4π-periodic anomalous Josephson effect between Majorana zero modes

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Usually the Josephson current is driven by the superconducting phase difference φ between two superconducting leads. Here we propose an anomalous Josephson current between two Majorana nanowires, which can be driven by both the superconducting phase difference φ and the orientation difference ϕ between the Zeeman fields in two Majorana wires. We show that the orientation of the Zeeman field acts as an effective superconducting phase under appropriate conditions, which provides a new signature of Majorana zero modes. Due to the presence of Majorana zero modes, the anomalous Josephson current is 4π periodic in both φ and ϕ. When both φ and ϕ are periodically varying, we predict the existence of peaks in the average Josephson current, which are the analog to Shapiro steps in the ac Josephson effect. Similarly, the existence of Majorana zero modes can be confirmed by the missing of odd peaks. The findings provide a scheme to experimentally verify the 4π-periodic Josephson effect, as well as a platform to realize the anomalous Josephson junction.

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I. INTRODUCTION

The existence of Majonana zero modes (MZMs) at the boundary of topological superconductors (TSCs) has been attracting a lot of interest for potential applications in topological quantum computations due to the non-Abelian exchange property of MZMs [1–10]. Unfortunately, the conclusive evidence of MZMs has not yet been observed in experiments. The latest progress in semiconductor-superconductor heterostructures shows that the observation of quantized Majorana conductance (QMC) has proved to be a hard task [11,12]. More importantly, the QMC has been shown to possibly also arise due to topologically trivial bound states [13–16] and the exclusion of these non-Majorana origins seems not easy [17–23] and still far from experimental verification. On the other hand, in quantum anomalous Hall (QAH)-superconductor junctions, which host chiral Majorana edge modes, the experimental observation of a half-integer quantized conductance plateau [24] has also been shown both theoretically [25,26] and experimentally [27] to be possibly attributed to non-Majorana origins, such as a good electric contact between the QAH film and the superconductor film [25].

Another important transport signature of MZMs is the 4π-periodic Josephson effect [28–31]. Although the 4π-periodic supercurrent is not specific to MZMs or even topological superconductors [32–34], it is still an important evidence of MZMs in experimentally interested systems, which may host MZMs. Nevertheless, the present observation of the missing first Shapiro step is not yet sufficient to conclude the existence of Majorana modes [35–46], because the missing of the $n = 1$ step can also be observed in the high-power oscillatory regime of the conventional 2π Josephson effect [35]. Therefore, the suppression of other odd Shapiro steps is important to confirm the 4π Josephson effect. Unfortunately, the observation of missing higher odd steps remains still a challenge in experiments. In addition, the most recent developments in the experiments on both Shapiro steps [45] and dc [47] Josephson currents show that the 4π-periodic components are very weak. Even worse, the observation of missing Shapiro steps can also be attributed to non-Majorana origins [48]. Therefore, the search for new schemes for observing the 4π-periodic Josephson effect and new signatures of MZMs is still highly desirable.

On the other hand, the anomalous Josephson junction [49–52], namely, the so-called φ_0 junction with an unconventional current-phase relation (CPR) $I(\varphi) = I_c \sin(\varphi - \varphi_0)$, has important applications in superconducting computer memory components [53], superconducting phase batteries and rectifiers [54], as well as flux- or phase-based quantum bits [55]. The anomalous Josephson junction can be realized via the coexistence of spin-orbit coupling and Zeeman field [50,56–59], noncoplanar ferromagnets [60–62], unconventional superconductors [63–67], and the manipulation of topological edge or surface states [68–78]. To our knowledge, the anomalous Josephson junction via MZMs has not been discussed.

In this work, we propose a 4π-periodic anomalous Josephson current between two Majorana nanowires coupled by a ferromagnetic wire. The anomalous Josephson current can be driven by both the orientation difference ϕ between the Zeeman fields in two Majorana wires and the superconducting

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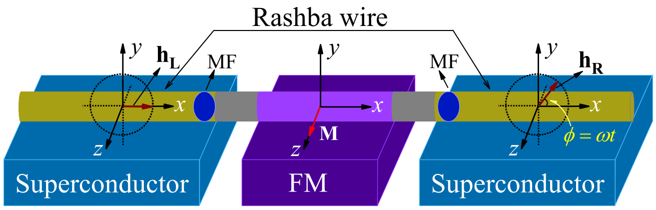

FIG. 1. Sketch of the 4π-periodic anomalous Josephson junction between two Majorana nanowires (a Rashba wire in proximity to an s-wave superconductor and in a Zeeman field) coupled by a ferromagnetic wire. The rotating Zeeman field $\mathbf{h_R}$ in the x-y plane can be induced by the proximity to a ferromagnet with a precessing magnetization driven by microwaves. The Zeeman field $\mathbf{h_L}$ is along the x direction. The magnetization orientation of the ferromagnetic wire is along the z direction.

phase difference φ. The orientation of the Zeeman field acts as an effective superconducting phase under appropriate conditions, which provides a signature of MZMs and a scheme to realize the anomalous Josephson junction. Moreover, the anomalous Josephson current is 4π periodic in both φ and ϕ. When both φ and ϕ are periodically varying, we predict the existence of peaks in the average Josephson current, which are the analog to Shapiro steps in the ac Josephson effect. Similarly, the existence of MZMs can be confirmed by the missing of odd peaks. The findings provide a scheme to experimentally verify the 4π-periodic Josephson effect, as well as a platform to realize the anomalous Josephson junction.

II. MODEL

The 4π-periodic anomalous Josephson junction is sketched in Fig. 1. The left and right Majorana wires are made by Rashba wires in proximity to s-wave superconductors and in the Zeeman fields $\mathbf{h_L}$ and $\mathbf{h_R}$, respectively. $\mathbf{h_L}$ is along the x direction and $\mathbf{h_R}$ is rotating with a frequency ω in the x-y plane and makes an angle $\phi = \omega t$ with the x axis. Two Majorana wires are coupled by a central ferromagnetic (FM) wire with the magnetization orientation along the z direction. The tight-binding Hamiltonian of the junction can be given by $H = H_{L,R} + H_{FM} + H_C$. Here the Hamiltonians of the left and right Majorana wires are

$$\begin{aligned} H_{L,R} = \sum_{i,\sigma,\sigma'} \Big\{ & (-\mu' + \mathbf{h_{L,R}} \cdot \boldsymbol{\sigma})_{\sigma\sigma'} c^\dagger_{i,\sigma} c_{i,\sigma'} \\ & - \Big[\Big(t_M + i\frac{\alpha}{a}\sigma_z \Big)_{\sigma\sigma'} c^\dagger_{i,\sigma} c_{i+1,\sigma'} + \text{H.c.} \Big] \Big\} \\ & + \sum_i [\Delta e^{i\varphi_{L,R}} c^\dagger_{i,\uparrow} c^\dagger_{i,\downarrow} + \text{H.c.}], \end{aligned} \tag{1}$$

where $c^\dagger_{i,\sigma}$ ($c_{i,\sigma}$) creates (destroys) an electron at site i with spin σ, t_M is the hopping energy in the wire, $\mu' = \mu - 2t_M$ with μ being the chemical potential measured from the bottom of electronic band, $\boldsymbol{\sigma}$ are three Pauli matrices for spin, $\mathbf{h_L} = h\hat{x}$ and $\mathbf{h_R} = h\cos\omega t\hat{x} + h\sin\omega t\hat{y}$ with h the Zeeman field strength, α is the Rashba coupling strength and a is the lattice constant, Δ is the superconducting pairing potential and $\varphi_{L,R}$ is the superconducting phase in the left or right Majorana wire. The Hamiltonian of the central ferromagnetic wire is $H_{FM} = \sum_{i,\sigma,\sigma'} \{(U - \mu' + m_z\sigma_z)_{\sigma\sigma'} c^\dagger_{i,\sigma} c_{i,\sigma'} - [t_F c^\dagger_{i,\sigma} c_{i+1,\sigma} + \text{H.c.}]\}$, where U is the electrostatic potential, m_z is the Zeeman splitting energy along the z axis, and t_F the hopping energy. H_C describes the coupling between the two Majorana wire leads and the central ferromagnetic wire at two interfaces $H_C = -t_C \sum_{i,\sigma} c^\dagger_{i,\sigma} c_{i+1,\sigma} +$ H.c. with t_C the coupling strength.

By using nonequilibrium Green's functions, the Josephson current through site l in the central FM wire is calculated by [79–81]

$$J = \frac{1}{h} \int_{-\infty}^{\infty} \text{Tr}\big[t_F \check{e} G^<_{l,l-1} - t_F \check{e} G^<_{l-1,l} \big] dE, \tag{2}$$

where $\check{e} = -e\tau_3 \otimes \sigma_0$ denotes the charge matrix, τ_3 is the third Pauli matrix in Nambu space, and σ_0 is the unit matrix in spin space. In equilibrium, the lesser-than Green's function is calculated by $G^< = f(E)[G^a - G^r]$, where $f(E)$ is the Fermi-Dirac distribution function. The retarded and advanced Green's functions read

$$G^r(E) = [G^a(E)]^\dagger = \frac{1}{E - H_{FM} - \Sigma^r_L(E) - \Sigma^r_R(E)}, \tag{3}$$

where H_{FM} is the Hamiltonian of the FM wire. The retarded self-energy $\Sigma^r_{L(R)}(E)$ due to coupling with the superconducting Majorana wires L (R) can be calculated numerically by the recursive method [82–84].

In addition, the Andreev bound state (ABS) spectra can also be numerically calculated through the Green's function technique. The ABSs result in peaks of particle density within the superconducting gap. By searching the peaks of particle density at site l in the FM wire

$$\rho_l = -\frac{1}{\pi} \text{Im}\{\text{Tr}[G^r(l,l)]\} \tag{4}$$

at a given phase difference φ, the energies of ABS levels can be located. Then the ABS spectra can be obtained by scanning φ, which is helpful for understanding the behavior of Josephson current.

III. ADDITIONAL PHASE IN ANDREEV REFLECTION

We consider a normal wire/superconducting Majorana wire interface. The right Majorana wire is described by the effective model

$$H = \begin{pmatrix} \epsilon_k + \alpha k & h e^{-i\phi} & 0 & \Delta e^{i\varphi} \\ h e^{i\phi} & \epsilon_k - \alpha k & -\Delta e^{i\varphi} & 0 \\ 0 & -\Delta e^{-i\varphi} & -\epsilon_k - \alpha k & -h e^{i\phi} \\ \Delta e^{-i\varphi} & 0 & -h e^{-i\phi} & -\epsilon_k + \alpha k \end{pmatrix} \tag{5}$$

in the basis $(C_{k\uparrow}, C_{k\downarrow}, C^\dagger_{-k\uparrow}, C^\dagger_{-k\downarrow})^T$ with $\epsilon_k = \hbar^2 k^2/2m - \mu$ denotes the kinetic energy measured from the chemical potential μ. For simplicity, we make a basis transform to remove the phase factors ϕ and φ. The new basis is $(e^{i\psi_-/2} C_{k\uparrow}, e^{-i\psi_+/2} C_{k\downarrow}, e^{-i\psi_-/2} C^\dagger_{-k\uparrow}, e^{i\psi_+/2} C^\dagger_{-k\downarrow})^T = U(C_{k\uparrow}, C_{k\downarrow}, C^\dagger_{-k\uparrow}, C^\dagger_{-k\downarrow})^T$ with the transform $U = \text{diag}(e^{i\psi_-/2}, e^{-i\psi_+/2}, e^{-i\psi_-/2}, e^{i\psi_+/2})$ and $\psi_\pm = \phi \pm \varphi$. The Hamiltonian under the transformation is

$$H' = UHU^\dagger = \epsilon_k \tau_z + \alpha k \sigma_z \tau_z + h\sigma_x \tau_z - \Delta \sigma_y \tau_y. \tag{6}$$

This Hamiltonian has a chiral symmetry $\Gamma H'(k)\Gamma^{-1} = -H'(k)$ with $\Gamma = \tau_x$ that protects the MZMs at two ends of the Majorana wire. We dub the eigenvalue of $\Gamma = \pm 1$ as chirality. This chiral symmetry requires that only couplings between states with opposite chiralities are possible [85]. Without loss of generality, we assume that the MZM at the normal wire/Majorana wire interface γ_1 has the chirality $+1$. In fact, the scattering matrix at the interface is the same in the case of chirality -1 of the MZM. In general, the coupling between the normal wire and the MZM at the interface can be written as

$$H_C = \sum_{k,\sigma} (t_\sigma C_{k\sigma} - t_\sigma^* C_{-k\sigma}^\dagger)\gamma_1. \tag{7}$$

Because the normal wire also has the chiral symmetry Γ, only the states with $\Gamma = -1$ in the normal wire are coupled to the MZM. The two independent eigenstates with $\Gamma = -1$ are $(1, 0, -1, 0)^T$ and $(0, 1, 0, -1)^T$. Therefore, the coupling at the interface can also be written as

$$H_C = \sum_{k,\sigma} [\tilde{t}_\sigma \big(e^{i(\sigma\phi-\varphi)/2} C_{k\sigma} - e^{-i(\sigma\phi-\varphi)/2} C_{-k\sigma}^\dagger \big)\gamma_1. \tag{8}$$

By comparing Eq. (7) with Eq. (8), we have $t_\sigma = \tilde{t}_\sigma e^{i(\sigma\phi-\varphi)/2}$ and $\tilde{t}_\sigma^* = \tilde{t}_\sigma$. Thus, the scattering matrix at the interface can be given by [86]

$$S(E) = \hat{1} - 2\pi i W^\dagger (E + i\pi W W^\dagger)^{-1} W, \tag{9}$$

with $W = (e^{i\psi_-/2} w_\uparrow, e^{-i\psi_+/2} w_\downarrow, -e^{-i\psi_-/2} w_\uparrow, -e^{i\psi_+/2} w_\downarrow)$ the coupling matrix. Here $w_\sigma = \sqrt{\pi\rho_0}\tilde{t}_\sigma$ and ρ_0 is the density of states at the Fermi level. We focus on the matrix block that describes the Andreev reflection process and relates incoming electrons to outgoing holes as $(C_{-k\uparrow}^\dagger, C_{-k\downarrow}^\dagger)^T = S_{he}(C_{k\uparrow}, C_{k\downarrow})$. In Eq. (7), we assume that the coupling strength is the same for spin-up and spin-down quasiparticles, i.e., $w_\uparrow = w_\downarrow = w_0$, and let $\Gamma_0 = 2\pi w_0^2$. Then we have

$$S_{he} = \frac{\Gamma_0 e^{-i\varphi}}{2\Gamma_0 - iE} \begin{pmatrix} e^{i\phi} & 1 \\ 1 & e^{-i\phi} \end{pmatrix}. \tag{10}$$

From Eq. (10), it is found that spin-up (spin-down) electrons will be reflected as spin-up (spin-down) holes with an additional phase ϕ ($-\phi$). While for spin-flipped Andreev reflection processes, there is no additional phase ϕ. Note that the spin basis is along the z direction in the left normal wire, which is always perpendicular to the rotating Zeeman field $\mathbf{h_R}$ in the x-y plane in the right Majorana wire. Therefore, both equal-spin and opposite-spin Andreev reflections are permitted, which is not in conflict with the triplet nature of MZM and the selective equal-spin Andreev reflection in the same spin axis with $\mathbf{h_R}$ [87]. The additional phase $\pm\phi$ in the equal-spin Andreev reflection amplitudes along the spin axis z are consistent with the results in the limit $\mu = \alpha = 0$ in the Supplemental Material of Ref. [17], and will be also confirmed numerically in the next section. This additional phase should be compensated by the superconducting phase difference φ in the formation of Andreev bound states. Thus, the additional phase in Andreev reflections will result in a phase shift in Andreev bound states and the Josephson current-phase relation.

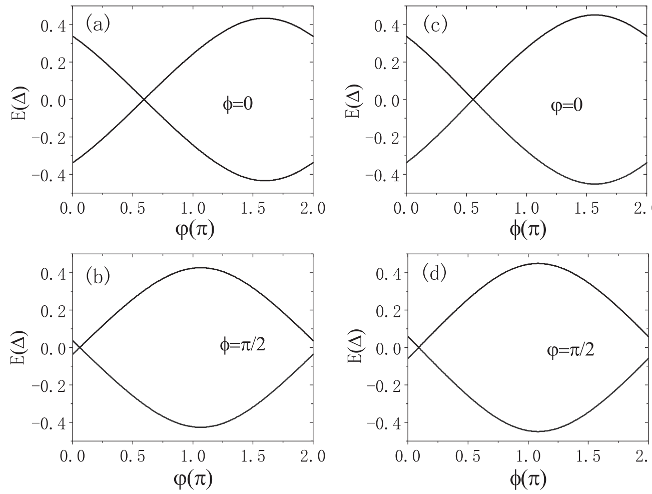

FIG. 2. The ABS levels as functions of both the superconducting phase difference φ [(a) and (b)] and the Zeeman field orientation difference ϕ of two Majorana wires [(c) and (d)]. $\phi = 0$ for (a) and $\pi/2$ for (b). $\varphi = 0$ for (c) and $\pi/2$ for (d). The junction parameters are: $\Delta = 0.1875$ meV, $t_M = t_F = t_C = 38\Delta$, $\alpha = 400$ meVÅ, $\mu = 8\Delta$, $U = 1.9\Delta$, $m_z = 7\Delta$, $h = 1.2h_c$ with $h_c = \sqrt{\mu^2 + \Delta^2}$, and the FM wire length $L = 10a$.

For a nonmagnetic wire with spin-degenerate electron occupation in the central region, neither 0-π transition nor anomalous Josephson effect occurs. When the central normal wire is replaced by an FM wire with the magnetization $\mathbf{M}$ along the z direction, the anomalous Josephson current can be obtained. We focus on the case of $m_z > \mu - U$ where only the spin-down band is occupied. This case is relatively simple in that only equal-spin Andreev reflection is permitted. The more important point is that only the additional phase $-\phi$ takes place, which leads to a scheme to observe the 4π-periodic Josephson effect.

We consider the case of $m_z > \mu - U$. Figure 2 shows the ABS levels as functions of both the superconducting phase difference φ and the Zeeman field orientation difference ϕ of two Majorana wires. It is clearly seen that ϕ is equivalent to φ, also acts as an effective superconducting phase difference and can drive the Josephson current. Note that even in the case of $\varphi = \phi = 0$, the ABS levels have an anomalous phase shift, which is attributed to the Rashba spin-orbit coupling.

IV. 4π-PERIODIC ANOMALOUS JOSEPHSON EFFECT

At first, we discuss the 2π-periodic dc Josephson effect in the stationary case of fixed φ and ϕ. From the above discussions, we know that ϕ has the same effect on driving the supercurrent as φ. Therefore, the Josephson current can be written as

$$I = I_c \sin(\varphi + \phi + \varphi_0), \tag{11}$$

where φ_0 is the Rashba coupling induced anomalous phase shift. Figure 3 shows the numerical results of Josephson currents as functions of φ and ϕ at $T = 0.5T_C$, which is consistent with the ABS levels and Eq. (11). It is shown that the first harmonic approximation is a good approximation, and the more sinusoidal behavior can be expected for higher temperatures

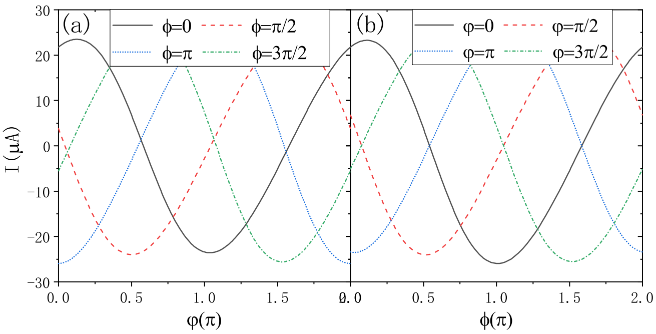

FIG. 3. (a) The Josephson currents as functions of φ for various ϕ, and (b) as functions of ϕ for various φ. The temperature is $T = 0.5T_C$ with T_C the superconducting critical temperature. The other parameters are the same as those in Fig. 2.

By applying a periodically varying magnetic flux through the Josephson circuit, the superconducting phase difference can be periodically modulated as $\varphi = \widetilde{\varphi} \sin \omega t$, where $\widetilde{\varphi}$ is the oscillating amplitude of φ and ω is the frequency. The Zeeman field orientation difference ϕ of two Majorana wires can be also periodically modulated by coupling the right Majorana wire to a ferromagnet with a precessing magnetization driven by microwaves as $\phi = \omega_0 t$ [88]. Then, the Josephson current can be written as

$$\begin{aligned} I &= I_c \sin(\varphi_0 + \omega_0 t + \widetilde{\varphi} \sin \omega t) \qquad (12) \\ &= I_c \sum_{m=-\infty}^{\infty} (-1)^m B_{-m}(\widetilde{\varphi}) \sin(\varphi_0 + \omega_0 t - m\omega t), \end{aligned}$$

where $B_{-m}(\widetilde{\varphi})$ is the $-m$th first type Bessel function and m is an integer. The situation is similar to the case of ac Josephson effect where a voltage with an ac component is applied. The dc component, i.e., the average of the Josephson current over time, can be found to be

$$\bar{I} = \begin{cases} 0, & \text{when } \omega_0 \neq n\omega \\ I_c(-1)^n B_{-n}(\widetilde{\varphi}) \sin \varphi_0, & \text{when } \omega_0 = n\omega \end{cases}, \qquad (13)$$

where n is an arbitrary positive integer.

In fact, due to the presence of two MZMs at interfaces, the fermion number parity guarantees that two ABS levels cross each other. Therefore, the Josephson effect should be 4π periodic and written as

$$\begin{aligned} I &= I_c \sin\left(\frac{\varphi_0}{2} + \frac{\omega_0 t}{2} + \frac{\widetilde{\varphi}}{2} \sin \omega t\right) \qquad (14) \\ &= I_c \sum_{m=-\infty}^{\infty} (-1)^m B_{-m}\left(\frac{\widetilde{\varphi}}{2}\right) \sin\left(\frac{\varphi_0}{2} + \frac{\omega_0 t}{2} - m\omega t\right). \end{aligned}$$

Now the average Josephson current becomes

$$\bar{I} = \begin{cases} 0, & \text{when } \omega_0 \neq 2n\omega \\ I_c(-1)^n B_{-n}(\frac{\widetilde{\varphi}}{2}) \sin \frac{\varphi_0}{2}, & \text{when } \omega_0 = 2n\omega \end{cases}. \qquad (15)$$

It implies that the missing odd peaks in the average Josephson current can be a new signature of 4π-periodic Josephson effect and MZMs. Figure 4 plots the numerically averaged Josephson current expressed in Eq. (14). It is clearly shown that the odd peaks are missing due to the 4π-periodic Josephson effect. Besides the even peaks, the small oscillations are attributed to the finite length of time over which the current is averaged.

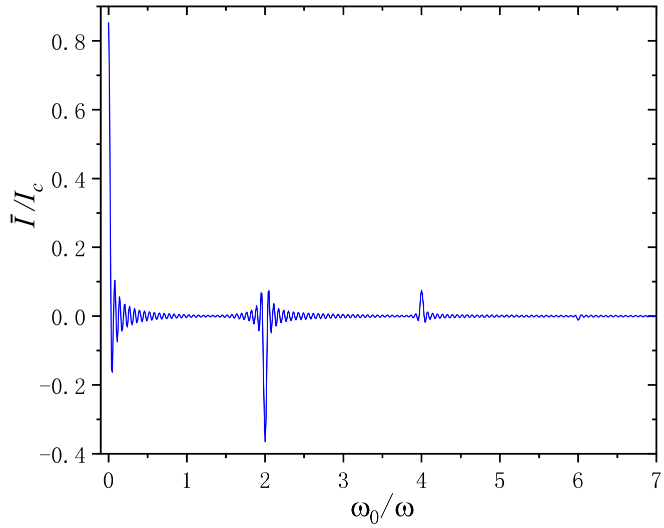

FIG. 4. The numerically averaged Josephson current expressed in Eq. (14) when both φ and ϕ are periodically modulated. The time interval over which the average is made is $[-31.415T, 31.415T]$ with $T = 2\pi/\omega$ being the period of the superconducting phase difference φ. $\phi_0 = \pi$, $\widetilde{\varphi} = \pi/2$. The other parameters are the same as those in Fig. 2.

V. EXPERIMENTAL FEASIBILITY

Finally, we comment on the experimental feasibility of this scheme to observe the 4π-periodic Josephson effect. The Majorana wires have been well established in experiments. The rotating Zeeman field can be induced by the proximity to a ferromagnet with a precessing magnetization driven by microwaves [88]. The precession frequency can be up to the order of GHz [88], which is higher than the quasiparticle poisoning frequency [89]. The ferromagnetic wire can be realized by depositing Fe/Au [90] or EuS [91] on the semiconductor nanowire. The superconducting phase difference φ can be periodically modulated by a periodically varying magnetic flux through the circuit. It is also noticeable that the Majorana wire considered in this Josephson junction is narrow enough to exclude the multi-sub-band effect. The length of the FM wire has ignorable effect on the measured supercurrent. Besides, the Josephson current is also insensitive to the electrostatic potential U in the FM wire. The missing odd peaks in the averaged Josephson current should be observable if only there remains one spin sub-band (along the z direction) occupied in the FM wire.

VI. CONCLUSIONS

In conclusion, we propose an anomalous Josephson junction, which involves two Majorana wire leads coupled by an FM wire. The Zeeman field in the left Majorana wire is along the x direction while that in the right Majorana wire is rotating in the x-y plane. The Zeeman field in the central FM wire is along the z direction. In this configuration, when only one spin sub-band is occupied in the FM wire, the orientation angle of the Zeeman field in the right Majorana wire ϕ has the same effect with the superconducting phase difference φ. This results in a 4π-periodic anomalous Josephson effect, which provides a signature of MZMs. When both φ and ϕ are periodically varying, we predict the existence of peaks in the average Josephson current, which are the analog to Shapiro steps in the ac Josephson effect. Similarly, the odd peaks should be missing due to the 4π period in the Josephson current. The findings provide a scheme to experimentally verify the 4π-periodic Josephson effect and the existence of MZMs, as well as a platform to realize the anomalous Josephson junction.

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