Electron-electron scattering and conductivity of disordered systems with a Galilean-invariant spectrum

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The electron-electron scattering does not affect the electrical current in Galilean-invariant systems. We show that, nevertheless, electron-electron collisions may contribute to the electric resistivity of systems with parabolic spectrum provided that they have multiply connected Fermi surfaces, and there is an additional mechanism of scattering. To this end, we calculate the resistivity of a two-dimensional electron gas with two filled transverse subbands in a presence of electron-electron and impurity scattering. Although the collisions between the electrons do not directly affect the current in such systems, they cause a redistribution of the electrons between the Fermi contours, which results in a noticeable change in resistivity for realistic mechanisms of impurity scattering.

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I. INTRODUCTION

In systems with a simple parabolic spectrum, electron-electron scattering does not contribute to the resistivity because of their Galilean invariance. In other words, the electron-electron collisions do not change the current because it is proportional to the total momentum of electrons, which is conserved in such collisions. The situation is more interesting if the spectrum of electrons has several branches, which may break the Galilean invariance. The effects of electron-electron scattering on the transport properties of different conductors that lack Galilean invariance have been a subject of interest for many years. Typically, the systems under investigation included two types of charge carriers with different charges or effective masses. First of all, a number of papers theoretically addressed scattering between electrons and holes in semimetals [1–4]. Another group of papers dealt experimentally and theoretically with mutual scattering of holes from spin-split subbands in GaAs heterostructures [5–7] in order to explain the unusual temperature of resistivity and magnetoresistance in them. The cyclotron resonance in Si inversion layers in a presence of collisions between electrons in different valleys was investigated in Ref. [8]. Several years ago, the electrical conductivity was calculated for non-Galilean electron systems with several geometries of Fermi surface [9,10]. Very recently, it was calculated for a two-dimensional (2D) electron gas with Rashba spin-orbit coupling [11]. Most theoretical papers predicted an increase in resistivity proportional to the square of temperature. The experiment [6] also revealed a decrease in the temperature derivative of resistivity at sufficiently high temperatures. All these papers have in common that individual two-particle collisions do not conserve the current despite momentum conservation. The reason is that the colliding particles have either opposite charges or different effective masses, so the current is not proportional to the total momentum of charge carriers.

In this paper, we show that the electron-electron collisions contribute to the resistivity even if the electrons are Galilean invariant provided that there is an additional mechanism of scattering. To this end, we consider a 2D electron gas formed in a semiconductor heterostructure with two populated subbands of transverse quantization. The Fermi surface of such systems is doubly connected, but the effective masses are equal in both subbands, hence, any individual electron-electron collision cannot change the current. Nevertheless, these collisions affect the resistivity if either the electron-impurity scattering rates on the two Fermi contours are different or there is a sufficiently strong intersubband impurity scattering. The reason is that the two-particle collisions redistribute nonequilibrium electrons between the two contours. As a result, the resistivity increases with temperature to a new finite value. We analyze different mechanisms of impurity scattering and determine the optimal conditions for observing this effect.

Except Ref. [11], all cited calculations of the electron-electron contribution to the resistivity used certain simplifying assumptions. In particular, they neglected the scattering processes that involve only one type of charge carriers. Here we take into account several possible scattering channels and show that the collisions involving only electrons at the same Fermi contour are essential at intermediate temperatures.

The paper is organized as follows. Section II presents the model, the kinetic equation, and general expressions for the electron-electron and electron-impurity collision integrals. In Sec. III, the kinetic equation is solved for arbitrary mechanism of impurity scattering. In Sec. IV, the results for different types of impurities are compared, and Sec. V contains the

summary of the results. Some lengthy expressions are given in the Appendix.

II. THE MODEL AND KINETIC EQUATION

Consider a 2D semiconductor heterostructure with parabolic dispersion law and Fermi level crossing two subbands of transverse quantization. In what follows, we will designate the lower and upper subbands by $n=1$ and $n=2$, respectively. Hence, the dispersion laws of electrons in these subbands are $\varepsilon_n(\boldsymbol{p}) = p^2/2m + \delta_{n2}\,\Delta_0$, where Δ_0 is the subband splitting. At low temperatures, the resistivity of this system is dominated by electron-impurity and electron-electron scatterings, so the kinetic equation for the nonequilibrium electron distribution is of the form

$$-\frac{e\boldsymbol{E}\boldsymbol{v}_n}{T}\bar{f}(1-\bar{f}) = I_n^{\rm imp} + I_n^{ee}, \tag{1}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{v}_n = \partial\varepsilon_n/\partial\boldsymbol{p}$, and $\bar{f}(\varepsilon_n)$ is the equilibrium Fermi function. The electron-impurity collision integral with account taken of intersubband scattering was obtained in many papers [12–15] and in the Born approximation is given by the equation,

$$I_n^{\rm imp}(\boldsymbol{p}) = \frac{2\pi}{\hbar}\sum_{n'}\int\frac{d^2p'}{(2\pi\hbar)^2}\delta(\varepsilon_n-\varepsilon_{n'}) \times |U_{nn'}(\boldsymbol{p}-\boldsymbol{p}')|^2[f_{n'}(\boldsymbol{p}')-f_n(\boldsymbol{p})], \tag{2}$$

where $U_{nn'}(\boldsymbol{p}-\boldsymbol{p}')$ is the matrix element of the impurity potential between electron states $(\boldsymbol{p}, n)$ and $(\boldsymbol{p}', n')$. The specific form of these matrix elements for different kinds of impurities will be discussed in Sec. IV.

The electron-electron collision integral may be written in the standard form as

$$I_n^{ee}(\boldsymbol{p}) = \sum_{n_1}\sum_{n_2}\sum_{n_3}\int\frac{d^2p_1}{(2\pi\hbar)^2}\int\frac{d^2p_2}{(2\pi\hbar)^2}\int d^2p_3 \times\delta(\boldsymbol{p}+\boldsymbol{p}_1-\boldsymbol{p}_2-\boldsymbol{p}_3)\delta(\varepsilon_n+\varepsilon_{n_1}-\varepsilon_{n_2}-\varepsilon_{n_3}) \times W_{n\cdots n_3}(\boldsymbol{p}\boldsymbol{p}_1,\boldsymbol{p}_2\boldsymbol{p}_3) \times[(1-f)(1-f_1)f_2f_3 - ff_1(1-f_2)(1-f_3)]. \tag{3}$$

In the limit of weak electron-electron interaction, the scattering probabilities may be calculated in the Born approximation. For simplicity, we assume that the interaction potential V is short ranged due to the presence of a nearby screening gate. If the distance to gate d_0 is smaller than the Fermi wavelength,

$$W_{n\cdots n_3} \equiv \frac{2\pi}{\hbar}\langle nn_1|V|n_2n_3\rangle^2, \tag{4}$$

where $\langle nn_1|V|n_2n_3\rangle = 4\pi e_0^2 d_0\kappa^{-1}\delta_{nn_2}\delta_{n_1n_3}$, e_0 is the electron charge, and κ is the dielectric constant. It was taken into account here that for wave vectors smaller than $1/d_0$, the longitudinal Fourier transform of V weakly depends on transverse coordinates and that the transverse wave functions for $n=1$ and $n=2$ are orthogonal.

As the system is rotationally symmetric in the plane of the electron gas, it is convenient to seek the solution of Eq. (1) in the form

$$f_n(\boldsymbol{p}) = \bar{f}(\varepsilon) + \bar{f}(\varepsilon)[1-\bar{f}(\varepsilon)]C_n(\varepsilon)\cos\varphi, \tag{5}$$

where the energy ε is measured from E_F and φ is the angle between $\boldsymbol{E}$ and $\boldsymbol{p}$. With this substitution, the electron-impurity collision integral is easily brought to the form

$$I_n^{\rm imp}(\varepsilon,\varphi) = (\Gamma_x C_{3-n} - \Gamma_n C_n)\cos\varphi\bar{f}(1-\bar{f}), \tag{6}$$

where the intrasubband scattering rates,

$$\Gamma_n = \frac{m}{2\pi\hbar^3}\int_{-\pi}^{\pi}d\chi[|U_{12}(\boldsymbol{p}-\boldsymbol{p}')|^2 + |U_{nn}(\boldsymbol{p}-\boldsymbol{p}')|^2(1-\cos\chi)] \tag{7}$$

are always positive, but the intersubband scattering rate,

$$\Gamma_x = \frac{m}{2\pi\hbar^3}\int_{-\pi}^{\pi}d\chi|U_{12}(\boldsymbol{p}-\boldsymbol{p}')|^2\cos\chi \tag{8}$$

may be, in general, of either sign. Note that $\Gamma_n > |\Gamma_x|$ regardless of the scattering potential.

The electron-electron collision integral is treated in the way similar to Ref. [11]. A substitution of Eq. (5) into Eq. (3) brings it to the form

$$\begin{aligned} I_n^{ee}(\varepsilon,\varphi) &= m\sum_{n_1}\sum_{n_2}\sum_{n_3}W_{n\cdots n_3}\int d\varepsilon_1\int d\varepsilon_2\int d\varepsilon_3 \\ &\times\delta(\varepsilon+\varepsilon_1-\varepsilon_2-\varepsilon_3)(1-\bar{f})(1-\bar{f}_1)\bar{f}_2\bar{f}_3 \\ &\times\int d\varphi_1\int\frac{d^2p_2}{(2\pi\hbar)^2}\int\frac{d^2p_3}{(2\pi\hbar)^2}\delta(\varepsilon_{n_2}-\varepsilon_2)\delta(\varepsilon_{n_3}-\varepsilon_3) \\ &\times\delta(\boldsymbol{p}+\boldsymbol{p}_1-\boldsymbol{p}_2-\boldsymbol{p}_3)[C_{n_2}(\varepsilon_2)\cos\varphi_2 + C_{n_3}(\varepsilon_3)\cos\varphi_3 \\ &- C_n(\varepsilon)\cos\varphi - C_{n_1}(\varepsilon_1)\cos\varphi_1]. \end{aligned} \tag{9}$$

Furthermore, the cosines in the last factor of the integrand may be expressed in terms of $p\cdots p_3$, φ, and φ_1. Assume that all the quantities except the distribution functions $\bar{f}$ and C_n are energy independent near the Fermi level. Subsequently, integrating in Eq. (9) over $\boldsymbol{p}_3$, $\boldsymbol{p}_2$, and $d\varphi_1$, one obtains

$$I_n^{ee} = \cos\varphi\frac{\Gamma_{ee}}{T^2}\int d\varepsilon' K(\varepsilon,\varepsilon')\left\{Q_n[C_n(\varepsilon')-C_n(\varepsilon)] + \Psi_n\frac{p_nC_{3-n}(\varepsilon') - p_{3-n}C_n(\varepsilon')}{p_1+p_2}\right\}, \tag{10}$$

where the effective electron-electron scattering rate is given by $\Gamma_{ee} = 8\pi e_0^4 d_0^2 m^3T^2/\kappa^2\hbar^5 p_1p_2$ and

$$K(\varepsilon,\varepsilon') = [1-\bar{f}(\varepsilon)]\frac{\varepsilon-\varepsilon'}{e^{(\varepsilon-\varepsilon')/T}-1}\bar{f}(\varepsilon'). \tag{11}$$

It is assumed that C_n are even functions of ε.

The first term in Eq. (10) is similar to the expression for the 2D conductors with the singly connected Fermi surface. The coefficients,

$$Q_n = 2\frac{p_n+2p_{3-n}}{p_n}\ln\left(\frac{p_n^2}{2mT}\right) \tag{12}$$

exhibit a logarithmic singularity at $T\to 0$ that results from head-on or small-angle collisions [16,17]. However, this term

vanishes identically if $C_n(\varepsilon) = \text{const}$, and for this reason it does not contribute to the electric resistivity if taken alone. Note that it originates not only from the collisions in which all the initial and final states belong to the same Fermi contour, but also from the scattering processes in which both the initial and the final states are at different contours.

The second term in Eq. (10) is nonzero even for energy-independent C_n and arises only from collisions of electrons with both initial and final states at different Fermi contours. The quantities Ψ_n are given by the equations,

$$\Psi_n = \frac{p_1 + p_2}{p_n} \ln\left(\frac{p_1 + p_2}{p_1 - p_2}\right). \tag{13}$$

The structure of this term emanates from the fact that the collision integral must be turned into zero by a distribution $f_n(\boldsymbol{p}) = \bar{f}(\varepsilon_n) + \boldsymbol{up}\,\bar{f}(\varepsilon_n)[1 - \bar{f}(\varepsilon_n)]$, i.e., with $p_1 C_2 = p_2 C_1$ [18]. Another condition $p_1\Psi_1 = p_2\Psi_2$ stems from the Galilean invariance of the system and the current conservation by the electron-electron collisions. The second term in Eq. (10) is of crucial importance for the effects considered here.

III. ELECTRICAL CONDUCTIVITY

Equation (1) results in a system of two integral equations in $C_n(\varepsilon)$. To solve it, one may use the method proposed by Brooker and Sykes [19] for calculating thermal conductivity of Fermi liquid. Upon a substitution of Eq. (5) into Eq. (1), reduce both parts by $\cos\varphi\sqrt{\bar{f}(1-\bar{f})}$ and introduce new variable,

$$\rho_n(\varepsilon) = \sqrt{\bar{f}(1-\bar{f})}\,C_n(\varepsilon). \tag{14}$$

As a result, the kernel $K(\varepsilon, \varepsilon')$ of the integral in Eq. (10) is replaced by a function of $\varepsilon' - \varepsilon$, and the integral equation may be brought to the differential form by a Fourier transform in ε with the parameter u. A subsequent replacement of the independent variable $\xi = \tanh(\pi T u)$ brings these equations to the form

$$\Gamma_{ee} Q_n(\hat{L} + 2)\rho_n(\xi) + 2\Gamma_{ee}\Psi_n \frac{p_n\rho_{3-n} - p_{3-n}\rho_n}{p_1 + p_2} - \frac{1}{\pi^2}\frac{\Gamma_n\rho_n - \Gamma_x\rho_{3-n}}{1-\xi^2} = -\frac{eE v_n}{\pi\sqrt{1-\xi^2}}, \tag{15}$$

where $\hat{L}$ is the differential operator,

$$\hat{L}\psi = \frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial\psi}{\partial\xi}\right] - \frac{\psi}{1-\xi^2}. \tag{16}$$

As the solutions of Eq. (15) are even functions of ξ, they may be presented in the form of a series,

$$\rho_n(\xi) = \sum_{r=0}^{\infty} \gamma_{nr}\psi_{2r}(\xi), \tag{17}$$

where $\psi_r(\xi)$'s are the eigenfunctions of operator $\hat{L}$ with eigenvalues $-(r+1)(r+2)$ [20]. A substitution of these expansions into Eq. (15) results in an infinite system of equations for the coefficients γ_{nr} of the form

$$2\Gamma_{ee}\left[r(2r+3)Q_n\gamma_{nr} + \Psi_n\frac{p_{3-n}\gamma_{nr} - p_n\gamma_{3-n,r}}{p_1+p_2}\right] + \frac{1}{\pi^2}\sum_{r=0}^{\infty} Y_{rs}(\Gamma_n\gamma_{ns} - \Gamma_x\gamma_{3-n,s}) = \frac{eE v_n}{\pi}X_r, \tag{18}$$

where Y_{rs}'s are the matrix elements of $(1-\xi^2)^{-1}$ between ψ_{2r} and ψ_{2s}, and X_r's are the projections of $(1-\xi^2)^{-1/2}$ on ψ_{2r}. The explicit expressions for these quantities are given in the Appendix. Once the coefficients γ_{nr} are found, the current density is calculated as the sum,

$$j = \frac{e}{2\pi\hbar^2}\sum_n \int d\varepsilon \bar{f}(1-\bar{f})p_n C_n(\varepsilon) = \frac{e}{4\pi^2\hbar^2}\sum_n p_n \sum_r X_r\gamma_{nr}. \tag{19}$$

In general, the system (18) may be solved only numerically, but an analytical solution is possible in the limiting cases of vanishing or very strong electron-electron scattering. If $\Gamma_{ee} = 0$, Eq. (1) with the electron-impurity collision integral Eq. (6) is easily solved in C_n, and the first part of Eq. (19) readily gives

$$\sigma_0 = \frac{e^2}{2\pi\hbar^2}\frac{\Gamma_1 p_2 v_2 + \Gamma_2 p_1 v_1 + \Gamma_x(p_1 v_2 + p_2 v_1)}{\Gamma_1\Gamma_2 - \Gamma_x^2}, \tag{20}$$

in agreement with previous results for two-subband conductors. In the opposite limit of a very strong electron-electron scattering, it is sufficient to keep in the system (18) only the pair of equations with $r = 0$. However, one cannot just neglect the electron-impurity scattering because the system would be satisfied by any solution with $p_2\gamma_{10} = p_1\gamma_{20}$ and, hence, be degenerate. To lift the degeneracy, one has to keep the electron-impurity scattering rates nonzero and isolate the most singular contribution in them to γ_{n0}. The resulting coefficients are inversely proportional to a linear combination of the impurity-scattering rates and satisfy the above ratio so that the currents carried by the subbands are proportional to the electron densities in them. Making use of the explicit expressions for Ψ_n Eq. (13) and the expression for j Eq. (19), one easily obtains the corresponding conductivity,

$$\sigma_\infty = \frac{e^2}{2\pi\hbar^2}\frac{(p_1 v_1 + p_2 v_2)^2}{\Gamma_1 p_1 v_1 + \Gamma_2 p_2 v_2 - \Gamma_x(p_1 v_2 + p_2 v_1)}. \tag{21}$$

The $\sigma_0(E_F)$ and $\sigma_\infty(E_F)$ curves vary in shape depending on the ratios of Γ_n and Γ_x, but the difference of these conductivities,

$$\sigma_0 - \sigma_\infty \propto \left[(\Gamma_1 - \Gamma_2)p_1 p_2 + \Gamma_x\left(p_1^2 - p_2^2\right)\right]^2 \tag{22}$$

is positive regardless of impurity type. The quadratic dependence of the conductivity difference Eq. (22) on the difference of the scattering rates is the consequence of the structure of the second term of the collision integral Eq. (10) and the general relation between Ψ_1 and Ψ_2, so it does not depend on the particular choice of the interaction potential V. For arbitrary temperatures and arbitrary relation between the electron-electron and impurity scattering, Equation (18) may

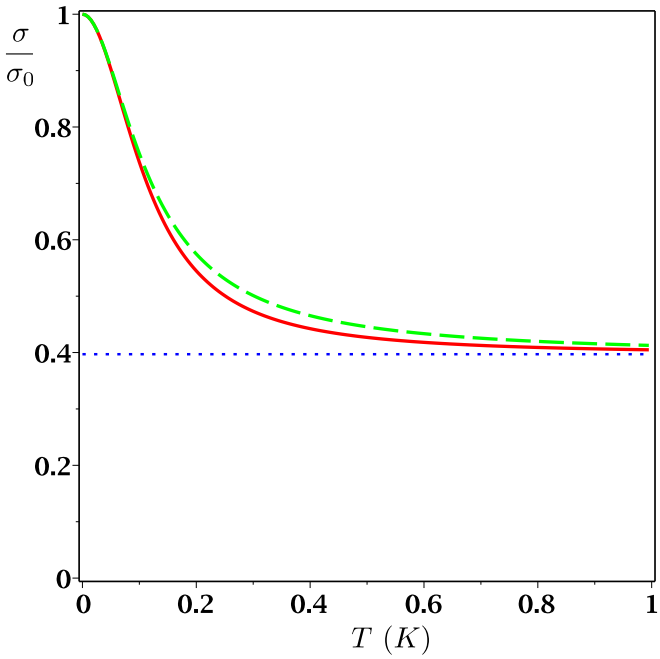

FIG. 1. The $\sigma(T)/\sigma_0$ dependence for a GaAs heterostructure with $\Delta_0 = 10$ meV, $E_F = 15$ meV, $d_0 = 20$ nm, $\Gamma_1 = 2.5 \times 10^{11}$ s^{-1}, $\Gamma_2 = 0.1\Gamma_1$, and $\Gamma_x = 0$. The dashed line shows the same quantity calculated with $Q_n = 0$.

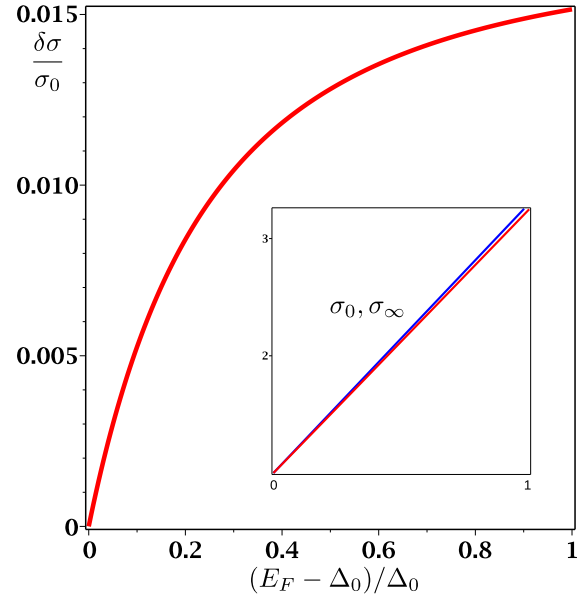

FIG. 2. The dependence of the relative difference $\Delta\sigma/\sigma_0 = (\sigma_0 - \sigma_\infty)/\sigma_0$ on the position of the Fermi-level E_F for pointlike impurities. The inset separately shows $\sigma_0(E_F)$ (blue curve) and $\sigma_\infty(E_F)$ (red curve) normalized to $\sigma_0(\Delta_0)$.

be solved only numerically. The $\sigma(T)$ curve for a GaAs heterostructure with $\Delta_0 = 10$ meV, $E_F = 15$ meV, $d_0 = 20$ nm, $\Gamma_1 = 2.5\times10^{11}$ s^{-1}, $\Gamma_2 = 0.1\Gamma_1$, and $\Gamma_x = 0$ is shown in Fig. 1. The dashed line shows the same curve calculated for $Q_n = 0$, i.e., in the absence of the first term in the collision integral Eq. (10). Although the solid and the dashed curves almost coincide in the low-T and high-T limits, they are noticeably different at intermediate temperatures. This suggests that the intrasubband electron-electron scattering also contributes to the electric resistivity, in contrast to the claim made in many previous papers [2,4,5,7–10]. The difference between the two calculated conductivities does not appear to be parametrically small.

IV. ESTIMATES FOR DIFFERENT TYPES OF IMPURITIES

Estimate now the effect of electron-electron scattering for different types of impurities. The intersubband scattering rate Γ_x is typically much smaller than the intrasubband rates Γ_n, and, therefore, to maximize the difference $\sigma_0 - \sigma_\infty$, one has to find the type of impurities with maximum difference $|\Gamma_1 - \Gamma_2|$.

First of all, consider neutral pointlike impurities, which may be formed by atomic vacancies in the semiconductor. In this case, the scattering potential may be written in the form

$$U(\boldsymbol{r}, z) = \Omega_0 \sum_i \delta(\boldsymbol{r} - \boldsymbol{r}_i)\delta(z - z_i), \tag{23}$$

where i labels impurities, $\boldsymbol{r}$ stands for the in-plane coordinates, and z is the transverse coordinate. Therefore, the impurity-averaged square of the matrix element is

$$|U_{nn'}(\boldsymbol{p} - \boldsymbol{p}')|^2 = \Omega_0^2 n_{\rm imp} \int dz\, \phi_n^2(z)\phi_{n'}^2(z), \tag{24}$$

where $n_{\rm imp}$ is the three-dimensional concentration of impurities and $\phi_n(z)$'s are the transverse components of the electron wave functions corresponding to the subband n. It is immediately seen that $\Gamma_x = 0$ in this case. The estimates made using ϕ_n for a triangular transverse confining potential give $\Gamma_1/\Gamma_2 \approx 1.3$, which results only in a slight decrease in σ with temperature about 1% (Fig. 2).

Consider now a δ-doped semiconductor heterostructure where the 2D electron gas is located between the impurities and the metallic gate in such a way that the impurity-2D gas distance h_0 is much larger than the 2D gas-gate separation d_0 [21]. The 2D Fourier transform of the unscreened potential of a single impurity $U_0(q) = 2\pi e_0^2 \exp(-qh_0)/q$ is well known. Therefore, in a presence of the gate, it becomes

$$\begin{aligned}\bar{U}_0(q) &= 2\pi e_0^2 q^{-1}\{\exp(-qh_0) - \exp[-q(h_0 + 2d_0)]\} \\ &\approx 4\pi e_0^2 d_0 \exp(-qh_0)\end{aligned} \tag{25}$$

provided that $qd_0 \ll 1$. As $\bar{U}_0(q)$ is independent of z, the matrix elements U_{12} and Γ_x are zero in this approximation. Following Ref. [22], one may bring Eq. (7) for the intrasubband scattering rates to the form

$$\Gamma_n = 64\pi \frac{mN_{\rm imp}e_0^4 d_0^2}{\hbar^3} \int_0^1 d\xi \frac{\xi^2}{\sqrt{1-\xi^2}} e^{-4p_n h_0 \xi/\hbar}, \tag{26}$$

where $N_{\rm imp}$ is the 2D concentration of impurities. If $p_n h_0 \ll \hbar$, the integral over ξ equals $\pi/4$. In the opposite limit, it scales as $(p_n h_0/\hbar)^{-3}$. If the Fermi level crosses the upper subband near its bottom and $p_2 \ll p_1$, it is quite possible that $p_1 h_0 > \hbar$ whereas $p_2 h_0 < \hbar$ and $p_1 d_0/\hbar < 1$. In this case, $\Gamma_1 \ll \Gamma_2$ and

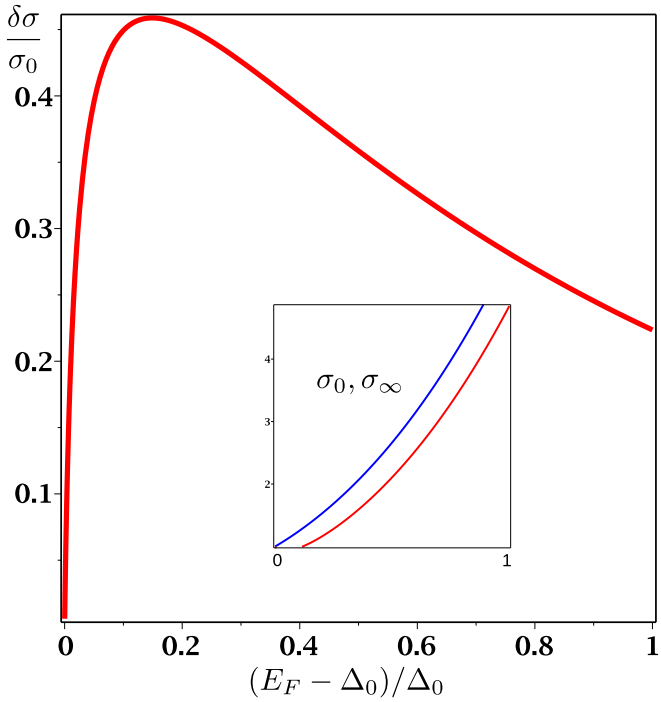

FIG. 3. The dependence of the relative difference $\delta\sigma/\sigma_0 = (\sigma_0 - \sigma_\infty)/\sigma_0$ on the position of the Fermi-level E_F for δ-doped Coulomb impurities. The distance between the electron gas and the impurity layer is $h_0 = 3\hbar/p_1(\Delta_0)$. The inset shows $\sigma_0(E_F)$ (blue curve) and $\sigma_\infty(E_F)$ (red curve) normalized to $\sigma_0(\Delta_0)$.

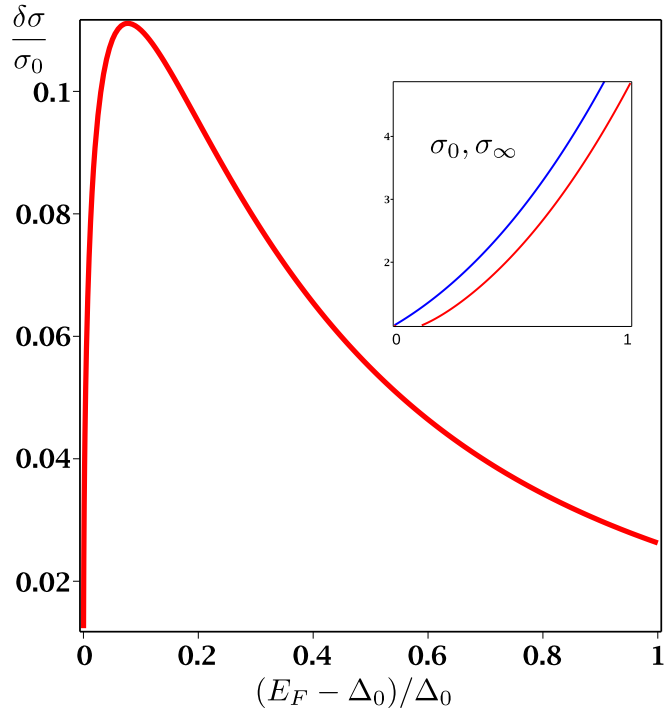

FIG. 4. The dependence of the relative difference $\delta\sigma/\sigma_0 = (\sigma_0 - \sigma_\infty)/\sigma_0$ on the position of the Fermi-level E_F for uniformly distributed Coulomb impurities. The inset shows $\sigma_0(E_F)$ (blue curve) and $\sigma_\infty(E_F)$ (red curve) normalized to $\sigma_0(\Delta_0)$.

the suppression of conductivity by electron-electron collisions may be about 100% if $E_F - \Delta_0 < \Delta_0$ (Fig. 3).

Finally, if the charged impurities with three-dimensional concentration $n_{\rm imp}$ are uniformly distributed over the volume of the semiconductor, the intrasubband scattering rates may be obtained by integrating Eq. (26) over h_0, which readily gives

$$\Gamma_n = 16\pi m n_{\rm imp} e_0^4 d_0^2/\hbar^2 p_n. \tag{27}$$

Therefore, $\Gamma_1/\Gamma_2 = p_2/p_1$, and the maximum difference of σ_0 and σ_∞ is about 10% at $E_F \approx 1.1\,\Delta_0$ (Fig. 4).

A good candidate for observing the suppression of conductivity are AlGaAs/GaAs heterostructures. Currently, these systems reached such a quality that the electron-electron scattering is much stronger than the impurity scattering at temperatures as low as 2 K [23,24]. Transport measurements in GaAs quantum wells with two populated subbands are also possible [25,26]. Figure 5 shows the relative correction to the conductivity calculated as a function of electron concentration and the width of a rectangular GaAs quantum well, which was used in experiments [25,26]. In this approximation, the maximum value of $\delta\sigma/\sigma_0$ does not depend on the width of the well and corresponds to the electron concentration slightly above the threshold for the population of the second subband. Wider wells require lower electron concentrations for observing the effect, but for narrow wells the range of concentrations where the effect exists is wider. The suppression of conductivity by electron-electron scattering could be also observed in double quantum wells where the overlap between the wave functions in them results in a formation of symmetric and antisymmetric states with a small energy separation, so the population of two subbands may be achieved at lower electron densities [27].

The above effects may be expected to take place at the temperatures about several Kelvins or lower, so the temperature-dependent correction to the resistivity from

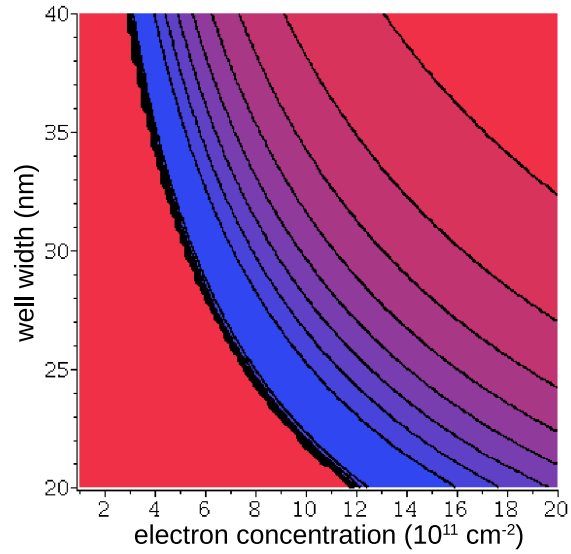

FIG. 5. Contour plot of $\delta\sigma/\sigma_0$ as a function of electron concentration and the width of quantum well for the case of uniformly distributed Coulomb impurities. The blue color corresponds to higher values of $\delta\sigma/\sigma_0$, and the red color corresponds to its smaller values.

electron-electron scattering should saturate long before the electron-phonon scattering comes into play. Note that this effect is purely semiclassical, and, therefore, it is much larger than the correction the the conductivity from quantum interference between electron-electron interaction and impurity scattering [28]. For two-dimensional systems, this correction is on the order of $e_0^2/\hbar \sim 10^{-4}\ \Omega^{-1}$, whereas for high-quality GaAs-AlGaAs heterosructures, the conductivity is on the order of $10^{-1}\ \Omega^{-1}$ and the semiclassical correction may be on the order of this conductivity.

V. CONCLUSIONS

In conclusion, it was shown that the electron-electron collisions may contribute to the resistivity of Galilean-invariant systems in a presence of other mechanisms of scattering. In a 2D electron gas with a parabolic spectrum and two populated subbands, it results in a significant increase in resistivity with temperature provided that the elastic-scattering rates in these subbands are essentially different. The reason is that these collisions redistribute nonequilibrium electrons that carry the current between the subbands in favor of the subband with stronger elastic scattering. The effect is maximal if the elastic scattering is caused by δ-doped Coulomb impurities located from the 2D electrons further than the screening length, and the Fermi level is not too high above the bottom of the upper subband.

As the expressions for σ_0 and σ_∞ do not contain any parameters of electron-electron scattering, the comparison of the low-temperature and high-temperature conductivities allows a separate determination of the impurity-scattering rates of electrons Γ_1 and Γ_2 in both subbands.

It was also shown that the standard assumption that the intraband electron-electron scattering does not affect the resistivity of multiband systems holds only in the limit of very frequent two-particle collisions. Actually, this scattering is essential at intermediate temperatures where the energy dependence of nonequilibrium electron distribution deviates from the simple derivative of the Fermi function.

Although the analytical expressions and numerical results were obtained for the particular case of the Coulomb interaction of electrons screened by a metallic gate, the general conclusions of this paper remain valid for arbitrary interactions because they stem only from the Fermi statistics and conservation laws.

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APPENDIX: EXPLICIT EXPRESSIONS FOR SOME QUANTITIES

The normalized eigenfunctions of differential operator $\hat{L}$ Eq. (16) are given by the expressions,

$$\psi_r(\xi) = \sqrt{\frac{(2r+3)(r+2)}{8(r+1)}}\sqrt{1-\xi^2}P_r^{(1,1)}(\xi), \tag{A1}$$

where $P_r^{(1,1)}(\xi)$ are Jacobi polynomials. The coefficients of expansion of $(1-\xi^2)^{-1/2}$ in these functions are given by

$$X_r = \int_{-1}^{1} d\xi \frac{\phi_{2r}(\xi)}{\sqrt{1-\xi^2}} = \sqrt{\frac{4r+3}{(2r+1)(r+1)}}. \tag{A2}$$

The matrix elements of $1/(1-\xi^2)$ between the eigenfunctions of $\hat{L}$ are given by the equation,

$$Y_{rs} = \int_{-1}^{1} d\xi \frac{\phi_{2r}(\xi)\phi_{2s}(\xi)}{1-\xi^2} = \frac{\min(r,s)+1/2}{\max(r,s)+1} \times \sqrt{\frac{(4r+3)(r+1)(4s+3)(s+1)}{(2r+1)(2s+1)}}. \tag{A3}$$

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