

## Limits of a rechargeable spin battery

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We discuss how the ideal rechargeable energy accumulator can be made and what the limits for solid-state energy storage are. We show that in theory, spin batteries based on heavy fermions can surpass chemical ones in terms of energy capacitance. The absence of chemical reactions in spin batteries makes them more stable, and also they do not need to be heated in cold conditions. We study how carrier statistics and the density of states affect the energy capacity of the battery. Also, we discuss hypothetical spin batteries based on neutron stars.

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### I. INTRODUCTION

Rechargeable electric batteries are one of the most important devices of modern civilization. It is obvious that their role will only increase in the future. Unfortunately, the existing chemical rechargeable batteries (based on reversible electrochemical reactions) are far from ideal. This manifests itself, for example, in their inevitable irreversible degradation, slow charging, relatively low energy capacitance per unit mass, the need for heating when the temperature drops, etc. Of course, progress does not stop; however, most efforts now are applied to chemical and physicochemical properties of batteries (see Refs. [1,2]). The relatively new physical idea of a quantum battery explores quantum states and entanglement properties [3–6]. The idea of using the spin degree of freedom to store energy has attracted a lot of attention in recent years [7–10]. Particularly, in a recent article [11] the authors proposed a spin battery (SB) which is a half-metal spin valve with suppressed spin flips of conducting electrons. This solution would allow us to store the electric energy reversibly without any chemical reactions during the charging process using nonequilibrium states of quasiparticles in a conductor instead.

Hence the following questions appear: Is it possible for a SB to surpass a chemical battery? What are the properties of the ideal SB? In this paper we theoretically “test” possible limits of the solid state for SBs and the more exotic matter of neutron stars as well.

### II. CHEMICAL POTENTIAL

The energy in a SB is stored in the spin carriers’ density deviation from its equilibrium value under the condition that spin relaxation is suppressed in this conductor. In other words, such a battery is just a certain volume filled with spin particles,

and energy accumulation appears due to a development of the nonequilibrium spin state. Such spin particles can be electrons in a conductor [11], while at the same time being charge carriers, and their spin direction,  $\pm$ , can be determined by the external magnetic field or magnetic contacts. In order to charge such a battery containing charged spin carriers or to transfer the accumulated nonequilibrium spin concentration into charge current, one can use, for instance, antiparallel magnetized half-metal [12–14] electrodes which pass only  $+$  or  $-$  spin correspondingly [11]; see Fig. 1.

In this situation the charging potential difference  $\delta\varphi$  induces variations of chemical potentials of  $\pm$  components (after the charging time when equilibrium is established):

$$\mu_{\pm} = \mu_{0,\pm} + \eta_{\pm},$$

where  $\mu_{0,\pm}$  are the equilibrium electrochemical potentials of the discharged battery determined by the density of the corresponding  $\pm$  components. Chemical potentials  $\eta_{\pm}$  are induced by the charging process, and their values could be found from the conditions  $\delta\varphi = \eta_{+}/q_{+} - \eta_{-}/q_{-}$  (where  $q_{\pm}$  are the electric charges of the corresponding carriers), together with spatial electroneutrality, which follows from the Poisson equation  $\Delta\varphi = -4\pi\{q_{+}[\rho_{+}(\mu_{+}) - \rho_{+}(\mu_{0})] + q_{-}[\rho_{-}(\mu_{-}) - \rho_{-}(\mu_{0})]\}$  (with the right-hand side equal to zero):

$$q_{+}[\rho_{+}(\mu_{+}) - \rho_{+}(\mu_{0})] + q_{-}[\rho_{-}(\mu_{-}) - \rho_{-}(\mu_{0})] = 0, \quad (1)$$

where  $\varphi$  is the electrical potential,  $\rho_{\pm}$  are the densities, and  $q_{\pm}$  are charges of the corresponding spin  $\pm$  carriers. Here, for simplicity, we assume  $\mu_{0,+} = \mu_{0,-} = \mu_{0}$ , as it is obvious that the equilibrium value of the electrochemical potential does not affect the basic principles of energy storage.

For a one-band conductor we have  $q_{+} = q_{-} = e$ , where  $e$  is the electron charge. Moreover, we can consider the usage of two-band conductors, which contain not only the electrons but also “holes” with opposite charge. What is more, the carriers are polarized in such a way that spin  $\pm$  is connected to charge  $\pm$ . In this situation the charges  $q_{\pm}$  have the opposite sign:  $q_{+} = -q_{-} = e$ . This is possible when the interaction

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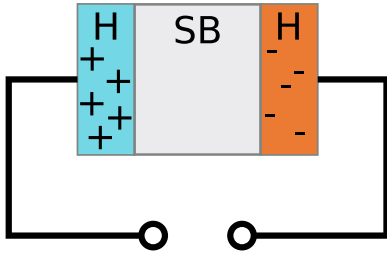


FIG. 1. Schematic illustration of a spin battery (SB) which is a conductor between half-metal electrodes (denoted by “H”) with opposite spin polarization.

between carriers from different bands is weak [15] or by using electron-hole pairing methods [16–18] for the spin-flip suppression as described in detail in Ref. [11]. When the one-band battery is being charged, the number of certain spin carriers increases, and this necessarily leads to the number of opposite spin carriers decreasing in order to preserve electroneutrality (1). In a two-band battery we always chose the polarity of the charging voltage in such a way as to have an increase in the number of carriers of both spins ( $\eta_+ > 0$ ). This means that we connect the positive-guided contact to the electrode with “+” polarity (corresponding to “holes” with charge  $q_+$ ) and the negative-guided contact to the electrode with “-” polarity (corresponding to electrons with charge  $q_-$ ). Nonequilibrium states caused by deviations of spin densities in one-band and two-band SBs are shown in Fig. 2. Here we show schematically the filling energy levels  $\varepsilon$  as functions of the corresponding densities of states (DOSs)  $D_{\pm}$  for spins  $\pm$ .

As can be seen, the two-band battery has polarity, and such a SB is equal to a chemical battery but with the following difference. In a chemical battery we have the concentrations changing, and correspondingly the changing of chemical potentials with respect to electrodes, while  $q_{\pm}$  corresponds to the one ion charge:  $-q_+ = q_- = -zF/N_a$ , where  $z$  is the positive valency,  $F$  is the Faraday constant, and  $N_a$  is Avogadro’s number (for definiteness we chose the sign to be the same

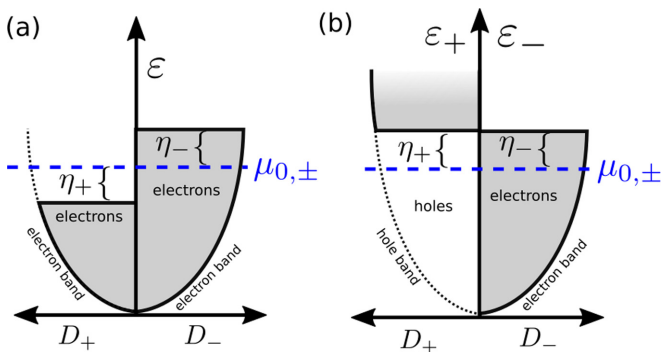


FIG. 2. (a) One-band “purely electronic” spin battery. (b) Two-band “electron-hole” spin battery. Filled levels are shaded gray. The number of carriers of one spin decreases when the number of carriers of another spin increases for the one-band battery, and polarity is not important. We chose the polarity of the two-band battery in such a way as to have an increase in the number of both types of carriers during the charging process.

as that of an elementary charge, and we assume the same absolute value of valency of all ions). In the absence of a charging potential difference in the circuit, the presence of nonequilibrium chemical potentials leads to the appearance of diffusion forces, which pull in or push out charges into the circuit. This happens on electrodes of the opposite “affinity” (such an affinity is related to chemical reactions in a usual chemical battery, or it is related to the presence of a conduction band only for certain spin on  $\pm$  electrodes in the SB). An asymmetry in the charge moving during relaxation into the thermodynamic equilibrium state causes an electric current in the full circuit [11]. Also, we can consider a SB with spin or charge carriers that are not being usual conductive electrons, but quasiparticles. Such quasiparticles can even have zero electric charge; however, in this situation the movement of such quasiparticles does not cause an electric current, and thus energy extraction from this battery is difficult. A SB does not require chemical reactions, and therefore it does not suffer from chemical degradation. As we show below, a SB does not require heating when it consists of a degenerate gas of charge or spin carriers. Obviously, a SB can be a source not only of charge but also of spin current [19–22]. Finally, a SB can be charged without electrodes by using polarized electromagnetic radiation [23].

### III. GENERAL FORMULAS FOR THE ENERGY OF A CHARGED SB

Let us denote  $E_{\pm}(\mu_{\pm})$  as the total internal energy of carriers of  $\pm$  components for a given value of the electrochemical potential  $\mu_{\pm}$ . The energy stored in the battery is the difference between the internal energies of charged and discharged states

$$\delta E = E_+(\mu_0 + \eta_+) + E_-(\mu_0 + \eta_-) - E_+(\mu_0) - E_-(\mu_0). \quad (2)$$

At the microscopic level the value of  $E$  in a SB as well in a chemical battery is determined by the equilibrium energy distribution of carriers  $n_{\pm}(\varepsilon, \mu)$ , by the DOS  $D_{\pm}(\varepsilon)$ , and by the volume  $\Omega$ :

$$E_{\pm}(\mu_{\pm}) = \Omega \int_0^{\infty} d\varepsilon \varepsilon D_{\pm}(\varepsilon) n_{\pm}(\varepsilon, \mu_{\pm}). \quad (3)$$

Also, we can write the following expression for  $\rho_{\pm}$  in order to substitute it in (1):

$$\rho_{\pm}(\mu_{\pm}) = \int_0^{\infty} d\varepsilon D_{\pm}(\varepsilon) n_{\pm}(\varepsilon, \mu_{\pm}). \quad (4)$$

As can be seen from (2) and (3), the energy distribution determines which parts of the DOS dependencies  $D_{\pm}(\varepsilon)$  give the main contribution. Of course, the energy distribution can be described by classical (Boltzmann), Bose, Fermi, or, even, so-called fractional statistics.

Note that usually,  $\varepsilon D(\varepsilon)$  increases with  $\varepsilon$  and, on the other hand, Bose distributions collect particles in states with lower energies where the DOS has its minimal values. Of course, the DOS of Bose distributions can have certain singularities (see Ref. [24]); thus some low-dimensional systems require special investigation. However, in the general case, if we do not consider nonphysical DOSs which diverge at zero faster than  $\varepsilon^{-1}$ , then in order to have maximal capacitance of the

SB, particles should occupy more states with larger DOSs. In this regard, fermions are the best choice for SBs. As two fermions cannot occupy the same state, they have to fill levels with higher energy with the number of particles on the rise. Below we discuss the influence of quantum statistics in detail. Intuitively, we understand that a battery is more sensitive to the temperature in the case of classical statistics. In the general case, when  $\mu_0$  and  $q_{\pm}$  are given, the search for a battery with maximal energy capacitance is reduced to a variational problem: the search for the conditional maximum of the functional  $\delta E[D_{\pm}, n_{\pm}]$ . This functional is linear on  $D_{\pm}$  and  $n_{\pm}$ , while the restrictive condition is Eq. (1). In reality, distributions and densities of states are limited by the physical states mentioned above.

#### IV. BRIEF DISCUSSION OF CLASSICAL STATISTICS

Boltzmann statistics corresponds to sufficiently low densities  $\rho_{\pm}$ , when  $\exp\{-\mu/T\} \gg 1$ , where  $T$  is the temperature in energy units. Note that in the case  $q_+ = q_-$  the electroneutrality condition (1) is broken when  $|\delta\varphi| \sim T$ . In this situation, an increase in the  $\pm$  component (depending on the sign of  $\delta\varphi$ ) cannot be compensated via a decrease in the  $\mp$  component due to the insufficient amount of it, and thus Eq. (1) does not have a solution. Since the temperature 300 K approximately corresponds to a potential difference of only 0.03 V, the case  $q_+ = q_-$  is not of interest in the classical limit. Particularly, this is why there is no sense in using chemical accumulators with different types of cations or anions in such a way that the opposite electrodes make an imbalance in the number of different ions of the same sign during the charging, because electroneutrality is broken at relatively low voltages. Generally, electroneutrality is not broken when  $q_+ = -q_-$  with an increase in  $\delta\varphi$ . In such a case only the average value of the DOS matters. Really, the classical distribution  $n_{\pm}(\varepsilon, \mu) \approx \exp\{\mu/kT\} \exp\{-\varepsilon/kT\}$ , where the constant in  $\mu$  is chosen to satisfy the normalization condition for the density of particles. From (1)–(4) we have the following:

$$\delta E \approx \Omega \left[ \exp\left\{\frac{\eta_+}{T}\right\} - 1 \right] (E_+(\mu_0) + E_-(\mu_0)). \quad (5)$$

Accordingly, the value of  $\delta E$  in the leading approximation with respect to  $T|q\delta\varphi|^{-1} \ll 1$  is

$$\delta E \sim \Omega T \left[ \rho_0\left(T, \mu_0 + \frac{q\delta\varphi}{2}\right) - \rho_0(T, \mu_0) \right],$$

where  $\rho_0(T, \mu_0)$  is the equilibrium concentration for a given temperature. As we can see, singularities of  $D(\varepsilon)$  do not play a role in the case of classical statistics, and only the average of the DOS appears in (5). Also, one can see that  $\delta E$  strongly depends on  $T$ , and on concentrations as well, which reduces the application of such accumulators at low temperatures without additional heating. Let us show that Fermi statistics changes the situation dramatically.

#### V. “FERMI” BATTERIES

Quantum Fermi statistics can be realized in SBs made from degenerate conductors such as metals where the equilibrium electrochemical potential is approximately equal to the Fermi

energy  $\mu_0 \simeq \varepsilon_F$ . Usually, in metals the Fermi energy is of the order of  $10 \text{ eV} \sim 10^5 \text{ K}$ . Therefore we can consider energies  $|q_{\pm}\delta\varphi| \sim |\eta_{\pm}| < \mu_0$  and neglect thermal blurring. The energy of the nonequilibrium state with splitting of spin components and the corresponding Lagrange function for this approximation can be found in Ref. [25]. Here we write it using our notation:

$$\delta E \approx \Omega \left[ \int_{\mu_0}^{\mu_0+\eta_+} d\varepsilon \varepsilon D_+(\varepsilon) + \int_{\mu_0}^{\mu_0+\eta_-} d\varepsilon \varepsilon D_-(\varepsilon) \right], \quad (6)$$

$$q_+[\rho_+(\mu_+) - \rho_+(\mu_0)] + q_-[\rho_-(\mu_-) - \rho_-(\mu_0)]$$

$$\cong q_+ \int_{\mu_0}^{\mu_0+\eta_+} d\varepsilon D_+(\varepsilon) + q_- \int_{\mu_0}^{\mu_0+\eta_-} d\varepsilon D_-(\varepsilon) = 0. \quad (7)$$

As can be seen from (6) and (7) the Fermi level shift determines the limits of integration, and as we show below, the DOS singularities can play an important role in the properties of a “Fermi” battery. Note that in a one-band battery ( $q_+ = q_-$ ) the integrand in (6) coincides up to multiplier  $\varepsilon$  with the integrand on the right-hand side of (7), which is set to zero. Thus, in this case, we have a certain compensation when we make a small parameter expansion for  $\eta_{\pm}/\mu_0$ . The physical meaning of this is the following: For a one-band battery we lose in energy when the number of quasiparticles of a certain kind decrease during charging, and for a two-band battery the number of quasiparticles of both kinds only increases during charging (when polarity is correctly chosen).

Firstly, we consider the case when the DOS does not have any singularities, i.e.,  $D_{\pm}(\varepsilon)$  are analytical functions which can be expanded in Taylor series near the initial Fermi level  $\mu_0$ :

$$D_{\pm}(\varepsilon) \approx D_{\pm}(\mu_0) + (\varepsilon - \mu_0)D'_{\pm}(\mu_0) + \frac{(\varepsilon - \mu_0)^2}{2}D''_{\pm}(\mu_0) + \dots,$$

where the prime means differentiation according to the only argument.

In the case of a smooth DOS, a SB based on a one-band normal metal and a SB based on a two-band normal metal have quite different energy storage properties. In a one-band metal we have  $D_+(\mu_0) = D_-(\mu_0) = D(\mu_0)$ , and the same relations for derivatives  $D', D''$ . The electroneutrality condition for the case  $q_+ = q_-$  with second-order accuracy with respect to  $\eta_{\pm}/\mu_0$  gives the following equation:

$$\eta_+ + \eta_- + \frac{D'(\mu_0)}{2D(\mu_0)}(\eta_+^2 + \eta_-^2) = 0, \quad (8)$$

which leads to

$$\eta_- \approx -\eta_+ - \frac{D'(\mu_0)}{D(\mu_0)}\eta_+^2 + \dots \quad (9)$$

Thus, from (6), (7), and (8), the energy of a one-band SB is

$$\delta E \approx \Omega D(\mu_0)\eta_+^2 + \dots \quad (10)$$

Here, the dots mean high-order terms with respect to  $\eta_{\pm}/\mu_0$ . As can be seen from (10), the linear terms (with respect to  $\eta_{\pm}$ ) are canceled, and the energy has only a quadratic dependence on the potential difference. This fact leads to the appearance of a small multiplier,  $\sim e\delta\varphi/\mu_0$ , compared with a two-band

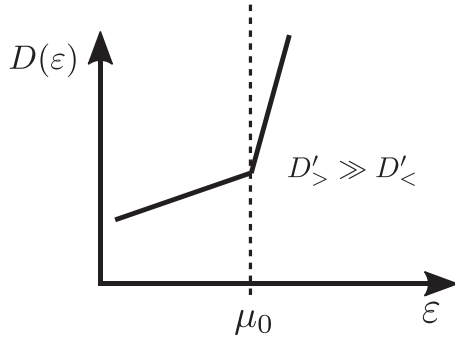


FIG. 3. Model DOS cusp at the Fermi level with a high value of the DOS derivative at the right.

SB. Indeed, in a two-band SB the linear approximation with respect to  $\eta_{\pm}/\mu_0$  is enough, which gives  $\eta_+ = \eta_-$ , and we have

$$\delta E = 2\Omega D(\mu_0)\mu_0\eta_+, \quad \frac{\delta E_{\text{one-band}}}{\delta E_{\text{two-band}}} \sim \frac{\eta_+}{\mu_0}. \quad (11)$$

As can be seen from (11) the energy is linear and greater than in the one-band case. In both cases the higher the Fermi energy (i.e., the density of carriers), the higher the energy of the SB. In the one-band SB the energy is proportional to the DOS at  $\mu_0$ , while in the two-band SB the energy is proportional to the product of the DOS at  $\mu_0$  and  $\mu_0$  itself. As can be seen, in both case (10) and case (11) we have the DOS at the Fermi level without averaging over the energy as opposed to the classical case.

Let us discuss how DOS singularities can increase the power capacity of a one-band Fermi SB based on the high-value parameter discussed above which contains the DOS derivative. Really, it is not unusual to have cusps in  $D(\varepsilon)$ , for instance, when Van Hove singularities are present [26,27], etc. For the sake of simplicity we consider the model case of linear dependence to the right and left of the cusp, and also we assume that the cusp of  $D(\varepsilon)$  is exactly aligned at  $\mu_0$ .

$$D(\varepsilon) = D(\mu_0) + D'(\mu_0 + 0)(\varepsilon - \mu_0)\theta(\varepsilon - \mu_0) + D'(\mu_0 - 0)(\varepsilon - \mu_0)\theta(\mu_0 - \varepsilon). \quad (12)$$

Note that expression (12) is not an expansion with respect to the small parameter, but the model DOS. Let us denote  $D = D(\mu_0)$ ,  $D'_> = D'(\mu_0 + 0)$ , and  $D'_< = D'(\mu_0 - 0)$ ; see Fig. 3. After taking integrals in Eqs. (7) and (6) we obtain

$$D\eta_+ + D'_>\frac{\eta_+^2}{2} + D\eta_- + D'_<\frac{\eta_-^2}{2} = 0, \quad (13)$$

$$\delta E = \left[ D\frac{\eta_+^2}{2} + D'_>\frac{\eta_+^3}{3} + D\frac{\eta_-^2}{2} + D'_<\frac{\eta_-^3}{3} \right] \Omega. \quad (14)$$

Assume that the value of  $D'_>$  is big enough to have  $D'_>\eta_+/D \gg 1$ , while the derivative  $D'_<$  is small (for simplicity we can take it to be zero). In such a case,  $\eta_- \approx -2^{-1}D^{-1}D'_>\eta_+^2$ , and the energy  $\delta E \sim \Omega D'_>\eta_+^3$ . Using the condition  $e\delta\varphi = \eta_+ - \eta_-$ , we can estimate the derivative value of  $D'_>$  which corresponds to the situation when the energy of a Fermi battery with a cusp exceeds estimation (10) for the same value of carrier density and the same DOS at the

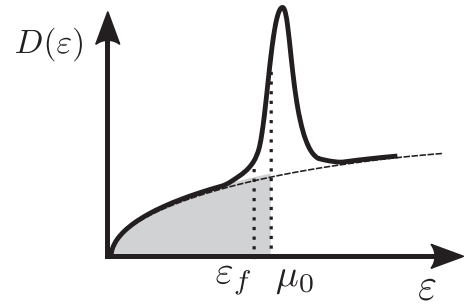


FIG. 4. Schematic illustration of the DOS in intermetallic alloys with  $f$  electrons. The dashed curve corresponds to  $D(\varepsilon)$  in the absence of  $s$ - $f$  interaction. “Heavy fermions” form states near the peak.  $\varepsilon_f$  is the electron bound energy in the  $f$  shell.

Fermi level. Let us take  $\varphi \sim 1$  V,  $\mu_0 \sim \varepsilon_F \sim 10$  eV, and then

$$D'_>\eta_+^3 > D\eta_+^2 \Rightarrow \frac{D'_>\eta_+}{D} > 1 \Rightarrow \frac{D}{D'_>} < 1 \text{ eV},$$

which looks experimentally feasible. The possibility of such a power capacity increasing follows from the fact that the DOS derivative gives the *main contribution* to the energy instead of being a correction.

## VI. HEAVY FERMIONS

The density of states of conduction electrons in metals is high as a result of high density  $\rho$ . Obviously, the DOS in normal metals is greater than the DOS in liquid electrolytes, despite the small effective electron mass in metals. In fact, the smallness of the effective electron mass is compensated by the high Fermi velocity, which is several orders greater than, for example, thermal ion velocities in dilutions.

One can imagine even more effective conductors for SBs, which contain so-called “heavy fermions” [28–33], such as some intermetallic antiferromagnetic alloys with  $f$  electrons. Indeed, in such correlated conductors the effective masses  $m_{\text{heavy}}^*$  of carriers are 100–1000 times greater than effective masses in normal metals  $m_{\text{normal}}^* \sim (1–10)m$ , where  $m$  is the free-electron mass.

The DOS of heavy fermions has irregularity; see Fig. 4. If we substitute the heavy-fermion DOS in expressions (10) and (11) taking into account that  $\eta_{\pm} \sim eV$ ,  $\mu_0 \sim \varepsilon_F$ , and  $D_{\text{heavy}} \sim (m_{\text{heavy}}^*/m_{\text{normal}}^*)^{3/2} D_{\text{normal}} \sim 10^4 D_{\text{normal}}$ , we get the following rough estimation:

$$\delta E_{\text{heavy}} \sim 10^4 \cdot \frac{\Omega}{\lambda_F^3} \frac{eV}{\varepsilon_F} \begin{cases} eV, & \text{one-band battery} \\ \varepsilon_F, & \text{two-band battery.} \end{cases} \quad (15)$$

Given that in metals we have  $\lambda_F \sim 10^{-10}$  m and the Fermi energy is in the range 1–10 eV, then for  $V \sim 1$  V/cm<sup>3</sup> we estimate the maximal power capacity of such a SB:  $W_{\text{max}} = 3600^{-1} \delta E_{\text{max}}/V \sim 10^4$  A h. For comparison, the battery power capacity of an Apple iPhone 12 Max Pro indicated in the specifications is 3.7 A h, i.e., three orders of magnitude less than the 1 cm<sup>3</sup> power capacity of a hypothetical heavy-fermion SB. Given that the metal density is approximately 10<sup>4</sup> kg/m<sup>3</sup>, we obtain that the energy density of a heavy-fermion metallic SB is about 100–1000 MJ/kg. This value can

be one order of magnitude greater than the normalized energy density of gasoline as a fuel for an internal combustion engine (which, in its turn, is about 46 MJ/kg [34,35]). At the same time, the energy density of chemical batteries is two orders of magnitude worse than that of gasoline, and this fact is an important problem of modern electric transport.

## VII. THE NEUTRON STAR AS A RECORD SB

Since neutrons in neutron stars are compressed by gravitational force to huge density, and due to the fact that they are fermions, they gain huge DOS values. A neutron has spin but does not have a charge, and starting from this fact, we can imagine a supercivilization which could store energy by “charging” neutron stars via polarized radiation, i.e., producing the difference in concentrations of spins referred to a certain direction in space. Here we do not discuss the problem of extraction of this energy. It is impossible to transform this energy into electric current due to the electrical neutrality of neutrons. Also, it is obvious that it is impossible to make some contacts with a neutron star. Thus the supercivilization has to investigate a noncontact method. Nevertheless, let us here estimate the colossal energy which can be stored in a neutron star in this fantastic scenario.

If a neutron star has a Sun mass  $M_{\odot} \approx 2 \times 10^{30}$  kg, its radius should be  $R \approx 10$  km =  $10^4$  m, and correspondingly the density  $\rho = 3M_{\odot}/4\pi R^3 \approx 1.4 \times 10^{18}$  kg/m<sup>3</sup>. These numbers are taken just for understanding the scale of such a fantastic “device”; the mass of a real neutron star should be greater than the Tolman-Oppenheimer-Volkoff limit of  $2.17M_{\odot}$  [36,37]. In order to estimate the DOS and Fermi energy, one can use simple Thomas-Fermi formulas with relativistic corrections [38,39] because the Fermi velocity is close to the speed of light  $c$  for such huge densities and, moreover, the momentum  $p \gg m_n c$ , where  $m_n$  is the mass of a neutron. Thus we have

$$\varepsilon = c\sqrt{p^2 + m_n^2 c^2} - m_n c^2 \approx cp, \quad p \gg m_n c, \quad (16)$$

$$D(\varepsilon) = \frac{\varepsilon^2}{\pi^2 c^3 \hbar^3}, \quad (17)$$

$$\varepsilon_F \approx \pi c \hbar \left( \frac{3\rho}{\pi m_n} \right)^{\frac{1}{3}}. \quad (18)$$

The electroneutrality condition is automatically satisfied in neutron stars. Spin concentrations just change accordingly: When some spin flips, it increases the concentration of one type and decreases the concentration of the opposite type. Therefore the energy of a “one-band” SB (10) based on a neutron star’s matter, which is stored in chemical potential spin splitting  $2\eta$ , is

$$\delta E_n \approx \Omega D(\varepsilon_F) \eta^2 \approx \frac{3^{\frac{2}{3}} \Omega}{c \hbar} \left( \frac{\rho}{\pi m_n} \right)^{\frac{2}{3}} \eta^2. \quad (19)$$

The maximal energy which can be reversibly stored in a “spin battery” based on such a neutron star corresponds to 100% polarization of its neutrons, i.e.,  $\eta = \varepsilon_F$ . This allows us to estimate the maximal energy capacitance of 1 mm<sup>3</sup> of neutron star matter  $\delta E_{n,\max}(\Omega = 10^{-9} \text{ m}^3) \sim 10^{42}$  J, which is 25 orders of magnitude more than the energy of the most powerful thermonuclear bomb [40] tested by mankind,  $2.4 \times 10^{17}$  J. The colossal energy reversibly stored for the whole neutron star with the above parameters is  $\sim 10^{63}$  J. Note that due to neutrons having zero electric charge, spin-orbit coupling is absent, and full spin polarization of the star does not lead to rotation of the star.

## VIII. CONCLUSION

We have theoretically shown that spin batteries can surpass modern chemical accumulators by orders of magnitude of energy capacitance due to the high density of states of electrons in metals, especially in the case of metals with “heavy fermions.” The fantastic scenario of using a neutron star’s matter would allow reversible storage of a colossal amount of energy. We did not find such estimations for neutron stars in the literature.

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