Driven strongly correlated quantum circuits and Hall edge states: Unified photoassisted noise and revisited minimal excitations

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We study the photoassisted shot noise (PASN) generated by time-dependent (TD) or random sources and transmission amplitudes. We show that it obeys a perturbative nonequilibrium (NEQ) fluctuation relation (FR) that fully extends the lateral-band transmission picture in terms of many-body correlated states. This FR holds in NEQ strongly correlated systems such as the integer or fractional quantum Hall regime as well as in quantum circuits formed by a normal or a Josephson junction (JJ) strongly coupled to an electromagnetic environment, with a possible temperature bias. We then show that the PASN is universally super-Poissonian, giving an alternative to a theorem by L. Levitov *et al.* which states that an ac voltage increases the noise. We show that this theorem is restricted to a linear dc current and that it does not apply to a nonlinear SIS (superconductor-insulator-superconductor) junction. Then we characterize minimal excitations in nonlinear conductors as those which ensure a Poissonian PASN, and show that these can carry a nontrivial charge value in the fractional quantum Hall regime. We also propose methods for shot noise spectroscopy and for a robust determination of the fractional charge which complement those we have proposed previously and that have been implemented experimentally [M. Kapfer *et al.*, Science **363**, 846 (2019) and R. Bisognin *et al.*, Nat. Commun. **10**, 2231 (2019)].

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I. INTRODUCTION

Time-dependent (TD) transport presents a powerful probe of quantum phenomena by introducing multiple parameters or functions under control: frequencies for emitted noise generated by constant forces, or TD forces generating current or noise at low or finite frequencies [1–6]. It has been analyzed in a mesoscopic context through seminal theoretical approaches, such as the Tien-Gordon theory [7–10] or the Landauer-Büttiker scattering approach, associated with the Floquet theory [11–17]. The effect of a periodic ac voltage $V_{ac}(t)$ at a frequency Ω_0 is often addressed within the so-called lateral-band transmission scheme, where $V_{ac}(t)$ is viewed as a coherent radiation with translates one electron energy by $l\Omega_0$ for each integer number l of exchanged photons. This yields a relation between the induced low-frequency shot noise and a superposition of duplicates over l of the noise in the dc regime. By linking current fluctuations, we coin it as a fluctuation relation (FR), which we distinguish from fluctuation-dissipation relations that involve current or conductance. The noise induced by V_{ac} is called photoassisted shot noise (PASN); it should be higher than its value in the dc regime according to a theorem by L. Levitov *et al.* [18]. While Poissonian shot noise in the dc regime is common to classical and quantum particles, the PASN has the interest to provide a signature of a quantum behavior through rectified current fluctuations.

The PASN is also an important tool to explore remarkable collective phenomena and macroscopic manifestation of quantum physics when strong correlations play a crucial role, but for which the lateral-band transmission picture has been claimed to be inappropriate [19]. Nonetheless, such a picture was recovered within specific models. For instance, within the Tomonaga-Luttinger Liquid (TLL) model, relevant to strongly correlated 1-D systems, unexpected Tien-Gordon type relations were obeyed either by the photo-assisted current (PAC) in Refs. [20,21] or by the PASN in Refs. [22,23] though not compared to its Tien-Gordon form. It turns out that these works were unified and enlightened by our nonequilibrium (NEQ) perturbative approach [24–27].

Here the same approach is adopted to extend fully the lateral-band transmission picture for PASN to many-body correlated states. Contrary to a majority of studies restricted to periodic voltages, we also extend it to nonperiodic tunneling amplitudes and voltages, which can then be generated by fluctuating sources or pseudo-random Lorentzian pulses [28,29]. Thus the NEQ approach cannot be coined as Tien-Gordon theory. It unifies many previous works based on specific models [5,22,23], beyond which it extends to a larger universality class of strongly correlated circuits and situations. Let us mention a quantum point contact (QPC) in incompressible edge states in the integer quantum Hall effect (IQHE) or the fractional quantum Hall effect (FQHE) (see Fig. 1), as well as a quantum circuit formed by a QPC (Fig. 2), a Josephson junction (JJ) [30] (Fig. 3) or a dual phase-slip JJ (Fig. 4) strongly coupled to an electromagnetic environment. Another strength of the NEQ approach is that it goes beyond initial thermalized many-body states to NEQ ones. It covers for instance the SIN (superconductor-insulator-normal) junction in a NEQ diffusive wire studied in Ref. [31], or a quantum circuit with a temperature bias we have studied recently [32] (see Fig. 2). In addition, this approach led to some NEQ FRs not derived so far [33], even for independent electrons. Although

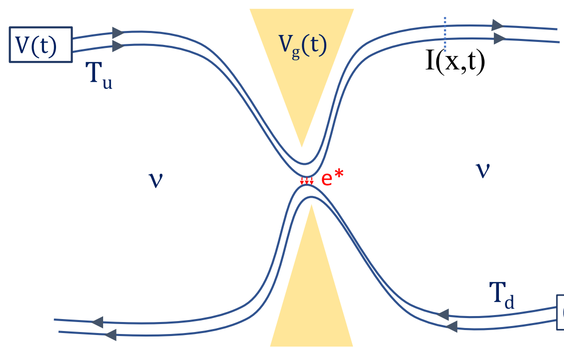

FIG. 1. First example. A QPC in the quantum Hall regime at an integer or fractional filling factor ν. One can include arbitrary profile, range and inhomogeneities of interactions between edge states. It is possible to have simultaneous time dependence of the voltage reservoirs and the gate, as well as different upper and down temperatures T_u, T_d or imperfect equilibration between edge states. $I(x,t)$ denotes the average chiral current at a position x along the upper edge.

we consider here the PASN of a current operator, the latter could refer, depending on the model, to a generalized force such as a voltage operator in the dual JJ junction [34] or a spin current in a magnetic junction.

The PASN is especially relevant for two rapidly growing and fascinating domains where injection and manipulation of controlled quantum electronic or photonic states is a challenge: electronic quantum optics and quantum electrodynamics of mesoscopic circuits.

On the one hand, an ideal test-bed for the former is offered by quantum Hall states. There, Coulomb interactions are fundamental to understand the FQHE and the emergence of fractional charges [35–37], while they couple edge states in the IQHE [38,39]. Electronic quantum optics is associated with the injection of on-demand electronic excitations and their time evolution through an interacting region. A first theoretical step to address this problematic was initiated by the author [40,41] by implementing a scattering approach for plasmon modes with time dependent boundary conditions. The corresponding works showed charge fractionalization [38,40–43], which plays, for instance, an important role in decoherence [44,45], and laid the foundation of NEQ bosonization [46]. Electronic quantum optics has become an independent field owing to subsequent pioneering experimental and theoretical achievements (for a review, see Ref. [39]). We mention for instance the analog for electrons of a single photon gun based on a mesoscopic capacitor [47], and implementation of minimal excitations generated by Lorentzian pulses [18,48,49]. In interferometers such as Hanbury-Brown and Twiss or Hong-Ou-Mandel (HOM) type setups, PASN has offered a tool to explore the charge fractionalization [43,50], to characterize minimal excitations and their statistics [6,48,51–53], or to perform electronic tomography [54,55].

On the other hand, quantum electrodynamics of mesoscopic circuits, based for instance on macroscopic atoms such as JJs, requires understanding of radiation-matter interactions, where the radiation corresponds to photons in the electromagnetic environment (for a recent review, see Ref. [56]). Such interactions give rise to the dynamical Coulomb blockade phenomena [57], which, in the strong back-action regime, was shown to offer a quantum simulation of strongly correlated one dimensional conductors [58–60]. Addressing the statistics of quantum states for both photons and electrons and the generation of squeezed photonic states has been based on finite-frequency noise in an ac driven circuit [61–64]. In particular, minimal excitations might offer an interesting basis in this framework [65].

It is indeed in an ac driven quantum circuit that some of the NEQ FRs we have obtained at finite frequencies [24] have been first tested experimentally [25]. They have been also used to achieve a robust determination of the fractional charge [35,36], or for analyzing experimental investigation of two-particle collisions in a HOM type geometry in the IQHE and the FQHE [6,66].

The present paper is focused on the PASN at zero frequency, while finite-frequency noise is reported to a separate one. Here we present some consequences and applications of the NEQ FRs for the PASN. We express the PASN in terms of current cumulants of a non-Gaussian source, such as a quantum conductor in the classical regime we have studied in Ref. [67]. We also derive relations for the PASN's differentials with respect to the ac voltage, then apply them to propose novel methods for charge determination and shot noise spectroscopy. We also derive an important universal inequality, showing that the PASN is super-Poissonian. This allows us to state that minimal excitations in nonlinear conductors ensure Poissonian PASN. We therefore provide an alternative characterization to that by L. Levitov *et al.* [18] which is rather restricted to a linear system. This gives a more thorough analysis than the one we presented in Ref. [24], and which was recovered in the specific model of a TLL [23].

The paper is organized as follows. In Sec. II, we recall the family of models and the minimal conditions required by the NEQ perturbative approach, discussing specifically its validity and limitations for quantum Hall edge states. We derive NEQ FRs for the PASN and its differentials in Sec. III, then specify to random sources or an initial thermal equilibrium. In Sec. IV, we show that the universal lower bound on the PASN is given by the PAC, and not necessarily by the noise in the dc regime, rather shown to be higher than the PASN in a SIS (superconductor-insulator-superconductor) junction. This leads us to revisit the criteria for minimal excitations in Sec. V. We finally discuss, in Sec. VI, two other applications based on differentials of the PASN with respect to the ac voltage: shot-noise spectroscopy and determination of the fractional charge.

II. THE PERTURBATIVE APPROACH

A. The model and minimal conditions

We consider the Hamiltonian underlying the NEQ perturbative approach [24,26,27],

$$\mathcal{H}(t) = \mathcal{H}_0 + e^{-i\omega_J t} p(t) A + e^{i\omega_J t} p^*(t) A^\dagger, \tag{1}$$

where the unperturbed and perturbing terms $\mathcal{H}_0$ and A are not specified, nor is the complex function $p(t)$, which can be nonperiodic, and whose phase $\varphi(t)$ as well as its modulus can depend on time,

$$p(t) = |p(t)| e^{-i\varphi(t)}. \tag{2}$$

We adopt the convention that any constant part of a global phase derivative is incorporated into ω_J, so that $\int dt \partial_t \varphi(t) = 0$.

We focus on transport associated with a given charge operator $\hat{Q}$ assumed to commute with $\mathcal{H}_0$ and to be translated through A by e^*,

$$[A, \hat{Q}] = e^* A, \tag{3}$$

where e^* is a model-dependent charge parameter. Thus the associated current operator, in view of Eq. (1), reads

$$\hat{I}(t) = \partial_t \hat{Q}(t) = -i\frac{e^*}{\hbar}(e^{-i\omega_J t} p(t)\, A - e^{i\omega_J t} p^*(t)\, A^\dagger). \tag{4}$$

Other charge operators not conserved by $\mathcal{H}_0$ might enter and couple to other independent constant forces, such as those associated with an electromagnetic environment. Indeed the operator A can be a superposition of terms associated with many positions, channels or circuit elements, $A = \sum_i A_i$, or a continuous integral over spatially extended processes. Nonetheless, contrary to the dc regime, this generalization is constrained by the fact that all TD fields must be incorporated into the single complex function $p(t)$.

The main other conditions for the approach are: (i) A is weak, with respect to which second order perturbative theory is valid and (ii) only correlators implying A and its hermitian conjugate are finite [see Eq. (A2)]. The condition (ii) leads, for a family of initial distributions [27], to a vanishing dc current at $\omega_J = 0$; in particular, in superconducting junctions, supercurrent must be negligible by coupling them to a dissipative environment or magnetic fields.

Indeed, the approach is not restricted to an initially thermalized system [27,33], but extends to any initial stationary NEQ density matrix ρ_0 obeying $[\rho_0, \mathcal{H}_0] = 0$. Thus ω_J can be superimposed on other constant independent forces, or one can consider a quantum circuit with a temperature bias [32] (see Fig. 2).

Generically, although not systematically, the coupling to a voltage $V(t)$ can be included into a term $\hat{Q}V(t)$ that can be absorbed by a unitary transformation [26] so that ω_J [Eq. (1)] and $\varphi(t)$ [Eq. (2)] obey the following Josephson-type relations, determined by e^* [in view of Eq. (3)],

$$\omega_J = \frac{e^*}{\hbar} V_{dc}, \tag{5a}$$

$$\partial_t \varphi(t) = \frac{e^*}{\hbar} V_{ac}(t), \tag{5b}$$

where $V_{ac}(t), V_{dc}$ are the ac and dc parts of $V(t)$. But more generally, the common charge e^* could be replaced by two different effective charges, and the above relations can even be broken for NEQ states, as is the case for the anyon collider [33,68,69]. For generality, we leave ω_J and $p(t)$ (with its amplitude and phase) as unspecified parameters of the model [70].

We have previously shown that the average current induced by $p(t)$, $\langle \hat{I}_{\mathcal{H}}(t) \rangle$, can be, at any time, fully expressed in terms of the dc characteristics only, $I_{dc}(\omega_J)$, whether $p(t)$ is periodic [26] or not [24,27]. The subscript $\mathcal{H}$ refers to the Heisenberg representation with respect to the Hamiltonian $\mathcal{H}(t)$. In the zero-frequency limit, one gets the PAC

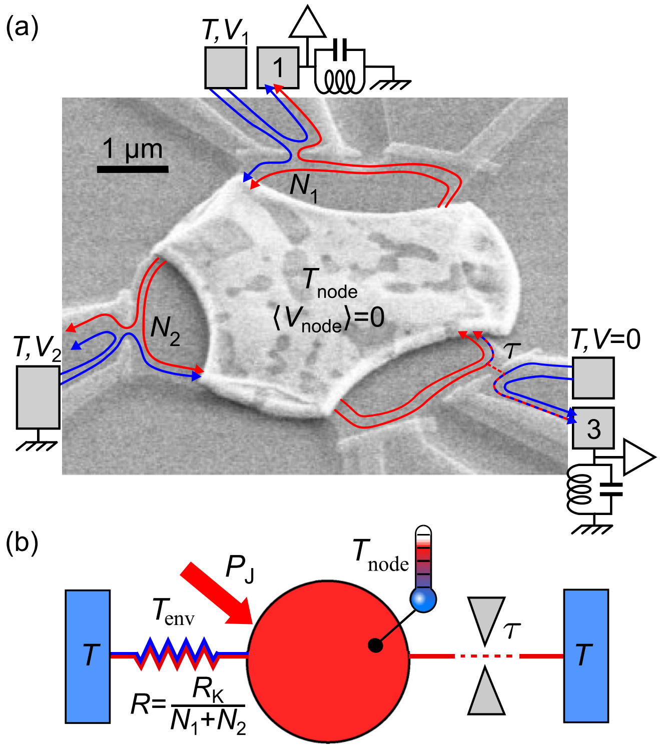

FIG. 2. Second example. A quantum circuit formed by a QPC (on the right side of the lower scheme) coupled to an electromagnetic environment and with a temperature bias, studied in Ref. [32] in the dc regime to address dynamical Coulomb blockade. The present NEQ FR extends to the two opposite conducting and insulating regimes of the quantum phase transition and yields PASN through the QPC in case both the potential drop and gate voltage are time dependent.

$$I_{ph}(\omega_J) = \int_{-T_0/2}^{T_0/2} \frac{dt}{T_0} \langle \hat{I}_{\mathcal{H}}(t) \rangle, \tag{6}$$

whose expression will be recalled in Eq. (24). Only dependence on the dc frequency ω_J is made explicit, while that on $p(t)$ is implicit through the subscript ph. Here T_0 is the period for periodic $p(t)$, and is a long measurement time for nonperiodic $p(t)$ for which it forms the key of a regularization procedure we have proposed in Ref. [27]. We think that this solves a divergency problem compared by Lee and Levitov [71] to the orthogonality catastrophe problem, for instance when $V(t)$ is formed by a single Lorentzian pulse. A similar procedure can be carried on for the PASN, defined by

$$S_{ph}(\omega_J) = \int_{-T_0/2}^{T_0/2} \frac{dt}{T_0} \int_{-\infty}^{\infty} d\tau \left\langle \delta\hat{I}_{\mathcal{H}}\left(t - \frac{\tau}{2}\right) \delta\hat{I}_{\mathcal{H}}\left(t + \frac{\tau}{2}\right) \right\rangle, \tag{7}$$

where $\delta\hat{I}_{\mathcal{H}} = \hat{I}_{\mathcal{H}}(t) - \langle \hat{I}_{\mathcal{H}}(t) \rangle$. We will nonetheless, for simplicity, assume that the Fourier transform of $p(t)$, $p(\omega)$, is regular at zero frequency, and we refer to Ref. [27] if not (replacing current by noise). We will show that $S_{ph}(\omega_J)$ is determined, through a universal FR given by Eq. (10), by

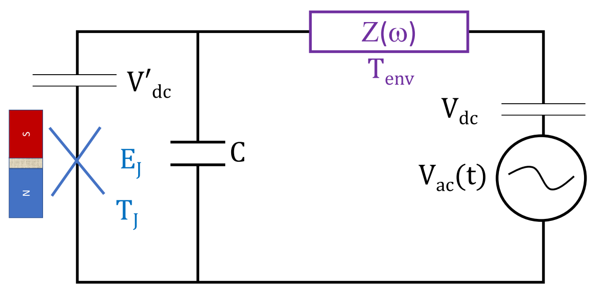

FIG. 3. Third example. A JJ with a small Josephson energy E_J or a NIS junction strongly coupled to an electromagnetic environment. An additional dc voltage V'_{dc} can enter into the Hamiltonian in Eq. (1) or in the NEQ stationary density matrix ρ_0. Also the temperature junction T_J can be different from that of the environment, $T_{\rm env}$.

$S_{dc}(\omega_J)$, the NEQ shot noise in the dc regime (which will be coined as the dc noise). It is only when the initial density matrix is thermal that $S_{dc}(\omega_J)$ is determined by $I_{dc}(\omega_J)$ [see Eq. (13)] and so is the PASN.

Some examples of models for which these relations hold are detailed in Ref. [27] and are illustrated in Figs. 1–4. For instance, $\hat{I}(t)$ is a tunneling current in case A refers to a tunneling operator between strongly correlated conductors with mutual Coulomb interactions. It is the Josephson current in a JJ at energies below the superconducting gap Δ (Fig. 3), for which one has $e^* = 2e$. Let us now discuss in more details the validity and limitations of the approach for a QPC in the quantum Hall regime.

B. Validity of the approach in quantum Hall states

For a QPC in the FQHE or IQHE at a filling factor ν, the perturbative approach applies to two opposite regimes: the weak backscattering one (when the QPC is almost open, see Fig. 1), where $\hat{I}(t)$ in Eq. (4) is a backscattering current, and the strong backscattering regime (when the QPC is pinched off), where $\hat{I}(t)$ is an electron tunneling current. While one has $e^* = e$ in the latter regime, one expects e^*/e to be a fraction in the former when one deals with the FQHE. Many theoretical approaches are based on effective bosonized theories, such as the chiral TLL description for interacting edges in the IQHE

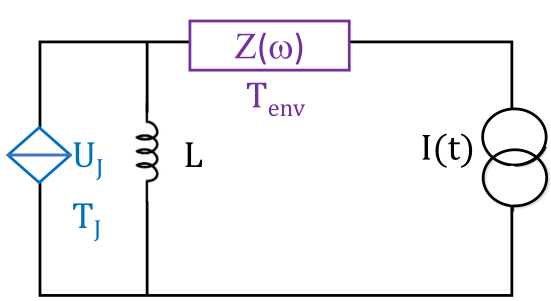

FIG. 4. Fourth example. A dual-phase JJ with a small effective parameter U_J. The roles of voltage and current are permuted, so that one imposes a time dependent current, while the voltage noise across the junction obeys the NEQ FR. Average voltage was computed in Ref. [34] and found to obey the relation provided by the perturbative approach [26,27] [see Eq. (B1)].

or Laughlin series in the FQHE given by $\nu = 1/(2n+1)$ with integer n, for which $e^* = \nu e$. For other ν belonging to hierarchical series, effective theories yield various models [72] with, possibly, different values of the dominant charge e^* (that for which the quasiparticle field has the smallest scaling dimension δ [73]). It is also frequent that two or more different quasiparticle fields with the same charge and dimension enter into A, a situation to which the approach can still be adapted.

Such effective theories predict a power-law behavior and a crossover energy scale $k_B T_B$ between the weak and strong backscattering regimes, leading to a vanishing dc conductance when both voltages and temperatures vanish. This delimits the validity of the perturbative approach in both regimes with respect to T_B.

Nonetheless, in experimental works aiming to determine fractional charge [35–37] and statistics [74], the measured dc current is not in accordance with this power-law behavior. Our approach has the advantage to be valid without specific Hamiltonian nor voltage dependence of the dc current. This explains why the NEQ FRs we obtained [24,33,76] provided robust methods to determine $e^* = e/5$ at $\nu = 2/5$ in Ref. [35] and $e^* = e/3$ at $\nu = 2/3$ in Ref. [36].

Although bosonization is not even necessary for the Hamiltonian in Eq. (1), one might require, to end up with this form, additional conditions. For instance, absorption of inhomogeneous couplings to ac sources into the function $p(t)$ might require that $\mathcal{H}_0$ is a quadratic functional of bosonic fields (a condition which is not required in the dc regime).

In order to implement such couplings, one might exploit a useful framework we have initiated [40,41], and which has been largely adopted in electronic quantum optics [5,43]. It describes the electronic charge propagation in terms of plasmon dynamics dictated by Coulomb interactions, inducing charge fractionalization. By developing the equation of motion method for bosonic fields, dynamics is solved for given time dependent boundary conditions dictated by the sources. On the one hand, a classical ac source injects a classical plasmon wave whose time evolution is determined through a scattering matrix for plasmon modes, providing the ac outgoing electronic currents. On the other hand, for a non-Gaussian source, such as another QPC different from the central one (replacing the voltage source in Fig. 1), the NEQ bosonization we have initiated in Ref. [41] has been extended to take into account statistical fluctuations of the injected current [46]. Our present NEQ approach applies to such non-Gaussian sources in the dc regime [33], and it is plausible that one can still end up with Eq. (1) in the ac regime, as we allow for a NEQ stationary density matrix and a time dependent modulus of $p(t)$ that could incorporate ac boundary conditions. For a more rigorous justification and determination of $p(t)$, one needs to combine our modelization of ac voltages [41] with that of dc non-Gaussian sources [46], a step not yet achieved to our knowledge.

The TD chiral current at a point x along the upper edge reads (see Ref. [77] gives a derivation in the dc regime):

$$I(x,t) = \nu\frac{e^2}{h}V(t) - \theta(x)\int dt'\lambda(x,t-t')\langle\hat{I}_{\mathcal{H}}(t')\rangle, \quad (8)$$

where $\theta(x)$ is the Heaviside function if the QPC is located at $x = 0$. Recall that $\langle\hat{I}_{\mathcal{H}}(t)\rangle$ is the average of the backscattering current in Eq. (4) in the Heisenberg representation. The function $\lambda(x,t)$ is determined by $\mathcal{H}_0$, and describes chiral plasmonic propagation between the QPC and x. Denoting its zero-frequency limit by λ, one gets $I(x,\omega=0)=\nu e^2/hV_{dc}-\theta(x)\lambda I_{ph}(\omega_J)$. One expects $\lambda=\nu$ for simple fractions, but it could be renormalized by nonuniversal features such as edge reconstruction [78].

It is frequent that one measures rather correlations or cross-correlations between chiral currents, which contain supplementary terms, similarly to Refs. [80–82] in the dc regime. This is also the case when sources are formed by additional QPCs, such as the anyon collider studied in the dc regime [33,68,69,74] and where application of two ac voltages with a time delay would form a HOM interferometer (as suggested in Ref. [53]). It turns out that the perturbative approach is still useful for the supplementary terms [79].

III. UNIVERSAL FLUCTUATION RELATIONS

In this section, we first derive the central NEQ FR for the PASN in Eq. (7), then apply it to non-Gaussian random sources, and finally deduce FRs for the differentials of the PASN with respect to the ac phase, which we will exploit for the other applications in Sec. VI.

A. Fluctuation relations between the ac and dc driven regimes

The derivation of the NEQ FR follows two steps, detailed in Appendix B. The first one yields a second order perturbative expression for the PASN in terms of two correlators [see Eq. (A2)], which are evaluated with the Hamiltonian $\mathcal{H}_0$ and the initial NEQ density matrix ρ_0, so that they depend only on the time difference τ. Their Fourier transforms at ω_J, denoted by $I_\rightarrow(\omega_J), I_\leftarrow(\omega_J)$, correspond to dc currents induced by ω_J which flow in two opposite directions. They determine average current and noise in the dc regime,

$$I_{dc}(\omega_{\rm J})=I_\rightarrow(\omega_{\rm J})-I_\leftarrow(\omega_{\rm J}), \tag{9a}$$

$$S_{dc}(\omega_J)/e^*=I_\rightarrow(\omega_J)+I_\leftarrow(\omega_J). \tag{9b}$$

Notice that the NEQ noise $S_{dc}(\omega_J)$ is given by $S_{ph}(\omega_J)$ in Eq. (7) whenever $p(t)=1$ in Eq. (1). In general, $I_\rightarrow(\omega_{\rm J})\neq I_\leftarrow(-\omega_J)$, thus one has not necessarily an odd dc current nor an even dc noise.

The second step consists into reversing the two above expressions, so that, alternatively, only the two functions $I_{dc}(\omega_{\rm J}), S_{dc}(\omega_J)$ determine completely time dependent transport. In particular, we can show that the PASN in Eq. (7) is fully determined by $S_{dc}(\omega_J)$ in Eq. (9b),

$$S_{ph}(\omega_J)=\int_{-\infty}^{\infty}\frac{d\omega'}{\Omega_0}\bar{P}(\omega')S_{dc}(\omega'+\omega_J), \tag{10}$$

where $\bar{P}(\omega)=|p(\omega)|^2$ and $\Omega_0=2\pi/T_0$.

Thus we obtain a universal FR between the ac and dc regimes, which, to our knowledge, has not been derived so far within the present large context of strongly correlated circuits and NEQ initial states. The PASN is a superposition of the noise evaluated at effective dc voltages $\omega_J+\omega'$ for all finite frequencies ω' of the driving photons, modulated by $\bar{P}(\omega')$. Even at $\omega_J=0$, the PASN is determined by the NEQ dc noise $S_{dc}(\omega')$ [indeed even $S_{dc}(\omega'=0)$ is a NEQ noise for initial NEQ states]. The above NEQ FR is independent on the form, range and force of Coulomb interactions or strong coupling to an electromagnetic environment; all these intervene only through the NEQ dc noise. The external ac or classical noise sources enter through $\bar{P}(\omega')$, which can be viewed as the transfer rate for the many-body eigenstates of $\mathcal{H}_0$ to exchange an energy $\hbar\omega'$ with the ac sources, as can be checked through a spectral decomposition [83].

Experimentally, one gets rid of undesirable contributions by considering the excess PASN. Here we define it by subtracting the dc noise in presence of the same dc voltage $S_{dc}(\omega_J)$ obtained when one switches off the ac source,

$$\underline{\Delta}S_{ph}(\omega_J)=S_{ph}(\omega_J)-S_{dc}(\omega_J). \tag{11}$$

Let us notice already that $\underline{\Delta}S_{ph}(\omega_J)$ was shown to be always positive by L. Levitov *et al.* [18] [see Eq. (22)], but this is not true in a nonlinear SIS junction as shown in Sec. IV, leading us to revisit minimal excitations in Sec. V.

Let us now specify to a periodic $p(t)$ with a frequency Ω_0 (see Appendix B for more details),

$$S_{ph}(\omega_J)=\sum_{l=-\infty}^{+\infty}P_lS_{dc}(\omega_J+l\Omega_0). \tag{12}$$

Here $P_l=\bar{P}(l\Omega_0)$ are the transfer rates for many-body states to exchange l photons with the source. It is only when $|p(t)|=1$ that P_l are probabilities, as $\sum_l P_l=1$ [see Eq. (B3)].

In the case of an initial thermal density matrix $\rho_0\propto e^{-\beta H_0}$ at a temperature $T=1/\beta$ [see Eq. (1)], the dc noise obeys the general relation, valid even when $I_{dc}(\omega_J)\neq -I_{dc}(-\omega_J)$ [24,33,76,84],

$$S_{dc}(\omega_J)=e^*\coth\left(\frac{\hbar\omega_J}{2k_BT}\right)I_{dc}(\omega_J). \tag{13}$$

The PASN is than detailed in Appendix C. Focusing on a periodic $p(t)$, on locking values $\omega_J=N\Omega_0$ with an integer N and on $\Omega_0\gg k_BT/\hbar$, we get, from Eq. (12),

$$S_{ph}(N\Omega_0)=\sum_{l\neq -N}P_l\,|I_{dc}[(N+l)\Omega_0]|+2P_{-N}k_BTG_{dc}(T). \tag{14}$$

Here $G_{dc}(T)=dI_{dc}(\omega_J)/dV_{dc}$ at $e^*V_{dc}\ll k_BT$, where temperature dependence, generic in nonlinear systems, is made explicit, while it is implicit in NEQ current average and PASN. Thus we get a mixture between the NEQ and thermal contributions (see Appendix C), similarly to the NEQ finite-frequency noise in the dc regime [76]. Taking the excess noise in Eq. (11) does not cancel the thermal contribution, even though we are in the NEQ quantum regime.

The FR in Eq. (14) unifies and goes beyond previous works restricted to $|p(t)|=1$ and to independent electrons scattered by a linear QPC [5,48] or to a TLL in Ref. [22]. It allows us to localize and regularize a divergency in the latter work [79].

Thus the universal FRs in Eqs. (10) and (12) extend fully the lateral-band transmission for the PASN to TD tunneling amplitudes and periodic, nonperiodic, or fluctuating sources. In the large family of strongly correlated circuits covered by our approach, NEQ many-body states replace thermal one-electron states. These FRs are also suited to address two-particle collisions in a symmetric or asymmetric HOM type geometry where two ac sources, periodic or not, operate with a time delay (as we noticed briefly in Ref. [27]). They have been used in a recent experimental analysis of two-electron collisions [6,66] in chiral quantum Hall edges.

B. Fluctuating sources

One advantage of considering nonperiodic $p(t)$ is that one can deal with classical states of radiations. Indeed, if we assume that $|p(t)| = 1$, one has $\int d\omega' \bar{P}(\omega')/\Omega_0 = 1$, so that $\bar{P}(\omega)$ becomes a probability [27]. It plays a similar role to the $P(E)$ function, the probability for a tunneling electron to exchange photons at a frequency $\omega = E/\hbar$ with an electromagnetic environment [57]. Indeed, this is precisely the meaning of $\bar{P}(\omega)$ if $\varphi(t)$ is associated with a Gaussian or non-Gaussian electromagnetic environment in the classical limit, formed for instance by a quantum conductor we have studied in Ref. [67].

More generally, if classical fluctuations of $\varphi(t)$ have a distribution $\mathcal{D}(\varphi)$, one has to take into account averages over $\mathcal{D}(\varphi)$, denoted by $< ... >_{\mathcal{D}}$,

$$\bar{P}(\omega) = \int_0^{T_0} \frac{dt}{T_0} e^{i\omega t} < e^{i(\varphi(t)-\varphi(0))} >_{\mathcal{D}} . \tag{15}$$

Notice that we assume here the stationarity of the distribution for φ so that $< e^{i(\varphi(t)-\varphi(t'))} >_{\mathcal{D}}$ depends only on $t - t'$, and the integral over $t + t'$ can be dropped. One can further write $< e^{i(\varphi(t)-\varphi(0))} >_{\mathcal{D}}$ as an exponential of cumulants of $\varphi(t)$ at order m (m is an integer; see Ref. [85] for the full expression),

$$J_m(t) = \frac{1}{m!} \langle (\varphi(t) - \varphi(0))^m \rangle_{\mathcal{D}}. \tag{16}$$

If we expand it up to $m = 3$, justified in the limit of weak coupling, we obtain

$$S_{ph}(\omega_J) = \iint \frac{d\omega dt}{\Omega_0} S_{dc}(\omega + \omega_J) e^{i\omega t - J_2(t) + iJ_3(t)}. \tag{17}$$

There might be various ways, depending on regimes and setups, to exploit this link, in particular to use the PASN as a way of detection of cumulants of the quantum conductor, as done with the PAC [26,27]. Compared to previous works proposing Tunnel junctions or JJs as cumulant detectors [85,86], the present model opens the path to exploit a larger family of strongly correlated detectors, which are not necessarily disconnected, and to drive both the detector and the non-Gaussian source in stationary NEQ states.

We insist nonetheless that a quantum environmental phase operator $\hat{\varphi}(t)$ whose dynamics is dictated by the Hamiltonian $\mathcal{H}_0$ can be also encoded into A through $e^{i\hat{\varphi}(t)}$, whose correlations affect the PASN through the dc noise $S_{dc}(\omega_J)$ according to Eq. (10).

C. Fluctuation relations for differentials of the PASN

An alternative to excess noise, aimed to remove undesirable noisy sources, consists into the derivative of the PASN with respect to the dc voltage, which, in view of the FRs in Eqs. (10) and (12), is determined through the differential dc noise.

It is also interesting, for some potential applications, to differentiate the PASN with respect to the ac components of the voltage $V_{ac}(\omega)$, or for more generality, $\varphi(\omega)$ [as Eq. (5b) is not systematic]. Given a nonperiodic or random $p(t)$, one can show that $\delta p(\omega')/\delta\varphi(\omega) = -ip(\omega' - \omega)$, so that

$$\frac{\delta S_{ph}(\omega_J)}{\delta\varphi(\omega)} = -i \int \frac{d\omega'}{\Omega_0} p(\omega') p^*(\omega' + \omega) \times [S_{dc}(\omega_J + \omega' + \omega) - S_{dc}(\omega_J + \omega')]. \tag{18}$$

Let us now take a second differential with respect to $\varphi(-\omega)$. Then we are back to the PASN through an interesting closed relation

$$\frac{\delta^2 S_{ph}(\omega_J)}{\delta\varphi(\omega)\delta\varphi(-\omega)} = S_{ph}(\omega_J + \omega) + S_{ph}(\omega_J - \omega) - 2S_{ph}(\omega_J). \tag{19}$$

A similar relation holds for the PAC in Eq. (24) [26], as well as for periodic drives, taking $\delta^2 S_{ph}(\omega_J)/\delta\varphi_k\varphi_{-k}$ with $\varphi_k = \varphi(k\Omega_0)$ for any integer k. We notice however that an experimental difficulty can arise in case one lacks a precise knowledge of the effective phase (for instance at the level of the QPC in the Hall regime), which makes noise spectroscopy discussed in Sec. VI A useful.

It is indeed easier to consider the limit of the stationary regime, defined by $p(t) = 1$ thus $p(\omega') = \delta(\omega')$ (i.e., the limit of a vanishing phase, up to multiple of 2π, and of a unit modulus). Then the first derivative in Eq. (18) vanishes, and one can replace, on the right-hand side (rhs) of Eq. (19), $S_{ph} \to S_{dc}$. In this limit, we can also show that $\delta^2 S_{ph}(\omega_J)/\delta\varphi(\omega)\delta\varphi(-\omega') \to 0$ for all $\omega' \neq \omega$. Thus, assuming furthermore that $|p(t)| = 1$ so that only functional dependence with respect to $\varphi(t)$ enters, the excess PASN defined through Eq. (11) can be expanded through the main quadratic correction due a small ac phase:

$$\underline{\Delta} S_{ph}(\omega_J) = \int_{-\infty}^{+\infty} d\omega |\varphi(\omega)|^2 [S_{dc}(\omega_J + \omega) + S_{dc}(\omega_J - \omega) - 2S_{dc}(\omega_J)] + o(\varphi^4). \tag{20}$$

This universal relation is coherent with the fact that only one or zero photon exchange processes enter at a low ac modulation. It is interesting to notice that, according to the NEQ FR we derived in the dc regime in Ref. [33], the sum $S_{dc}(\omega_J + \omega) + S_{dc}(\omega_J - \omega)$ corresponds to twice the symmetrized finite-frequency noise. For periodic drives the integral is replaced by a discrete sum. In particular, in case $\varphi(t) = \varphi_{ac} \cos \Omega_0 t$ we get

$$\underline{\Delta} S_{ph}(\omega_J)/[S_{dc}(\omega_J + \Omega_0) + S_{dc}(\omega_J - \Omega_0) - 2S_{dc}(\omega_J)] = \varphi_{ac}^2. \tag{21}$$

As will be discussed in Secs. VI A and VI B, the above relations offer methods for shot noise spectroscopy and for a robust determination of the fractional charge, based on determining ω_J and the Josephson-type relation in Eq. (5a) whenever it holds. Indeed there are also situations where ω_J implies other unknown parameters, which can then be determined consequently. Two have been addressed in Ref. [33]: either ω_J is linked to a nonuniversal parameter of fractional statistics that enters in analyzing the anyon collider, or to the

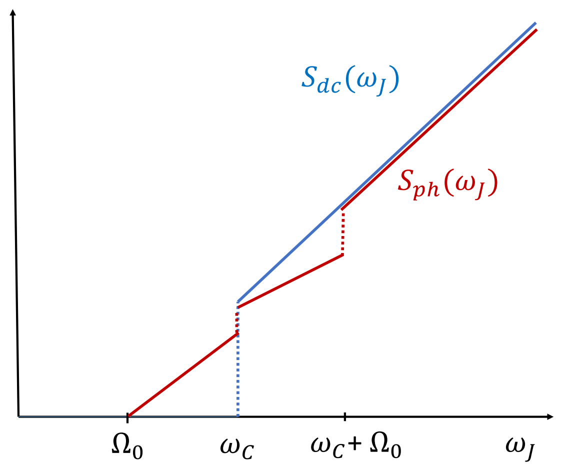

FIG. 5. The dc noise associated with the quasiparticle current in a SIS junction, as a function of the dc frequency ω_J (in blue), and the PASN under a small sine voltage (in red). S_{dc} vanishes below a threshold $\omega_c = 2\Delta/\hbar$ where Δ is the superconducting gap, and has a linear behavior above, $S_{dc}(\omega_J) = e^*(a\omega_J - b)$. The PASN behavior is sketched (units are arbitrary) by choosing $b/a = \Omega_0 = \omega_C/2$. One has $S_{ph}(\omega_J) < S_{dc}(\omega_J)$ for all $\omega_J > \omega_C$.

voltage drop generated by a temperature bias, which yields the Seebeck coefficient.

IV. UNIVERSAL LOWER BOUNDS ON THE PASN

This section gives crucial features that will allow us to revisit minimal excitations in Sec. V. We will first show that the universal lower bound provided by L. Levitov *et al.* [18] is restricted to linear conductors, by giving the counterexample of a nonlinear SIS junction with an initial thermal distribution. Then we show that the PASN is super-Poissonian, as it is rather the PAC which provides its universal lower bound.

A. Breakdown of the dc noise bound in a nonlinear SIS junction

In an independent-electron picture, the choice for the excess noise, $\underline{\Delta} S_{ph}(\omega_J)$ in Eq. (11) is motivated by the fact that it arises from the cloud of electron-hole excitations generated by the ac voltage [5,87], thus inducing a positive excess noise, $\underline{\Delta} S_{ph}(\omega_J) > 0$. Indeed, in a more general framework of strongly correlated systems, the ac voltage was shown to increase the noise through a theorem by L. Levitov *et al.* [18,49],

$$S_{ph}(\omega_J) \geqslant S_{dc}(\omega_J). \tag{22}$$

Nonetheless, we show now that adding an ac voltage to a dc one could decrease the PASN in a nonlinear SIS junction, so that these inequalities are reversed.

We adopt, in a similar context as these works, an initial thermalized distribution in the zero temperature limit, so that the dc noise is Poissonian [see Eq. (13)]. We also consider a quasiparticle current I_{dc} [7,8] with a voltage gap $2\Delta/e$, thus a dc frequency gap $\omega_C = 2\Delta/\hbar$ (here $e^* = e$), and a linear behavior above, $I_{dc}(\omega_J > 0) = \theta(\omega_J - \omega_c)(a\omega_J - b)$ where a, b are positive coefficients (see Fig. 5). This gives in particular $G_{dc}(T) = 0$. Now we choose the dc and ac frequencies such that $\Omega_0 < \omega_C < \omega_J$ and $b/a < \omega_J - \Omega_0 < \omega_C$. We consider a weak enough sine voltage so that we can use the second-order expansion in Eq. (21). As the dc noise is Poissonian, the sign of $\underline{\Delta} S_{ph}$ is that of $I_{dc}(\omega_J + \Omega_0) + I_{dc}(\omega_J - \Omega_0) - 2I_{dc}(\omega_J) = a(\Omega_0 - \omega_J) + b < 0$ [one has $I_{dc}(\omega_J - \Omega_0) = 0$ as $\omega_J - \Omega_0 < \omega_C$]. Therefore the PASN is decreased in this dc voltage range,

$$S_{ph}(\omega_J) < S_{dc}(\omega_J), \tag{23}$$

which is at odd with the inequality in Eq. (22). We can indeed plot the PASN for all dc voltages (see Fig. 5), which is also slightly below the dc noise at $\omega_J > \omega_C + \Omega_0$ where it is given by: $S_{ph}(\omega_J) = (P_0 + 2P_1)S_{dc}(\omega_J)$. This is due to the weak sine voltage which yields $P_0 + 2P_1 \lesssim 1$.

Indeed, for such a weak ac voltage, we can show that the PASN is Poissonian for all $\omega_J > \Omega_0$, $S_{ph}(\omega_J) = eI_{ph}(\omega_J)$ [see Eq. (B1)]. Then our result is coherent with the known fact that $I_{ph}(\omega_J) < I_{dc}(\omega_J) = S_{dc}(\omega_J)/e$ in the range $\omega_C < \omega_J < \omega_C + \Omega_0$ [88].

B. Super-Poissonian PASN

Considering again a NEQ density matrix ρ_0 and nonperiodic $p(t)$, let us first recall the relation obtained for the PAC in Eq. (6) [26,27],

$$I_{ph}(\omega_J) = \int_{-\infty}^{\infty} \frac{d\omega'}{\Omega_0} \bar{P}(\omega') I_{dc}(\omega' + \omega_J). \tag{24}$$

Similarly to Eq. (10), it is also interpreted within a lateral-band transmission picture for correlated NEQ many-body states. Now we have shown that the dc noise is super-Poissonian [33]

$$S_{dc}(\omega_J) \geqslant e^*|I_{dc}(\omega_J)| \tag{25}$$

due to the fact that $I_{\rightarrow}(\omega_J)$ and $I_{\leftarrow}(\omega_J)$ are positive [see Eqs. (9a) and (9b)]. This is obviously verified by Eq. (13) for an initial thermal equilibrium, which yields a Poissonian dc noise at low temperatures. Nonetheless, the super-Poissonian dc noise does not arise necessarily from thermal effects if the initial NEQ distribution is, for instance, generated by additional dc voltages (see Fig. 3) rather than by temperature gradients (as in Fig. 2).

Now by comparing Eq. (10) to Eq. (24), and using Eq. (25), we obtain also a super-Poissonian PASN [24]:

$$S_{ph}(\omega_J) \geqslant e^*|I_{ph}(\omega_J)|. \tag{26}$$

This is an important inequality, also valid when one has periodic drives, and even when the global system is in the ground state. Notice that this inequality suggests an alternative for the excess noise, given by $S_{ph}(\omega_J) - e^*|I_{ph}(\omega_J)|$, which yields always a positive sign, although it is not the most relevant experimentally as discussed in Appendix D.

Let us now comment on the case the dc current is linear with respect to ω_J and $|p(t)| = 1$. Then $I_{ph}(\omega_J) = I_{dc}(\omega_J)$, which becomes linear as well. In particular, if Eqs. (5a) and (5b) hold, one has simply

$$I_{ph}(\omega_J) = G_{dc}V_{dc}, \tag{27}$$

where $G_{dc} = I_{dc}(\omega_J)/V_{dc}$ is the linear conductance. Therefore, the lower bound on the PASN becomes given by the dc current, $S_{ph}(\omega_J) \geqslant e^*|I_{dc}(\omega_J)|$, exactly as is the case for the dc noise in Eq. (25). Nonetheless, it is only with an initial ground many-body state, that the dc current can be replaced by the dc noise [see Eq. (13)], so that one recovers Eq. (22).

The inequality in Eq. (26) offers an alternative to the one in Eq. (22), and is valid in the SIS junction we addressed above for all dc voltages. Although restricted to a perturbative regime, it covers a much larger family of nonlinear systems and quantum circuits. But an important difference from Eq. (22) is that the PAC, forming the universal lower bound, is also determined by the ac voltage.

V. REVISITING MINIMAL EXCITATIONS

In view of the above features, we address in this section the issue of characterizing minimal excitations, whose realization requires a ground many-body state, for instance the low-temperature limit of an initial thermal equilibrium.

A. L. Levitov's characterization: Limitation to linear conductors

Characterization of minimal excitations (we focus here on "electron" type ones) by L. Levitov *et al.* [18] through the PASN is based on the central inequality in Eq. (22).

First, the authors impose an injected charge per period $Q_{\rm cycle} = Ne$. As they assume that $I(t) = \partial_t Q(t) = e^2 V(t)/h$ (for a linear ballistic conductance with noninteracting electrons), they obtain $Q_{\rm cycle} = e^2 \int_0^{T_0} dt V(t)/h = e^2 T_0 V_{dc}/h$, controlled by the dc component of the voltage $V(t)$ only. This leads to the condition $V_{dc} = Nh/(T_0 e)$ or, taking $e^* = e$ in Eq. (5a), to $\omega_J = N\Omega_0$.

Secondly, according to Eq. (22), the TD voltage, which minimizes the PASN by injecting well-defined electronic excitations, must ensure the equality $S_{ph}(\omega_J) = S_{dc}(\omega_J)$, the lower bound of the PASN. This requires that the Fourier components p_l of $p(t) = e^{-i\varphi(t)}$ obey

$$l < -N \Rightarrow p_l = 0. \tag{28}$$

For that, the total voltage must be formed by a series of Lorentzian pulses centered at kT_0 with a width $2W$, so that the phase derivative verifies [see Eq. (5b), thus $\int dt \partial_t \varphi(t) = 0$],

$$\partial_t \varphi(t) = \frac{N\Omega_0}{\pi} \sum_{k=-\infty}^{\infty} \frac{1}{1 + (t - kT_0)^2/W^2} - N\Omega_0. \tag{29}$$

Nonetheless, such a characterization requires the current to be linear, thus does not apply to a QPC in the FQHE with a nonlinear dc current as claimed in Ref. [18].

Let us give three reasons for that. First, the injected charge corresponds to the PAC in Eq. (24), which, for a nonlinear dc current, has a nontrivial functional dependence on the ac voltage [27].

Second, let us adopt the Lorentzian pulses, and apply Eq. (28) to the FR in Eq. (12), so that:

$$S_{ph}(N\Omega_0) = \sum_{l \geqslant -N}^{\infty} P_l S_{dc}((N + l)\Omega_0), \tag{30}$$

In order to reach the equality $S_{ph}(N\Omega_0) = S_{dc}(N\Omega_0)$, given a dc Poissonian noise, one needs in general a linear dc current [notice that one has to add $2k_B G_{dc}(T) T P_{-N}$ on its rhs, in view of Eq. (14)].

Third, the authors were not aware of an implicit hypothesis underlying the inequality in Eq. (22), the linearity of the dc current. So it cannot be generalized to nonlinear conductors, such as the SIS junction addressed in Sec. IV A.

B. Super-Poissonian to Poissonian PASN: Minimal excitations

We have shown that the PASN is universally super-Poissonian [see Eq. (26)] whatever is the initial NEQ distribution. This is a first central ingredient of our alternative path. The second one is to define minimal excitations as those for which the PASN becomes Poissonian, thus equality is reached in Eq. (26). We focus, for simplicity, on a periodic $p(t)$ with $|p(t)| = 1$.

Instead of solving for the voltage, we gain generality by reasoning in terms of $\varphi(t)$ and the dc frequency ω_J [the relations in Eqs. (5a) and (5b) are not systematic]. Now ω_J, which does not fix the transferred charge, is not fixed but has rather to be determined, on the same level as $\varphi(t)$, by requiring equality in Eq. (26). For that, we write Eqs. (C2) and (B1) in the limit of a strictly zero temperature,

$$S_{ph}(\omega_J) = e^*[I_{ph,+}(\omega_J) - I_{ph,-}(\omega_J)], \tag{31a}$$

$$I_{ph}(\omega_J) = I_{ph,+}(\omega_J) + I_{ph,-}(\omega_J). \tag{31b}$$

We have separated $I_{ph,\pm}(\omega_J) = \sum_{\pm(\omega_J + l\Omega_0) \geqslant 0} P_l\, I_{dc}(\omega_J + l\Omega_0)$, the contributions to the PAC generated by either positive or negative effective dc drives. We have used the fact that $I_{dc}(\omega_J = 0) = 0$ and $\omega_J I_{dc}(\omega_J) \geqslant 0$ for a thermal distribution [27], so that $\pm I_{ph,\pm}(\omega_J) \geqslant 0$. Therefore the Poissonian limit is reached whenever $I_{ph,+}(\omega_J) = 0$ or $I_{ph,-}(\omega_J) = 0$. We focus here on the condition $I_{ph,-}(\omega_J) = 0$. In order to be ensured whatever the profile of I_{dc}, this requires that $P_l = 0$ for all l such that $\omega_J + l\Omega_0 < 0$. Then one can show, using similar arguments to those by L. Levitov *et al.* [18], that the phase must have the form in Eq. (29), and that $\omega_J = N\Omega_0$ due to analytic properties of $p(t)$ in the complex plane.

Therefore, we get, from Eq. (30)

$$S_{ph}(N\Omega_0) = e^*|I_{ph}(N\Omega_0)|. \tag{32}$$

This Poissonian regime indicates that the PASN reduces to the average charge given by $e^*|I_{ph}(N\Omega_0)|$, now generated only by photon absorption of the many-body ground state. Indeed, since temperatures are always finite, and even for the present NEQ quantum regime with $T \ll \hbar\Omega_0/k_B$, one has: $S_{ph}(N\Omega_0) = e^*|I_{ph}(N\Omega_0)| + 2k_B T P_{-N} G_{dc}(T)$ [see Eq. (14)].

Similarly, in case one superimposes a finite dc frequency ω_{dc} on top of $N\Omega_0$, one goes back to a super-Poissonian PASN. Let us give an example for $N = 1$ and decrease the dc drive by a frequency ω_{dc} verifying $k_B T/\hbar \ll \omega_{dc} < \Omega_0$. Then we get

$$S_{ph}(\Omega_0 - \omega_{dc}) = e^*|I_{ph}(\Omega_0 - \omega_{dc})| + 2e^* P_{-1}|I_{dc}(-\omega_{dc})|.$$

This analysis provides another example at odd with the inequality in Eq. (22), by considering again a SIS junction in the ground state (see Fig. 5). As mentioned in Sec. IV A, a sine voltage reduces the PAC in the range $\omega_C < \omega_J < \omega_C + \Omega_0$ compared to $I_{dc}(\omega_J)$ [88], a result we can extend to an arbitrary profile of the voltage. Since we showed that Lorentzian pulses generate a Poissonian PASN [we do not have any thermal contribution as $G_{dc}(T) = 0$], one has $S_{ph}(N\Omega_0) < S_{dc}(N\Omega_0)$ if N verifies $0 < N\Omega_0 - \omega_C < \Omega_0$. In particular, Lorentzian pulses with one electron per cycle ($N = 1$) reduce the PASN with respect to the dc noise whenever the frequency of the pulses is above the gap, $\Omega_0 > \omega_C$.

Recall also that a Poissonian PASN can be reached by a weak sine voltage applied to the SIS junction (see Sec. IV A), thus is not exclusive to Lorentzian pulses.

Our analysis can be extended to a nonperiodic $p(t)$ with a possible time-dependent modulus $|p(t)|$, where similar analytic properties of $p(\omega)$ lead to a Poissonian PASN.

We finally insist that for a NEQ initial distribution, the inequality in Eq. (26) remains strict even for Lorentzian pulses.

C. FQHE: Nontrivial charge of minimal excitations and super-Poissonian PASN

In the FQHE, the renormalization by a fractional charge e^* in front of the current arises from the fact that A translates the charge by e^*, which is chosen as the dominant process (that which has the lower scaling dimension [73]). But contrary to the initial claim of L. Levitov *et al.*, the Lorentzian pulses cannot carry $Q_{\text{cycle}} = Ne^*$ per cycle. It was shown, in Refs. [23,53], that one has still $Q_{\text{cycle}} = Ne$, but the proof is restricted to Laughlin states, $\nu = 1/(2n + 1)$, for which $e^* = \nu e$.

Let us consider hierarchical states for other incompressible filling factors. One needs to assume, in order to reach an almost Poissonian PASN, that the Lorentzian pulses are not deformed at the level of the QPC. We also assume that Eqs. (5a) and (5b) hold, where e^* enters, so that the condition $\omega_J = N\Omega_0$ means that the value of V_{dc} for a given frequency Ω_0 depends on e^*.

Given this condition, we would like to provide the charge carried by a minimal injected excitation in the region before the QPC (see Fig. 1), where the chiral current reduces to the first term on the rhs of Eq. (8), thus for $x < 0$. The charge per cycle is given by (as $\omega_J = N\Omega_0 = 2N\pi/T_0$)

$$Q_{\text{cycle}} = \nu\frac{e^2}{h}\int_0^{T_0} dt V(t) = Ne\frac{\nu e}{e^*}. \tag{33}$$

This suggests that Q_{cycle} gives a possible access to e^*, as one generally determines ν from conductance plateaus. Within effective theories, there are many models whose dominant backscattering process [73] can carry different charges e^*. For instance, for $\nu = 2/(2n + 1)$ with integer n, some models lead to $e^* = e/(2n + 1)$ [35,36,72], so that $Q_{\text{cycle}} = 2Ne$ is integer, which is the same charge as that in the IQHE at $\nu = 2$.

Indeed, the weak backscattering regime holds above k_BT_B where the thermal contribution to the PASN in Eq. (14) cannot be ignored. Therefore we get a strictly super-Poissonian PASN, contrary to the claim for the TLL in Ref. [23], $S_{ph}(N\Omega_0) = e^*|I_{ph}(N\Omega_0)| + 2k_BTP_{-N}G_{dc}(T)$, where $G_{dc}(T)$ is a power of T.

VI. OTHER APPLICATIONS

A. Shot-noise spectroscopy

In general, the transfer rates $\bar{P}(\omega)$ in Eq. (10) might be unknown as they can be affected, for instance, by interactions or by NEQ or fluctuating sources. Thus one possible advantage of the FR in Eq. (10) would reside in shot-noise spectroscopy. An interesting protocol has been proposed in Ref. [89], but is nonetheless restricted to noninteracting electrons, a linear dc current and periodic voltages. It should be more facilitated here by the compact form of the FR in Eq. (10) in terms of the dc noise S_{dc}, which has a nontrivial behavior in nonlinear systems. There are in addition situations where the sources to be probed are nonperiodic, such as a random non-Gaussian radiation [see Eqs. (15) and (17)]. Without knowledge of the underlying model, one could measure the noise both in absence and in presence of the sources, then extract $\bar{P}(\omega')$ by varying the dc drive ω_J.

Indeed as $\bar{P}(\omega') = |p(\omega')|^2$ in Eq. (10) hides the phase of $p(\omega')$, it would be more efficient to consider PASN at a finite frequency ω, where nondiagonal terms $p(\omega')p^*(\omega' + \omega)$ enter (see Ref. [24]). Interestingly, we have also obtained these nondiagonal terms in the differential of the PASN with respect to $\varphi(\omega)$, given by Eq. (18) [or the ac voltage in case Eq. (5b) holds]. In order to evaluate differentials, the phase of $p(t)$ has to be known, so that this procedure applies when one needs to determine its time-dependent amplitude (e.g., for tunneling or a Josephson energy). Nonetheless, one could superimpose a controlled phase $\varphi_a(t)$ on an unknown phase $\varphi(t)$, then take the differential in Eq. (18) in the limit $\varphi_a(t) = 0$, such that $p(\omega')$ on the rhs becomes determined only by $\varphi(t)$. Notice that one can also superimpose a periodic $\varphi_a(t)$ on top of a nonperiodic $\varphi(t)$.

Now one could probe directly a small enough $\varphi(t)$, using the second order expansion in Eq. (20). This is especially easier when one applies a sine phase without knowing its amplitude φ_{ac}, renormalized for instance by interactions while keeping the same form, $\varphi(t) = \varphi_{ac}\cos\Omega_0 t$. Then, given an arbitrary ω_J, one needs to measure both the PASN and the dc noise and to consider the ratio in Eq. (21).

Another spectroscopy scheme, valid in the case of a thermal distribution, could be based on exploiting the thermal contribution on the rhs of Eq. (14), $2P_{-N}k_BTG_{dc}(T)$. For each N (thus a dc voltage), looking at the unique term in the noise which depends on $T < \hbar\Omega_0/k_B$ provides P_{-N}, once one measures $G_{dc}(T)$.

B. Robust determination of the fractional charge

An important family of applications of our approach consists into robust methods we have proposed for the determination of the fractional charge in the FQHE [26,27,33,75], and implemented experimentally to determine $e^* = e/5$ at $\nu = 2/5$ in Ref. [35] and $e^* = e/3$ at $\nu = 2/3$ in Ref. [36]. Such methods are more advantageous than those based on the dc Poissonian noise [37] in Eq. (13). In particular they do not require thermalized states nor high voltages, which could induce heating. They are based on looking at the noise argument rather than a proportionality factor, as the key step is to determine the Josephson frequency ω_J, which yields the charge e^* in case the relation in Eq. (5a) holds. The method based on the NEQ FR in Eq. (10) works better if the dc noise has a singular behavior close to zero, which corresponds to a locking $\omega_J = N\Omega_0$. Such a singularity becomes more pronounced by taking the second derivative $\delta^2 S_{ph}(\omega_J)/\delta^2\omega_J$, formed by a series of peaks around $N\Omega_0$. Nonetheless, if one deals with an initial thermal ρ_0, a low enough temperature is required to preserve these peaks, which would be otherwise rounded by thermal effects when $|\omega_J - N\Omega_0| < k_B T/\hbar$ [see the second term on the rhs of Eq. (14)].

We propose here a more direct method, which does not rely on such a singular behavior nor low temperatures, and equally valid for a NEQ distribution ρ_0. It is based on the FR for the second differential of the PASN in Eq. (19). Assuming that $\varphi_{ac} = e^* v_{ac}/\hbar$, $\omega_J = e^* v_{dc}/\hbar$, one looks for the value of e^* for which both sides, determined only by PASN, become identical. This would be easier in the limit of a small cosine modulation, using Eq. (20).

We have also derived a similar relation for the PAC [26]. Nonetheless, the PAC becomes trivial for a linear dc current [see Eq. (27)], as is often the case for the experimental groups aiming to determine the fractional charge [35,36], thus motivating their recourse to the methods based on noise we had proposed [75,76]. Thus the measured dc current does not obey a power-law behavior as predicted by the effective theories. This illustrates precisely the power of the methods based on the NEQ approach, which are independent on the underlying microscopic description of the edge states as long as it can be cast in the form of Eq. (1).

VII. DISCUSSION AND CONCLUSIONS

We have studied the PASN generated by radiation fields operating in a large family of physical systems, such as a QPC in the FQHE or the IQHE with interacting edges, as well as quantum circuits formed by a JJ, NIS, or a dual phase slip JJ strongly coupled to an electromagnetic environment. We have related the PASN in a universal manner to its counterpart in a dc regime characterized by a NEQ distribution, similarly to relations obeyed by the finite-frequency current for ac drives [27] and finite-frequency noise in the dc regime [33]. The NEQ FRs unify higher dimensional and one-dimensional physics, although the latter is atypical as it is drastically affected even by weak interactions. They also unify previous works based on specific models and an initial thermal equilibrium [22,23,53,89].

We have discussed how can these NEQ FRs be potentially relevant to shot-noise spectroscopy. In addition, we can transpose to the PASN various methods based on the PAC and addressed in Refs. [26,27,67], in particular to probe the fractional charge or to detect current cumulants of a non-Gaussian source, though the implementation is not identical due to different properties of current and noise. Indeed, in case the dc current is linear and $|\bar{p}(t)| = 1$, the PAC becomes trivially equal to the dc current and the PASN offers a nontrivial alternative. This is precisely the case in two situations arising in the IQHE and FQHE.

On the one hand, interactions between edge states still play an important role in the IQHE, which is addressed in many works through the plasmon scattering approach [40,41,43,90,91]. But bosonized models justify the linearity of the dc current through a spatially local QPC, thus permitting the recourse to the scattering approach for independent electrons in those works. Notice that all these hypothesis are not required within our approach, as we can deal with a possible nonlinear dc current, which is the signature of the QPC.

On the other hand, in the FQHE, the dc measured current is quite often weakly nonlinear in experiments aiming to probe fractional charges [35,36] and statistics [74]. As the PAC is trivial, the FR for the PASN, already obtained in Ref. [24], has been fruitful for an experimental determination of the fractional charge at $\nu = 2/5$ [35] as well as for the analysis of two-particle collision experiments [6,66]. In those experimental works where effective theories such a the TLL are not in accordance with the observed dc current, our methods have the advantage to be robust with respect to the underlying microscopic description and nonuniversal features, such as edge reconstruction or absence of edge equilibration. In the present paper, we have proposed a more advantageous method based on a second differential of the PASN, or on its expansion for a weak sine voltage.

We have also found that the excess noise can be negative in a nonlinear SIS junction. Thereby the qualification of "photoassisted" is not universally relevant: the PASN can be reduced by an ac voltage superimposed on a dc one. This feature is at odd with a theorem by L. Levitov *et al.* [18], which is restricted to a linear dc current. Such a theorem was at the heart of characterizing minimal excitations for an initial thermal equilibrium at low temperatures. We have provided an alternative characterization. Showing that the PASN is super-Poissonian whatever is the NEQ initial distribution, the Lorentzian profile of the voltage is precisely the one which leads to a Poissonian PASN when the system is in the ground many-body state. For hierarchical states of the FQHE, we showed that the charge carried by minimal excitations is still depending on the fractional charge, and that the PASN is super-Poissonian.

Finally, compared to the dc regime, additional limitations of the approach arise. These are mainly due to possible couplings to time-dependent forces or boundary conditions, which have to be incorporated into a unique complex function, for instance through unitary transformations of the Hamiltonian. Thus the approach might be still justified in presence of one additional tunneling point between edges, but not for multiple mixing points (see for instance Ref. [92]). Also an interferometer with multiple QPCs driven by different ac voltages is not expected to enter within the domain of validity of the present approach, but the latter would still offer a test in the limiting cases of identical ac voltages, or of a dominant tunneling through one QPC.

In the quantum Hall regime with an almost open QPC, one usually measures correlations between chiral edge currents. Such correlations don't always reduce to the backscattering PASN, especially in NEQ setups such as the anyon collider. But the perturbative approach is still useful to express the additional terms, as will be addressed separately [79] (in the same spirit as Refs. [77,80–82,93,94] for the dc regime).

Finally, an important open question consists into characterizing minimal excitations beyond the second-order perturbation we have carried on.

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APPENDIX A: DERIVATION OF THE NEQ FR

This Appendix provides a detailed derivation of the relation obtained in Eq. (10). In order to express the PASN in Eq. (7) to second order of perturbation with respect to A, we do not need an expansion of the S matrix, as S is already of second order. Thus we can directly replace $\delta\hat{I}_{\mathcal{H}}(t)$ by $\hat{I}_{\mathcal{H}_0}(t)$, or, in Eq. (4), $A_{\mathcal{H}}(t)$ by $A_{\mathcal{H}_0}(t) = e^{i\mathcal{H}_0 t}A\, e^{-i\mathcal{H}_0 t}$. Then the effect of the ac drive factorizes,

$$\frac{S(\omega_J;t,\tau)}{e^*} = e^{-i\omega_J\tau}p\Big(t+\frac{\tau}{2}\Big)p^*\Big(t-\frac{\tau}{2}\Big)I_{\rightarrow}(-\tau) + e^{i\omega_J\tau}p^*\Big(t+\frac{\tau}{2}\Big)p\Big(t-\frac{\tau}{2}\Big)I_{\leftarrow}(\tau). \quad \text{(A1)}$$

The two correlators $I_{\rightarrow}(\tau), I_{\leftarrow}(\tau)$ determine all observables associated with the current in Eq. (4) to second-order perturbation; they keep track of unspecified Hamiltonian and initial stationary NEQ density matrix ρ_0, thus depend only on time difference τ [27,33],

$$\hbar^2 I_{\rightarrow}(\tau) = e^*\langle A^{\dagger}_{\mathcal{H}_0}(\tau)A_{\mathcal{H}_0}(0)\rangle,$$
$$\hbar^2 I_{\leftarrow}(\tau) = e^*\langle A_{\mathcal{H}_0}(0)A^{\dagger}_{\mathcal{H}_0}(\tau)\rangle. \quad \text{(A2)}$$

Notice that the Fourier transforms $I_{\rightarrow}(\omega), I_{\leftarrow}(\omega)$ are real because both functions verify: $X^*(\tau) = X(-\tau)$ [24,27,83]. Here $X(\omega) = \int d\tau e^{i\omega\tau}X(\tau)$, as the measurement time T_0 delimits only integration over t. This implements the first important step underlying the derivation of various NEQ perturbative relations.

As time-translation invariance is broken, double-Fourier transform introduces two frequencies, ω, Ω,

$$S(\omega_J;\omega,\Omega) = \int_{-T_0/2}^{T_0/2}\frac{dt}{T_0}\int_{-\infty}^{\infty}d\tau e^{i\Omega t}e^{i\omega\tau}S(\omega_J;\tau,t). \quad \text{(A3)}$$

Focussing here on $\Omega=\omega=0$, and letting $S_{ph}(\omega_J) = S(\omega_J;0,0)$, we obtain the PASN in terms of the Fourier transforms of the two correlators in Eq. (A2)

$$S_{ph}(\omega_J) = e^*\int\frac{d\omega'}{\Omega_0}|p(\omega')|^2[I_{\rightarrow}(\omega_J+\omega') + I_{\leftarrow}(\omega_J+\omega')]. \quad \text{(A4)}$$

We have defined $p(\omega) = \int_{-T_0/2}^{T_0/2}e^{i\omega t}p(t)dt/T_0$. An additional term, not considered here, might arise in presence of a singularity $p_{dc}\delta(\omega)$ (such as is the case for a single Lorentzian pulse). Such a term has a similar form as that appearing in the PAC in Ref. [27], replacing the current by the noise.

It is useful to recall that $I_{\rightarrow}, I_{\leftarrow}$ determine as well the expressions of current average and zero-frequency noise in the dc regime, i.e., at $p(t) = 1$, given by Eqs. (9a,9b) [24,33,76].

Thus, we can view $I_{\rightarrow}$ and $I_{\leftarrow}$ as transfer rates in opposite directions, whose difference yields the dc current, while their superposition yields the dc noise. For a JJ in series with an electromagnetic environment, $I_{\rightarrow}, I_{\leftarrow}$ offer the two counterparts, given initial NEQ states, of the $P(E)$ function which corresponds rather to initial thermal states.

We stress that, contrary to the majority of previous studies on TD transport, the two correlators $I_{\rightarrow}, I_{\leftarrow}$ are not necessarily linked: one can have $I_{\rightarrow}(\omega) \neq I_{\leftarrow}(-\omega)$ and they do not obey a detailed balance equation if we do not consider initial thermal states. Therefore, $I_{\rightarrow}, I_{\leftarrow}$ are, in full generality, two independent functions.

Next, in order to derive the FR relating the PASN under the drive $p(t)$ to that in the dc regime, S_{dc}, we compare the two expressions respectively given by Eqs. (A4) and (9b).

One can then see that the combination of the NEQ correlators in the integral of Eq. (A4) is nothing but the noise in the dc regime, evaluated at an effective dc drive given by $\omega_J + \omega'$. This leads to Eq. (10).

APPENDIX B: PERIODIC DRIVES

This Appendix is devoted to detail the PASN and the PAC in presence of a periodic $p(t)$ at a frequency Ω_0. Then the integral in Eq. (10) reduces to a sum over $\omega' = l\Omega_0$ for integer $l \geqslant 1$, which leads to Eq. (12).

This relation is similar to the PAC in Eqs. (6) and (24) [26,27],

$$I_{ph}(\omega_J) = \sum_{l=-\infty}^{+\infty}P_l I_{dc}(\omega_J + l\Omega_0). \quad \text{(B1)}$$

We notice that when $|p(t)| \neq 1$, we have $\sum_{l=-\infty}^{+\infty}p_l p^*_{l+k} = F.T.[|p(t)|^2]_k$, the Fourier transform at $k\Omega_0$ of $|p(t)|^2$. In particular, for $k=0$, we have

$$\sum_{l=-\infty}^{l=+\infty}P_l = \langle|p(t)|^2\rangle_{T_0}, \quad \text{(B2)}$$

where average refers to that over a period (see Ref. [27] for a nonperiodic $p(t)$). When $|p(t)| = 1$, we recover the orthogonality

$$|p(t)| = 1 \Rightarrow \sum_{l=-\infty}^{+\infty}p_l p^*_{l+k} = \delta_k, \quad \text{(B3)}$$

where δ_k is the Kronecker sign.

APPENDIX C: INITIAL THERMAL EQUILIBRIUM DISTRIBUTION

This Appendix gives a more detailed expression of the PASN for an initial thermal distribution. By injecting the expression of the dc noise in Eq. (13) into Eq. (10), the PASN becomes totally determined by the NEQ dc current,

$$S_{ph}(\omega_J) = e^*\int\frac{d\omega'}{\Omega_0}\bar{P}(\omega'-\omega_J)\coth\Big(\frac{\hbar\omega'}{2k_BT}\Big)I_{dc}(\omega'). \quad \text{(C1)}$$

Our expression is different from the one derived by L. Levitov *et al.* for a QPC in the FQHE [18], and which we recover only for a linear dc current $I_{dc}(\omega') = \hbar G_{dc}\omega'/e^*$. For a periodic drive, we get

$$S_{ph}(\omega_J) = e^* \sum_{l=-\infty}^{l=+\infty} P_l \coth\left[\frac{\hbar(\omega_J + l\Omega_0)}{2k_B T}\right] I_{dc}(\omega_J + l\Omega_0). \tag{C2}$$

Let us now specify further to the case $\omega_J = N\Omega_0$. Since we deal here with an equilibrium thermal distribution, we have $\omega_J I_{dc}(\omega_J) \geqslant 0$ even when I_{dc} is not odd (see Ref. [27]). If we consider now the NEQ quantum regime at $\Omega_0 \gg k_B T/\hbar$, we obtain Eq. (14) in the text. Though we have a NEQ PASN, an equilibrium thermal noise weighted by P_{-N} arises, given by $P_{-N}S_{dc}(0)$. It is due to a vanishing effective dc frequency when the many-body state at energy $N\Omega_0$ emits N photons. For independent electrons, it was interpreted as a reduced thermal contribution from the reservoirs [48]. Since the dc current becomes then linear, one has generically $G_{dc}(T) = G_{dc}$ constant, so that the contribution of $l = -N \pm 1$, proportional to $G_{dc}\hbar\Omega_0$, dominates $k_B T G_{dc}$.

Nonetheless one cannot neglect systematically $k_B T G_{dc}(T)$ in nonlinear conductors, where $G_{dc}(T)$ depends on T, as is the case of effective theories for the FQHE [79].

APPENDIX D: THREE CHOICES FOR THE EXCESS PASN

This Appendix aims to discuss the various choices and sign of the excess noise. In the dc regime, it is often defined as

$$\Delta S_{dc}(\omega_J) = S_{dc}(\omega_J) - S_{dc}(0). \tag{D1}$$

Recall that in a NEQ setup with couplings to dc voltages independent from ω_J, $S_{dc}(0)$ is finite even when all temperatures are set to zero, and is therefore different from the thermal equilibrium noise.

For the excess PASN, there is not a single convention as it depends on which reference is chosen in a given experimental context. In Eq. (11), we have chosen as a reference the dc noise in presence of the same dc voltage.

One could also adopt a second choice, by subtracting the same reference as that in Eq. (D1),

$$\Delta S_{ph}(\omega_J) = S_{ph}(\omega_J) - S_{dc}(0). \tag{D2}$$

This yields, focusing on a periodic drive at a frequency Ω_0 [see Eq. (12)],

$$\Delta S_{ph}(\omega_J) = \sum_{l=-\infty}^{+\infty} P_l \Delta S_{dc}(\omega_J + l\Omega_0) + \left(\sum_{l=-\infty}^{+\infty} P_l - 1\right) S_{dc}(0). \tag{D3}$$

The relations between the two choices is given by

$$\underline{\Delta} S_{ph}(\omega_J) = S_{ph}(\omega_J) - S_{dc}(\omega_J) = \Delta S_{ph}(\omega_J) - \Delta S_{dc}(\omega_J). \tag{D4}$$

Excess noise is expected to have a positive sign, as noise should increase with additional voltage sources. This is indeed not systematic in the dc regime, as we have shown for zero or finite frequency noise for nonlinear conductors [83,95]. In the text, Sec. IV A, we showed that the choice in Eq. (11) can lead to a negative sign in a SIS junction. In a separate paper, we will show that Eq. (D3) has a negative sign in the FQHE.

In view of the super-Poissonian noise in Eq. (26), a third choice guarantees a positive sign, $S_{ph}(\omega_J) - e^*|I_{ph}(\omega_J)|$. But such a choice is not so advantageous. If the dc current is nonlinear, one would need to measure the nontrivial PAC. In addition, subtracting a noise reference is more convenient to get rid of undesirable sources, which affect the PASN in a different manner from the PAC. For instance, if one takes the zero dc voltage limit, one has $I_{ph}(0) = 0$ in case I_{dc} is odd and $P(\omega') = P(-\omega')$, but has still a finite $S_{ph}(0)$.

A similar choice was given in Ref. [23]. Restricted to a thermal equilibrium and to the TLL model, that work recovered the super-Poissonian PASN of Ref. [24]. This motivated the authors to define the excess noise as $S_{ph}(\omega_J) - e^* \coth[\beta\omega_J/2] I_{ph}(\omega_J)$, whose sign is however not well determined. In case the dc current is linear [see Eq. (27)], this amounts to adopt the second choice, Eq. (11). Such a definition was intended to cancel thermal contributions, but indeed cancels only the contribution of $l = 0$ in Eq. (14), and not the term $P_{-N}S_{dc}(0)$.

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