# Comment on "Curved-space Dirac description of elastically deformed monolayer graphene is generally incorrect"

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In a recent paper [M. M. Roberts and T. Wiseman, Phys. Rev. B **105**, 195412 (2022)], based on methods typical of the condensed-matter literature (the real-space gradient expansion applied to the tight-binding model), conclude that the curved-space Dirac description of the low-energy conductivity electrons of monolayer graphene is incorrect when only strain is present (elastic monolayer graphene). In this Comment we point out that, in a much earlier paper [A. Iorio and P. Pais, Phys. Rev. D **92**, 125005 (2015)], basing our analysis on methods typical of quantum field theory in curved space-time, we had concluded the same. Unfortunately, the authors of [M. M. Roberts and T. Wiseman, Phys. Rev. B **105**, 195412 (2022)] have missed that result (even though it is very clearly stated in various places, starting from the Abstract), to the extent of citing our paper as one where strain is actually used for that wrong purpose. Since the paper in point is written with the declared intent of bringing clarity into the debate, we believe that this Comment of ours is very much due, precisely to give it another try to achieve the latter purpose.

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The work [1] clearly emphasizes, starting from the title, that the curved space Dirac description of elastically deformed monolayer graphene is not always correct. The authors arrive at that conclusion by analyzing in detail the real-space gradient expansion of the tight-binding model for $\pi$ electrons.

In the Introduction, there is a list of earlier works that, instead, use that wrong description. Henceforth, in those papers, the low-energy regime of the $\pi$ electrons of a membrane of monolayer graphene, subject *only* to strain, is taken there to be governed by a Dirac type of Hamiltonian on a curved background. Among those works, the authors include Ref. [2] (listed as Ref. [20] in Ref. [1]).

In fact, seven years earlier, in Ref. [2], we had the very same concern of Ref. [1]: is strain alone enough to produce measurable effects that one can describe by using a Dirac theory on curved space? or else, intrinsic curvature is actually necessary, for instance, through topological defects of the lattice? Note that the latter request is behind even earlier works of one of the authors (see, for instance, the review [3] and references therein).

Therefore, performing an analysis based on the Weyl symmetry of any Dirac theory describing massless excitations, that should indeed include graphene [4], we unambiguously reached the following conclusions: with strain alone one cannot "mimic" a Dirac field in curved space because the coupling of the latter with gravity has to happen through the spin connection, but (quoting from Ref. [2]) *"...when graphene is only subject to strain, the spin-connection gauge field that arises plays no measurable role..."* and *"... we see that the very well-known pseudomagnetic field (and, for that matter, even a putative pseudoelectric field) induced by pure strain, cannot be accounted for by the spin-connection/Weyl pure gauge field."* The first quotation is from the Abstract, the second from a Sec. titled "Zero curvature: No physical effects of strain through the spin connection."

The analysis of Ref. [2] started from an action for a massless Dirac field in a curved $(2+1)$-dimensional space,

$$\mathcal{A} = i \int d^3Q \sqrt{g} \bar{\psi} E_a^\mu \gamma^a (\partial_\mu + \Omega_\mu)\psi,$$

where $\Omega_\mu = \frac{1}{2}\omega_\mu^{ab} J_{ab}$ with $J_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$, the Lorentz generators, $E_a^\mu$ is the inverse of the three-dimensional vielbein $e_\mu^a$ (the dreibein), $\omega_\mu^{ab}$ is the spin connection. One wanted to see *under which conditions* such an action could account for the description of the low-energy $\pi$ electrons.

When only strain is present, the metric is bound to be of the form

$$g_{\mu\nu}(Q) = e^{2\Sigma(Q)} \eta_{\mu\nu},$$

where $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1)$, and the information about the metric being fully encoded in the conformal factor $\Sigma$. Applying Weyl symmetry, one finds that the field strength, associated with the Weyl connection $\Omega_\mu$,

$$F_{ab} = \partial_a \Sigma_b - \partial_b \Sigma_a = (\partial_a\partial_b - \partial_b\partial_a)\Sigma$$

is zero.

Therefore, when there are no effects produced by topological defects (encoded here in a nonzero intrinsic curvature of the membrane, something the strain can never give), then the effect is zero. It is only when such topological defects are there, that the Dirac description on curved space does have measurable effects, as discussed in Sec. III B of Ref. [2], and in many other papers that use this approach.

This does not mean that strain alone does not have a measurable effect on the conductivity properties of graphene! It only means that we have to employ a different kind of formalism, namely, the one based on the pseudomagnetic gauge field

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as known by many already and as confirmed in the analysis of Ref. [2].

Let us close by stating that to have two different approaches giving the same result should add clarity to the issue, and this is precisely the intent of the present Comment.

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