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Letter

Fixed point annihilation for a spin in a fluctuating field

Adam Nahum

Laboratoire de Physique, École Normale Supérieure, CNRS, Université PSL, Sorbonne Université, Université de Paris, 75005 Paris, France

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A quantum spin impurity coupled to a gapless free field (the Bose-Kondo model) can be represented as a (0+1)-dimensional field theory with long-range-in-time interactions that decay as $|t-t'|^{-(2-\delta)}$. This theory is a simpler analog of nonlinear σ models with topological Wess-Zumino-Witten terms in higher dimensions. In this Letter we show that the renormalization group (RG) flows for the impurity problem exhibit an annihilation between two nontrivial RG fixed points at a critical value δ_c of the interaction exponent. The calculation is controlled at large spin S. This clarifies the phase diagram of the Bose-Kondo model and shows that it serves as a toy model for phenomena involving fixed point annihilation and "quasiuniversality" in higher dimensions.

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The annihilation of a stable with an unstable fixed point is a generic possibility in renormalization group (RG) flows when a parameter such as the spatial dimensionality, which does not flow, is varied [1–7]. When this happens it leads to an interesting regime just beyond the annihilation point. No physical fixed point exists in this regime (although "annihilation" really means that the real fixed points disappear into the complex plane, where they may correspond to nonunitary conformal field theories [8]). Nevertheless, the RG flows become very slow. This can yield particles with anomalously small masses or weakly first-order phase transitions with extremely long correlation lengths [1] that show quasiuniversal [4,9] behavior below this scale.

One generic class of examples includes field theories with cubic terms that have continuous transitions in low dimensions, which become first order (as predicted by mean-field theory) in high dimensions. These include the Potts model [3] (which also undergoes annihilation in two dimensions as a function of the number of states [1,2,10–12]) as well as Landau theories for order parameters on complex or real projective space [13,14].

This Letter is motivated by a fixed point annihilation phenomenon that was proposed to resolve debates about Monte Carlo results for deconfined criticality [15] in (2+1)-dimensional (2+1D) antiferromagnets [9,16]. In Refs. [17,18] this was put in terms of a dimensional hierarchy of nonlinear σ models in d space-time dimensions with SO$(d+2)$ global symmetry [19]. These σ models have a topological Wess-Zumino-Witten (WZW) term in the action. The case $d=2$ is the well-known WZW theory with a conformal fixed point [20], and $d=3$ is an effective field theory for various competing order parameters in 2+1D magnets [21,22]. It was argued that fixed point annihilation occurs between two and three dimensions.

Unfortunately, this example of fixed point annihilation, like the others mentioned above, requires an integer-valued parameter (here d) to be treated as continuously variable. An annihilation that takes place at a noninteger critical dimensionality may be useful conceptually for understanding nearby values of d, but it cannot be realized physically (and there may be ambiguities in defining the continuous d theory). It would be instructive to have a toy model that retained basic features of the WZW example without the unphysical feature of noninteger d.

Here we show that the simplest member of the "WZW" hierarchy, in $d=1$, provides such a model if we augment it with a long-range interaction. This is a model of a spin impurity in a gapless environment [23–27] and was suggested as a model for fixed point annihilation in [18]. We find that many key features of the higher-dimensional example are retained (fixed point annihilation, quasiuniversality, emergent symmetry). But since the fixed point annihilation occurs in $d=1$ the model is accessible to numerical simulations and

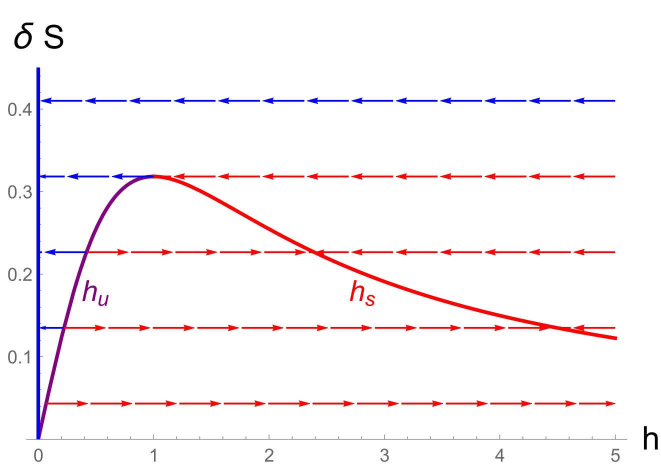

FIG. 1. Fixed points and flows as a function of the exponent δ in the memory kernel $K \sim 1/|t-t'|^{2-\delta}$. h is the dimensionless coupling of the 1D nonlinear σ model. $h=0$ is the ordered fixed point, and $h=\infty$ (not shown) is a noninteracting spin with $2S+1$ degenerate ground states. $h_{s,u} = Sg_{s,u}$ label the branches of stable and unstable fixed points.

perhaps to experiment. The model is analytically tractable at large spin.

The $d = 1$ theory without a long-range interaction is simply the quantum mechanics of a spin S (or, more generally, a rotor), described using the spin path integral with its well-known Berry phase term [28]. The version with a power-law interaction $\sim 1/|t - t'|^{2-\delta}$ describes a spin with a retarded interaction, physically representing an interaction with a gapless zero-temperature bath that has been integrated out. This is known as the Bose-Kondo model [23–27,29–39]. It falls into the larger family of quantum impurity models describing a local quantum-mechanical degree of freedom interacting with a bath of critical fluctuations [26,27,40–42].

We study the model in a large spin limit that allows the RG equation to be obtained to all orders in the coupling. Using the background field method, the calculation is a simple extension (to include the Berry phase term) of the analysis by Kosterlitz of the long-range classical Heisenberg model in one dimension [43]. The β function shows an interesting structure, with two nontrivial fixed points that annihilate each other when the interaction power law $2 - \delta$ is varied through a critical value. The flows as a function of δ are qualitatively like those suggested for the WZW model as a function of ϵ (in $2 + \epsilon$ dimensions) [17,18], except in the behavior of one of the nontrivial fixed points (the stable one) when $\delta \to 0$.

Model. We consider a Euclidean action for a spin of size S with a long-range temporal interaction:

$$\mathcal{S} = \frac{1}{2g} \int dt dt' \, K(t - t')[\vec{n}(t) - \vec{n}(t')]^2 - iS\,\Omega[\vec{n}]. \qquad (1)$$

Here $\vec{n} = (n_1, n_2, n_3)$, with $\vec{n}^2 = 1$, is the field appearing in the coherent state path integral. This is a formulation of the SO(3)-symmetric Bose-Kondo model, in which the spin is coupled to a local magnetization $\vec{m}$ (associated with additional "bulk" degrees of freedom) via a Hamiltonian $H_{\text{int}} = J\vec{S} \cdot \vec{m}$ [23–27]. If $\vec{m}$ has SO(3)-invariant autocorrelations obeying Wick's theorem, then integrating $\vec{m}$ out yields Eq. (1) with $g^{-1} \propto J^2S^2$ and with a kernel $K(t - t')$ that is proportional to the autocorrelator of $\vec{m}$. We assume this is a power law at large $\tau = t - t'$, $K(t - t') \propto |t - t'|^{-(2-\delta)}$ with $-1 < \delta < 1$. For convenience we normalize K as

$$K(\tau) = \frac{C\Lambda^\delta}{|\tau|^{2-\delta}}, \quad C = \frac{(1-\delta)}{4\Gamma(\delta)\sin(\pi\delta/2)}. \qquad (2)$$

The constant C is chosen so that the Fourier transform of $K(\tau)$ has a simple normalization [43,44], and a power of the UV cutoff frequency Λ is included in $K(\tau)$ so that g is dimensionless. Finally, the Berry phase term $\Omega[\vec{n}]$ is the solid angle on the sphere traced out by the trajectory, written in terms of an extension of the field $\vec{n}(t)$ to a field $\vec{n}(t, u)$ defined on a strip with $u \in [0, 1]$ as [28]

$$\Omega[\vec{n}] = \frac{1}{2} \int_0^1 du \int dt \epsilon^{\mu\nu} \vec{n} \cdot (\partial_\mu \vec{n} \times \partial_\nu \vec{n}), \qquad (3)$$

or, more simply, as $\Omega[\vec{n}] = \int dt (1 - \sin\psi)\dot{\phi}$ in the coordinates $\vec{n} = (\cos\psi\cos\phi, \cos\psi\sin\phi, \sin\psi)$.

Before calculating the β function, let us ask what we can expect from stability considerations.

The action Eq. (1) has two trivial fixed points, at $g = 0$ and at $g = \infty$. That at $g = 0$ is a perfectly ordered state, with no local fluctuations in $\vec{n}(t)$. The fixed point at $g = \infty$ describes a quantum spin with $2S + 1$ degenerate ground states.

By counting dimensions we see that when δ is negative, the ordered fixed point at $g = 0$ is unstable, and the $g = \infty$ fixed point is stable. The simplest expectation (confirmed in the large-S calculation below) is that in this $\delta < 0$ regime the model flows, for any positive g, to $g = \infty$. On the other hand when δ is positive, the ordered fixed point becomes stable, so the model is in a stable ordered phase for small enough g. At the same time the $g = \infty$ fixed point becomes unstable.

The flows for *infinitesimal* g are similar to those in a classical one-dimensional (1D) model without the Berry phase term because the Berry phase term in the action is subleading in the limit $g \to 0$. As in the classical model, the change in stability of the ordered fixed point is accompanied by the appearance of a nontrivial unstable fixed point, representing a phase transition, at a coupling g_u that is of order δ for small positive δ [43].

However, the Berry phase term plays a role for noninfinitesimal g. In particular, the $g = \infty$ fixed point is unstable for $\delta > 0$, unlike a simple classical disordered fixed point. The simplest consistent hypothesis is therefore that for sufficiently small positive δ there is another nontrivial fixed point g_s, with $g_s > g_u$, which is *stable*. This fixed point governs a stable large-g phase with power-law correlations. Heuristically, the Berry phase term has prevented $\vec{n}(t)$ from being trivially disordered at large g, leading instead to a stable "critical phase". At small δ, with fixed S, this stable fixed point can be studied by perturbative RG in the strength of the impurity-spin coupling [26].

What happens to these fixed points as δ is increased? A simple guess (in analogy to the higher-dimensional problem) is that at some critical value δ_c they merge and annihilate, meaning that for a sufficiently long range interaction the model is always in the ordered phase (Fig. 1). We will confirm this directly when S is large.

RG results. At large S the interesting regime is where the coupling g and the exponent δ are both of order $1/S$, so we will write

$$h = gS, \quad \widetilde{\delta} = \delta S. \qquad (4)$$

This scaling of the coupling ensures that the two terms in the action are of comparable size in the limit of large S. (If this is not the case, then one of the two terms dominates the action for the "fast" modes that we integrate out in the RG step, leading to a more trivial RG equation.) The spin size S itself is quantized and does not flow, but it serves as a large parameter that justifies a one-loop calculation [20]. This calculation can be done with the background field method [43,45–47] and is described in the Supplemental Material [44].

Our basic result is the RG equation,

$$\frac{dh}{d\tau} = \frac{1}{S}\left(-\widetilde{\delta}\,h + \frac{2}{\pi}\frac{h^2}{1+h^2}\right) + O\left(\frac{1}{S^2}\right), \qquad (5)$$

where the RG time τ is the logarithm of a physical timescale. The topology of the associated flows is shown in Fig. 1.

The value

$$\delta_c = \frac{1}{\pi S} \tag{6}$$

for the interaction exponent separates two regimes. For larger δ, all flows lead to the ordered phase, as noted above, but for $0 < \delta < \delta_c$ there is a stable nontrivial phase (governed by the fixed point at g_s), separated by a second-order phase transition (governed by g_u) from the ordered phase. The RG eigenvalue of the coupling at $g_{u,s}$ is $y_g = \pm\delta\sqrt{1-\pi^2\widetilde{\delta}^2}$, with the + sign for g_u.

The scaling dimension of the field $\vec{n}$ is $x_1 = \delta/2$ at both nontrivial fixed points, so that the spin autocorrelator decays as $|t-t'|^{-\delta}$. This exponent value is expected to be exact, like for other long-range models, since the two local operators appearing in the long-range term renormalize independently when $t - t'$ is large [33,43,48–50]. Below we will also need the scaling dimension x_k of the symmetric k-index tensor $X^{(k)}_{a_1,\dots,a_k}$ that is obtained as the traceless part of the operator $n_{a_1}\cdots n_{a_k}$. At one-loop order this obeys [51] (see the Supplemental Material [44])

$$x_k = \frac{k(k+1)}{2}x_1. \tag{7}$$

We conjecture that the topology of the flows found here at large S applies for all values of the spin, including $S = 1/2$. It would be interesting to study this numerically. The partition function for the spin has a Monte Carlo sign-free diagrammatic formulation, with propagators of $\vec{m}$ represented as arcs connecting points t and t' on the spin's world line [52], and the model may also be studied with numerical RG [53,54].

Let us return to the analogy with higher-dimensional models for deconfined criticality and competing orders. In the WZW hierarchy, two key features are (1) quasiuniversality in the regime just beyond the fixed point annihilation ($\epsilon \gtrsim \epsilon_c$ in $2+\epsilon$ dimensions) and (2) the emergence of the full symmetry of the σ model from a smaller microscopic symmetry group, thanks to the irrelevance of operators analogous to $X^{(k)}$ for large enough k. We examine analogs of these phenomena in the present system.

Quasiuniversality. The quasiuniversality phenomenon will occur in this 1D model when $\delta \gtrsim \delta_c$. The spin will ultimately be ordered even if the bare coupling h is large, but this will not be apparent until a timescale ξ that diverges *exponentially* with $(\delta-\delta_c)^{-1/2}$ because the flows spend a large amount of RG time close to $h = 1$ [1,2].

At small $\delta - \delta_c$ we can continue to classify operators as relevant or irrelevant, and the long RG time spent close to $h = 1$ means that irrelevant perturbations, which will be present in a generic microscopic model with the appropriate symmetry, become exponentially small in $(\delta-\delta_c)^{-1/2}$ [9]. This exponential suppression of differences between bare models underlies quasiuniversality. For example, we will have approximate universality in the functional form of the spin autocorrelator $\langle\vec{n}(t)\cdot\vec{n}(0)\rangle$, despite the fact that it is not a power law for $\delta > \delta_c$.

In fact, a simplifying feature of RG for the long-range model is that the flow of the renormalized coupling $h(\tau)$—obtained from running the RG up to a physical timescale $\Lambda^{-1}e^{\tau}$—can be plotted simply by plotting the spin autocorrelator, at least within the present large-S approximation. This is because the RG Eq. (5) can be expressed in terms of the running scaling dimension $x(h)$ of the σ model field $\vec{n}$ as $\dot{h} = [-\delta + 2x(h)]h$(see Eq. (15) of the Supplemental Material [44]). RG for the correlator then gives

$$\langle\vec{n}(t)\cdot\vec{n}(0)\rangle = \frac{h(0)}{(\Lambda t)^{\delta}\, h(\ln\Lambda t)}. \tag{8}$$

It would be interesting to use the correlator Eq. (8) to obtain a proxy for the β function from Monte Carlo simulations in the quasiuniversal regime.

As an aside, a curious feature of the model is that the two-point function Eq. (8) tends to a constant at large times both in the ordered ($h \to 0$) phase, which is stable for $\delta > 0$, and also in the free-spin ($h \to \infty$) phase that exists for $\delta < 0$. However, the two fixed points are different.

One concrete way to see the difference is in connected two-point functions $G^{(k)}(t)$ of operators with higher spin, $k > 2S$. For a completely free spin, nonvanishing operators exist only with spin $k \leqslant 2S$. However, in the microscopic theory of a spin coupled to a bath we can construct nonvanishing operators with any spin k, as discussed in the Supplemental Material [44]. Let $G^{(k)}(t)$ be a connected two-point function for such operators. In the free-spin phase $\lim_{t\to\infty} G^{(k)}(t)$ is nonzero for $k \leqslant 2S$ and vanishes for $k > 2S$ (because the spin and bath decouple at the governing IR fixed point; see [44]). In contrast, in the ordered phase we expect that $\lim_{t\to\infty} G^{(k)}(t)$ is nonzero for all k because the corresponding continuum operators $X^{(k)}$, defined above, have nonvanishing long-distance correlations at the ordered fixed point. [Correlation functions at the ordered fixed point are simple since only the zero mode of $\vec{n}(t)$ needs to be averaged over.]

Emergent symmetry. We can construct a simple toy model for the emergent symmetries [SO(4) in 1+1D and SO(5) in 2+1D] that arise in various higher-dimensional microscopic models for which the WZW σ models serve as effective field theories [9,21,22,55–60]. In these examples, the N-component σ model field $\vec{n}$ is viewed as the concatenation of two separate fields, $\vec{n} = (\vec{\Phi}^A, \vec{\Phi}^B)$. $\vec{\Phi}^A$ and $\vec{\Phi}^B$ are not related by microscopic symmetry but may be related by an emergent SO(N) symmetry at a critical point. In 2+1D, for example, $\vec{\Phi}^A$ and $\vec{\Phi}^B$ could be the Néel and Valence-Bond Solid (VBS) order parameters, with the critical point of interest separating Néel and VBS phases.

Here we take $\Phi^A = n_z$ and $\vec{\Phi}^B = (n_x, n_y)$. That is, we think of Eq. (1) as an effective field theory for a phase transition in an *anisotropic* microscopic Hamiltonian, with only $O(2) = \mathbb{Z}_2 \ltimes U(1)$ symmetry, which is promoted to emergent SO(3) at a critical point. The critical point lies at the boundary of a phase with easy-axis order for n_z.

For concreteness, consider simple O(2)-invariant Hamiltonians for spin 1/2 and spin 1. Nontrivial examples require at least two anisotropic couplings in the microscopic Hamiltonian, as will be clear below. For a spin 1 we could consider single-ion anisotropy and an anisotropic bath coupling: $H_{\text{aniso}} = J(S_x m_x + S_y m_y + \gamma S_z m_z) - \Delta S_z^2$. For a spin 1/2 the S_z^2 term trivializes, but we could consider a local anisotropy for the bath, $H'_{\text{aniso}} = J(S_x m_x + S_y m_y + \gamma S_z m_z) - \Delta m_z^2$. We assume that $\delta \leqslant \delta_c$ and that J is small enough that the isotropic models ($\Delta = 0, \gamma = 1$) flow to the fixed point at g_s, which is stable in the absence of anisotropy (Fig. 1).

Microscopic O(2) symmetry allows the perturbations $\delta\mathcal{S} = \int dt (uX^{(2)}_{33} + vX^{(4)}_{3333} + \cdots)$ to the continuum action. Here $X^{(2)}_{33} \propto n_z^2 - (n_x^2 + n_y^2)/2$ is the leading anisotropy which will drive the transition, and $X^{(4)}_{3333}$ is a subleading anisotropy. The scaling dimension formula Eq. (7) is reliable only at large S, but it suggests that for small spin there is a range of positive δ where $X^{(2)}$ is the only relevant anisotropy, with $X^{(4)}$ and higher anisotropies being irrelevant. We assume δ is in this range.

Then, the SO(3)-invariant fixed point governs a phase transition line in the $(\Delta/J, \gamma)$ plane. One point on this line, at (0, 1), has microscopic SO(3) symmetry, but at other points on the line SO(3) emerges only in the IR. One adjacent phase is the easy-axis phase, where $\mathbb{Z}_2$ is broken. The nature of the other phase will depend on the spin. For spin-1/2 it is likely a power-law phase [23] in which the easy-plane order parameter dominates.

It may be interesting to check for symmetry enhancement starting from other microscopic symmetry groups. For example S_4 (tetrahedral) symmetry allows the perturbation $X^{(3)}_{123}$. We may argue that the field theory with this symmetry breaking and with $S = 1$ is an effective theory for a long-range four-state Potts model in which the partition sum is weighted by (-1) for each domain wall.

Conclusions. The impurity model can be seen as the simplest member of a dimensional hierarchy of σ models with topological terms [19]. We have argued that some interesting features of the RG flows in higher dimensions are also present in 0+1D, giving a rich phase diagram for the Bose-Kondo model. The model yields an example of fixed point annihilation that is tractable both analytically and in simulations and also shows analogs of phenomena from higher-dimensional "non-Landau" phase transitions. It would be interesting to examine other variations, for example, models in large-N limits, with other symmetric spaces for the target space, or with coupling to fermions, and to explore physical realizations of the tunable interaction exponent δ (perhaps via a bosonic bath whose hopping parameters vary with distance from the impurity). Finally, it would also be interesting to look for the annihilation phenomenon in models relevant to impurities in critical magnets in which the bath is not Gaussian [29] or settings where the impurity arises as a self-consistent description of an interacting many-body system [38].

Note added. The impurity in the large-S limit was also analyzed recently in two other papers [61,62], with results for the β function consistent with those above. In addition, these papers make interesting connections with Wilson lines and line defects in conformal field theory. Quantum Monte Carlo results for spin 1/2 are now also available [63] and are consistent with the phase diagram obtained here.

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