## **Even-odd parity transition in strongly correlated locally noncentrosymmetric superconductors: Application to CeRh<sub>2</sub>As<sub>2</sub>**

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The discovery of the multiple  $H - T$  phase diagram of CeRh<sub>2</sub>As<sub>2</sub> offers a new route to designing topological superconductors. Although weak-coupling theories explain the experimental phase diagram qualitatively, a quantitative discrepancy between them has discouraged a conclusive interpretation. In this Letter, we thoroughly study the effect of the Coulomb interaction and the phase diagrams of locally noncentrosymmetric superconductors. We reveal an even-odd parity transition and the enhancement of the parity transition field in strongly correlated superconductors, and an issue of  $CeRh<sub>2</sub>As<sub>2</sub>$  is resolved.

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*Introduction.* Searching for odd-parity superconductors has been a central issue in designing topological materials [\[1–4\]](#page-4-0). Due to Fermi statistics, odd-parity superconductors are classified into spin-triplet superconductors within the theory by Bardeen, Cooper, and Schrieffer (BCS) [\[5\]](#page-4-0), which explains various phenomena of superconductivity. In topological science, this constraint on Cooper pairs has led to the renewed interest in uncommon spin-triplet superconductors such as UPt<sub>3</sub> [\[6–9\]](#page-4-0), UCoGe [\[10](#page-4-0)[–12\]](#page-5-0), and UTe<sub>2</sub> [\[13–16\]](#page-5-0). However, some internal degrees of freedom of Cooper pairs are overlooked in the canonical BCS theory.

Recently, the local noncentrosymmetry in superconductors has attracted much attention and shed light on the sublattice degrees of freedom in Cooper pairs [\[17–31\]](#page-5-0). Interestingly, a sublattice antisymmetric Cooper pair is allowed, leading to an odd-parity superconducting state without spin-triplet pairs. Therefore, sublattice degrees of freedom pave a new way to design topological odd-parity superconductors based on pairing in an ordinary spin-singlet channel [\[22,32\]](#page-5-0). In the high-magnetic-field phase of locally noncentrosymmetric superconductors, the sublattice antisymmetric superconducting state is theoretically predicted to be thermodynamically stable, and is called the pair-density-wave (PDW) state [\[19\]](#page-5-0).

The discovery of the two superconducting phases in the  $H - T$  phase diagram of  $CeRh<sub>2</sub>As<sub>2</sub>$  [\[33\]](#page-5-0) has focused attention on the sublattice degrees of freedom in superconductors [\[32–](#page-5-0)[46\]](#page-6-0). In fact, due to the two-sublattice crystalline structure of  $CeRh<sub>2</sub>As<sub>2</sub>$ , the inversion symmetry is locally broken at the Ce site but globally preserved [\[33\]](#page-5-0). The qualitative similarity of the phase diagrams between the weak-coupling theory  $[19,33]$  and experiment  $[33,40]$  strongly suggests an essential role of local inversion symmetry breaking in  $CeRh<sub>2</sub>As<sub>2</sub>$ , and the two superconducting phases were interpreted based on the even-odd parity transition within the superconducting state [\[33,40\]](#page-5-0).

In contrast to the preceding argument, there are two issues regarding the nature of  $CeRh<sub>2</sub>As<sub>2</sub>$ . First, the microscopic mechanism of superconductivity in  $CeRh<sub>2</sub>As<sub>2</sub> remains$ unsolved. The unconventional superconductivity mediated by quantum critical fluctuations is studied in this Letter. Second, the parity transition field of  $Cerh<sub>2</sub>As<sub>2</sub>$  significantly exceeds the Pauli-Clogston-Chandrasekhar limit and is larger than the prediction of weak-coupling theory by a factor of five [\[19,33\]](#page-5-0). Although two scenarios have been proposed within the weak-coupling theory [\[44,](#page-5-0)[45\]](#page-6-0), we try to resolve the issue by verifying an intrinsic phase diagram of strongly correlated locally noncentrosymmetric superconductors.

To tackle the above-mentioned problems, we focus on the electronic correlation effect of Ce *f* electrons, which have a localized character. Indeed, the large electronic specific heat coefficient  $\gamma \sim 1000 \text{ mJ/mol K}^2$  supports the presence of heavy-fermion bands near the Fermi level, and non-Fermi-liquid behaviors suggest quantum criticality in  $CeRh<sub>2</sub>As<sub>2</sub>$  [\[33,37,39\]](#page-5-0). These experimental observations indicate that the Coulomb interaction crucially impacts the electronic state of CeRh<sub>2</sub>As<sub>2</sub>. Hence, theoretical studies of strong correlation effects in locally noncentrosymmetric superconductors have been desired.

In this Letter, we conduct a thorough study on quantum critical multipole fluctuations, the resulting superconductivity, and the superconducting phase diagrams of locally noncentrosymmetric strongly correlated electron systems. To clarify these properties, the fluctuation exchange (FLEX) approximation which appropriately reproduces the critical behaviors of self-consistent renormalization theory [\[47\]](#page-6-0) is adopted. In the FLEX scheme, the Green function and the self-energy depend on each other, and therefore the retardation effect, quasiparticle scattering, and internal field are taken into account (see Supplemental Material Sec. S1 [\[48\]](#page-6-0)). Theoretical results are compared with the superconducting phase diagrams of  $CeRh<sub>2</sub>As<sub>2</sub>$ , and the origin of superconductivity is discussed. As a result, the *XY* -type antiferromagnetic fluctuation consistent with nuclear magnetic resonance (NMR) [\[39\]](#page-5-0) and superconductivity with a dominant  $d_{x^2-y^2}$ -wave pairing are revealed, and the obtained enhanced parity transition field resolves the issue of the phase diagram in  $CeRh<sub>2</sub>As<sub>2</sub>$ .

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<span id="page-1-0"></span>

FIG. 1. Schematic figure of the bilayer Rashba-Hubbard model. Yellow circles represent the Ce atoms of  $CeRh<sub>2</sub>As<sub>2</sub>$ . We introduce first- and second-neighbor intralayer hopping integrals. We also introduce an interlayer hopping integral as*t*⊥. The staggered Rashbatype antisymmetric spin-orbit coupling is included in the model as it arises from the asymmetric potential by  $Rh<sub>2</sub>As<sub>2</sub>$  layers. The magnetic field is applied parallel to the *z* axis.

*Bilayer Rashba-Hubbard model.* We focus on the lowenergy coherent heavy-fermion band and construct the bilayer Rashba-Hubbard model, in which Coulomb correlation and spin-orbit coupling are taken into account [\[49,50\]](#page-6-0). Note that all parameters of our model are renormalized through the Kondo effect. The model is given by

$$
\hat{H} = \sum_{k} \varphi^{\dagger}(k) \mathcal{H}_{0}(k) \varphi(k) + U \sum_{i,\sigma} n_{i\uparrow\sigma} n_{i\downarrow\sigma}, \qquad (1)
$$

where *U* is the on-site Coulomb repulsion,  $\mathcal{H}_0(\mathbf{k}) =$  $\varepsilon(\boldsymbol{k})s_0\otimes\sigma_0+\alpha\boldsymbol{g}(\boldsymbol{k})\cdot\boldsymbol{s}\otimes\sigma_z-\mu_BHs_z\otimes\sigma_0+\tilde{t}_{\perp}(\boldsymbol{k})s_0\otimes\sigma_z$  $\sigma_+ + \tilde{t}_\perp(-\mathbf{k})s_0 \otimes \sigma_-, \ \varphi(\mathbf{k}) = (c_{\mathbf{k}\uparrow\mathrm{A}}, c_{\mathbf{k}\downarrow\mathrm{A}}, c_{\mathbf{k}\uparrow\mathrm{B}}, c_{\mathbf{k}\downarrow\mathrm{B}})^\top$ , and  $c_{kso}$  ( $c_{kso}^{\dagger}$ ) is an annihilation (creation) operator for an electron with momentum *k*, spin *s*, and sublattice  $\sigma \in \{A, B\}$ (Fig. 1). Here,  $s_{\mu}$  and  $\sigma_{\mu}$  consisting of a 2 × 2 unit matrix and three Pauli matrices represent spin and sublattice degrees of freedom, respectively.<sup>1</sup> The first term of  $\mathcal{H}_0$  represents intralayer hopping including the chemical potential and is given by  $\varepsilon(\mathbf{k}) = -2t(\cos k_x + \cos k_y) + 4t'\cos k_x\cos k_y - \mu$ . The vector  $g(k)$  describes the Rashba-type antisymmetric spinorbit coupling given by  $g(k) = [-\partial \varepsilon(k)/\partial k_{y}, \partial \varepsilon(k)/\partial k_{x}, 0]$ , and *H* represents the Zeeman magnetic field parallel to the *z* axis. The last two terms of  $\mathcal{H}_0$  describe interlayer hopping given as  $\tilde{t}_{\perp}(\mathbf{k}) = t_{\perp} (1 + e^{-ik_x})(1 + e^{-ik_y})$ . This model is a straightforward extension of the Rashba-Hubbard model for globally inversion-asymmetric strongly correlated electron systems [\[51–63\]](#page-6-0). Hereafter, we set  $t' = 0.3$ ,  $t_{\perp} = 0.1$ ,  $\mu_B = 1$ , and  $U = 3.9$  with a unit of energy  $t = 1$  and determine the chemical potential so that the electron density per site *n* is 0.85. In the present numerical study, we use  $64 \times 64$  *k* meshes, and 16384, 8192, or 4096 Matsubara frequencies for  $T = 0.004, 0.004 < T < 0.01$ , or  $0.01 \le T$ , respectively. By comparing with the quantum Monte Carlo method, the FLEX approximation was confirmed as a reliable method within the intermediate-coupling region in which the Coulomb interaction *U* is smaller than half of the bandwidth *W* , namely, the condition for justification of the FLEX approximation is  $U/W \leq 0.5$  [\[64–68\]](#page-6-0). Our choice of model parameters satisfies this condition and the FLEX

TABLE I. Classification of the multipole operators in the bi**laster I.** Classification of the multipole operators in the bi-<br>layer Rashba-Hubbard model. Here,  $\hat{s}_{\pm} = (\hat{s}_x \pm i\hat{s}_y)/\sqrt{2}$  are ladder operators for spin. E (O) represents the even-parity (odd-parity) multipole operators. C, L, and T represent charge, longitudinal spin, and transverse spin operators, respectively.  $\hat{\sigma}_0$  and  $\hat{\sigma}_z$  ( $\hat{\sigma}_x$  and  $\hat{\sigma}_y$ ) are intrasublattice (intersublattice) operators.

$\ell$	$\hat{\sigma}_{0}$	$\hat{\sigma}_r$	$\hat{\sigma}_{v}$	$\hat{\sigma}_z$
$\hat{s}_0$	E C intra	E C inter	O C inter	O C intra
$\hat{s}_z$	E L intra	$E L$ inter	O L inter	O L intra
$\hat{s}_+$	E T intra	E T inter	O T inter	O T intra

approximation is expected to give a good description of strongly correlated superconductors in this case.

*Multipole susceptibility.* First, we discuss quantum critical multipole fluctuations. A dynamical susceptibility tensor is given by the generalized susceptibility as

$$
\chi_{\hat{\mathcal{O}}}(q, i\nu_n) = \sum_{\xi_1\xi_2\xi_3\xi_4} \hat{\mathcal{O}}_{\xi_1\xi_2}\chi_{\xi_2\xi_1\xi_3\xi_4}(q, i\nu_n)\hat{\mathcal{O}}_{\xi_3\xi_4},\qquad(2)
$$

where  $iv_n$  are bosonic Matsubara frequencies and  $\xi$  is abbreviated notation for  $\xi = (s, \sigma)$ . The operators  $\hat{\mathcal{O}}$  are (extended) multipole operators as  $\hat{\mathcal{O}} = \hat{s} \otimes \hat{\sigma}$  [\[69–71\]](#page-6-0), and classified into even-parity (odd-parity) multipoles for  $\hat{\sigma}_0$ ,  $\hat{\sigma}_x$  ( $\hat{\sigma}_y$ ,  $\hat{\sigma}_z$ ). In Table I, we summarize the classification of multipole operators in our system. We adopt the normalization convention  $tr[\hat{\mathcal{O}}^{\dagger}\hat{\mathcal{O}}]=1$ , and therefore operators are represented by the tr[*C*<sup>T</sup> C<sup>*I*</sup> = 1, and therefore operators are repress<br>Pauli matrices as  $\hat{s}_{\mu} = s_{\mu}/\sqrt{2}$  and  $\hat{\sigma}_{\mu} = \sigma_{\mu}/\sqrt{2}$ .



FIG. 2. (a) The magnetic field dependence of static multipole fluctuations. The maxima of the longitudinal and transverse magnetic susceptibilities are shown. Note that the even-parity and odd-parity multipole fluctuations are nearly degenerate. We assume  $\alpha/t_{\perp}$  = 2 and  $T = 0.01$ . (b) The momentum dependence of longitudinal magnetic susceptibility (left) and transverse magnetic susceptibility (right) at  $H = 0.15$ .

<sup>&</sup>lt;sup>1</sup>μ runs over {0, *x*, *y*, *z*}, and  $\sigma_{\pm}$  are given by ( $\sigma_x \pm i\sigma_y/2$ .

In Fig.  $2(a)$ , the magnetic field dependence of the maximum of transverse and longitudinal magnetic susceptibilities is shown. We see that the transverse (longitudinal) susceptibility is enhanced (reduced) by the magnetic field. Note that the even-parity magnetic multipole fluctuation for  $s \otimes \sigma_0$  and the odd-parity magnetic multipole fluctuation for  $s \otimes \sigma_z$  are nearly degenerate, while the odd-parity multipole is slightly favored. Other multipole susceptibilities are negligibly small. From Fig. [2\(b\),](#page-1-0) both transverse and longitudinal magnetic susceptibilities show the peak structure around  $\mathbf{Q} = (\pi, \pi)$ , and thus an antiferromagnetic spin fluctuation develops in the system. The momentum dependence is not qualitatively affected by the magnetic field. Therefore, we presume a dominant *XY* type antiferromagnetic fluctuation, $\frac{2}{3}$  which is consistent with the observation in  $CeRh<sub>2</sub>As<sub>2</sub>$  by the NMR measurement [\[39\]](#page-5-0). We confirmed that the behaviors of the multipole susceptibility do not qualitatively depend on the strength of spin-orbit coupling α/*t*<sup>⊥</sup> (see Supplemental Material Sec. S2) [\[48\]](#page-6-0).

*Superconductivity.* To investigate superconductivity, we adopt the linearized Eliashberg equation which is given by

$$
\lambda \Delta_{\xi\xi'}(k) = \frac{T}{N} \sum_{k'} \Gamma_{\xi\xi_1\xi_2\xi'}^a(k - k') F_{\xi_1\xi_2}(k), \tag{3}
$$

$$
F_{\xi_1\xi_2}(k) = G_{\xi_1\xi_3}(k')\Delta_{\xi_3\xi_4}(k')G_{\xi_2\xi_4}(-k'),\tag{4}
$$

where  $\Delta$  is the gap function and  $\Gamma^a$  is the particle-particle channel irreducible vertex function. Here, we adopted abbreviated notation  $k = (k, i\omega_n)$ . With the power method, we numerically evaluate  $\lambda$ , eigenvalues of the linearized Eliashberg equation and determine the critical temperature  $T_c$  from the criterion  $\lambda = 1$ .

The crystalline space group of CeRh<sub>2</sub>As<sub>2</sub> is *P4/nmm* (No. 129), and therefore the point group is *D*4*<sup>h</sup>*. Since we introduce the magnetic field parallel to the *z* axis, some symmetry operations are prohibited, and the point group reduces to *C*4*<sup>h</sup>*. Hence, the superconducting gap functions are classified based on irreducible representations of *C*4*<sup>h</sup>*. Using conventional notation, we decompose the superconducting gap functions into the spin-singlet component and spin-triplet component,

$$
\Delta^{\sigma\sigma'}(\boldsymbol{k}) = \{ \psi^{\sigma\sigma'}(\boldsymbol{k}) + \boldsymbol{d}^{\sigma\sigma'}(\boldsymbol{k}) \cdot \boldsymbol{s} \} \boldsymbol{i} s_{\boldsymbol{y}},
$$
(5)

for the intrasublattice and intersublattice pairing channels.<sup>3</sup> Here,  $\psi^{AA}(\mathbf{k})$  [ $\psi^{AB}(\mathbf{k})$ ] represents an intrasublattice (intersublattice) spin-singlet order parameter, while  $d^{AA}(k)$  $[d^{AB}(k)]$  is an intrasublattice (intersublattice) spin-triplet order parameter. In the following calculations, the order parameters of intersublattice pairing are negligibly small. The basis functions of the intrasublattice order parameter for each irreducible representation of  $C_{4h}$  are summarized in Table II.

In Fig.  $3(a)$ , the eigenvalues of the Eliashberg equation for each irreducible representation are shown (see Supplemental Material Secs. S3 and S4) [\[48\]](#page-6-0). The  $B_g$  and  $B_u$  representations are dominant. From the momentum dependence displayed

TABLE II. The basis functions for the intrasublattice superconducting order parameter. IR represents the irreducible representations of the point group *C*4*<sup>h</sup>*. We take into account the time-reversal symmetry breaking under the magnetic field, and the degeneracy of the  $E_{g/u}$  states is lifted. Thus, we distinguish them as  $E_{g/u}^1$  and  $E_{g/u}^2$ . Note that the spin-singlet component  $\psi(k)$  and spin-triplet in-plane component  $d_{x,y}(\mathbf{k})$  for the *E* representations are prohibited due to the *Cz* <sup>2</sup> rotation symmetry.

IR	$\psi(k)$	d(k)
$A_g, A_u$	$1, k_x k_y (k_x^2 - k_y^2)$	$k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}}, k_y \hat{\mathbf{x}} - k_x \hat{\mathbf{y}}$
$B_g, B_u$	$k_{x}k_{y}, k_{x}^{2} - k_{y}^{2}$	$k_x \hat{\mathbf{x}} - k_y \hat{\mathbf{y}}, k_y \hat{\mathbf{x}} + k_x \hat{\mathbf{y}}$
		$(k_x + ik_y)\hat{z}$
$E_g^1, E_u^1$ $E_g^2, E_u^2$	$\theta$	$(k_x - ik_y)\hat{z}$

in Fig. 3(b), both of these states contain spin-singlet  $d_{x^2-y^2}$ wave dominant pairing as well as spin-triplet subdominant pairing with *p*-wave symmetry. These unconventional Cooper pairs are stabilized by the antiferromagnetic fluctuations and spin-orbit coupling and not significantly changed against the magnetic field.

With almost the same momentum dependence, the different sublattice structures of the gap functions distinguish the  $B_g$  and  $B_u$  representations. In the even-parity  $B_g$  representation, the spin-singlet (spin-triplet) gap function has the same (opposite) sign between the sublattices A and B, as  $\psi^{AA}(\mathbf{k}) = \psi^{BB}(\mathbf{k})$  and  $\mathbf{d}^{AA}(\mathbf{k}) = -\mathbf{d}^{BB}(\mathbf{k})$ . On the other hand,  $\psi^{AA}(\mathbf{k}) = -\psi^{BB}(\mathbf{k})$  and  $\mathbf{d}^{AA}(\mathbf{k}) = \mathbf{d}^{BB}(\mathbf{k})$  in the odd-parity  $B_u$  representation. Therefore, the  $B_g$  and  $B_u$  representations correspond to the BCS and PDW states predicted in the weak-coupling theory  $[19]$ . As shown in Fig.  $3(a)$ , eigenvalues of the Eliashberg equation for both  $B_g$  and  $B_u$ 



FIG. 3. (a) The magnetic field dependence of eigenvalues of the Eliashberg equation for each irreducible representation. We assume  $\alpha/t_{\perp} = 2$  and  $T = 0.01$ . (b) The momentum dependence of intrasublattice spin-singlet and spin-triplet gap functions,  $\psi^{AA}$ (k) and  $d^{AA}(k)$ , of the  $B_g$  representation for  $H = 0.15$ . Note that  $d^{g\sigma}_z(k) = 0$ . Results for the  $B_u$  representation are almost the same as the figures.

 $2 \text{An } XY$ -type fluctuation means that transverse fluctuations dominate over longitudinal ones in the paramagnetic state.

<sup>&</sup>lt;sup>3</sup>In our system, the intrasublattice pairing corresponds to the intralayer pairing while the intersublattice pairing corresponds to the interlayer pairing.



FIG. 4. (a)–(d) *H*-*T* phase diagrams of the bilayer Rashba-Hubbard model for  $\alpha/t_{\perp} = 0, 1, 2, 3$ . We show the superconducting transition lines of the even-parity *B<sub>u</sub>* and odd-parity *B<sub>u</sub>* states, on which eigenvalues of the Eliashberg equation become unity. The BCS and PDW states correspond to the  $B_g$  and  $B_u$  superconducting phases, respectively.  $H^*$  denotes the magnetic field at the parity transition point.

representations are weakened by the magnetic field due to the Pauli depairing effect. However, the  $B_u$  state is more robust compared with the  $B_g$  state, and therefore at  $H = 0.24$  the parity transition from the even-parity  $B_g$  state to the odd-parity  $B_u$ state occurs. The transition can be understood from the viewpoint of the intrinsic magnetic response of these states. Indeed, the  $B_{\varrho}$  state is Pauli limited, but the  $B_{\mu}$  state mostly avoids the Pauli limiting because the magnetic susceptibility does not decrease through the superconducting transition [\[18,30\]](#page-5-0).

*Phase diagram.* Let us discuss the *H*-*T* phase diagram of the bilayer Rashba-Hubbard model. In Figs.  $4(a) - 4(d)$ , we show the phase diagrams for  $\alpha/t_{\perp} = 0$ , 1, 2, and 3, which is known as a control parameter of local noncentrosymmetricity  $[18]$ <sup>4</sup>. We show the superconducting transition lines of the  $B_g$  and  $B_u$  states, which correspond to the BCS and PDW states as mentioned before.

From the obtained phase diagrams, the zero-field superconducting transition temperature  $T_c$  and the magnetic field at the parity transition point  $H^*$  are estimated as  $(T_c, H^*)$  = (0.0141, 0.195), (0.0135,0.212), and (0.0124,0.192) for  $\alpha/t_{\perp} = 1$ , 2, and 3, respectively. From these estimations, we evaluate  $H^*$  with a unit of  $T_c$ ,

$$
\frac{H^*}{T_c} = \begin{cases}\n13.8 & (\alpha/t_\perp = 1), \\
15.7 & (\alpha/t_\perp = 2), \\
15.5 & (\alpha/t_\perp = 3).\n\end{cases}
$$
\n(6)

Thus, we conclude that the parity transition fields scaled by the transition temperature are approximately  $H^*/T_c \simeq 15$  and universal against a variation of  $\alpha/t_{\perp}$ . Although a much smaller value  $H^*/T_c \simeq 2$  was predicted by the mean-field theory [\[19\]](#page-5-0), in CeRh<sub>2</sub>As<sub>2</sub> the experimental values  $H^* \simeq 3.9$  T and  $T_c \simeq$ 0.26 K lead  $(H^*/T_c)_{exp} \simeq 10$  [\[33\]](#page-5-0). Hence, our theoretical result is quantitatively consistent with the phase diagram of  $CeRh<sub>2</sub>As<sub>2</sub>$  and resolves the discrepancy between the weakcoupling theory and experiment.

Now we discuss the origin of the enhancement in the parity transition field  $H^*$ . First, a possible origin is the nonsymmorphic crystalline symmetry as proposed in Ref. [\[45\]](#page-6-0). Actually, an indicator of local noncentrosymmetricity  $\alpha/\tilde{t}_{\perp}(\boldsymbol{k})$  diverges at the Brillouin zone faces in nonsymmorphic crystals because of  $\tilde{t}_{\perp}(\mathbf{k}_{\text{face}}) = 0$ , and therefore the effect of the spin-orbit coupling is more essential than the symmorphic case [\[45,72\]](#page-6-0). However, our analysis of the bilayer Rashba-Hubbard model does not support this possibility, because the ratio  $H^*/T_c$  is universal against a variation of  $\alpha/t_1$ . Second, we may expect that the spin-triplet component in the gap function changes the spin state of Cooper pairs and increases *H*<sup>∗</sup>. However, this is also unlikely because we observe a strong dependence on  $\alpha/t_{\perp}$  of the parity mixing parameter *r* (Fig. [5\)](#page-4-0), which is defined as

$$
r = \frac{\max_{k} |d^{\text{AA}}(k)|^{2}}{\max_{k} |\psi^{\text{AA}}(k)|^{2}}.
$$
 (7)

From Eq. (6) and Fig. [5,](#page-4-0) we find that  $H^*/T_c$  and *r* are almost uncorrelated. Third, we rule out the field dependence of the effective interaction as the main origin. In the bilayer Rashba-Hubbard model, the effective pairing interaction is field dependent because of the field-enhanced magnetic anisotropy (see Fig. [2\)](#page-1-0). Figure [6](#page-4-0) shows eigenvalues of the Eliashberg

<sup>&</sup>lt;sup>4</sup>While the case  $\alpha/t_{\perp} = 0$  corresponds to the bilayer system without spin-orbit coupling, for  $\alpha/t_{\perp} = \infty$  the system is equivalent to a set of monolayer systems with Rashba-type spin-orbit coupling [\[18\]](#page-5-0).

<span id="page-4-0"></span>

FIG. 5. The parity mixing parameter *r* as a function of  $\alpha/t_{\perp}$ . The values for the  $B_g$  and  $B_u$  representations are nearly the same. We assume  $T = 0.01$  and  $H = 0.15$ .

equation with and without the field dependence of magnetic anisotropy, $5$  and we find that the field-enhanced magnetic anisotropy weakens the superconducting instabilities. However, the parity transition field *H*<sup>∗</sup> is hardly affected. Thus, field-enhanced magnetic anisotropy is also irrelevant for the enhancement of  $H^*$ . We conclude from these considerations that a large parity transition field consistent with  $Cerh<sub>2</sub>As<sub>2</sub>$  is due to the internal field arising from the quantum critical antiferromagnetic fluctuation. Near the antiferromagnetic critical point, the spin correlation between electrons significantly develops and electrons give an internal field to other electrons. In our system, the external Zeeman field applied to an electron is screened by the surrounding electrons, and therefore, the scale of magnetic fields is remarkably increased [\[73\]](#page-6-0). This is consistent with the large upper critical fields of  $CeRh<sub>2</sub>As<sub>2</sub>$  which significantly exceeds the Pauli-Clogston-Chandrasekhar limit even for the in-plane direction [\[33\]](#page-5-0).

*Conclusion.* We investigated the nature of quantum critical multipole fluctuation and superconductivity in the bi-



FIG. 6. The magnetic field dependence of eigenvalues of the Eliashberg equation for the  $B_g$  and  $B_u$  representations.  $B_g^{\text{iso}}$  and  $B_{\mu}^{\text{iso}}$  represent the eigenvalues without the field-enhanced magnetic anisotropy. The parity transition points are indicated by the gray dashed line. We assume  $\alpha/t_{\perp} = 2$  and  $T = 0.01$ .

layer Rashba-Hubbard model, a minimal model of locally noncentrosymmetric strongly correlated electron systems. Experimental observations in a recently discovered superconductor  $Cerh<sub>2</sub>As<sub>2</sub>$  [\[33\]](#page-5-0) were discussed. The *XY*-type antiferromagnetic fluctuations are shown consistent with the NMR study [\[39\]](#page-5-0), and the applied magnetic field along the *z* axis enhances the fluctuations. Due to the critical antiferromagnetic fluctuation, superconductivity with dominant *d<sub>x</sub>*<sub>2−*y*</sub>2 -wave pairing and subdominant *p*-wave pairing is stabilized, and the two superconducting phases with different space inversion parity appear in the *H*-*T* phase diagram, consistent with  $Cerh_2As_2$ . The parity transition field is enhanced by the quantum critical fluctuation, and the obtained value is in quantitative agreement with the experimental value. These results support the parity transition in the superconducting state of  $Cerh<sub>2</sub>As<sub>2</sub>$  [\[33\]](#page-5-0) and indicate topological superconductivity  $[32]$ . Our theory not only solves the issues of CeRh<sub>2</sub>As<sub>2</sub> but also elucidates general behaviors of the family of locally noncentrosymmetric strongly correlated superconductors.

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<sup>5</sup>To investigate the effect of the field-induced change of effective interaction, we solve the Eliashberg equation with the zero-field vertex functions while the Green function at  $H \neq 0$  are adopted. The eigenvalue without the field-enhanced magnetic anisotropy  $\lambda^{iso}$  is defined by the equations  $\lambda^{iso} \Delta_{\xi \xi'}(k) = \frac{T}{N} \sum_{k'} \Gamma^{a, H=0}_{\xi \xi_1 \xi_2 \xi'}(k - k') F^{H \neq 0}_{\xi_1 \xi_2}(k)$ and  $F_{\xi_1\xi_2}^{H\neq 0}(k) = G_{\xi_1\xi_3}^{H\neq 0}(k')\Delta_{\xi_3\xi_4}(k')G_{\xi_2\xi_4}^{H\neq 0}(-k').$ 

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