Shot noise in resonant tunneling: Role of inelastic scattering

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We study the influence of inelastic processes on shot noise and the Fano factor for a one-dimensional double-barrier structure, where resonant tunneling takes place between two terminals. Most studies to date have found, by means of various approximate or phenomenological methods, that shot noise is insensitive to dephasing caused by inelastic scattering. In this paper, we explore the status of this statement by deriving a general Landauer-Büttiker-type formula that expresses the current noise and Fano factor in a one-dimensional conductor through inelastic scattering amplitudes. For a double-barrier structure, exact scattering amplitudes are calculated in the presence of a time-dependent potential that acts in the region between the barriers. This allows us to rigorously analyze the role of dephasing in the current noise generated by applying a finite bias voltage to the resonant level. As an example of the dephasing potential, we consider the one induced by equilibrium phonons. We show that for phonons propagating in one dimension, the random phase of the electron wave function, which is induced by the electron-phonon coupling, exhibits diffusionlike dynamics. At the same time, for higher-dimensional phonons, the electron phase dynamics turns out to be nondiffusive, such that the average square of the phase grows logarithmically with time. We calculate transmission coefficients of a double-barrier structure for these two types of phonon-induced dephasing. In the case of diffusive phase relaxation, the resonant level has a Lorentzian shape with the broadening determined by a sum of the elastic linewidth and the phase breaking rate. Logarithmic dephasing leads to an unusual shape of the size-quantized level: the transmission coefficient is characterized by the two energy scales, one governed by the transparency of barriers and the other by the phonon correlation time. We further calculate the Fano factor for these types of dephasing, using exact expressions for inelastic transmission and reflection amplitudes. It turns out that when an integer number of levels falls into the energy window of width eV, where V is the voltage applied to the structure, the Fano factor is really insensitive to inelastic processes inside the structure and coincides with the prediction of phenomenological models with an accuracy of small corrections depending on these processes. On the contrary, at low voltages, when the eV window is smaller than the level width, this dependence is particularly pronounced and the phenomenological formula does not work.

I. INTRODUCTION

Phase coherence of electron waves plays an essential role in low-dimensional transport [1]. Inelastic electron scattering processes, such as electron-electron or electron-phonon collisions, break down the phase coherence, leading to dephasing of electron waves. With decreasing intensity of such collisions (e.g., with lowering temperature), an electron system exhibits a crossover from the classical regime to the quantum one, where interference effects become prominent.

One of the simplest and most important for practice interference phenomena is the so-called resonant tunneling through a quantum dot with tunnel contacts. At very low temperatures, the role of inelastic collisions is negligible and the conductance of the dot demonstrates a narrow high peak whenever the Fermi level coincides with one of the size-quantization levels. The appearance of these conductance peaks is due to interference of electron waves inside the quantum dot. The width of the resonant peaks is determined by the transparency of the contacts that represent tunneling barriers for electrons. With increasing temperature, the interference breaks down because of the inelastic collisions, the peaks widen and gradually disappear. When the phase coherence is completely destroyed, the conductance of the double-barrier structure is given by a classical formula containing only the probabilities of tunneling through the contacts.

This phenomenon of resonant tunneling was studied experimentally in various systems, such as carbon nanotubes [2–6] and nanowires with tunnel contacts, bulk structures with double barriers [7–11], and many others. Theoretically, the effect of inelastic scattering on resonant tunneling was considered in detail in Ref. [12]. It was shown there, in particular, that although the width of the resonant levels is determined both by the transparency of the barriers and by the rate of inelastic processes, the conductance at temperatures exceeding this width is determined solely by the transparency of the barriers.

Another important characteristic of such structures is the current-noise power [13,14]. Current noise consists of two terms: thermal and nonequilibrium noise. The Landauer-Büttiker scattering formalism, considering a system attached to Fermi leads and characterizing the structure by the

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scattering amplitudes, is a common tool for the analysis of the current noise. The thermal (or Johnson-Nyquist) noise is proportional to the product of the temperature T and the conductance G of the structure (proportional to its transparency):

$$S_T(\omega = 0) = 2TG.$$

The nonequilibrium noise at zero temperature (shot noise) is proportional to the product of the transmission coefficient $\mathcal{T}$ and the reflection coefficient $\mathcal{R} = 1 - \mathcal{T}$ of the scatterer:

$$S \propto \mathcal{T}(1 - \mathcal{T}).$$

The convenient measure of shot noise is the Fano factor—the ratio of zero-frequency noise power to the Poissonian noise:

$$F = S(\omega = 0)/S_P. \tag{1}$$

The Poissonian noise is proportional to the average current:

$$S_P = e\langle I \rangle \tag{2}$$

(here, we considered spinless particles with charge e).

For resonant tunneling through a double-barrier structure in the nonlinear transport regime (the temperature is smaller than the applied bias voltage), the Fano factor is given by [13]

$$F = \frac{\Gamma_L^2 + \Gamma_R^2}{(\Gamma_L + \Gamma_R)^2} = \frac{\mathcal{T}_L^2 + \mathcal{T}_R^2}{(\mathcal{T}_L + \mathcal{T}_R)^2}. \tag{3}$$

Here, $\Gamma_L \propto \mathcal{T}_L$ and $\Gamma_R \propto \mathcal{T}_R$ quantify the strength of the left and right barriers in energy units. The width of the resonant level in the absence of inelastic processes is given by $\Gamma_L + \Gamma_R$: the stronger the barrier, the smaller its contribution to the level width. The Fano factor given by Eq. (3) can take values between 1/2 and 1. Expression Eq. (3) was obtained in Refs. [15,16] using complementary approaches.

Remarkably, when written in terms of the transmission coefficients $\mathcal{T}_{L,R}$, the expression for the Fano factor can be interpreted both quantum-mechanically and classically. Thus, an interesting feature of the resonant double-barrier structure is the apparent insensitivity of the Fano factor to inelastic processes in the system. This statement has been widely discussed in literature and tested by various methods. The classical tunneling picture based on the master equation (describing fully incoherent transport) employed in Refs. [17,18] or the approach based on the Langevin equation [13] led to Eq. (3). Within the quantum coherent picture of transport, a solution of the quantum master equation [19] and calculations based on nonequilibrium Green's functions [20,21] yielded the same result Eq. (3) for the Fano factor.

Since in both the limits of fully coherent and fully incoherent transport, the Fano factor is given by the same formula, it is tempting to argue that coherence and its breaking by inelastic processes does not play any role here. However, this does not, of course, prove that, in the intermediate case of finite dephasing, the result remains unchanged. Indeed, considering all the interference-induced terms separately, one can imagine the situation when this coherent contribution is exactly zero (complete destructive interference). Any finite dephasing, destroying such a cancellation, would lead to corrections to Eq. (3), whereas extremely strong dephasing would kill all the interference terms, thus restoring Eq. (3) in the classical limit.

Several works indeed found nonuniversality of the Fano factor for the double-barrier structure. In particular, Ref. [22]

found corrections to this expression within the sequential-tunneling picture. Another work, Ref. [23], demonstrated strong deviation from result Eq. (3) by phenomenologically adding random (completely uncorrelated) phases to the quantum amplitudes that describe electron propagation between the barriers. However, importantly, these papers used various assumptions and simplifications in the consideration of inelastic scattering. Thus, the question of the influence of inelastic scattering on the Fano factor for resonant tunneling is still open. This calls for the analysis of inelastic processes within an exact quantum-mechanical model. The main goal of this paper is to develop such a formalism and to finally resolve the question about the influence of inelastic processes on the Fano factor of a double-barrier structure.

Another interesting current noise contribution is so-called delta-T noise [24]. The latter is similar to shot noise but exists at zero applied voltage and nonequal temperature of the leads. Derived in this paper, the shot noise equation can also help analyze such experiments with a more complex scatterer. As shown in Ref. [25], delta-T noise can be used to probe the nature of collective excitations in interacting one-dimensional (1D) systems, which have been intensively studied theoretically [26] and experimentally [27] on fractional quantum Hall edges. Another interesting application of shot noise is probing Majorana physics. In Refs. [28,29], current noise through quantum dots coupled to Majorana zero modes was studied. Also, tunneling currents through coupled quantum dots and current noise are sensitive to correlated electronic states [30]. An alternative way to investigate nonstationary processes in low dimension transport is the Keldysh Green's function formalism, for example, a current through a localized state in a phonon bath was analyzed in Ref. [31]. Spin-orbit interaction in quantum dots also leads to changes in the shot noise. In Ref. [32], the suppression of shots with increasing spin-orbit interactions was found. The study of dephasing processes is also important in the frame of quantum processing. As shown in Ref. [33], bilayer graphene is a promising material for this task, The authors performed a single spin reading in a graphene quantum dot.

In this paper, we obtain a general expression for the noise power for transport through a 1D scatterer in the presence of inelastic processes. The current noise is expressed in terms of inelastic transmission and reflection amplitudes, which allows us to take dephasing into account exactly. Next, we obtain the inelastic transmission amplitudes for a double-barrier structure with a random time-dependent potential acting between the barriers. Within this scattering formalism, we calculate shot noise for resonant tunneling in the presence of inelastic scattering. After averaging over the random potential, we obtain the structure's Fano factor and analyze it for different types of dephasing of the electron wave function. Specifically, we investigate phonons as a source of the random potential leading to dephasing. For dephasing caused by 1D phonons, the dispersion of the random wave-function phase φ_f possesses a diffusion growth with time t, characterized by the dephasing time τ_φ: $\langle \varphi_f^2 \rangle \sim t/\tau_\varphi$. Remarkably, for two-dimensional (2D) and three-dimensional (3D) phonons, dephasing is logarithmic: $\langle \varphi_f^2 \rangle \sim \ln(Tt/\hbar)$.

In the case of diffusive phase dynamics, the resonant levels in a double-barrier structure have a Lorentzian shape with the
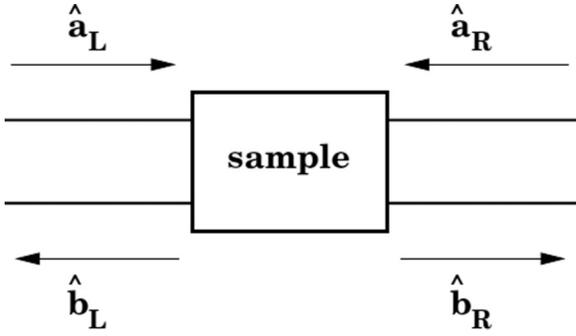
FIG. 1. Schematics of the structure. The four electron operators of the right- and left-moving plain waves (shown by arrows) on both sides of the sample are related by the scattering matrix, Eq. (41). There are only two independent operators, $\hat{a}_L$ and $\hat{a}_R$ supplied by the terminals; the other two, $\hat{b}_L$ and $\hat{b}_R$, are expressed through them by means of scattering amplitudes.

broadening determined by a sum of the elastic linewidth and the phase breaking rate. Logarithmic dephasing leads to an unusual shape of the size-quantized level: the transmission coefficient is characterized by two energy scales, one governed by the transparency of barriers and the other by the phonon correlation time. When the voltage exceeds the characteristic resonant level width, the leading contribution to the Fano factor, in agreement with the predictions of phenomenological models, becomes insensitive to the inelastic scattering; the influence of inelastic processes is only reflected in the presence of small corrections to this result. On the contrary, when the voltage is smaller than the level width, the Fano factor depends strongly on the type of dephasing and the phenomenological formula does not work.

II. CURRENT NOISE AND FANO FACTOR

We start by deriving general formulas for the current in the 1D geometry within the scattering theory approach. In this section, we obtain the noise power in a 1D conductor with an inelastic scatterer, see Fig. 1. The current noise is determined by the current fluctuations,

$$S(t - t', x, x') = \tfrac{1}{2}\langle \delta \hat{I}(x, t)\delta \hat{I}(x', t') + \delta \hat{I}(x', t')\delta \hat{I}(x, t)\rangle,$$
(4)

where

$$\delta \hat{I}(x, t) = \hat{I}(x, t) - \langle \hat{I}(x, t)\rangle;$$
(5)

$\hat{I}$ is the current operator, which takes the form

$$\hat{I}(x, t) = \frac{\hbar e}{2mi}\left[\hat{\Psi}^\dagger(x, t)\frac{\partial \hat{\Psi}(x, t)}{\partial x} - \frac{\partial \hat{\Psi}^\dagger(x, t)}{\partial x}\hat{\Psi}(x, t)\right]$$
(6)

for a single parabolic band characterized by mass m, $\hat{\Psi}$ is the electron field operator, and $\langle \ldots \rangle$ denotes the expectation value determined by the statistical operator.

On the left of the scatterer, the wave-function operator for free waves is written as

$$\hat{\Psi}_L(x, t) = \int_0^\infty \frac{dk}{\sqrt{2\pi}}e^{-iE_k t}[e^{ikx}\hat{a}_L(k) + e^{-ikx}\hat{b}_L(k)],$$
(7)

where $\hat{a}_L(k)$ and $\hat{b}_L(k)$ are fermionic annihilation operators of the right- and left-moving waves with wave vector k and energy E_k on the left (hence index L) part of the structure. Similarly, we define the the fermion field operator $\hat{\Psi}_L(x, t)$ to the right of the sample through the operators $\hat{a}_R(k)$ and $\hat{b}_R(k)$ describing the right- and left-moving waves, respectively, in the right (hence R) part of the setup.

To obtain the expectation value of the current, we need to express the field operators through the independent operators corresponding to the incident waves, $\hat{a}_L(k)$ and $\hat{a}_R(k)$, supplied by the two terminals. The scattered waves are expressed through those incident waves by means of the scattering matrix describing the sample (see Fig. 1). Below, we will first consider the case of elastic scattering and then generalize the scattering approach to include inelastic processes.

A. Elastic scatterer

To set the stage and introduce the formalism, we first analyze the case of elastic scattering and reproduce the known results for the current noise. In this case, the scattering matrix connects the incident and outgoing waves with the same wave vector and is given by

$$\begin{pmatrix} \hat{b}_L(k) \\ \hat{b}_R(k) \end{pmatrix} = \begin{pmatrix} \mathtt{r}_L(k) & \mathtt{t}(k) \\ \mathtt{t}(k) & \mathtt{r}_R(k) \end{pmatrix}\begin{pmatrix} \hat{a}_L(k) \\ \hat{a}_R(k) \end{pmatrix}.$$
(8)

Here, the amplitudes $\mathtt{r}_R(k)$ and $\mathtt{r}_L(k)$ describe the reflection of the waves arriving from the right and left terminals, respectively. In the general case, these complex amplitudes differ by the phase, leading to the same reflection coefficient $\mathcal{R} = |\mathtt{r}_R|^2 = |\mathtt{r}_L|^2$. The transmission amplitudes $\mathtt{t}(k)$ for the scattering of waves on both sides of the sample are identical when the time-reversal symmetry is preserved. The unitarity of the scattering matrix requires

$$\mathcal{T} + \mathcal{R} = 1, \quad \mathtt{r}_R\mathtt{t}^* + \mathtt{r}_L^*\mathtt{t} = 0,$$
(9)

where $\mathcal{T} = |\mathtt{t}|^2$ is the elastic transmission coefficient. The scattering amplitudes for the elastic scatterer may depend on wave-vector k labeling the incoming waves (the same k labels the outgoing waves). In what follows, for simplicity, we have assumed here that the right and left reflection amplitudes have the same phase and omit their subscripts (R/L). In fact, the phases of reflection amplitudes are of no importance for the averaged current and noise in the elastic case. We will return to this point in the case of inelastic scattering below.

Using scattering matrix Eq. (8), we can write the current operator to the left of the scatterer through the operators $\hat{a}_L(k)$ and $\hat{a}_R(k)$ that describe the left and right terminals (Fig. 1), respectively:

$$\begin{aligned} \hat{I}(x, t) = \frac{\hbar e}{4\pi m}\int dk\, dk'\, e^{i(E_{k'} - E_k)t} \\ \times [\hat{a}_L^\dagger(k')\hat{a}_L(k)C_{LL}(k', k; x) \\ + \hat{a}_R^\dagger(k')\hat{a}_R(k)C_{RR}(k', k; x) \\ + \hat{a}_L^\dagger(k')\hat{a}_R(k)C_{LR}(k', k; x) \\ + \hat{a}_R^\dagger(k')\hat{a}_L(k)C_{RL}(k', k; x)]. \end{aligned}$$
(10)

Here, the coefficients $C_{ij}(k, p; x)$ are expressed in terms of the scattering amplitudes as follows:

$$C_{LL} = (k + k')[e^{i(k-k')x} - \mathtt{r}^*(k')\mathtt{r}(k)e^{-i(k-k')x}]$$
$$- (k - k')[\mathtt{r}^*(k')e^{i(k+k')x} - \mathtt{r}(k)e^{-i(k+k')x}], \quad (11)$$

$$C_{RR} = -(k + k')e^{i(k'-k)x}\mathtt{t}^*(k')\mathtt{t}(k), \quad (12)$$

$$C_{LR} = -(k + k')e^{i(k'-k)x}\mathtt{r}^*(k')\mathtt{t}(k)$$
$$+ (k - k')e^{-i(k'+k)x}\mathtt{t}(k), \quad (13)$$

$$C_{RL} = -(k + k')e^{i(k'-k)x}\mathtt{t}^*(k')\mathtt{r}(k)$$
$$+ (k' - k)e^{-i(k'+k)x}\mathtt{t}^*(k'). \quad (14)$$

The current operator is represented as a double integral over the wave vectors. Averaging over the states emanating from the left and right leads (independent terminals, $i, j = L, R$) produces

$$\langle \hat{a}_i^\dagger(k)\hat{a}_j(p) \rangle = \delta_{ij}\delta(k - p)f_i(k), \quad (15)$$

where f_i is the electron distribution function of terminal i. The delta function in Eq. (15) removes one of the integrals over the wave vectors in Eq. (10). In what follows, we will focus on the case of equilibrium terminals described by the Fermi-Dirac distribution. Using Eq. (15), one obtains the average current, which is now a single integral over the energy E of the scattering states:

$$\langle \hat{I}(x, t) \rangle = \frac{e}{h} \int dE \, \mathcal{T}(E)[f_L(E) - f_R(E)]. \quad (16)$$

Next, we turn to the calculation of the current-noise power. The statistical average of the fourth power of creation/annihilation operators reads

$$\langle \hat{a}_i^\dagger \hat{a}_j \hat{a}_n^\dagger \hat{a}_m \rangle - \langle \hat{a}_i^\dagger \hat{a}_j \rangle \langle \hat{a}_n^\dagger \hat{a}_m \rangle = \delta_{im}\delta_{jn}f_m(1 - f_n), \quad (17)$$

where, for brevity, we denote by a single index the wave vector and the terminal. The averaged correlation of the current fluctuations can be then written as

$$\langle \delta \hat{I}(x, t)\delta \hat{I}(x', t') \rangle$$
$$= \left(\frac{\hbar e}{4\pi m}\right)^2 \int dk \, dp \, e^{-i(E_k - E_p)(t - t')}$$
$$\times [f_L(p)[1 - f_L(k)] C_{LL}(p, k; x)C_{LL}(k, p; x')$$
$$+ f_R(p)[1 - f_R(k)] C_{RR}(p, k; x)C_{RR}(k, p; x')$$
$$+ f_L(p)[1 - f_R(k)] C_{LR}(p, k; x)C_{RL}(k, p; x')$$
$$+ f_R(p)[1 - f_L(k)] C_{RL}(p, k; x)C_{LR}(k, p; x')]. \quad (18)$$

The noise power for *elastic scattering*, i.e., the zero-frequency noise [34], is calculated from the current correlation function Eq. (18) with both current fluctuations are taken on one side of the scatterer:

$$S = \frac{e^2}{h} \int dE \, \{\mathcal{T}(E)[f_L(1 - f_L) + f_R(1 - f_R)]$$
$$+ \mathcal{T}(E)[1 - \mathcal{T}(E)](f_L - f_R)^2\}. \quad (19)$$

In this formula, the first term describes thermal noise. In the limit of zero applied bias voltage, the fluctuations are proportional to the temperature multiplied by the susceptibility—the conductance in this case, in accordance with the fluctuation-dissipation theorem. The second term is the shot noise, which is sensitive to the energy dependence of the transmission coefficient. If the temperature is zero, only the shot-noise contribution is nonzero and one gets [35]

$$S = \frac{e^2}{h} \int_\mu^{\mu+eV} dE \, \mathcal{T}(E)[1 - \mathcal{T}(E)], \quad (20)$$

where the energy integral is restricted to the window determined by the bias voltage V and μ is the chemical potential of one of the terminals. Shot noise is quantitatively characterized by a single number—the Fano factor, Eq. (1),

$$F = \frac{\int dE \, \mathcal{T}(E)[1 - \mathcal{T}(E)]}{\int dE \, \mathcal{T}(E)}, \quad (21)$$

where the integration is performed over the energy difference between the left and right terminals.

B. Inelastic scatterer

Let us now turn to the case of an inelastic scatterer. To take into account inelastic processes (caused, for example, by the electron-phonon interaction), we consider a general form of the scattering matrix:

$$\hat{b}_L(k) = \int dk' [\mathtt{r}_L(k, k')\hat{a}_L(k') + \mathtt{t}_L(k, k')\hat{a}_R(k')], \quad (22)$$

$$\hat{b}_R(k) = \int dk' [\mathtt{t}_R(k, k')\hat{a}_L(k') + \mathtt{r}_R(k, k')\hat{a}_R(k')]. \quad (23)$$

Here, the scattering amplitudes, which are now functions of the two wave vectors, are labeled by the indexes corresponding to the outgoing waves. Dependence on two wave vectors will be in the case of a time-dependent potential of scatter, which we calculate in the next section. In this case, inelastic processes break down the time-reversal symmetry. Hence, we introduce distinct transmission amplitudes for the waves scattered to the right and to the left. Scattering amplitudes satisfy the unitary relations, which are now of the integral form

$$\delta(k - k') = \int dp \, [\mathtt{r}_L^*(k, p)\mathtt{r}_L(k', p) + \mathtt{t}_L^*(k, p)\mathtt{t}_L(k', p)], \quad (24)$$

and the same with $R \leftrightarrow L$.

The current operator to the left of the scatterer can be written as

$$\hat{I}(x, t) = \frac{\hbar e}{4\pi m} \int dk dk' dp dp' e^{i(E_{k'} - E_k)t}$$
$$\times [\hat{a}_L^\dagger(p)\hat{a}_L(p')C_{LL}(k, k'; p, p'; x)$$
$$+ \hat{a}_R^\dagger(p)\hat{a}_R(p')C_{RR}(k, k'; p, p'; x)$$
$$+ \hat{a}_L^\dagger(p)\hat{a}_R(p')C_{LR}(k, k'; p, p'; x)$$
$$+ \hat{a}_R^\dagger(p)\hat{a}_L(p')C_{RL}(k, k'; p, p'; x)], \quad (25)$$

where the coefficients $C_{ij}(k, k'; p, p'; x)$ read

$$C_{LL} = (k + k')[e^{i(k-k')x}\delta(k' - p)\,\delta(k - p') \\ - e^{-i(k-k')x}\mathtt{r}_L^*(k', p)\,\mathtt{r}_L(k, p')] \\ + (k' - k)\big[e^{-i(k+k')x}\delta(k' - p)\,\mathtt{r}_L(k, p') \\ - e^{i(k+k')x}\delta(k - p')\,\mathtt{r}_L^*(k', p)\big], \quad (26)$$

$$C_{RR} = -(k + k')e^{-i(k-k')x}\mathtt{t}_L^*(k', p)\,\mathtt{t}_L(k, p'), \quad (27)$$

$$C_{LR} = -(k + k')e^{-i(k-k')x}\mathtt{r}_L^*(k', p)\,\mathtt{t}_L(k, p') \\ + (k' - k)e^{-i(k+k')x}\delta(k' - p)\,\mathtt{t}_L(k, p'), \quad (28)$$

$$C_{RL} = -(k + k')e^{-i(k-k')x}\mathtt{r}_L(k, p')\,\mathtt{t}_L^*(k', p) \\ - (k' - k)e^{i(k+k')x}\delta(k - p')\,\mathtt{t}_L^*(k', p). \quad (29)$$

For the current to the right of the scatterer, one obtains an analogous expression with $R \leftrightarrow L$. After the statistical averaging, the current in the left lead takes a form

$$\langle \hat{I}(x, t)\rangle = \frac{\hbar e}{4\pi m}\int dk\,dk'\,dp\,e^{i(E_{k'} - E_k)t}[f_L(p)C_{LL}(k, k'; p, p; x) + f_R(p)C_{RR}(k, k'; p, p; x)]. \quad (30)$$

Note that, in the presence of inelastic scattering, the expectation value of the current depends on time. In real experiments, to obtain the average current, one also performs the time averaging. We assume that this time averaging is done with a function $g_T(t)$ defining a wide time window τ_{av}—the characteristic time of averaging. This time is the largest timescale in the problem. The window function $g_T(t)$ has the following properties:

$$\int_{-\infty}^{\infty} dt\, g_T(t) = 1, \quad (31)$$

$$\tau_{\text{av}}\int_{-\infty}^{\infty} dt\, g_T(t)e^{i(E_k - E_{k'})t} \sim \delta(E_k - E_{k'}). \quad (32)$$

We will denote the averaging with $g_T(t)$ as $\langle \ldots \rangle_t$. With the help of this averaging, a useful relation can be obtained from the flow conservation for each incident p:

$$\int dk\,dk'\,\langle e^{i(E_k - E_{k'})t}\rangle_t[C_{LL}(k, k'; p, p; x) + C_{RR}(k, k'; p, p; x)] = 0. \quad (33)$$

Let us now define time-dependent scattering amplitudes ($j = R, L$):

$$\int dk\, e^{-iE_k t}\mathtt{t}_j(k, p) = \tilde{\mathtt{t}}_j(t, p), \quad (34)$$

$$\int dk\, e^{-iE_k t}\mathtt{r}_j(k, p) = \tilde{\mathtt{r}}_j(t, p). \quad (35)$$

The flow conservation Eq. (33) can then be cast into the following form [cf. Eq. (24)]:

$$1 = \langle \tilde{\mathtt{r}}_L^*(t, p)\tilde{\mathtt{r}}_L(t, p)\rangle_t + \langle \tilde{\mathtt{t}}_L^*(t, p)\tilde{\mathtt{t}}_L(t, p)\rangle_t. \quad (36)$$

Using this, we can find the average current,

$$\langle\langle \hat{I}(x, t)\rangle\rangle_t = \frac{e}{h}\int dp\, v(p)\,\langle \tilde{\mathtt{t}}_L^*(t, p)\tilde{\mathtt{t}}_L(t, p)\rangle_t \cdot [f_L(p) - f_R(p)], \quad (37)$$

where $v(p)$ is the velocity at momentum p. From now on, we will assume, for simplicity, a linearized dispersion relation for electrons, so v will not depend on p. The transmission coefficient in the inelastic case does not depend on whether the waves are supplied by the left terminal or by the right one:

$$\mathcal{T}(p) = \langle \tilde{\mathtt{t}}_R^*(t, p)\tilde{\mathtt{t}}_R(t, p)\rangle_t = \langle \tilde{\mathtt{t}}_L^*(t, p)\tilde{\mathtt{t}}_L(t, p)\rangle_t. \quad (38)$$

With this transmission coefficient, we get the conventional Landauer formula for the average current:

$$\langle\langle \hat{I}(x, t)\rangle\rangle_t = \frac{ev}{h}\int dp\, \mathcal{T}(p)[f_L(p) - f_R(p)]. \quad (39)$$

Next, we derive the expression for the correlation function of current fluctuations in the left lead in the *inelastic scattering* case:

$$\langle \delta\hat{I}(x, t)\delta\hat{I}(x', t')\rangle = \left(\frac{\hbar e}{4\pi m}\right)^2 \int dk\,dk'\,dp\,dp'\,dq\,dq'\,e^{i(E_{k'} - E_k)t + i(E_{q'} - E_q)t'} \\ \times [f_L(p)[1 - f_L(p')]C_{LL}(k, k'; p, p'; x)C_{LL}(q, q'; p', p; x') \\ + f_R(p)[1 - f_R(p')]C_{RR}(k, k'; p, p'; x)C_{RR}(q, q'; p', p; x')$$

$$+ f_L(p)[1 - f_R(p')]C_{LR}(k, k'; p, p'; x)C_{RL}(q, q'; p', p; x')$$
$$+ f_R(p)[1 - f_L(p')]C_{RL}(k, k'; p, p'; x)C_{LR}(q, q'; p', p; x')]. \tag{40}$$

The noise power in the inelastic case is define through the time-average of the correlation function Eq. (40) as follows:

$$S(\omega = 0) = \int dt' \langle \langle \delta \hat{I}(x, t + t') \delta \hat{I}(x', t) \rangle \rangle_t. \tag{41}$$

Substituting Eq. (40) into Eq. (41) and using Eqs. (26)–(29), (34), and (35), we obtain a general expression for the current noise in the left lead in terms of inelastic scattering amplitudes:

$$S_L = S_{\text{therm}} + S_{\text{shot}}, \tag{42}$$

$$S_{\text{therm}} = \frac{e^2}{h} \int dk \, v \, \langle \tilde{\mathfrak{t}}_L^*(t, k) \tilde{\mathfrak{t}}_L(t, k) \rangle_t \{ f_L(k)[1 - f_L(k)] + f_R(k)[1 - f_R(k)] \}, \tag{43}$$

$$S_{\text{shot}} = \frac{e^2}{h} \int dp \, dp' \, dk \, v \, \{ f_L(p)[1 - f_R(p')] \, \mathfrak{r}_L^*(k, p) \mathfrak{t}_L(k, p') \, \langle \tilde{\mathfrak{t}}_L^*(t, p') \tilde{\mathfrak{r}}_L(t, p) \rangle_t$$
$$+ f_R(p)[1 - f_L(p')] \, \mathfrak{t}_L^*(k, p) \mathfrak{r}_L(k, p') \, \langle \tilde{\mathfrak{r}}_L^*(t, p') \tilde{\mathfrak{t}}_L(t, p) \rangle_t$$
$$- f_L(p)[1 - f_L(p')] \, \mathfrak{r}_L^*(k, p) \mathfrak{r}_L(k, p') \, \langle \tilde{\mathfrak{t}}_L^*(t, p') \tilde{\mathfrak{t}}_L(t, p) \rangle_t$$
$$- f_R(p)[1 - f_R(p')] \, \mathfrak{r}_L^*(k, p) \mathfrak{r}_L(k, p') \, \langle \tilde{\mathfrak{t}}_L^*(t, p') \tilde{\mathfrak{t}}_L(t, p) \rangle_t \}. \tag{44}$$

Here we split the total noise into the thermal part S_{therm}, which vanishes exactly when the temperatures of the reservoirs are sent to zero, and the remaining shot-noise part S_{shot}, which remains finite in this limit. When the scattering is elastic, i.e., when $\mathfrak{t}_j(k, p) = \delta(k - p)\mathfrak{t}(p)$ and the same holds for the reflection amplitudes, Eqs. (42)–(44) reduce to Eq. (19). In particular, when contacts are kept at $T = 0$, Eq. (44) becomes, for elastic scattering, Eq. (20).

For the noise in the right lead, one performs a replacement $R \leftrightarrow L$, as usual. Equations (42)–(44) are the central result of this paper. In what follows, they will be used to calculate the shot noise in a double-barrier structure in the presence of inelastic electron-phonon scattering.

III. TRANSMISSION COEFFICIENT WITH DEPHASING

In Sec. III, we have derived a general formula, Eq. (38), for the transmission coefficient describing an inelastic scatterer. In this section, we will obtain an expression for the transmission coefficient for a 1D double-barrier structure in the presence of a time-dependent potential. We will then use this result to compute the transmission coefficient for the case of inelastic scattering due to electron-phonon interaction. This will allow us to calculate the conductance of the structure and to analyze the role of phonon-induced dephasing in resonant tunneling. As a warmup, we analyze in Appendix A the case of a time-dependent single barrier modeled by a delta-function potential. Next, we generalize that consideration to the spatially extended double-barrier setup and introduce two microscopic models of phonon-induced dephasing.

A. Transmission coefficient for a double barrier structure with time-dependent potential

We have considered a simple example of transmission through a time-dependent delta barrier in Appendix A and derived the inelastic transmission amplitude $\mathfrak{t}(p, k)$ for this toy model. In this section, we obtain the transmission coefficient

$\mathcal{T}(p)$ of a 1D quantum dot with tunnel contacts in the presence of a random time-dependent potential $V(x, t)$ and then perform averaging over realizations of the phonon-induced potential.

We assume that the random potential is applied only inside the quantum dot formed by the two barriers. We further assume that the magnitude of the potential is smaller than the electron's kinetic energy. Next, the potential is considered to be smoothly varying both in space and in time. This will allow us to neglect the electron backscattering induced by the potential ($r_c k \gg 1$, where k is the wave vector of an electron and r_c is a characteristic spatial scale of the potential) and transitions between the levels of size quantization inside the dot,

$$\partial V(x, t)/\partial t \ll V(x, t)/\tau_f,$$

where $\tau_f = 2L/v$ is the time of flight back and forth between the barriers separated by distance L.

Under these assumptions and upon linearization of the electron's dispersion, $E_k \approx \hbar v k$, the influence of the random potential on the electron wave function reduces to the appearance of a random phase factor,

$$\psi(x, t) = \exp \left(ikx - i\frac{E}{\hbar}t \right)$$
$$\times \exp \left[-\frac{i}{\hbar} \int^t d\tau \, V(x - vt + v\tau, \tau) \right], \tag{45}$$

$$\text{for} \quad \Delta \tau_c \gg \hbar, \quad k r_c \gg 1, \quad |V(x, t)| \ll \hbar v k, \tag{46}$$

where

$$\Delta = 2\pi \hbar/\tau_f$$

is the interlevel energy spacing inside the quantum dot formed by the barriers. We can introduce here the already mentioned random addition to the electron wave function phase:

$$\varphi_f = \frac{1}{\hbar} \int^t d\tau \, V(x - vt + v\tau, \tau).$$

With this form for the wave function, we obtain a general expression for transmission amplitude for a double barrier structure in the presence of a given realization of weak random potential Eq. (46). The two pointlike barriers labeled by $i = 1, 2$ are located at $x = 0$ and $x = L$ and are characterized

by their individual transmission and reflection amplitudes, t_i and r_i, respectively (for each of the barriers, its right and left reflection amplitudes are equal). As shown in Appendix B, the inelastic transmission amplitude t_R for electrons transmitted to the right lead is given by

$$\mathsf{t}_R(p, k) = \int_{-\infty}^{\infty} dy \, e^{i(k-p)y} \mathsf{t}_1 \mathsf{t}_2 \sum_{n=0}^{\infty} \left(\mathsf{r}_1 \mathsf{r}_2 e^{2ikL} \right)^n \exp\left[-\frac{i}{\hbar} \int_{-\tau_f n}^{0} d\tau \, V(x_{\mathrm{in}}(\tau), \tau - y/v) \right], \quad \tau_f = 2L/v, \qquad (47)$$

where $x_{\mathrm{in}}(t)$ is the trajectory of a particle between the barriers, which consists of ballistic segments of length L. Using Eq. (47), the transmission coefficient (38) takes the following form:

$$\mathcal{T}(k) = \langle \mathsf{t}_R^*(t, k) \mathsf{t}_R(t, k) \rangle_t = |\mathsf{t}_1|^2 |\mathsf{t}_2|^2 \sum_{n_1, n_2} \left(\mathsf{r}_1 \mathsf{r}_2 e^{2ikL} \right)^{n_1} \left(\mathsf{r}_1^* \mathsf{r}_2^* e^{-2ikL} \right)^{n_2} \left\langle \exp\left[-\frac{i}{\hbar} \int_{-\tau_f n_1}^{-\tau_f n_2} d\tau \, V(x_{\mathrm{in}}(\tau), \tau + t) \right] \right\rangle_t, \qquad (48)$$

The transmission coefficient Eq. (48) in the absence of dephasing potential reduces to the well-known expression

$$\mathcal{T}(k) = \frac{|\mathsf{t}_1 \mathsf{t}_2|^2}{|1 - \mathsf{r}_1 \mathsf{r}_2 e^{2ikL}|^2}.$$

Now the task is to perform the averaging of the exponential factor in Eq. (48) over the random field. For the Gaussian distribution of $V(x, t)$, the averaging can be done exactly. Furthermore, assuming that the characteristic correlation time τ_c for the variation of $V(x, t)$ is longer than the flight time τ_f, one can first average $V(x_{\mathrm{in}}(\tau), \tau + t)$ over the position between the barriers, yielding an effective time-dependent potential $U(\tau + t)$. Then the averaging over fluctuations of this potential gives rise to the dephasing factor,

$$f(t) \equiv \langle e^{i \int_0^t d\tau \, U(\tau)} \rangle \qquad (49)$$

$$= \exp\left\{ \frac{\left[\int_0^t d\tau \, K(\tau) \right]^2}{2K(0)} - \frac{1}{2} \int_0^t \int_0^t d\tau d\tau' K(\tau - \tau') \right\}$$

$$= \begin{cases} 1, & t \ll \tau_c \\ e^{-\int_0^t \int_0^t d\tau d\tau' K(\tau - \tau')/2}, & t \gg \tau_c, \end{cases} \qquad (50)$$

expressed through the correlation function

$$K(\tau - \tau') = \langle U(\tau) U(\tau') \rangle_U, \qquad (51)$$

where averaging is performed over the random potential fluctuations. In the case of the phonon potential, this implies averaging over the random phases of phonon modes (in the classical approach) or with the phonon density matrix (quantum).

Dephasing processes can affect the transmission coefficient only when the correlation time of the potential fluctuations is smaller than time τ_{in} that the particle spends between the barriers, i.e., time flight multiplied by resonator quality factor. Therefore, we further assume the following time hierarchy:

$$\tau_f \ll \tau_c \ll \tau_{\mathrm{in}},$$

and use, in what follows, the second line of Eq. (50) for $f(t)$.

The applicability of Eq. (47) in a parabolic spectrum is limited by the use of a quasiclassical wave function corresponding to the linearized dispersion and classical external potential $V(x, t)$. This assumes that the change of particle's

velocity v is small in the process of the backscattering off the barriers in the presence of a fluctuating field, leading to the heating or cooling of the particle in addition to dephasing. The corresponding condition can be expressed through the quality factor Q of the resonator, as follows:

$$Q \ll \frac{v \tau_c}{\min[r_c, L]} \left(\frac{E}{V} \right)^2,$$

where V is the characteristic magnitude of the fluctuating potential. Indeed, if the resonator's quality factor is very high and the particle is influenced by the random potential for a very long time, its speed (and, hence, energy) may change. When considering the case of electron-phonon interaction, a rather small rate of quantum spontaneous phonon emission is also required to justify the quasiclassical approach.

B. Transmission for the diffusion type of dephasing

Equation (49) can be simplified in the case of diffusion-type phase dynamics when $K \tau_c^2 / \hbar^2 \ll 1$, where $K = K(0)$ is the square of a characteristic magnitude of the effective random potential, see Eq. (51). In this regime, the random-sign changes, $\delta \varphi_f$, of the phase of the electron wave function, caused by the potential during time τ_c, are small: $\delta \varphi_f \ll 1$. The total change of the electronic phase φ_f, i.e., the sum of these random elementary changes, will then show a phase diffusion at long times:

$$\langle [(\varphi_f(t) - \varphi_f(0)]^2 \rangle \propto t.$$

For $t \gg \tau_c$ in the diffusion regime of dephasing, we can rewrite the dephasing factor, Eq. (50), in the conventional exponential form:

$$f(t) \approx e^{-t/\tau_\varphi}. \qquad (52)$$

Here we introduced the dephasing time τ_φ according to

$$\frac{1}{\tau_\varphi} = \int_0^{\infty} \frac{d\tau}{\hbar^2} K(\tau). \qquad (53)$$

The inverse dephasing time is the phase diffusion coefficient in this case. It may turn out that the time integral of the correlator Eq. (51) that enters Eq. (53) vanishes. We will see

below that this situation is realized for the phonon-induced potential. In this case, as we will show below, one needs to calculate exactly the double time integral entering Eq. (49).

$$\mathcal{T}(k) = \frac{|t_1|^2 |t_2|^2 [1 - |r_1|^2 |r_2|^2 \exp(-2\tau_f/\tau_\varphi)]}{(1 - |r_1|^2 |r_2|^2)[1 - r_1 r_2 \exp(2ikL - \tau_f/\tau_\varphi)][1 - r_1^* r_2^* \exp(-2ikL - \tau_f/\tau_\varphi)]}. \tag{54}$$

Equation (54) can be expanded near one resonant level, where it acquires the following, valid for an arbitrary dephasing rate:

$$\mathcal{T}(\delta E) = \frac{\Gamma_1 \Gamma_2}{\Gamma_\Sigma} \frac{\frac{\hbar}{\tau_f} \sinh(\tau_f/\tau_\varphi) + \Gamma_\Sigma e^{-\tau_f/\tau_\varphi}}{\left[\frac{2\hbar}{\tau_f} \sinh\left(\frac{\tau_f}{2\tau_\varphi}\right) + \Gamma_\Sigma e^{-\tau_f/2\tau_\varphi}\right]^2 + \delta E^2}. \tag{55}$$

Here, $\Gamma_1 = |t_1|^2 \hbar/\tau_f$ and $\Gamma_2 = |t_2|^2 \hbar/\tau_f$ are the transmission coefficients of individual barriers, $\Gamma_\Sigma = (\Gamma_1 + \Gamma_2)/2$, and δE is the deviation of the electron's energy from the resonance (we assumed a small transparency for both barriers, $|t_i| \ll 1$).

Transmission across the double barrier structure can be considered in three regimes depending on the dephasing rate. If the dephasing rate is small, $\tau_{in} \sim \tau_\varphi$, the resonant coherent tunneling is realized and we can transform Eq. (55) into the conventional Breit-Wigner form:

$$\mathcal{T}(\delta E) = \frac{\Gamma_1 \Gamma_2}{\Gamma_\Sigma} \frac{\Gamma_\Sigma + \hbar/\tau_\varphi}{(\Gamma_\Sigma + \hbar/\tau_\varphi)^2 + \delta E^2}. \tag{56}$$

Such a dephasing-broadened Breit-Wigner resonance was first obtained for this model by Stone and Lee [12]. It is seen that the resulting quasilevel's width consists of two parts:

$$\tilde{\Gamma} = \Gamma_\Sigma + \hbar/\tau_\varphi, \tag{}$$

governed by the transparency of barriers and the dephasing rate.

In the case of strong dephasing $\hbar/\tau_\varphi \sim \Delta$, the regime of classical tunneling sets in. In this case, particles transmit across the double-barrier structure like a classical object and the total transmission coefficient is determined by the classical transmission probabilities of individual barriers. It is worth noting that in this regime Eq. (54) is formally not applicable, since transitions between the size-quantization levels start to play an important role in the quantum-mechanical description. However, Eq. (54) still yields a correct classical result for $\mathcal{T}(k)$:

$$\mathcal{T} = \frac{|t_1|^2 |t_2|^2}{1 - |r_1|^2 |r_2|^2} = \frac{\mathcal{T}_1 \mathcal{T}_2}{\mathcal{T}_1 + \mathcal{T}_2 - \mathcal{T}_1 \mathcal{T}_2}. \tag{57}$$

The result is obtained from Eq. (54) and is valid for an arbitrary transparency of the barriers. For strong barriers, one can directly use Eq. (55), which, in the limit of strong dephasing, yields

$$\mathcal{T} \approx \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \frac{2\pi}{\Delta}. \tag{58}$$

For intermediate values of the dephasing rate, $1/\tau_f > 1/\tau_\varphi > 1/\tau_{in}$, the regime of coherent sequential tunneling is realized, where the transmission coefficient is described by Eq. (55).

Now let us calculate the transmission coefficient Eq. (48) for diffusion-type phase dynamics Eq. (52). In this case, the transmission coefficient can be calculated exactly:

C. Transmission for nondiffusive dephasing

In some cases, the diffusion approximation for the phase dynamics may fail. In particular, for the case of electron-phonon interaction considered in detail in Sec. V below, logarithmic phase dynamics,

$$\langle [\varphi_f(t) - \varphi_f(0)]^2 \rangle = \gamma \ln(t/\tau_c), \tag{}$$

occurs for $t > \tau_c$ instead of phase diffusion. This gives rise to a power-law time dependence of the dephasing factor in Eq. (49):

$$f(t) \approx e^{-\gamma \ln(t/\tau_c)} = \left(\frac{\tau_c}{t}\right)^\gamma. \tag{59}$$

We will refer to this type of dephasing as logarithmic dephasing.

Now we calculate the transmission coefficient for logarithmic-type phase dynamics Eq. (59). The calculation is a somewhat less straightforward. First, Eq. (48) with the power-law dephasing factor Eq. (59) yields

$$\mathcal{T}(k) = |t_1|^2 |t_2|^2 \sum_{n_1, n_2} (r_1 r_2 e^{2ikL})^{n_1} (r_1^* r_2^* e^{-2ikL})^{n_2}$$

$$\times \left(\frac{1}{|n_1 - n_2|\tau_f/\tau_c + 1}\right)^\gamma. \tag{60}$$

Near the resonance, the summation over n_1 and n_2 in Eq. (60) can be performed, leading to

$$\mathcal{T}(\delta E) = \frac{\Gamma_1 \Gamma_2 \tau_c}{2\hbar} [e^\lambda \mathcal{E}_\gamma(\lambda) + e^{\lambda^*} \mathcal{E}_\gamma(\lambda^*)], \tag{61}$$

where we introduced

$$\lambda = \frac{\Gamma_\Sigma + i\delta E}{\hbar/\tau_c}, \tag{62}$$

and

$$\mathcal{E}_\gamma(\lambda) = \int_1^\infty dx \, x^{-\gamma} e^{-\lambda x} \tag{63}$$

is the exponential integral function. Specifically, to calculate the sum in Eq. (60) we used the following integral representation of a power-law function:

$$\frac{1}{z^\gamma} = \frac{1}{\Gamma(\gamma)} \int_0^\infty dx \, x^{\gamma-1} e^{-zx}, \tag{64}$$

with $\Gamma(\gamma)$ the gamma function. This amounts to introducing an effective exponential dephasing factor Eq. (52) with τ_φ depending on x, followed by averaging over x. By using this, we calculated the sum in Eq. (60) for a given value of x, exactly as was done for the exponential dephasing factor in Eq. (54). The last integral over x in Eq. (64) can be calculated exactly for one size quantization level. We expanded the integrand
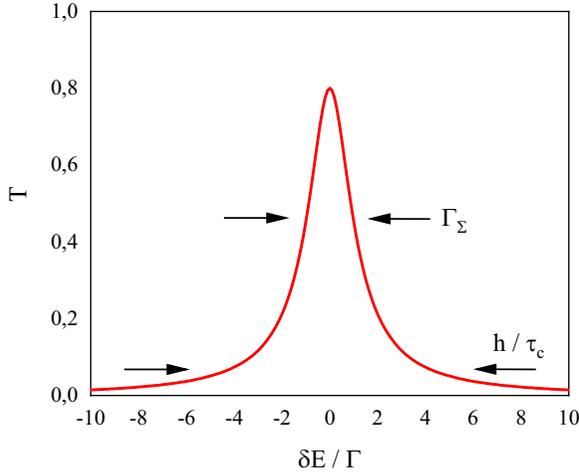
FIG. 2. Transmission coefficient Eq. (61) for the logarithmic dephasing has two energy scales: one is determined by the transparency of barriers through Γ_Σ and the other by the correlation time τ_c.

near the resonant level Eq. (56), multiply it with $x^{-\gamma}$, and obtain Eq. (61) after the integration over x. The integral can be converted to the integral Eq. (63) which starts from unity instead of zero.

The transmission coefficient Eq. (61) for logarithmic dephasing is not described by a standard Lorentzian Breit-Wigner energy dependence. Instead, it is characterized by two energy scales, Γ_Σ and $\hbar/\tau_c$, see Fig. 2. Assuming $\gamma \ll 1$, we can simplify Eq. (61):

$$\mathcal{T}(\delta E) \approx \frac{\Gamma_1 \Gamma_2 \tau_c}{2\hbar} \left[\left(\frac{\hbar/\tau_c}{\Gamma_\Sigma + i\delta E} \right)^{1-\gamma} + \left(\frac{\hbar/\tau_c}{\Gamma_\Sigma - i\delta E} \right)^{1-\gamma} \right].$$

(65)

For energies $\delta E \gg \Gamma_\Sigma$, the transmission coefficient gradually decreases on scale $\hbar/\tau_c$.

When $K\tau_c^2/\hbar^2 \gg 1$, yet another regime of wave function phase growing is realized, which we term the ballistic regime. This means that the random change of the wave-function phase is large, $\delta\varphi_f \gg 1$, on the potential correlation timescale. In the ballistic case, the dephasing exponent in the dephasing factor is given by

$$\ln f(t) \approx -\frac{t^2 K(0)}{2\hbar^2},$$

which leads to the characteristic phase decay rate:

$$1/\tau_\varphi \approx \sqrt{K(0)}/\hbar.$$

D. Conductance

The linear conductance across a system is calculated by means of the Landauer formula that follows from Eq. (39),

$$G = -\frac{e^2}{h} \int dE \, \frac{\partial f}{\partial E} \langle \mathcal{T}(E) \rangle,$$

(66)

where f is the Fermi-Dirac distribution function. At low temperatures $T \ll \tilde{\Gamma} \ll \Delta$, one finds

$$G = \frac{e^2}{h} \sum_n \frac{\Gamma_1 \Gamma_2}{\Gamma_\Sigma} \frac{\tilde{\Gamma}}{\tilde{\Gamma}^2 + (E_n - \mu)^2},$$

(67)

where μ is the Fermi level in the leads and E_n is the energy of the nth quasilevel. Here, one needs to make a remark that, for logarithmic dephasing, the result may, in general, depend on the ratio of T and $\hbar/\tau_c$. If $T \ll \hbar/\tau_c$, one has to use in Eq. (67) the transmission coefficients in the form of Eq. (65). However, for the relevant case of phonon-induced dephasing, as we show later in Sec. V, the correlation time τ_c is determined by the temperature, $\hbar/\tau_c = T$. As a consequence, for $T \ll \Gamma_\Sigma$ the transmission coefficient Eq. (61) results in Eq. (67) with replacing $\tilde{\Gamma} \to \Gamma_\Sigma$.

If the temperature exceeds the level width, $\tilde{\Gamma} \ll T \ll \Delta$, we get

$$G = \frac{e^2}{h} \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \sum_n \frac{\pi}{2T \cosh^2 \left(\frac{E_n - \mu}{2T} \right)}.$$

(68)

Note that the conductance is independent of the dephasing rate, and hence on the mechanism of dephasing, already in this regime. For temperatures that are higher than the level spacing, $\Delta \ll T$, the conductance reads

$$G = \frac{e^2}{h} \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \frac{2\pi}{\Delta}.$$

(69)

This result can also be obtained by means of the classical consideration that corresponds to Eq. (58) for the classical transmission coefficient.

IV. SHOT NOISE

In this section, we analyze shot noise in the double barrier structure. As we mentioned before, the Fano factor is defined by shot noise at zero temperature. Therefore, in this section we will assume that the leads are kept at $T = 0$. Also, to simplify equations we assume that applied voltage is $2V$. We start from the fully coherent case:

$$F = \frac{\int_{\mu-eV}^{\mu+eV} dE \, \mathcal{T}(E)[1 - \mathcal{T}(E)]}{\int_{\mu-eV}^{\mu+eV} dE \, \mathcal{T}(E)}$$

$$= \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} - \frac{2\Gamma_1\Gamma_2}{(\Gamma_1 + \Gamma_2)^2} \frac{eV\Gamma_\Sigma}{\Gamma_\Sigma^2 + (eV)^2} \frac{1}{\arctan(eV/\Gamma_\Sigma)}.$$

(70)

Here and in what follows, we have assumed that the Fermi level is exactly at one of the size quantization levels. This equation is valid irrespective of the relation between eV and Δ.

For the inelastic case, in the presence of random potential, it is useful to write the numerator of Eq. (70), i.e., shot noise Eq. (44), using the sum representation. As shown in Appendix B, reflection amplitudes for the double barrier structure can be represented as

$$r_{R,L}(k, p) = \frac{r_2 e^{2ikL} - r_1^*}{t_1^* t_2} t_{L,R}(k, p).$$

(71)

Using this, we have for $T = 0$:

$$
\begin{aligned}
S_L &= \frac{e^2}{h} \int_{(\mu - eV)/v}^{(\mu + eV)/v} dk\, v\, |\mathbf{r}_1 e^{ikL} - \mathbf{r}_2^* e^{-ikL}|^2 \\
&\quad \times \sum_{\{n_i\}} (\mathbf{r}_1 \mathbf{r}_2 e^{2ikL})^{n_1 + n_3} (\mathbf{r}_1^* \mathbf{r}_2^* e^{-2ikL})^{n_2 + n_4} \\
&\quad \times \left\langle \exp\left(-\frac{i}{\hbar} \int_{\tau_f(n_4 - n_3)}^{\tau_f(n_1 - n_2)} d\tau\, U(\tau) \right) \right\rangle_t.
\end{aligned}
\tag{72}
$$

For $U(\tau) = 0$, we immediately arrive at Eq. (70).

Let us now analyze the shot noise for diffusion-type dephasing:

$$
\begin{aligned}
&\left\langle \exp\left(-\frac{i}{\hbar} \int_{\tau_f(n_4 - n_3)}^{\tau_f(n_1 - n_2)} d\tau\, U(\tau) \right) \right\rangle_t \\
&= \exp\left(-\frac{\tau_f}{\tau_\varphi} |n_1 - n_2 + n_3 - n_4| \right).
\end{aligned}
\tag{73}
$$

We can represent this exponential factor as follows:

$$
\exp(-|n|\tau_f/\tau_\varphi) = \int \frac{dy}{\pi} \frac{\tau_f/\tau_\varphi}{(\tau_f/\tau_\varphi)^2 + y^2} e^{iny}.
\tag{74}
$$

Next, we use $\Gamma_\varphi = \hbar/\tau_\varphi$ to simplify equations. With this representation, we can use the expression for shot noise in the coherent case [i.e., Eq. (72) with $U(\tau) = 0$], replace there $k \to k + y$, and perform the integration over $\delta = y\hbar/\tau_f$:

$$
\begin{aligned}
S_L &= \frac{e^2}{h} \int_{\mu - eV}^{\mu + eV} dE \int_{-\infty}^{\infty} \frac{d\delta}{\pi} \frac{\Gamma_\varphi}{\Gamma_\varphi^2 + \delta^2} \\
&\quad \times \mathcal{T}(E + \delta)[1 - \mathcal{T}(E + \delta)].
\end{aligned}
\tag{75}
$$

This results in the following Fano factor for diffusive dephasing:

$$
\begin{aligned}
F_{\text{diff}} &= \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} \\
&\quad - \frac{2\Gamma_1 \Gamma_2}{(\Gamma_1 + \Gamma_2)^2} \frac{eV\Gamma_\Sigma}{(\Gamma_\Sigma + \Gamma_\varphi)^2 + (eV)^2} \frac{1}{\arctan\left(\frac{eV}{\Gamma_\Sigma + \Gamma_\varphi}\right)}.
\end{aligned}
\tag{76}
$$

Compared with the result for the coherent case, Eq. (70), the diffusive dephasing leads to the replacement $\Gamma_\Sigma \to \Gamma_\Sigma + \Gamma_\varphi$ everywhere except for the numerator of the second term.

Now we calculate the shot noise for the case of the logarithmic dephasing. For this, we use the same trick as for the calculation of the transmission coefficient:

$$
\begin{aligned}
&\left\langle \exp\left(-\frac{i}{\hbar} \int_{\tau_f(n_4 - n_3)}^{\tau_f(n_1 - n_2)} d\tau\, U(\tau) \right) \right\rangle_t \\
&= \left(\frac{1}{|n_1 - n_2 + n_3 - n_4|\tau_f/\tau_c + 1} \right)^\gamma = \int_0^\infty dy \frac{y^{\gamma - 1} e^{-y}}{\Gamma(\gamma)} \\
&\quad \times \int \frac{dx}{\pi} \frac{y\tau_f/\tau_c}{(y\tau_f/\tau_c)^2 + x^2} e^{i(n_1 - n_2 + n_3 - n_4)x}.
\end{aligned}
\tag{77}
$$

For logarithmic dephasing, we can introduce the dephasing strength, similarly to the diffusion type:

$$
\Gamma_\varphi = \hbar/\tau_c.
$$

Then, for the calculation of the Fano factor, we can perform the same procedure as for diffusion-type phase dynamics:

$$
F_{\log} = \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} - \frac{2\Gamma_1 \Gamma_2}{(\Gamma_1 + \Gamma_2)^2} \frac{G}{H},
\tag{78}
$$

$$
G = \int \frac{dy}{\Gamma(\gamma)} y^{\gamma - 1} e^{-y} \frac{eaV\Gamma_\Sigma}{(eV)^2 + (\Gamma_\Sigma + y\Gamma_\varphi)^2},
\tag{79}
$$

$$
H = \int \frac{dy}{\Gamma(\gamma)} y^{\gamma - 1} e^{-y} \arctan\left(\frac{eV}{\Gamma_\Sigma + y\Gamma_\varphi} \right).
\tag{80}
$$

Using these equations, we can distinguish three different regimes:

$$
\Gamma_\Sigma \ll \Gamma_\varphi \ll eV:
$$

$$
F_{\log} = F_{\text{diff}} = \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} - \frac{2\Gamma_1 \Gamma_2}{(\Gamma_1 + \Gamma_2)^2} \frac{2}{\pi}\left(\frac{\Gamma_\Sigma}{eV} \right),
\tag{81}
$$

$$
\Gamma_\Sigma \ll eV \ll \Gamma_\varphi:
$$

$$
F_{\log} = \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} - \frac{2\Gamma_1 \Gamma_2}{(\Gamma_1 + \Gamma_2)^2} \frac{2}{\pi}\left(\frac{\Gamma_\Sigma}{\Gamma_\varphi} \right)^\gamma \left(\frac{\Gamma_\Sigma}{eV} \right)^{1 - \gamma},
\tag{82}
$$

$$
F_{\text{diff}} = \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} - \frac{2\Gamma_1 \Gamma_2}{(\Gamma_1 + \Gamma_2)^2} \frac{\Gamma_\Sigma}{\Gamma_\varphi},
\tag{83}
$$

$$
eV \ll \Gamma_\Sigma \ll \Gamma_\varphi:
$$

$$
F_{\log} = \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} - \frac{2\Gamma_1 \Gamma_2}{(\Gamma_1 + \Gamma_2)^2},
\tag{84}
$$

$$
F_{\text{diff}} = \frac{\Gamma_1^2 + \Gamma_2^2}{(\Gamma_1 + \Gamma_2)^2} - \frac{2\Gamma_1 \Gamma_2}{(\Gamma_1 + \Gamma_2)^2} \frac{\Gamma_\Sigma}{\Gamma_\varphi}.
\tag{85}
$$

The main difference between the logarithmic and diffusion types of dephasing is now clearly seen. For the diffusion type of dephasing, the Fano factor is insensitive to the applied voltage when it is lower then the dephasing-induced contribution to the level width Γ_φ. For larger voltages, the Fano factor is given by the well-known result, with a small correction $\propto 1/eV$. The Fano factor in the case of logarithmic dephasing has three different regimes depending on applied voltage. If the voltage is lower than the dephasing width Γ_φ but higher than the elastic width Γ_Σ, the Fano factor depends on the voltage, in contrast to F_{diff}. This difference can be seen in Fig. 3. In both cases, when the voltage exceeds the elastic width Γ_Σ, the Fano factor approaches the universal value Eq. (3) that can be obtained in the classical consideration.

V. PHONON INDUCED DEPHASING

In this section, we will study the influence of acoustic phonons on the transmission and current fluctuations. We assume sufficiently low temperatures when both backscattering and electron transitions between levels of quantization in the double-barrier structure due to their interaction with phonons can be neglected [36]. Cases of 1D, 2D, and 3D phonons will be considered. In experiment, all three possibilities can be realized.
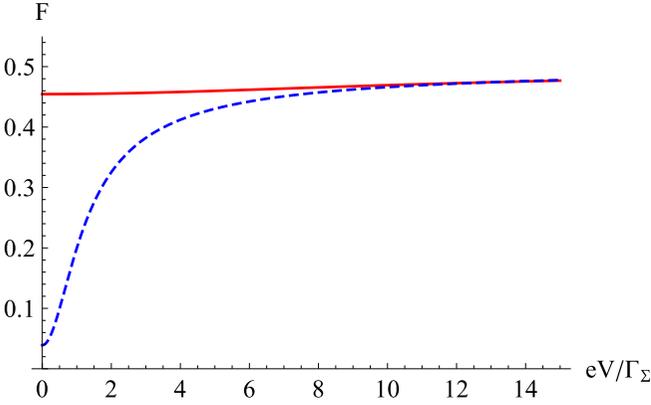
FIG. 3. The Fano factor for different types of dephasing as a function of applied voltage. Red solid line is for diffusion-type dephasing, Eq. (76); blue dashed line is for the logarithmic type, Eq. (78). Parameters for the plot: $\Gamma_1 = \Gamma_2 = \Gamma_\Sigma$, $\Gamma_\varphi/\Gamma_\Sigma = 10$, and $\gamma = 0.1$.

Interaction with acoustic phonons is governed by the Hamiltonian:

$$\hat{V}_{\text{e-ph}} = g\,\varphi(\mathbf{r}, t), \tag{86}$$

$$\varphi(\mathbf{r}, t) = \sum_{\mathbf{k}} \sqrt{\frac{\omega_k}{2V}} (ib_{\mathbf{k}} e^{i\mathbf{k}\mathbf{r} - i\omega_k t} - ib_{\mathbf{k}}^* e^{-i\mathbf{k}\mathbf{r} + i\omega_k t}), \tag{87}$$

where in the last equation only longitudinal phonons are included, s is the speed of longitudinal phonons, and $\omega_k = sk$. Note that g has different dimensions in the three cases. The average of phonon field amplitudes is given by

$$\langle b_{\mathbf{k}} b_{\mathbf{p}}^* \rangle = \langle b_{\mathbf{p}}^* b_{\mathbf{k}} \rangle = n_k \delta_{\mathbf{k}\mathbf{p}},$$

where n_k is the Bose distribution function. The correlators of the phonon fields for different spatial dimensions are

$$\text{1D:} \quad K_{\text{1D}}(x, t) = \frac{g^2 s}{4\pi r_c^2} \left[\frac{r_c^2}{(x + st)^2} - \frac{\pi^2}{\sinh^2\left(\frac{\pi(x+st)}{r_c}\right)} \right.$$

$$\left. + \frac{r_c^2}{(x - st)^2} - \frac{\pi^2}{\sinh^2\left(\frac{\pi(x-st)}{r_c}\right)} \right], \tag{88}$$

$$\text{2D:} \quad K_{\text{2D}}(x, t) = \frac{g^2}{2\pi} \int dk\, k\omega_k n_k \cos(\omega_k t) J_0(kx), \tag{89}$$

$$\text{3D:} \quad K_{\text{3D}}(x, t) = -\frac{1}{2\pi x} \frac{\partial K_{\text{1D}}(x, t)}{\partial x}, \tag{90}$$

where

$$\tau_c = \frac{\hbar}{T_{\text{ph}}}, \quad r_c = s\tau_c, \tag{91}$$

$x = |x_1 - x_2|$ and $t = |t_1 - t_2|$, and x_i is the coordinate along the 1D channel. In the case of 1D phonons, they propagate along the same direction as electrons. In general, phonons are characterized by the temperature T_{ph} that may differ from the electronic temperature in the leads. This allows us to consider the zero-temperature electronic noise at finite phonon temperature (finite τ_c).

A. Ballistic regime: $K\tau_c^2/\hbar^2 \gg 1$

Here we summarize the expressions for dephasing rate in the ballistic regime for different phonon dimensionalities:

$$\text{1D:} \sqrt{\frac{\pi s g^2}{12\hbar^2 r_c^2}}, \quad \text{2D:} \sqrt{\frac{\zeta(3)sg^2}{2\pi\hbar^2 r_c^3}}, \quad \text{3D:} \sqrt{\frac{\pi^2 sg^2}{60\hbar^2 r_c^4}}, \tag{92}$$

where $\zeta(x)$ is the Riemann zeta function.

B. Diffusive and logarithmic dephasing: $K\tau_c^2/\hbar^2 \ll 1$

1D case. The phonon-induced dephasing rate is be calculated using Eq. (53):

$$\frac{1}{\tau_\varphi} = \frac{g^2}{2\hbar^2 r_c} = \frac{g^2}{2\hbar^3 s} T_{\text{ph}}. \tag{93}$$

In this calculation, we assume that phonons could transmit across barriers. The 1D dephasing rate grows linearly with the phonon temperature. The transmission coefficient is given by Eq. (54). Interestingly, this dephasing rate has a peculiar limit of zero sound speed:

$$\frac{1}{\tau_\varphi} \sim \int d\tau\, K(\tau) \sim \frac{T}{s} \xrightarrow[s \to 0]{} \infty. \tag{94}$$

More accurately, there are two different cases:

$$\frac{s}{L} \gg T: \quad K(\tau) \sim \frac{T^2}{s} e^{-T\tau}, \quad \forall \tau, \tag{95}$$

$$\frac{s}{L} \ll T: \quad K(\tau) \sim \begin{cases} \frac{T}{L} e^{-s\tau/L}, & \tau \ll \frac{L}{s} \\ \frac{T^2}{s} e^{-T\tau}, & \tau \gg \frac{L}{s}. \end{cases} \tag{96}$$

Thus, if we start to decrease the sound speed to zero, eventually the second regime ($s/L \ll T$) is established. Then the correlator becomes time independent for $Q \ll v/s$, which leads to

$$\langle e^{i \int_0^t d\tau V(\tau)} \rangle \approx e^{-t^3 g^2 Ts/L^2}, \quad \frac{1}{\tau_\varphi} \xrightarrow[s \to 0]{} 0. \tag{97}$$

2D case. For phonons in the plane, the direct calculations of the dephasing rate by means of Eq. (53) gives zero,

$$\int_0^\infty d\tau\, K(\tau) = 0, \tag{98}$$

as can be easily seen from the phonon correlator in two dimensions, Eq. (89).

In this case, we need to analyze more accurately the double integral in Eq. (49), which depends on the ratio between the structure size and the phonon correlation length. For short structures, one obtains

$$\int_0^t \frac{d\tau}{\hbar} \int_0^\tau \frac{d\tau'}{\hbar} K(\tau') \approx \gamma_{\text{2D}} \ln\left(\frac{t}{\tau_c}\right), \quad \frac{L}{r_c} \ll 1, \tag{99}$$

$$\gamma_{\text{2D}} = \frac{g^2}{2\pi\hbar^2 r_c s} = \frac{g^2}{2\pi\hbar^3 s^2} T_{\text{ph}}. \tag{100}$$

For long structures, $L/r_c \gg 1$, at short times $t \ll L/s$, one gets a linear growth and for larger times a logarithmic grows,

same as in Eq. (99):

$$\int_0^t \frac{d\tau}{\hbar} \int_0^\tau \frac{d\tau'}{\hbar} K(\tau') \approx \begin{cases} \gamma_{2D}\frac{\pi s}{2L}t, & t \ll \frac{L}{s} \\ \gamma_{2D}\ln\left(\frac{t}{\tau_c}\right), & t \gg \frac{L}{s}. \end{cases} \quad (101)$$

This means that the random phase induced by the interaction with 2D phonons grows slower with time than for the diffusive phase dynamics. This is the reason why we get zero for the dephasing rate in Eq. (52). For $\gamma \ll 1$, we can use the logarithmic result regardless of the relation between L and r_c.

The typical value of electron-phonon coupling constant is $\lambda \sim 5 \times 10^{-11}$ erg, the material density is $\rho_{3D} = 5$ g/cm^3, and the surface phonon localization length κ is an order of atomic length: $\kappa \sim 1$ nm^{-1}. With these values, one estimates

$$\gamma_{2D}/T \sim 1 \times \text{K}^{-1}.$$

3D case. In this case, we have the same scenario as for 2D phonons, but γ_{3D} is now quadratic in the phonon temperature:

$$\gamma_{3D} = \frac{g^2}{2\pi^2\hbar^2 r_c^2 s} = \frac{g^2}{2\pi^2\hbar^4 s^3}T_{ph}^2. \quad (102)$$

The limit of zero sound velocity for 2D and 3D phonons is taken in the same way as for 1D phonons. The conductance has a non-Breit-Wigner form. It is quite close to e^2/h, when the phonon temperature is the same as that for electrons in the contacts, since the characteristic dephasing-induced level broadening is of the same order as temperature.

C. Fano factor for logarithmic dephasing

As was shown above, for 2D and 3D phonons the logarithmic dephasing takes place. To calculate the Fano factor in this case, we assume that the leads are at zero temperature, while the phonons between the barriers are kept at nonzero temperature $T_{ph} = T > 0$. We then apply Eq. (78) for the Fano factor for logarithmic dephasing. For example, in a system with the equal barriers, $\Gamma \equiv \Gamma_1 = \Gamma_2$, for 2D phonons at $\Gamma \ll eV \ll T$ we get from Eq. (82):

$$F = \frac{1}{2}\left[1 - \frac{2}{\pi}\left(\frac{\Gamma}{T}\right)^{Tg^2/\pi\hbar^3 s^2}\left(\frac{\Gamma}{eV}\right)^{1-Tg^2/\pi\hbar^3 s^2}\right]. \quad (103)$$

VI. CONCLUSION

In this paper, we have addressed the problem of influence of inelastic processes on the Fano factor for a double-barrier structure. We have derived a general expression for the shot-noise power for a 1D conductor with arbitrary inelastic scattering. Next, we have considered the transmission of electrons through a double-barrier resonator with a random nonstationary potential inside it. We have derived exact inelastic transmission and reflection amplitudes that depend on the random time-dependent potential. We have analyzed the transmission coefficient and the Fano factor for this structure, assuming two types of dynamics of electronic wave-function phases: diffusion type, $\langle\varphi_f^2\rangle \sim t/\tau_\varphi$, and logarithmic, $\langle\varphi_f^2\rangle \sim \ln(t/\tau_c)$. For the transmission coefficient, diffusion-type dynamics leads to a Lorentzian shape with the width determined by a sum of elastic and dephasing contributions. The logarithmic type of dephasing leads to an unusual shape of the transmission coefficient as a function of energy, with the

two scales given by the elastic width and the inverse correlation time of the fluctuating potential. The Fano factor for such a structure also depends on the dephasing type. For diffusion-type dephasing, the Fano factor is largely insensitive to dephasing up to small corrections at low bias voltage. For the logarithmic type, there is a strong dependence of the Fano factor on the dephasing rate at low voltages, when the bias is within a single broadened level. Finally, we have applied our general formalism to the case when the fluctuating potential is produced by phonons. For 1D phonons, the dephasing is of the diffusion type with the rate proportional to the phonon temperature. For 2D and 3D phonons, we have found that dephasing is of the logarithmic type, which leads to an unusual temperature dependence of the Fano factor. To conclude, a quantum-mechanical derivation of shot noise through inelastic transmission and reflection amplitudes and averaging over random potential reveals a dependence of the Fano factor on the type of wave-function phase dynamics.

APPENDIX A: EXAMPLE: TRANSMISSION ACROSS TIME-DEPENDENT DELTA BARRIER

We start from a simple example: Obtain transmission amplitudes across scattering from time-dependent delta barrier, assuming that this time dependence is random but a well-defined function in time. The Schrodinger equation is

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + f(t)\delta(x)\psi = i\hbar\frac{\partial\psi}{\partial t}. \quad (A1)$$

Defining the wave function in the basis of incident waves, we can write it in next form:

$$\psi_< = e^{ikx-iE_k t/\hbar} + \int dp\, r(p,k)e^{-ipx-iE_p t/\hbar}, \quad x<0, \quad (A2)$$

$$\psi_> = \int dp\, t(p,k)e^{ipx-iE_p t/\hbar}, \quad x>0, \quad (A3)$$

where we assume that the k wave incident from the left side and all energies contribute to transmission and backscattering. From the Schrodinger equation, we can find conditions at the delta function position:

$$\psi_<(0) = \psi_>(0), \quad (A4)$$

$$\frac{2m}{\hbar^2}f(t)\psi_<(0) = \frac{\partial\psi_>}{\partial x}(0) - \frac{\partial\psi_<}{\partial x}(0). \quad (A5)$$

These conditions should be valid for any time and we can obtain the system of equations by integrating with $e^{iE_q t/\hbar}$. Next we use one approximation which will allow us to solve the task exactly—linearizing spectra $E_k \approx v\hbar k$. With these

assumptions, we can find the next equations defining transmission and reflection amplitudes:

$$\delta(q - k) + r(q, k) = t(q, k), \tag{A6}$$

$$\int dp \, \frac{2mv}{\hbar^2} f(q - p) t(p, k)$$

$$= iq[t(q, k) - \delta(q - k) + r(q, k)], \tag{A7}$$

$$f(q - p) = \int dt \, f(t) e^{iv(q-p)t}. \tag{A8}$$

Solving these equations, we obtain transmission amplitude through a random but well time-defined function $f(t)$:

$$t(p, k) = -ik \int dx \int^x dy \, e^{ipx - iky} \exp\left(-\int_y^x dz \frac{m}{\hbar^2} f\left(\frac{z}{v}\right)\right). \tag{A9}$$

If a particle's energy change is small due to time-dependent potential $|p - k|/k \ll 1$, we can replace the prefactor on the right side of Eq. (A7) on $q \to k$ and obtain the next equation for the transmission amplitude:

$$t(p, k) = \int dx \, \frac{ik}{ik - f(x/v)/\hbar} e^{i(p-k)x}. \tag{A10}$$

For the case of stationary barrier $f(t) = \lambda$, this leads to the well-known result:

$$t(p, k) = \frac{ik}{ik - \lambda/\hbar} \delta(p - k). \tag{A11}$$

APPENDIX B: TRANSMISSION AND REFLECTION AMPLITUDE FOR A DOUBLE-BARRIER STRUCTURE WITH DEPHASING

In this Appendix, we obtain transmission coefficients through the double barrier structure with a random nonstationary potential. As we mentioned in the main text for the weak and smooth potential wave function, we take an additional phase only, see Eq. (45).

For a system with barriers, the general wave function moving from the left with incident wave vector k is

$$\psi_{\mathrm{I}} = e^{ik(x-vt)} + \sum_p r_{pk} e^{-ip(x+vt)},$$

$$\psi_{\mathrm{II}} = \alpha(x, t) \sum_p a_{pk} e^{ip(x-vt)} + \beta(x, t) \sum_p b_{pk} e^{-ip(x+vt)},$$

$$\psi_{\mathrm{III}} = \sum_p t_{pk} e^{ip(x-vt)},$$

$$\alpha(x, t) = \exp\left(-i \int^t d\tau V(x + vt - v\tau, \tau)\right),$$

$$\beta(x, t) = \exp\left(-i \int^t d\tau V(x - vt + v\tau, \tau)\right).$$

Here regions I, II, and III correspond to regions before, between, and after double barriers. To obtain transmission and reflection coefficients, we need to use scattering matrices of barriers linking ingoing and outgoing waves. At this step, we can assume that barriers are pointlike and add some extra

phases to reflection and transmission amplitudes without loss of generality. Barriers located at the 0 and L points. The condition on the left barrier is

$$\begin{pmatrix} t_1 & r_1 \\ r_1 & t_1 \end{pmatrix} \begin{pmatrix} \psi_{\mathrm{I}}^> \\ \psi_{\mathrm{II}}^< \end{pmatrix} = \begin{pmatrix} \psi_{\mathrm{II}}^> \\ \psi_{\mathrm{I}}^< \end{pmatrix}, \tag{B1}$$

where index $> (<)$ corresponds to the right (left)-moving parts of the wave function. We neglect the dependence of the barrier transmission/reflection amplitude on the wave vector, assuming that the random potential weak and transmitted particle wave vector changes are small $|p - k|/k \ll 1$. So, we can take it on incident wave vector, i.e., $t_i = t_i(k)$. From Eq. (B1) and the same one on the second barrier, we can obtain

$$\frac{\alpha(0, t)}{t_1} \sum_p a_{pk} e^{-ipvt} - \frac{r_1 \beta(0, t)}{t_1} \sum_p b_{pk} e^{-ipvt} = e^{-ikvt},$$

$$\sum_p t_{pk} e^{ip(L-vt)} = t_2 \alpha(L, t) \sum_p a_{pk} e^{ip(L-vt)},$$

$$r_2 \alpha(L, t) \sum_p a_{pk} e^{ip(L-vt)} = \beta(L, t) \sum_p b_{pk} e^{-ip(L+vt)}.$$

Using these equations, one can obtain the transmission coefficient:

$$t_{pk} = \int dt \, v e^{ipvt} t_1 t_2 (1 - r_1 r_2 F(t) e^{-\tau_f \frac{\partial}{\partial t}})^{-1} G(t) e^{-ikvt}, \tag{B2}$$

$$F(t) = \frac{\alpha(L, t + \tau_f/2) \beta(0, t)}{\alpha(0, t) \beta(L, t - \tau_f/2)}, \quad G(t) = \frac{\alpha(L, t + \tau_f/2)}{\alpha(0, t)}. \tag{B3}$$

By expanding this equation in a series of reflections between barriers, assuming that a random potential varying slowly on time flight scale $\partial V(x, t)/\partial t \ll V(x, t)/\tau_f$:

$$t_{pk} = \int dx \, e^{i(k-p)x} t_1 t_2 \sum_{n=0}^\infty (r_1 r_2 e^{i2kL})^n \tag{B4}$$

$$\times \exp\left(-i \int_{-\tau_f n}^0 d\tau \, V(x_{\mathrm{in}}(\tau), \tau - x/v)\right), \tag{B5}$$

with $x_{\mathrm{in}}(t)$ the trajectory of a particle between barriers, describing reflecting motion.

Using the same method, we can obtain the reflection amplitude:

$$r_{pk} = \int dt dq \, v e^{i(p-q)vt} \frac{r_2 F(t) e^{i2qL} - r_1^*}{G(t) t_1^* t_2} t_{qk}. \tag{B6}$$

For small potential $(K\tau_c^2 \ll 1)$, $F(t) \approx G(t) \approx 1$ and we obtain a simple relation between reflection and transmission amplitudes:

$$r_{pk} = \frac{r_2 e^{ipL} - r_1^* e^{-ipL}}{e^{-ipL} t_1^* t_2} t_{pk}. \tag{B7}$$

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