Erratum: Nonlinear sigma model with particle-hole asymmetry for the disordered two-dimensional electron gas [Phys. Rev. B 103, 125422 (2021)]

Georg Schwiete

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In the original paper, we erroneously neglected the influence of massive modes on the derivation of the four-gradient term in the nonlinear sigma model (NLσM) action. Explicit integration of the massive modes gives the following additional contribution:

$$S_M = \frac{\pi\nu}{16} DD'_\varepsilon \mathrm{Tr}[\nabla^2 \hat{Q}(\nabla\hat{Q})^2]. \tag{Er1}$$

This term should be included on the right-hand side of Eq. (13). When combined with the contribution originating from the gradient expansion without account of the massive modes, $S^{(2)}_{0,\eta,\varphi} = -\frac{\pi\nu}{8} DD'_\varepsilon \mathrm{Tr}[\nabla^2\hat{Q}(\nabla\hat{Q})^2]$, Eq. (18), the overall coefficient of the four-gradient term is halved, $S_M + S^{(2)}_{0,\eta,\varphi} = -\frac{\pi\nu}{16} DD'_\varepsilon \mathrm{Tr}[\nabla^2\hat{Q}(\nabla\hat{Q})^2]$. Except for the coefficient of the four-gradient term, the calculations and conclusions presented in the original paper remain unchanged. In particular, the interaction corrections to the static part of the correlation function in the one-loop approximation presented in Sec. IV do not depend on the four-gradient term. The mechanism for producing the four-gradient term S_M through the coupling of soft and massive modes had been noted previously in Ref. [1] in the context of the quantum Hall effect.

In order to understand the origin of S_M, it is sufficient to focus on the noninteracting case as described in Sec. V B. Retracing the steps outlined in Appendix B, the parametrization of the matrix $\hat{Q}$, Eq. (B2), should be generalized to include massive fluctuations [2] $\hat{Q} \to \hat{Q}_M = \hat{U}\hat{P}_M\hat{\bar{U}}$. Here, $\hat{P}_M$ is a Hermitian matrix that is block diagonal in Keldysh space and $\delta\hat{P}_M = \hat{P}_M - \hat{\sigma}_3$ parametrizes massive fluctuations around the saddle point. Correspondingly, the Keldysh partition function is written as $Z = \int_{\Psi^\dagger,\Psi,\hat{P}_M,\hat{U}} I[\hat{P}_M]\exp(iS)$ with

$$S = \int \vec{\bar{\Psi}}\left(\hat{G}_0^{-1} + \frac{i}{2\tau}\hat{P}_M + \hat{\bar{U}}[\hat{G}_0^{-1}, \hat{U}]\right)\vec{\Psi} + \frac{i\pi\nu}{4\tau}\mathrm{Tr}\big[\hat{P}_M^2\big]. \tag{Er2}$$

In the expression for the partition function, $I[\hat{P}_M]$ is the Jacobian arising due the parametrization of $\hat{Q}_M$. With the definition $\hat{G}_M^{-1} = \hat{G}^{-1} + \frac{i}{2\tau}\delta\hat{P}_M$, the partition function after integration over the fermionic fields can be presented as $Z = \int_{\hat{P}_M,\hat{U}} e^{iS}$ with $S = S[\hat{U}, \delta\hat{P}_M] + S[\delta\hat{P}_M]$ and

$$S[\hat{U}, \delta\hat{P}_M] = -i\,\mathrm{tr}\,\ln\big[1 + \hat{G}_M\hat{\bar{U}}\big[\hat{G}_0^{-1}, \hat{U}\big]\big], \tag{Er3}$$

$$S[\delta\hat{P}_M] = -i\,\mathrm{tr}\,\ln\left[1 + \hat{G}\frac{i}{2\tau}\delta\hat{P}_M\right] + \frac{i\pi\nu}{4\tau}\mathrm{tr}\big[\hat{P}_M^2\big] - i\,\ln I[\hat{P}_M]. \tag{Er4}$$

Here, $S[\hat{U}, \delta\hat{P}_M]$ describes the coupling of soft and massive modes. The influence of the massive modes was entirely neglected in the expansion described in Sec. V B of the original paper, which was based on $S[\hat{U}, \delta\hat{P}_M = 0]$. This expansion led to S_1, Eq. (58), and S_2, Eq. (59) (which equals $S^{(2)}_{0,\eta,\varphi}$ in the notation of Sec. III A). The integration of the massive modes produces a contribution to the NLσM with four gradients, S_M [Eq. (Er1)], of the same form as S_2. To obtain this term, it is sufficient to integrate $\delta\hat{P}_M$ in the Gaussian approximation. Therefore, $S[\delta\hat{P}_M]$ should be expanded up to second order in $\delta\hat{P}_M$. Upon substituting $\hat{P}_M = \sigma_3 + \delta\hat{P}_M$, linear terms in $\delta\hat{P}_M$ cancel between the first two terms in Eq. (Er4) by virtue of the saddle-point approximation. Higher order terms in $\delta\hat{P}_M$ resulting from the expansion of the tr ln in Eq. (Er4) give subleading contributions (in the parameter $1/\varepsilon_F\tau$), since they involve a $\xi_{\mathbf{p}}$ integration over a product of only retarded (or only advanced) Green's functions. The Jacobian $I[\hat{P}_M]$ is not easily evaluated in a continuum model, as it requires a regularization. However, from diagrammatic considerations one expects deviations from the self-consistent Born approximation (which underlies the saddle-point equation), to be suppressed by powers of $(\varepsilon_F\tau)^{-1}$. In effect, we approximate the quadratic form in $\delta\hat{P}_M$ by $S[\delta\hat{P}_M] \approx \frac{i\pi\nu}{4\tau}\mathrm{tr}[\delta\hat{P}_M^2]$.

Corrections to the NLσM originating from the coupling of soft and massive modes in $S[\hat{U}, \delta\hat{P}_M]$ can be organized as a cumulant expansion in $\delta S = S[\hat{U}, \delta\hat{P}_M] - S[\hat{U}, \delta\hat{P}_M = 0]$. δS, in turn, is obtained by expanding G_M in powers of $\delta\hat{P}_M$. At first order, the cumulant expansion gives $\delta S^{(1)} = \langle\delta S\rangle$, where $\langle\cdots\rangle$ stands for a Gaussian average with the action $S[\delta\hat{P}_M]$. Such terms can be checked to give small corrections only. The contribution of interest originates from the second cumulant $\delta S^{(2)} = \frac{i}{2}\langle\langle(\delta S)^2\rangle\rangle$

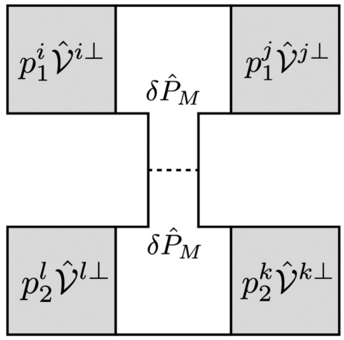

FIG. 1. Generation of the four-fermion term S_M through the coupling of soft and massive modes. The dashed line stands for an impurity line connecting two retarded or two advanced Green's functions.

by replacing $\hat{U}[\hat{G}_0^{-1}, \hat{U}] \to \mathcal{O} = \frac{1}{2m}[\hat{\mathcal{V}}^i \overrightarrow{\nabla}^i - \overleftarrow{\nabla}^i \hat{\mathcal{V}}^i]$ in Eq. (Er3), expanding the logarithm to second order in $\mathcal{O}$, and further expanding one of the two Green's functions in the resulting expression for δS to first order in $\delta \hat{P}_M$ as $\hat{G}_M \approx \hat{G} - \frac{i}{2\tau}\hat{G}\delta\hat{P}_M\hat{G}$. After averaging with respect to $\delta\hat{P}_M$, one finds

$$S_M = \frac{i}{4\pi\nu\tau}\int d\mathbf{r}\ \mathrm{tr}[(\hat{G}\mathcal{O}\hat{G}\mathcal{O}\hat{G})^{\parallel}_{\mathbf{r},\mathbf{r}}(\hat{G}\mathcal{O}\hat{G}\mathcal{O}\hat{G})^{\parallel}_{\mathbf{r},\mathbf{r}}]. \tag{Er5}$$

Figure 1 displays the corresponding diagram. Focusing only on the particle-hole asymmetric contribution, one obtains

$$S_M = -\pi\nu DD'_\varepsilon \mathrm{Tr}[\sigma_3 \hat{\mathcal{V}}^{i\perp}\hat{\mathcal{V}}^{i\perp}\hat{\mathcal{V}}^{j\perp}\hat{\mathcal{V}}^{j\perp}], \tag{Er6}$$

which results in Eq. (Er1). For a comparison with Ref. [1], notice the relation $\mathrm{Tr}[\nabla^2\hat{Q}(\nabla\hat{Q})^2] = -\mathrm{Tr}[(\nabla\hat{Q})^2(\nabla\hat{Q})^2\hat{Q}]$.

Finally, we would like to note that after incorporating S_M into the derivation, Eq. (66) should include the term $-\frac{\pi\nu}{16}DD'_\varepsilon\mathrm{Tr}[\nabla^2\hat{Q}(\nabla\hat{Q})^2]$ on the right-hand side. Two additional remarks: (i) in the last sentence of the abstract, the phrase "thermodynamic transport coefficient" should be replaced by "thermoelectric transport coefficient," and (ii) the definition of $\delta\hat{X}$ above Eq. (17) should read $\delta\hat{X} = \hat{X} - \hat{\sigma}_3$.

[1] X.-F. Wang, Z. Wang, C. Castellani, M. Fabrizio, and G. Kotliar, Nucl. Phys. B **415**, 589 (1994).

[2] A. M. Pruisken and L. Schäfer, Nucl. Phys. B **200**, 20 (1982).