless of its topological charge. Because we used the lattice square to simulate the system, we regard the vortices sitting at the same circumference of a square to have the same distance to the centering vortex. The distributions of  $P(S)$  estimated from  $10^4$  realizations are in good agreement with the theoretical prediction in Eq. (9) in two spatial dimensions. Numerically estimated distributions are slightly narrower than the corresponding Wigner-Dyson distribution. In two dimensions the interactions between topological defects are logarithmic with the distance between topological defects [61], excluding the validity of a short-range hard-disk model as shown in the Supplemental Material [46]. As the deviations from the ideal distribution are more pronounced for slower quenches, we attribute them to coarsening.

**Conclusions.** Combining the scaling theory of continuous phase transitions, the Kibble-Zurek mechanism, and elements of stochastic geometry, we have described the spontaneous formation of topological defects across a continuous phase transition as a Poisson point process with KZM density. Using this framework, we have predicted the universal form of the defect spacing distribution as a function of the quench time and spatial dimension. In one-dimensional systems, defect-defect correlations are enhanced and can be taken into account by considering the finite size of defects. The theory is expected to accurately reproduce the spacing distribution in higher dimensions, as we have shown in a two-dimensional setting, where remaining deviations are attributed to coarsening. Our results are amenable to experimental test with established technology, exploiting any of the platforms previously used to probe KZM scaling, provided it is endowed with spatial resolution, as is the case with trapped-ion systems [8,9], colloidal monolayers [62], multiferroics [11,12], ultracold gases in various geometries [6,13–15], and quantum simulators, to name some prominent examples.

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