Reply to "Comment on 'Curved-space Dirac description of elastically deformed monolayer graphene is generally incorrect' "

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We respond to the Comment on our paper [Phys. Rev. B **105**, 195412 (2022)]. The authors of the Comment claim our paper arrived at similar conclusions to their earlier paper [Phys. Rev. D **92**, 125005 (2015)]. In this response to their Comment, we outline why we believe our paper is fundamentally different in nature, we review its conclusions, and detail how these are different to those of their earlier work.

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We are pleased that others are actively interested in this topic and definitely appreciate different perspectives on it. However, we feel that our results [1] and those of Ref. [2] are different. In their comment [3], the authors claim that in our paper we "conclude that the curved-space Dirac description of the low-energy conductivity electrons of monolayer graphene is incorrect when only strain is present (elastic monolayer graphene)." We absolutely do not restrict to the case of only having strain but also treat curvature too, which is, in fact, where the most interesting physics lies. It is worth noting that freely suspended monolayer graphene will generally have both locally varying strain and curvature.

Long ago [4] it was argued that the low-energy continuum description of elastically distorted and bent monolayer graphene is massless Dirac fermions coupled to a generally nontrivial geometry (via a frame e^i_A and torsion-free spin connection Ω_i) as well as a "strain" gauge field A_i (a result reviewed in the Appendix of Ref. [3]). From the point of view of curved space quantum field theory, the strain gauge field and spin connection couple very differently to the Dirac fermion ψ via a covariant derivative (see, for instance, (12) of Ref. [2])

$$\nabla_i \psi = \left(\partial_i - iA_i + \frac{i}{2}\sigma^3\Omega_i\right)\psi,$$

where σ^3 is the usual third Pauli matrix. The different couplings imply different and distinct physics results from the spin connection and strain gauge field.

We note here that even in this early paper [4] it was pointed out that in-plane distortions do not introduce intrinsic curvature, and so the physical effect of this strain viewed from the Dirac description must be felt purely through the strain gauge field. This is similar to what is discussed in Sec. III A of Ref. [3] and the latter part of the Comment although they make this argument in a different way using the curved space Dirac equation (which is why we cited their work originally in Ref. [2], writing "Based on these analyses much work has assumed a curved space Dirac description exists [16–25]." where theirs is Ref. [20]).

Our work is focused on the validity of this continuum Dirac description derived from the microscopic description. Following Ref. [4] we take this tight-binding model with general spatially dependent hopping functions which are slowly varying and study its continuum limit. For small perturbations of the hopping functions this can be interpreted as arising from *both* strain *and* curvature of a graphene lattice embedded as a curved membrane in $\mathbb{R}^3$. We find that generically one does *not* recover the gauged curved Dirac theory in the low-energy continuum limit since higher derivative corrections cannot be neglected. Although naive power counting of derivatives would suggest we can ignore them, the gauge invariance associated with the strain gauge field together with the lattice scale appearing in the strain gauge field, schematically going as $A \sim u/a$ (where u is the strain and a is the lattice scale), cancels the naive suppression of higher covariant derivative terms. It is worth noting that these higher derivative terms are not coordinate invariant but carry a memory of the lattice (see the discussion around Eq. (23) of Ref. [2]). We have shown that it is possible to fine-tune the tight-binding model hopping functions so that the emergent strain gauge field is parametrically small, and in this case one can obtain a consistent curved space Dirac description even for nonlinear curvature. However, using simple mechanical arguments we have argued that this fine-tuning looks very unnatural from the perspective of physical monolayer graphene embedded into $\mathbb{R}^3$.

We note that it had been previously observed in Ref. [5] that the large gauge field meant the spin connection should be ignored. However, there they argued the perturbation to the frame should still be kept (leading to "Weitzenbock geometry"). Our conclusion is different, namely, that both the frame and the spin connection perturbations are generically on the same order as the higher derivative corrections due to the gauge invariance associated with the strain gauge field which is inherited from the lattice theory.

We note that for perturbative distortions it is consistent to truncate to Dirac if we ignore corrections to the frame (and, hence, also spin connection), leaving us with the flat-space gauged Dirac theory. But to go beyond this approximation and

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include generic corrections to the frame (even those that are pure diffeomorphism, i.e., pure strain) one must also include higher derivative terms. We also believe this consistent truncation does not apply to nonlinear distortions, even at quadratic order in the perturbation, and even in the case of pure in-plane strain, so no curvature.

[1] M. M. Roberts and T. Wiseman, Curved-space Dirac description of elastically deformed monolayer graphene is generally incorrect, Phys. Rev. B **105**, 195412 (2022).

[2] A. Iorio and P. Pais, Revisiting the gauge fields of strained graphene, Phys. Rev. D **92**, 125005 (2015).

[3] A. Iorio and P. Pais, Comment on "Curved-space Dirac description of elastically deformed monolayer graphene is generally incorrect," Phys. Rev. B **106**, 157401 (2022).

[4] F. de Juan, M. Sturla, and M. A. H. Vozmediano, Space Dependent Fermi Velocity in Strained Graphene, Phys. Rev. Lett. **108**, 227205 (2012).

[5] M. A. Zubkov and G. E. Volovik, Emergent gravity in graphene, J. Phys.: Conf. Ser. **607**, 012020 (2015).