Nonparaxiality-triggered Landau-Zener transition in spoof plasmonic waveguides

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Photonic lattices have been widely used for simulating quantum physics, owing to the similar evolutions of paraxial waves and quantum particles. However, nonparaxial wave propagations in photonic lattices break the paradigm of the quantum-optical analogy. Here we reveal that nonparaxiality exerts stretched and compressed forces on the energy spectrum in the celebrated Aubry-André-Harper model. By exploring the minigaps induced by the finite size of the different effects of nonparaxiality, we experimentally present that the expansion of one band gap supports the adiabatic transfer of boundary states while Landau-Zener transition occurs at the narrowing of the other gap, whereas identical transport behaviors are expected for the two gaps under paraxial approximation. Our results not only leverage nonparaxial transitions as a new degree of freedom, but also serve as a foundation for future studies of dynamic state transfer.

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I. INTRODUCTION

Photonic lattices such as waveguide arrays provide a versatile platform for investigating fundamental physics [1–9]. Over the past two decades, many intriguing phenomena have been demonstrated in evanescently coupled waveguide arrays, including Landau-Zener (LZ) transition [10,11], topological end modes [12–16], Anderson localization [17–20] in disordered lattices, etc. The underlying principle relies on the analogy between the paraxial Helmholtz equation for electromagnetic waves and the Schrödinger equation describing quantum particles. However, the nonparaxial approximation cannot be always strictly satisfied, e.g., in waveguide systems conducting microwaves with long wavelengths, where the spatial evolution of the field envelope is comparable to the variation of the structural parameters. In fact, nonparaxial light is quite ubiquitous in natural photonic systems, such as spin-orbit interaction of nonparaxial light [21,22], nonparaxial Airy beams [23–28] and other nonparaxial accelerating beams [29–31], etc. Recently, nonparaxiality has attracted growing interests, which has been shown to play an important role in third-harmonic generation [32] and asymmetric topological pumping [33]. Nevertheless, the study of nonparaxial wave propagation and related phenomena is still in its infancy, and it still remains elusive how nonparaxiality can benefit the field of photonics. To this end, demonstrating the largely overlooked functionality of nonparaxility by well-known fundamental physical phenomena would be highly desired.

As a typical fundamental dynamics, LZ transition, a transition between two states in a quantum system driven under a time-dependent Hamiltonian, is frequently encountered in different physical fields. Classical analogs of LZ transitions have also been investigated in platforms such as atomic-optical system [34–36], coupled cavities [37,38], and other two-state systems [39–41]. Interestingly, LZ transition in topological transport of edge states has recently been demonstrated in driven acoustic cavity systems, where an on-site term is implemented with the frequency in each waveguide cavity of varying height [42]. In addition, since an adiabatic condition is usually required in topological photonics, such as the topological pumping in periodically modulated lattice systems [6,43–51], the nonadiabatic LZ transition is expected to break the topological transport [52,53]. It is thus highly appealing to study the LZ transition in nonparaxial topological photonic systems, where the interplay between topology, adiabaticity, and nonparaxiality could give rise to interesting phenomena.

In this work we study the LZ transition in a spatially modulated waveguide array designed according to the celebrated topological Aubry-André-Harper (AAH) model, which conducts microwaves in a nonparaxial way. The two topological boundary states (TBSs) localized at opposite boundaries in each of the bulk gap act as two avoided-crossing levels due to the finite-size effect. The time-dependent linear driving through the avoided-crossing points was realized by tuning the structural configurations of the microwave waveguide. It is found that when taking the paraxial approximation, the TBSs in the two bulk gaps have nearly equivalent minigaps, and

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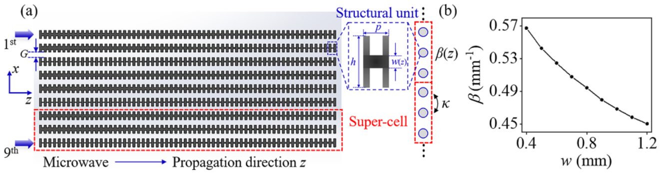

FIG. 1. Schematic diagram of a finite Aubry-André-Harper (AAH) lattice realized by well-designed periodically modulated ultrathin metallic waveguides. (a) The constant nearest-neighboring (NN) hopping comes from NN coupling between equally spaced waveguides, while the modulation of on-site potential (propagation constant) can be realized by changing the girder width w of the H-shape unit of each waveguide. (b) Propagation constant β as a function of girder width w.

exhibit identical behaviors, namely, LZ transition or adiabatic pumping between TBSs, depending on the energy gap and driving frequency. Intriguingly, when the realistic nonparaxial modifications are considered, one TBS minigap gets enlarged, while the other is reduced. Remarkably, LZ transition was observed around the significantly reduced minigap induced by nonparaxiality, instead of both gaps behaving identically under the paraxial approximation. Our experimental results are nicely consistent with both numerical simulations and theoretical analysis of the exact Helmholtz wave equation. Our work may provide further insights for the study of unexplored dynamics triggered by the nonparaxiality of light.

II. PHOTONIC AAH MODEL

We start from the one-dimensional AAH model [43,45,54] with a periodic spatial modulation of the on-site potential, which is described by the tight-binding Hamiltonian

$$H_A = \sum_{m=1}^{N} \beta_m \hat{c}_m^\dagger \hat{c}_m + \sum_{m=1}^{N-1} [\kappa \hat{c}_m^\dagger \hat{c}_{m+1} + \text{ H.c.}]. \quad (1)$$

Here m labels the lattice site with a total number of N, $\hat{c}_m^\dagger$ ($\hat{c}_m$) creates (annihilates) a particle at the mth site, κ is the nearest-neighbor (NN) hopping coefficient, and $\beta_m = \beta_0 + \Delta\beta \cos(2\pi bm + \phi)$ represents the spatially modulated potential of the mth site. In the modulation term, $\Delta\beta$ is the modulation amplitude, b controls the periodicity, which is set representatively as $b = 1/3$ throughout the paper, resulting in a supercell structure with three sublattices. $\phi \in [0, 2\pi]$ is the modulation phase, which plays a similar role as a momentum variable.

To realize a photonic counterpart of the above AAH model, we have experimentally fabricated well-designed ultrathin metallic waveguides composed of the "H-shape" structural unit [55,56], as schematically shown in Fig. 1(a), where the x direction denotes the spatial dimension and the propagation direction z acts as the synthetic time dimension. Through the coupled-mode theory, the constant NN hopping κ can be simulated by the coupling between equally spaced NN waveguides in the whole array (see Fig. S1 in the Supplemental Material (SM) [57]). For the later realization of LZ transition, a time-dependent modulation of phase along the propagation direction is introduced as $\phi = \phi_0 + \Omega z$, where ϕ_0 represents the initial phase and the "frequency" Ω is defined as the ratio between the total change of the phase $\Delta\phi$ and the waveguide length L. The desired instantaneous on-site potential profile can then be obtained by modulating the propagation constant of each waveguide along both the spatial dimension x and propagation direction z, which is achieved by changing the girder width w of the H-shape unit; see in Fig. 1(b).

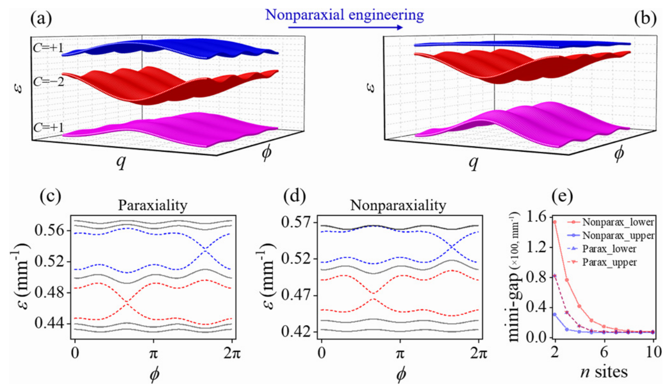

FIG. 2. The band structures in the two-dimensional momentum space (q, ϕ) under (a) paraxial approximation and (b) nonparaxial modification. The calculated eigenvalue as a function of modulation phase ϕ under (c) paraxial approximation and (d) nonparaxial correction. The parameters are $N = 9$, $\beta_0 = 0.501$ mm^{-1} ($w_0 = 0.8$ mm), $\kappa = 0.0316$ mm^{-1} ($G = 1.8$ mm), $\Delta\beta = 0.0493$ mm^{-1} ($\Delta w = 0.35$ mm), and $\kappa_{\text{NNN}} = -0.00632$ mm^{-1} ($2kn_0 \sim 0.158$). (e) The lower and upper gaps as a function of the number of lattice sites.

By carrying out the Fourier transform $\hat{c}_m = (1/\sqrt{N}) \sum_q e^{iqm} \hat{c}_q$ under periodic boundary conditions and treating ϕ as a momentum dimension, the band structure in the two-dimensional (2D) (q, ϕ) momentum space can be obtained [58,59]; see Fig. 2(a). The topological properties of the three bands can be characterized by the Chern number of $C = 1, -2$, and 1, respectively, guaranteeing the emergence of chiral edge states (henceforth referred to as TBSs) within both bulk band gaps in the open boundary condition. The TBSs are localized at the boundary waveguides, exhibiting a nearly exponential decay into the bulk. Consequently, for sufficiently large lattices, the TBSs from opposite boundaries within the same gap have negligible spatial overlap and coupling, thus forming a gapless crossing in each gap (see the SM for details [57]). However, when reducing the lattice size, the spatial overlap between the TBSs gradually increases, and becomes non-negligible for small lattice sizes. This will cause considerable coupling and level repulsion with a minigap between the two TBSs.

Interestingly, when treating the TBSs in each bulk gap as a two-level system, the potential modulation along the z direction practically acts as a time-dependent driving of the two-level system [60,61]. When the modulation frequency Ω is comparable to the minigap between the two TBSs, LZ transition may happen between them. In the following we will go beyond the coupled-mode theory, and consider how the nonparaxiality of microwave modifies the energy spectrum and affects the LZ transition.

III. NONPARAXIAL MODIFICATIONS

In the derivation of the above photonic AAH model from the coupled-mode theory, paraxial approximation has been assumed, which requires the condition of $|\partial^2\psi/\partial z^2| \ll 2k|\partial\psi/\partial z|$ with the wave vector k. Specially, since our waveguide system conducts microwaves with much smaller wave vectors, the above condition will not be fully satisfied, and thus we need to go beyond the coupled-mode theory and turn back to the original Helmholtz equation for the waveguide system,

$$i\frac{\partial\psi}{\partial z} - \frac{1}{2kn_0}\frac{\partial^2\psi}{\partial z^2} = H\psi. \tag{2}$$

Here $H = \frac{1}{2kn_0}\nabla_x^2 + \frac{k}{2}(\frac{n^2-n_0^2}{n_0})$ is the Helmholtz-Hamiltonian operator which can be replaced by the above structurally modeled tight-binding AAH Hamiltonian H_A, n_0 is the reference refractive index, and the wave vector is given as $k = \omega/c$ for monochronic electromagnetic wave $E = \Re\{\psi e^{i\omega t - ikn_0 z}\}$. To achieve an intuitive understanding of the nonparaxial effect, we can rewrite Eq. (2) in an effective Schrödinger-type form that absorbs the second derivative of z in a self-consistent way [33]:

$$i\frac{\partial\psi}{\partial z} = \frac{H_A}{1 + i\frac{1}{2kn_0\partial z}}\psi = H_{\text{eff}}\psi, \tag{3}$$

where the effective Hamiltonian under Padé approximation is approximately given by

$$H_{\text{eff}}^{(1,1)} \approx H_A - \frac{1}{2kn_0}H_A^2. \tag{4}$$

Interestingly, apart from the paraxial Hamiltonian H_A, the consideration of nonparaxiality leads to an additional term proportional to H_A^2 (see the SM for details [57]). Based on the previous work [33], we could conclude that the nonparaxial term $-\frac{1}{2kn_0}H_A^2$ contributes a negative next-nearest-neighboring (NNN) coupling, accordingly, $\kappa_{\text{NNN}} \approx -\kappa^2/(2kn_0)$. For our microwave system with a very small wave vector k, such negative NNN couplings cannot be neglected and reshape the energy spectrum. For a much smaller vector, we need to take the higher-order approximant into account, in which longer-range couplings appear and the coupling matrix becomes more complex. In contrast, for larger wave vector (e.g., the optical region), the negative NNN couplings can be ignored, and the effective Hamiltonian is reduced to the paraxial one.

Next, we study how the nonparaxial term modifies the band structure. Thanks to the commutation relation $[H_{\text{eff}}^{(1,1)}, H_A] = 0$, the eigenstate $|\phi\rangle$ of H_A with eigenvalue ε is also an eigenstate of H_{eff}, but the effective energy is shifted to $\varepsilon - \frac{1}{2kn_0}\varepsilon^2$. This modification results in a deformation of the original band structure; see Fig. 2(b).

However, the Chern number for each band also keeps invariant because of the same eigenstates as the original one. It should be noted that for the experimentally relevant parameter regime with a small but non-negligible fitting parameter $1/(2kn_0)$, the first (third) band obviously gets compressed (stretched) with significantly enlarged (reduced) band dispersion and bandwidth; see Fig. 2(b).

IV. LZ TRANSITION BETWEEN TBSS

The realization of LZ transition relies on two conditions, one is a two-level system with an avoided energy crossing determined by a control parameter, and the other is a z-dependent driving to sweep the parameter across the avoided crossing point, where the changing rate of the energy should be comparable to the minimum gap at the avoided crossing. To achieve LZ transition, we resort to z-dependent driving of a small-size system at a finite rate. As a concrete example we consider a small system with $N = 9$ lattices and calculate the energy spectra as a function of modulation phase ϕ for both the paraxial Hamiltonian H_A (without NNN hopping terms) and nonparaxial effective Hamiltonian H_{eff} (with NNN hopping terms $\kappa_{\text{NNN}} = -0.00632\ \text{mm}^{-1}$); see Figs. 2(c) and 2(d), respectively. The other parameters are chosen as $\beta_0 = 0.501\ \text{mm}^{-1}$ ($w_0 = 0.8$ mm), $\kappa = 0.0316\ \text{mm}^{-1}$, and $\Delta\beta = 0.0493\ \text{mm}^{-1}$ ($\Delta w = 0.35$ mm). In the paraxial case [Fig. 2(c)], two nearly equivalent minigaps between two TBSs are opened at the crossing points around $\phi = 2\pi/3$ and $\phi = 5\pi/3$, respectively, which results from almost the same energy dispersion and hence the group velocity of the first and third bulk bands. In the nonparaxial case, however, the upper (lower) minigap is reduced (enlarged).

The different minigap can be understood in the following way. Assume the energy spacing around ε is $\delta\varepsilon$ for the paraxial case. According to the effective energy $\varepsilon - \frac{1}{2kn_0}\varepsilon^2$, the energy spacing in the nonparaxial case is given by $[1 - 1/(kn_0)\varepsilon]\delta\varepsilon$, resulting in that the higher energy has smaller energy spacing. In Fig. 2(e) we show the lower and upper minigaps of TBSs at the avoided crossing points as a function of system size. As the system size increases, the general decrease of two minigaps can be explained by the weaker and weaker higher-order couplings between TBSs. It means that the nonparaxial effects on LZ transition are more prominent in a small-size system.

The time-varying control parameter is engineered by z-dependent modulation phase $\phi(z)$ along the synthetic direction z in our system. In the vicinity of the avoided crossing of the TBSs, LZ transition can be modeled by a two-level effective Hamiltonian [62,63],

$$H_{\text{LZ}}(\delta\phi) = \begin{pmatrix} \alpha\delta\phi & \frac{\Delta}{2} \\ \frac{\Delta}{2} & -\alpha\delta\phi \end{pmatrix}. \tag{5}$$

Here the coupling term Δ can be obtained by the minimum gap size at $\delta\phi = 0$, and α is a fitting parameter characterizing the slope of the crossing. H_{LZ} in Eq. (5) is the two-level effective Hamiltonian for describing the process of two edge states coupling. Obviously we naturally consider $(|\psi_T\rangle, |\psi_B\rangle)$ as a complete set of basis of H_{LZ}. The TBSs of $|\psi_T\rangle$ and $|\psi_B\rangle$ are defined as the top boundary state and the bottom boundary state. We show the eigenvalues of H_{LZ} around the two avoided crossings at $\phi = 2\pi/3$ and $\phi = 5\pi/3$ (solid curves) in Figs. 3(a) and 3(c), respectively.

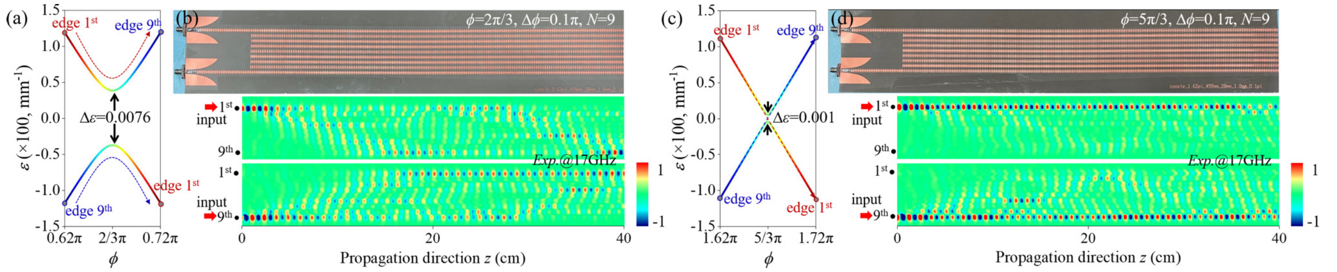

FIG. 3. Experimental results of topological pumping and LZ transition in a microwave waveguide array. (a) and (c) Energy spectrum around $\phi = 2\pi/3$ and $\phi = 5\pi/3$, respectively. (b) and (d) Schematic diagram of the waveguide array and the observation of pumping process around $\phi = 2\pi/3$ and $\phi = 5\pi/3$, respectively. The other parameters are chosen as $N = 9$, $\Delta\phi = 0.1\pi$, $G = 1.8$ mm, $L = 40$ cm.

The eigenstates turn out to be hybridized states of $|\psi_T\rangle$ and $|\psi_B\rangle$ as a result of the coupling. The degree of hybridization is characterized by different colors, where the red and blue colors represent the two unhybridized limits of wave functions $|\psi_T\rangle$ and $|\psi_B\rangle$ localized at top and bottom boundaries, respectively.

Intriguingly, by taking advantage of the significantly enlarged (reduced) gap at $\phi = 2\pi/3$ ($\phi = 5\pi/3$) induced from the nonparaxial effect, we now show that both adiabatic transport behavior and nonadiabatic LZ transition can be realized in our system under the same modulation frequency Ω, which is given by $\Delta\phi/Z_{\max}$. This can be achieved by choosing $\Omega = \pi/4000$ mm^{-1} in the range smaller than the gap at $\phi = 2\pi/3$ but larger than the gap at $\phi = 5\pi/3$. Such a choice makes it possible that the state evolves adiabatically along the eigenstates around $\phi = 2\pi/3$ while it undergoes LZ transition around $\phi = 5\pi/3$ (see the SM for details [57]). In the adiabatic transport we assume that the initial states at $\phi_0 = 0.62\pi$ are sufficiently far away from the avoided crossing of $|\psi_T\rangle$ and $|\psi_B\rangle$ at $\phi = 2\pi/3$. The initial top boundary state (red) could evolve adiabatically with increasing ϕ into the final state (blue) at the bottom boundary, and vice versa; see Fig. 3(a). In the LZ transition we also consider the initial states of $|\psi_T\rangle$ and $|\psi_B\rangle$ at $\phi_0 = 1.62\pi$ away from the avoided crossing at $\phi = 5\pi/3$. When approaching the avoided crossing with increasing ϕ, the top boundary state (red) in the upper energy level tunnels to the top boundary state in the lower level, and so does the bottom boundary state (blue); see Fig. 3(c). This means that the LZ transition renders each boundary state localized at where they start with negligible transfer to the other boundary.

In experiments, the samples of microwave waveguides have the same parameters ($N = 9$, $\Delta\phi = 0.1\pi$, $G = 1.8$ mm, and $L = 400$ mm) as the above discussion; see Fig. 3(b) for $\phi_0 = 0.62\pi$ and Fig. 3(d) for $\phi_0 = 1.62\pi$. After injecting the electric field into the samples, we detect the propagation of E_z component of electric fields on the microwave near field platform (see the SM for the case of $N = 6$ [57]). As expected, in the case of $\phi_0 = 0.62\pi$, we observe that the electric field injected from the first (or ninth) boundary waveguide gradually transfers across the bulk to the ninth (or first) boundary waveguide. Note that the two pumping processes from the top-port excitation and bottom-port excitation are symmetric, as a result of the symmetric energy structure around the avoided-crossing points in Fig. 3(a). The electric fields around the avoided crossing point are dominated in both the top and bottom ports, which can be viewed as a effect of beam splitter. In the case of $\phi_0 = 1.62\pi$, the electric field injected from the first (or ninth) waveguide keeps rather stable during the pumping process, so that the electric field is still concentrated on the initial waveguide first (or ninth) at the output end. Our results already demonstrate the successful transfer of TBS and the localization of isolated TBS, which is based on the nonparaxial condition in the LZ model realized in the microwave system.

V. DISCUSSION

In our system, the nonparaxial effects are significant with a small number of waveguides. We have already shown that as the number of waveguides increases, the coupling effect between the TBSs weakens so that the nonparaxial modifications at the avoided-crossing point become smaller. For a fixed modulation frequency, we may observe that the field propagation changes from adiabatic evolution to LZ transition with increasing waveguide number N. To confirm our argument, the CST simulations have been shown in Fig. 4 for the three cases: $N = 6,\ 12,\ 18$. From the results of simulations, in the cases of $N = 6,\ 12$, the adiabatic tunneling from the right to left boundaries could be observed, but the coupling between

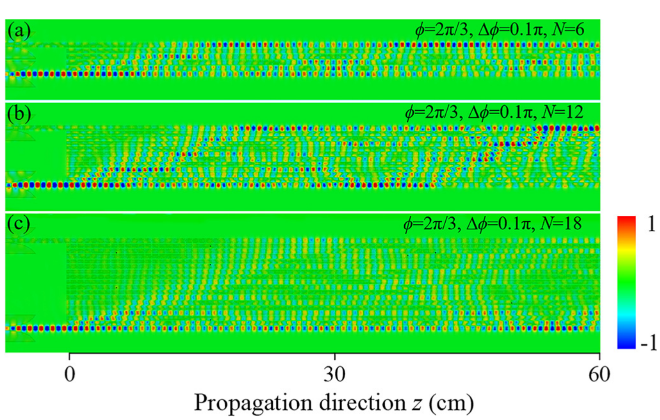

FIG. 4. CST simulation of pumping processes with different sites (a) $N = 6$ and (b) $N = 12$, and (c) $N = 18$. The injected light propagates along the input boundary waveguide without scattering. The other parameters are chosen as $L = 60$ cm, $\phi = 2\pi/3$, $\Delta\phi = 0.1\pi$, Spacing $G = 1.8$ mm between adjacent waveguides.

TBSs is blocked as the N increases to 18, which demonstrates the negligible gap and complete LZ transition.

VI. SUMMARY

We have theoretically predicted and experimentally demonstrated nonparaxiality-triggered LZ transition in microwave waveguide arrays with spatial modulations. Nonparaxiality of microwaves plays a crucial role in modifying spectrum and the adiabatic condition, that is, one gap is enlarged and supports the adiabatic transfer of boundary states while the other gap is narrowed to make the LZ transition. Our approach of microwave nonparaxial engineering can be extended to higher dimensions and can benefit other physical systems, such as mechanical vibrations, elastic waves, electrical circuitries, and thermal transfers.

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