Letter

Non-Abelian half-quantum vortices in $^3P_2$ topological superfluids

Yusuke Masaki,[1,2,*] Takeshi Mizushima,[3] and Muneto Nitta[1,4]

[1]*Research and Education Center for Natural Sciences, Keio University, Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan*
[2]*Department of Applied Physics, Graduate School of Engineering, Tohoku University, Sendai, Miyagi 980-8579, Japan*
[3]*Department of Materials Engineering Science, Osaka University, Toyonaka, Osaka 560-8531, Japan*
[4]*Department of Physics, Keio University, Hiyoshi 4-1-1, Japan*

(Received 12 July 2021; revised 25 January 2022; accepted 27 May 2022; published 14 June 2022)

$^3P_2$ superfluids realized in neutron stars are the largest topological quantum matters in our universe. We establish the existence and stability of non-Abelian half-quantum vortices (HQVs) in $^3P_2$ superfluids with strong magnetic fields. Using a self-consistent microscopic approach, we find that a singly quantized vortex is energetically destabilized into a pair of two non-Abelian HQVs owing to the strongly spin-orbit-coupled pairing. We find a topologically protected Majorana fermion on each HQV, thereby providing twofold non-Abelian anyons characterized by both Majorana fermions and a non-Abelian first homotopy group.

DOI: 10.1103/PhysRevB.105.L220503

Introduction. Quantum physics tells us that all particles are either fermions or bosons under certain assumptions; a wave function of multiparticle states is symmetric (asymmetric) under the exchange of two bosons (fermions). However, an exception, anyons, exists. The exchange of two anyons causes the wave function to acquire a phase factor [1,2]. Such anyons explain the physics of fractional quantum Hall states [3,4] and have been experimentally observed for $\nu = 1/3$ fractional quantum Hall states [5]. Recently, another option has attracted a great deal of attention, that is, non-Abelian anyons. The exchange of two non-Abelian anyons leads to a unitary matrix acting on a set of wave functions as a generalization of the phase factor for Abelian anyons. Although non-Abelian anyons have yet to be observed, they have been predicted theoretically to exist in $\nu = 5/2$ fractional quantum Hall states [6], topological superconductors [7,8], and spin liquids [9,10]. Non-Abelian anyons have attracted significant interest owing to the possibility for a platform of topological quantum computation [11–13] which is robust against noises, in contrast to the conventional quantum computation methods.

There are two apparently different origins of non-Abelian anyons, one fermionic and the other bosonic. The fermionic origin is based on Majorana fermions realized in topological superconductors [8,14–20]. Majorana fermions are particles that coincide with their own antiparticles [21]. This is the main route for topological quantum computation. By contrast, non-Abelian anyons are also realized in bosonic systems, the statistics of which are due to non-Abelian vortices supported by a non-Abelian first homotopy group of order parameter (OP) manifolds, giving noncommutativity under the exchange of two vortices [22,23]. Examples can be found in liquid crystals [24,25] and spinor Bose-Einstein condensates (BECs) [23,26–28]. Two apparently different non-Abelian anyons have been discussed separately thus far and their relation has yet to be clarified.

The aim of this Letter is to present vortices simultaneously accompanied by the two different non-Abelian natures, that is, fermionic and bosonic origins of non-Abelian anyons. A system that realizes such vortices is a neutron superfluid expected to occur in neutron star cores. This is called the $^3P_2$ superfluid (spin-triplet p-wave pairings) of neutrons [29–34], which has been recently shown to be the largest topological quantum matter in our universe [35] (a class DIII in the classification of topological insulators and superconductors [36,37]), allowing a gapless Majorana fermion on its boundary [35] and vortex cores [38]. From the Ginzburg-Landau (GL) theory [33,39–44], this matter was found to admit non-Abelian half-quantum vortices (HQVs) [45,46] in addition to integer vortices [33,39–42,47,48], coreless vortices [49], domain walls [50], and boojums on the surface [51]. Such topological defects may play a crucial role in the dynamics and evolution of neutron stars.

Unlike the Feynman-Onsager quantization of circulation, HQVs, or more generally fractionally quantized vortices [52–54], appear ubiquitously in diverse systems with multiple components. The topological stability of HQVs (or fractional quantum vortices) has been predicted in the A phase [55,56] of superfluid ^{3}He, unconventional superconductors [57–62], spinor BECs [26–28,63–67], multicomponent superconductors [68–73], and BECs [74–83] and even high-energy physics such as quantum chromodynamics [84–88] and physics beyond the standard model of elementary particles [89,90]. Abelian HQVs were experimentally confirmed in the uniaxially disordered superfluid ^{3}He [91,92] and in a spinor BEC [93]. However, no systems admitting non-Abelian HQVs, which have both bosonic and fermionic origins of non-Abelian anyons, are known thus far. The existence of HQVs in $^3P_2$ superfluids was proposed to explain a longstanding unsolved problem of neutron stars: the origin of the pulsar glitch phenomena, that is, sudden speedup events of neutron

*yusuke.masaki.c1@tohoku.ac.jp

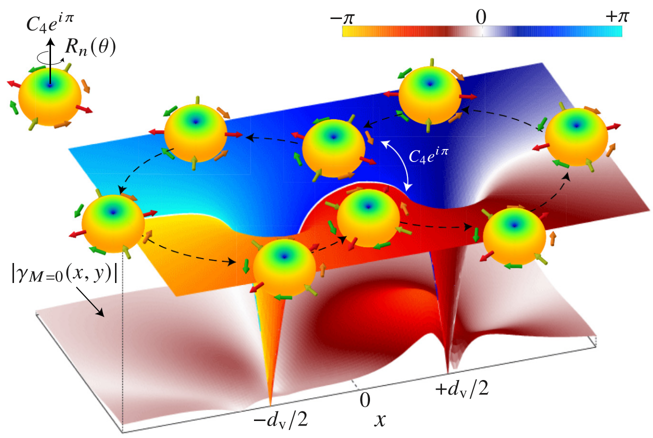

FIG. 1. Schematic of a pair of non-Abelian HQVs in a D_4-BN state at a cross section perpendicular to the two parallel vortex lines, characterized by $(\kappa, n) = (1/2, +1/4)$ at $x = d_v/2$ and $(1/2, -1/4)$ at $x = -d_v/2$. Its spin momentum structure is shown by objects with colored arrows representing d vectors.

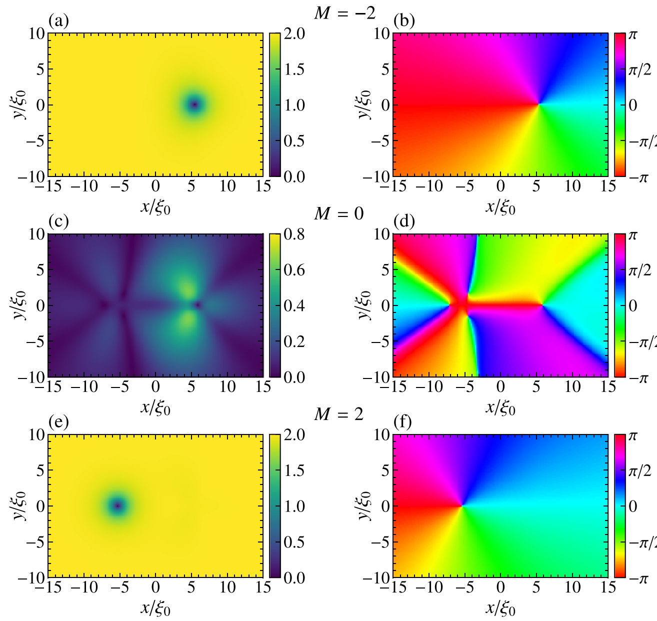

FIG. 2. Two HQVs with $d_v \simeq 10.7\xi_0$. (a), (c), and (e) Spatial profiles of the amplitude $|\gamma_{M=-2,0,2}(\boldsymbol{R})|$ and (b), (d), and (f) spatial profiles of the phase $\arg[\gamma_{M=-2,0,2}(\boldsymbol{R})]$, with (c) and (d) representing the induced component.

stars [94]. However, the energetic stability of the HQVs has not been investigated thus far, e.g., it remains as an important unsolved problem whether a singly quantized vortex can split into two HQVs or not.

In this study, we microscopically establish the existence and stability of non-Abelian HQVs, along each of which we find a topologically protected gapless Majorana fermion. In the presence of a strong magnetic field relevant for magnetars, i.e., neutron stars accompanied by extraordinary large magnetic fields, the ground state is in a dihedral-four biaxial nematic (D_4-BN) phase [35,47,95]. There a singly quantized vortex is shown to be split into two non-Abelian HQVs. Each HQV admits a gapless Majorana fermion, thereby being a different type of non-Abelian anyon. We also calculate the interaction energy between HQVs and find an intrinsic mechanism of their thermodynamic stability due to the uniaxial nematic pairing induced around the cores.

Non-Abelian HQVs. Here we focus on non-Abelian HQVs in the D_4-BN phase of a ${}^3P_2$ superfluid. Let us consider systems invariant under a U(1) gauge transformation and SO(3) spin momentum rotation. A ${}^3P_2$ superfluid is the condensation of spin-triplet Cooper pairs with a total angular momentum of $J = 2$, the OP of which is given by a 3×3 traceless symmetric tensor $\mathcal{A}_{\mu\nu(\mu,\nu=x,y,z)}$ with spin index μ and momentum index ν. The continuous symmetries act as $\mathcal{A} \to e^{i\varphi} g \mathcal{A} g^{\rm tr}$, $e^{i\varphi} \in$ U(1) and $g \in$ SO(3). The homogeneous OP of the D_4-BN state has a diagonal form [40] $\mathcal{A} = \Delta \mathrm{diag}(1, -1, 0)$,[1] which is invariant under a C_4 rotation around the z axis in a point node direction, combined with the π phase rotation. Its spin momentum structure is schematically shown by d vectors, $d_\mu(\boldsymbol{k}) = \sum_\nu \mathcal{A}_{\mu\nu} k_\nu$, using arrows in the top left object in Fig. 1. A large magnetic field relevant to magnetars thermodynamically stabilizes the D_4-BN state with point nodes along the direction of the magnetic field [35,43,47,95].

The OP manifold in the D_4-BN state, $R = [\mathrm{U}(1) \times \mathrm{SO}(3)]/D_4$, leads to rich topological charges of line defects supported by the first homotopy group $\pi_1(R) = \mathbb{Z} \times_h D_4^*$ [98]. This includes non-Abelian HQVs with noncommutative topological charges, which behave as non-Abelian anyons with a bosonic origin [46]. An asymptotic form for an isolated vortex is given by $\mathcal{A}(\theta) = e^{i\kappa\theta} R_n(\theta) \mathcal{A} R_n^{\rm tr}(\theta)$, where $\theta \equiv \tan^{-1}(y/x)$ is the azimuthal angle and $R_n(\theta) \in$ SO(2) is a rotation matrix around the z axis by the angle $n\theta$. The integer vortices are characterized by $\kappa \in \mathbb{Z}$ and $n = 0$. In the D_4-BN state, the π phase jump arising from $\kappa \in \mathbb{Z} + 1/2$ is compensated by the C_4 rotation with $n \in \mathbb{Z} \pm 1/4$. Thus, HQVs are topologically allowed and a singly quantized vortex is predicted to be split into a pair of HQVs, as illustrated in Fig. 1.

Structure and Stability of the HQVs. To microscopically discuss the stability of non-Abelian HQVs, we utilize quasiclassical theory [99–101]. Assuming uniformity along the z direction, we determine the spatial profile of $\mathcal{A}_{\mu\nu}(\boldsymbol{R} = (x, y))$ by self-consistently solving the Eilenberger equation complemented by a gap equation for interacting neutrons through a zero-range attractive ${}^3P_2$ force (see Ref. [101] for details). Below we show the numerical results at temperature $T = 0.4T_c$ and the Zeeman magnetic field $V_Z = 0.5T_c$ with the critical temperature T_c. For this parameter set, the D_4-BN state is the most stable uniform state. A unit length is given by the coherence length $\xi_0 = v_F/2\pi T_c$ with the Fermi velocity v_F.

In Fig. 2 we show a pair of HQVs with finite intervortex distance $d_v \simeq 10.7\xi_0$. It is convenient to expand $\mathcal{A}_{\mu\nu}$ as $\mathcal{A}_{\mu\nu} = \sum_{M=-2}^{2} \gamma_M(\boldsymbol{R}) \Gamma_{M,\mu\nu}$, where Γ_M is a 3×3 basis tensor of the z component of the total angular momentum J_z such

[1]This OP takes a form similar to that of the planar state of superfluid ^{3}He [96,97], but these states admit different topological defects because of their different OP manifolds.

that $J_z\Gamma_M = M\Gamma_M$ and $\gamma_M(\boldsymbol{R})$ is the complex OP projected onto the sector $J_z = M$. The D_4-BN state is represented by $|\gamma_{M=2}| = |\gamma_{M=-2}|$ and $\gamma_{M=-1,0,1} = 0$. For an isolated HQV, the aforementioned asymptotic form is recast into $\mathcal{A}_{\mu\nu}(\theta) = \sum_{M=-2,2}\gamma_M e^{i(\kappa\theta - M\varphi)}\Gamma_{M,\mu\nu}$ with the vorticity $\kappa = 1/2$ and the rotation angle of the triad $\varphi = n\theta = \pm\theta/4$. We set $\kappa > 0$ without a loss of generality, whereas the choice of $n = +1/4$ ($-1/4$) corresponds to the clockwise (counterclockwise) texture of the gap structure. The two HQVs shown in Figs. 1 and 2 are characterized by a pair of $(\kappa, n) = (1/2, -1/4)$ at $x = -d_v/2$ and $(1/2, +1/4)$ at $x = d_v/2$, where the magnetic mirror symmetry perpendicular to the y direction is imposed [101]. The amplitudes (phases) of $\gamma_M(\boldsymbol{R})$ are shown in the left (right) panels of Fig. 2. In each $M = \pm 2$ sector, a single winding structure is realized [Figs. 2(b) and 2(f)], and in the $M = 0$ sector, a structure with winding $2 = 3 - 1$ is induced, as indicated in Fig. 2(d). Note that in the bulk region, $\gamma_{M=0}$ moves toward zero.

The isolated HQV for $n = +1/4$ ($-1/4$) consists of three components, that is, a singular vortex component for $M = -2$ ($+2$), an almost uniform unwinding component for $M = +2$ (-2), and the induced component for $M = 0$. It can be regarded as a chiral p-wave superconducting vortex with the spin parallel to the chirality, and the phases of the induced components have winding -1 for $(\kappa, n) = (1/2, 1/4)$ and 3 for $(1/2, -1/4)$ [102,103]. In the former (latter) case, the vorticity is antiparallel (parallel) to the chirality. However, the amplitude of the induced component $|\gamma_{M=0}(\boldsymbol{R})|$ breaks the axial symmetry to a threefold symmetry for $n = 1/4$ and fivefold symmetry for $n = -1/4$ (see Ref. [101]). The axial symmetry is also broken by the boundary conditions.

The two types of internal structures in the $M = 0$ component induced for HQVs with $n = \pm 1/4$ are modulated by the connection of these two HQVs. We find that this modulation causes an interaction between the two HQVs and binds them together. To reveal the interaction between HQVs, we compute the Luttinger-Ward energy functional $\mathcal{J}_{\rm sn}$ from the self-consistently determined quasiclassical propagator [104]. The interaction energy denoted by $\Delta\mathcal{J}_{\rm sn}(d_v)$ is the difference between the energy of a pair of two HQVs with separation d_v and the energies of two isolated HQVs (see Ref. [101]). In this definition, the boundary effects due to the long-tailed flows of the mass and spin currents are properly eliminated. In Fig. 3 we show the interaction energies of the two HQVs. The triangles (circles) are calculated by considering (neglecting) the induced components $\gamma_{M=\pm 1}$. The difference appears only for $d_v = 0$, where singly quantized vortices are realized; the triangle at $d_v = 0$ stands for the double core vortex (d vortex), whose core is occupied with $\gamma_{M=\pm 1}$, as in the superfluid ^{3}He-B phase [105,106]. The d vortex (triangles) has lower energy than the vortex without $\gamma_{M=\pm 1}$ (circles) because condensation energy due to $\gamma_{M=\pm 1}$ is gained at the origin. Significantly, for finite d_v, the interaction energy decreases as d_v increases from zero and reaches the minimum at a finite intervortex distance d_v, which means that the d vortex is unstable for splitting into the two HQVs. The gain in the interaction energy is due to the deformations in $\gamma_{M=0}(\boldsymbol{R})$, and the two HQVs form a bound molecule with an optimal separation.

Molecules of HQVs are also discussed in regard to superfluid ^{3}He [55] and unconventional superconductors [58],

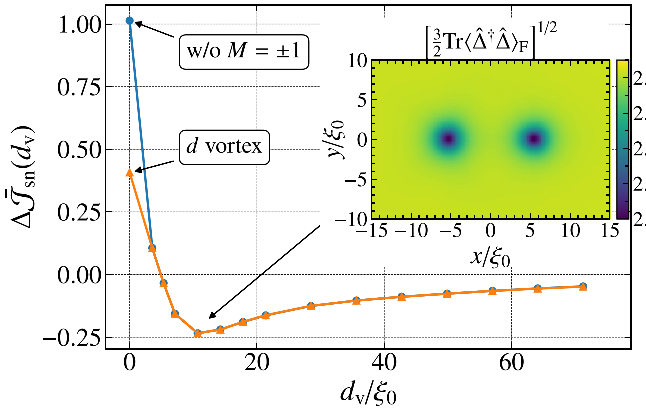

FIG. 3. Interaction energy of two HQVs as a function of their separation d_v. The triangles (circles) are calculated by considering (neglecting) the possibilities of the induced components of $M = \pm 1$. The inset shows the total amplitude of the OP shown in Fig. 2; the corresponding intervortex distance is indicated by the arrow. The free energy is scaled as $\bar{\mathcal{J}}_{\rm sn} = \mathcal{J}_{\rm sn}/\nu_n T_c^2 \xi_0^2 \Omega_z$, where Ω_z is the length of the system in the z direction and ν_n is the density of states at the Fermi energy in the normal state.

but their stabilization mechanisms are different from ours: In the superfluid ^{3}He-A phase, the spin mass correction was phenomenologically introduced to stabilize the HQV [55]; however, its realization remains controversial because the strong-coupling effects destabilize the HQV [107–110]. In the polar phases, the stability of the HQVs is supported by an extrinsic mechanism from strong anisotropic impurity effects using the GL theory [111–113]. There is no intrinsic interaction between the two HQVs in the weak-coupling limit because two spin sectors are independent. By contrast, in the present case, a different mechanism of the interaction originates from the deformation in the induced component γ_0 because of the strongly spin-orbit-coupled pairing.

Majorana zero modes in non-Abelian HQVs. Finally, we clarify the existence of topologically protected zero-energy states in HQVs, which behave as non-Abelian (Ising) anyons. Using the OP determined self-consistently for a separation of $d_v \simeq 10.7\xi_0$ and spatial uniformity along the z direction, we solve the Bogoliubov–de Gennes (BdG) equation $\mathcal{H}_{{\rm BdG},k_z}(\boldsymbol{R})\vec{u}_{\alpha,k_z}(\boldsymbol{R}) = \epsilon_{\alpha,k_z}\vec{u}_{\alpha,k_z}(\boldsymbol{R})$, where $\mathcal{H}_{{\rm BdG},k_z}$ is a 4×4 matrix in the spin and Nambu space, $\vec{u}_{\alpha,k_z} = [u_{\alpha,k_z,\uparrow}, u_{\alpha,k_z,\downarrow}, v_{\alpha,k_z,\uparrow}, v_{\alpha,k_z,\downarrow}]$ is the αth eigenvector of the axial momentum k_z, and ϵ_{α,k_z} is its eigenenergy [101]. We set the quasiclassical parameter $k_F\xi_0 = \varepsilon_F/\pi T_c = 5$ with the Fermi momentum (energy) k_F (ε_F). For the spectroscopy of the vortex-bound states, we show the fermionic local density of states for $k_z = 0$ in Fig. 4(a), $\nu_{k_z=0}(\boldsymbol{R};\omega) = \sum_{\alpha,\sigma}|u_{\alpha,k_z=0,\sigma}(\boldsymbol{R})|^2\delta(\omega - \epsilon_{\alpha,k_z=0})$ along $y = 0$. In the energy region below the bulk gap $\omega_g/T_c \sim 2.0$, the spectral weights are localized around the HQV cores and the edge (not shown). The energy levels of the vortex bound states are discretized with level spacing on the order of $\omega_g^2/\varepsilon_F \sim 0.255T_c$.

It is worth noting that each vortex hosts a single zero-energy state with numerical accuracy only at $k_z = 0$. Here we demonstrate that the zero modes are protected by two

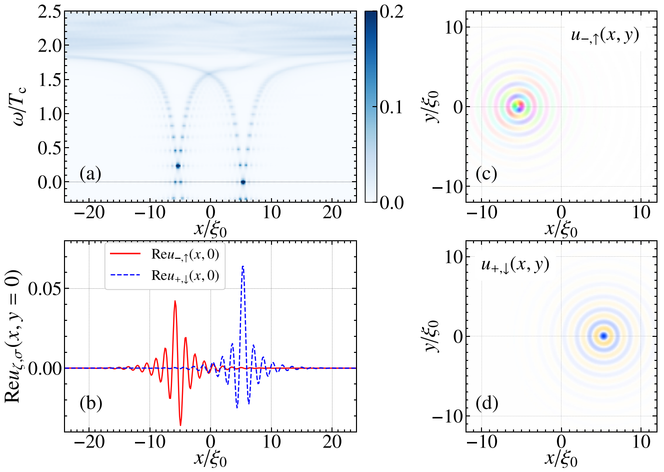

FIG. 4. (a) Local density of states $\nu_{k_z=0}(\boldsymbol{R};\omega)$ at $k_z=0$ and $y=0$ for a pair of HQVs located at $(x,y)=(\pm d_v/2,0)$ with $d_v \simeq 10.7\xi_0$. (b) Real parts of two Majorana wave functions $u_{-,\uparrow}$ and $u_{+,\downarrow}$ for $y=0$. Also shown are the two-dimensional spatial profiles of (c) $u_{-,\uparrow}$ and (d) $u_{+,\downarrow}$. The colormaps indicate their phase information for the same color bar as in Fig. 2(b). The saturation indicates their intensities.

discrete symmetries relevant to a three-dimensional vortex line, the mirror symmetry and the chiral symmetry, and yield a fermionic origin of the non-Abelian nature. To clarify this, we employ the semiclassical approximation as $\check{\mathcal{H}}_{\mathrm{BdG},k_z}(x,y)\mapsto \check{\mathcal{H}}(\boldsymbol{k},\theta)$, which varies slowly in real-space coordinates as a function of the azimuthal angle θ around the vortex line [110,114,115]. For the topological protection, consider the mirror reflection $\mathcal{M}_{xy}$ with respect to the xy plane. As demonstrated in Refs. [116,117], if the gap function is odd under $\mathcal{M}_{xy}$, the HQV may support a Majorana zero mode protected by the mirror symmetry. For the mirror reflection invariant momentum $\boldsymbol{k}_{\mathrm{M}}\equiv(k_x,k_y,k_z=0)$, the BdG Hamiltonian commutes with the mirror reflection operator $\check{\mathcal{M}}^{-}_{xy}$ as $[\check{\mathcal{H}}(\boldsymbol{k}_{\mathrm{M}},\theta),\check{\mathcal{M}}^{-}_{xy}]=0$, because $\mathcal{A}_{xz}=\mathcal{A}_{yz}=0$, i.e., $\gamma_{M=\pm1}=0$ in non-Abelian HQVs [101]. Hence, the Hamiltonian with $k_z=0$ is block diagonalized in terms of the eigenvalues of the mirror operator $\lambda=\pm i$, as $\check{\mathcal{H}}(\boldsymbol{k}_{\mathrm{M}},\theta)=\bigoplus_\lambda \tilde{\mathcal{H}}_\lambda(\boldsymbol{k}_{\mathrm{M}},\theta)$, where the 2×2 submatrix $\tilde{\mathcal{H}}_\lambda$ is still subject to the particle-hole symmetry. In terms of the Altland-Zirnbauer symmetry classes, each subsector at $k_z=0$ belongs to class D, similar to spinless chiral superconductors [36]. The topological invariant relevant to the class-D BdG Hamiltonian $\tilde{\mathcal{H}}_\lambda$ on the base space $(\boldsymbol{k}_{\mathrm{M}},\theta)\in S^2\times S^1$ is the $\mathbb{Z}_2$ number defined as $\nu_\lambda=(i/\pi)^2\int_{S^2\times S^1}\mathrm{tr}(AdA+2A^3/3)$ (mod 2) with the Berry connection A obtained from the occupied eigenstates of $\tilde{\mathcal{H}}_\lambda(\boldsymbol{k}_{\mathrm{M}},\theta)$ [118–120]. The non-Abelian HQV in the D_4-BN state has a nontrivial value of the $\mathbb{Z}_2$ invariant in each mirror subsector, $\nu_\lambda=+1$ (-1) for $\lambda=+i$ $(-i)$, which ensures the existence of a single Majorana zero mode at $k_z=0$ in each HQV that behaves as a non-Abelian (Ising) anyon [117,120].

In addition to the $\mathbb{Z}_2$ number, the chiral symmetry, which is a combination of the particle-hole symmetry and the magnetic π rotation, defines a $\mathbb{Z}$ topological number and protects the zero mode [101]. Hence, the zero mode accompanied by HQVs and its Majorana nature are guaranteed by the mirror symmetry and chiral symmetry, unless the three-dimensional vortex lines break these symmetries. In rotating neutron stars, HQVs are aligned with the rotation axis, thus accompanied by Majorana zero modes at $k_z=0$ that are the fermionic origin of the non-Abelian nature.

In Figs. 4(b)–4(d) we show the wave functions of the zero modes obtained by separating the edge mode and the vortex core mode through a linear combination of the two particle-hole symmetric eigenpartners. We assign the label $\zeta=+$ $(-)$ to the state localized around $\boldsymbol{R}_1$ $(\boldsymbol{R}_2)$ instead of $(\alpha,k_z=0)$. The Majorana condition $u_{\zeta,\sigma}(\boldsymbol{R})=[v_{\zeta,\sigma}(\boldsymbol{R})]^*$ is satisfied by choosing the global phase properly. The real parts of $u_{-,\uparrow}$ and $u_{+,\downarrow}$ along $y=0$ are shown in Fig. 4(b); for the other combinations of ζ and σ, the wave functions $u_{\zeta,\sigma}$ are zero. The phases of $u_{-,\uparrow}$ and $u_{+,\downarrow}$ have single and no winding, as indicated by the two-dimensional colormaps in Figs. 4(c) and 4(d), respectively. The two Majorana fermions in the two non-Abelian HQVs are in opposite spin sectors and have different structures in their phase winding.

Summary. We have found twofold non-Abelian anyons stably existing in a $^3P_2$ nematic superfluid, that is, non-Abelian HQVs characterized by a non-Abelian first homotopy group of a bosonic origin and Majorana fermions present inside their cores. Thus far only non-Abelian anyons with either bosonic or fermionic non-Abelian origin have been known in other systems. From the microscopic approach in a $^3P_2$ nematic superfluid, we have found that the HQVs are stabilized in the form of molecules through the interaction mediated by the uniaxial nematic component. This is a different stabilization mechanism of HQVs due to the gap functions with a strong spin-orbit coupling. Furthermore, we have found a single Majorana zero mode in each HQV. Their non-Abelian Ising nature is elucidated on the basis of topological invariants.

The existence of such HQVs is essential to explain a certain aspect of neutron stars [94]. Our finding presents the possibility for advances in the study of non-Abelian anyons, possibly applicable to alternative directions in topological quantum computation and neutron star physics.

Acknowledgments. Y.M. thanks Robert Regan for useful comments. Numerical calculations in this study were done using the facilities of the Supercomputer Center, the Institute for Solid State Physics, the University of Tokyo. This work was supported by a Grant-in-Aid for Scientific Research on Innovative Areas "Quantum Liquid Crystals" (Grants No. JP20H05163 and No. JP22H04480) from JSPS of Japan and JSPS KAKENHI (Grants No. JP18H01217, No. JP19K14662, No. JP20K03860, No. JP20H01857, No. JP21H01039, and No. JP22H01221).

[1] J. M. Leinaas and J. Myrheim, Nuovo Cim. B **37**, 1 (1977).
[2] F. Wilczek, Phys. Rev. Lett. **49**, 957 (1982).
[3] B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984); **52**, 2390(E) (1984).

[4] D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984).
[5] J. Nakamura, S. Liang, G. C. Gardner, and M. J. Manfra, Nat. Phys. **16**, 931 (2020).
[6] G. W. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991).
[7] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
[8] D. A. Ivanov, Phys. Rev. Lett. **86**, 268 (2001).
[9] A. Kitaev, Ann. Phys. (NY) **321**, 2 (2006).
[10] Y. Motome and J. Nasu, J. Phys. Soc. Jpn. **89**, 012002 (2020).
[11] A. Y. Kitaev, Ann. Phys. (NY) **303**, 2 (2003).
[12] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. **80**, 1083 (2008).
[13] S. D. Sarma, M. Freedman, and C. Nayak, npj Quantum Inf. **1**, 15001 (2015).
[14] A. Y. Kitaev, Phys. Usp. **44**, 131 (2001).
[15] J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012).
[16] M. Leijnse and K. Flensberg, Semicond. Sci. Technol. **27**, 124003 (2012).
[17] C. Beenakker, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013).
[18] S. R. Elliott and M. Franz, Rev. Mod. Phys. **87**, 137 (2015).
[19] M. Sato and S. Fujimoto, J. Phys. Soc. Jpn. **85**, 072001 (2016).
[20] C. W. J. Beenakker, SciPost Phys. Lect. Notes **15**, 15 (2020).
[21] E. Majorana, Nuovo Cim. **14**, 171 (1937).
[22] H.-K. Lo and J. Preskill, Phys. Rev. D **48**, 4821 (1993).
[23] T. Mawson, T. C. Petersen, J. K. Slingerland, and T. P. Simula, Phys. Rev. Lett. **123**, 140404 (2019).
[24] V. Poenaru and G. Toulouse, J. Phys. (Paris) **38**, 887 (1977).
[25] N. D. Mermin, Rev. Mod. Phys. **51**, 591 (1979).
[26] G. W. Semenoff and F. Zhou, Phys. Rev. Lett. **98**, 100401 (2007).
[27] M. Kobayashi, Y. Kawaguchi, M. Nitta, and M. Ueda, Phys. Rev. Lett. **103**, 115301 (2009).
[28] M. O. Borgh and J. Ruostekoski, Phys. Rev. Lett. **117**, 275302 (2016); **118**, 129901(E) (2017).
[29] M. Hoffberg, A. E. Glassgold, R. W. Richardson, and M. Ruderman, Phys. Rev. Lett. **24**, 775 (1970).
[30] R. Tamagaki, Prog. Theor. Phys. **44**, 905 (1970).
[31] T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. **46**, 114 (1971).
[32] T. Takatsuka, Prog. Theor. Phys. **47**, 1062 (1972).
[33] R. W. Richardson, Phys. Rev. D **5**, 1883 (1972).
[34] A. Sedrakian and J. W. Clark, Eur. Phys. J. A **55**, 167 (2019).
[35] T. Mizushima, K. Masuda, and M. Nitta, Phys. Rev. B **95**, 140503(R) (2017).
[36] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Phys. Rev. B **78**, 195125 (2008).
[37] S. Ryu, A. P. Schnyder, A. Furusaki, and A. W. W. Ludwig, New J. Phys. **12**, 065010 (2010).
[38] Y. Masaki, T. Mizushima, and M. Nitta, Phys. Rev. Research **2**, 013193 (2020).
[39] T. Fujita and T. Tsuneto, Prog. Theor. Phys. **48**, 766 (1972).
[40] J. A. Sauls and J. W. Serene, Phys. Rev. D **17**, 1524 (1978).
[41] P. Muzikar, J. A. Sauls, and J. W. Serene, Phys. Rev. D **21**, 1494 (1980).
[42] J. A. Sauls, D. L. Stein, and J. W. Serene, Phys. Rev. D **25**, 967 (1982).
[43] S. Yasui, C. Chatterjee, and M. Nitta, Phys. Rev. C **99**, 035213 (2019).
[44] S. Yasui, C. Chatterjee, M. Kobayashi, and M. Nitta, Phys. Rev. C **100**, 025204 (2019).
[45] J. A. Sauls, Anisotropic superfluidity in neutron stars and strong coupling effect in superfluid ^{3}He, Ph.D thesis, SUNY Stony Brook, 1980.
[46] K. Masuda and M. Nitta, Prog. Theor. Exp. Phys. **2020**, 013D01 (2020).
[47] K. Masuda and M. Nitta, Phys. Rev. C **93**, 035804 (2016).
[48] C. Chatterjee, M. Haberichter, and M. Nitta, Phys. Rev. C **96**, 055807 (2017).
[49] L. B. Leinson, Mon. Not. R. Astron. Soc. **498**, 304 (2020).
[50] S. Yasui and M. Nitta, Phys. Rev. C **101**, 015207 (2020).
[51] S. Yasui, C. Chatterjee, and M. Nitta, Phys. Rev. C **101**, 025204 (2020).
[52] E. Babaev, Phys. Rev. Lett. **89**, 067001 (2002).
[53] E. Babaev, A. Sudbo, and N. W. Ashcroft, Nature (London) **431**, 666 (2004).
[54] E. Babaev, A. Sudbo, and N. W. Ashcroft, Nat. Phys. **3**, 530 (2007).
[55] M. M. Salomaa and G. E. Volovik, Phys. Rev. Lett. **55**, 1184 (1985).
[56] M. M. Salomaa and G. E. Volovik, Rev. Mod. Phys. **59**, 533 (1987).
[57] H.-Y. Kee, Y. B. Kim, and K. Maki, Phys. Rev. B **62**, R9275 (2000).
[58] S. B. Chung, H. Bluhm, and E.-A. Kim, Phys. Rev. Lett. **99**, 197002 (2007).
[59] J. Garaud and E. Babaev, Phys. Rev. B **86**, 060514(R) (2012).
[60] A. A. Zyuzin, J. Garaud, and E. Babaev, Phys. Rev. Lett. **119**, 167001 (2017).
[61] S. B. Etter, W. Huang, and M. Sigrist, New J. Phys. **22**, 093038 (2020).
[62] P. T. How and S.-K. Yip, Phys. Rev. Research **2**, 043192 (2020).
[63] U. Leonhardt and G. E. Volovik, Pisma Zh. Eksp. Teor. Fiz. **72**, 66 (2000) [JETP Lett. **72**, 46 (2000)].
[64] T.-L. Ho, Phys. Rev. Lett. **81**, 742 (1998).
[65] T. Ohmi and K. Machida, J. Phys. Soc. Jpn. **67**, 1822 (1998).
[66] S.-H. Shinn and U. R. Fischer, Phys. Rev. A **98**, 053602 (2018).
[67] Y. Kawaguchi and M. Ueda, Phys. Rep. **520**, 253 (2012).
[68] J. Goryo, S. Soma, and H. Matsukawa, Europhys. Lett. **80**, 17002 (2007).
[69] Y. Tanaka, A. Crisan, D. D. Shivagan, A. Iyo, K. Tokiwa, and T. Watanabe, Jpn. J. Appl. Phys. **46**, 134 (2007).
[70] A. Crisan, Y. Tanaka, D. D. Shivagan, A. Iyo, L. Cosereanu, K. Tokiwa, and T. Watanabe, Jpn. J. Appl. Phys. **46**, L451 (2007).
[71] J. W. Guikema, H. Bluhm, D. A. Bonn, R. Liang, W. N. Hardy, and K. A. Moler, Phys. Rev. B **77**, 104515 (2008).
[72] Y. Tanaka, H. Yamamori, T. Yanagisawa, T. Nishio, and S. Arisawa, Physica C **538**, 12 (2017).
[73] Y. Tanaka, H. Yamamori, T. Yanagisawa, T. Nishio, and S. Arisawa, Physica C **548**, 44 (2018).
[74] D. T. Son and M. A. Stephanov, Phys. Rev. A **65**, 063621 (2002).
[75] K. Kasamatsu, M. Tsubota, and M. Ueda, Phys. Rev. Lett. **93**, 250406 (2004).
[76] M. Eto, K. Kasamatsu, M. Nitta, H. Takeuchi, and M. Tsubota, Phys. Rev. A **83**, 063603 (2011).

[77] M. Eto and M. Nitta, Phys. Rev. A **85**, 053645 (2012).
[78] M. Cipriani and M. Nitta, Phys. Rev. Lett. **111**, 170401 (2013).
[79] M. Tylutki, L. P. Pitaevskii, A. Recati, and S. Stringari, Phys. Rev. A **93**, 043623 (2016).
[80] K. Kasamatsu, M. Eto, and M. Nitta, Phys. Rev. A **93**, 013615 (2016).
[81] B. Mencia Uranga and A. Lamacraft, Phys. Rev. A **97**, 043609 (2018).
[82] M. Eto and M. Nitta, Phys. Rev. A **97**, 023613 (2018).
[83] M. Kobayashi, M. Eto, and M. Nitta, Phys. Rev. Lett. **123**, 075303 (2019).
[84] A. P. Balachandran, S. Digal, and T. Matsuura, Phys. Rev. D **73**, 074009 (2006).
[85] E. Nakano, M. Nitta, and T. Matsuura, Phys. Rev. D **78**, 045002 (2008).
[86] M. Eto, Y. Hirono, M. Nitta, and S. Yasui, Prog. Theor. Exp. Phys. **2014**, 012D01 (2014).
[87] M. Eto and M. Nitta, Phys. Rev. D **104**, 094052 (2021).
[88] Y. Fujimoto and M. Nitta, Phys. Rev. D **103**, 054002 (2021).
[89] G. R. Dvali and G. Senjanovic, Phys. Rev. Lett. **71**, 2376 (1993).
[90] M. Eto, M. Kurachi, and M. Nitta, J. High Energy Phys. 08 (2018) 195.
[91] S. Autti, V. V. Dmitriev, J. T. Mäkinen, A. A. Soldatov, G. E. Volovik, A. N. Yudin, V. V. Zavjalov, and V. B. Eltsov, Phys. Rev. Lett. **117**, 255301 (2016).
[92] J. T. Mäkinen, V. V. Dmitriev, J. Nissinen, J. Rysti, G. E. Volovik, A. N. Yudin, K. Zhang, and V. B. Eltsov, Nat. Commun. **10**, 237 (2019).
[93] S. W. Seo, S. Kang, W. J. Kwon, and Y.-I. Shin, Phys. Rev. Lett. **115**, 015301 (2015).
[94] G. Marmorini, S. Yasui, and M. Nitta, arXiv:2010.09032.
[95] T. Mizushima, S. Yasui, and M. Nitta, Phys. Rev. Research **2**, 013194 (2020).
[96] Y. Makhlin, M. Silaev, and G. E. Volovik, Phys. Rev. B **89**, 174502 (2014).
[97] M. A. Silaev and G. E. Volovik, J. Exp. Theor. Phys. **119**, 1042 (2014).
[98] S. Kobayashi, M. Kobayashi, Y. Kawaguchi, M. Nitta, and M. Ueda, Nucl. Phys. B **856**, 577 (2012).
[99] G. Eilenberger, Z. Phys. **214**, 195 (1968).
[100] J. W. Serene and D. Rainer, Phys. Rep. **101**, 221 (1983).
[101] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.105.L220503 for details of the microscopic formalism and topological protection of the zero mode.
[102] R. Heeb and D. F. Agterberg, Phys. Rev. B **59**, 7076 (1999).
[103] M. Matsumoto and R. Heeb, Phys. Rev. B **65**, 014504 (2001).
[104] A. B. Vorontsov and J. A. Sauls, Phys. Rev. B **68**, 064508 (2003).
[105] E. V. Thuneberg, Phys. Rev. Lett. **56**, 359 (1986).
[106] M. M. Salomaa and G. E. Volovik, Phys. Rev. Lett. **56**, 363 (1986).
[107] T. Kawakami, Y. Tsutsumi, and K. Machida, Phys. Rev. B **79**, 092506 (2009).
[108] T. Kawakami, Y. Tsutsumi, and K. Machida, J. Phys. Soc. Jpn. **79**, 044607 (2010).
[109] T. Kawakami, T. Mizushima, and K. Machida, J. Phys. Soc. Jpn. **80**, 044603 (2011).
[110] T. Mizushima, Y. Tsutsumi, T. Kawakami, M. Sato, M. Ichioka, and K. Machida, J. Phys. Soc. Jpn. **85**, 022001 (2016).
[111] N. Nagamura and R. Ikeda, Phys. Rev. B **98**, 094524 (2018).
[112] M. Tange and R. Ikeda, Phys. Rev. B **101**, 094512 (2020).
[113] R. C. Regan, J. J. Wiman, and J. A. Sauls, Phys. Rev. B **104**, 024513 (2021).
[114] K. Shiozaki and M. Sato, Phys. Rev. B **90**, 165114 (2014).
[115] Y. Tsutsumi, T. Kawakami, K. Shiozaki, M. Sato, and K. Machida, Phys. Rev. B **91**, 144504 (2015).
[116] Y. Ueno, A. Yamakage, Y. Tanaka, and M. Sato, Phys. Rev. Lett. **111**, 087002 (2013).
[117] Y. Tsutsumi, M. Ishikawa, T. Kawakami, T. Mizushima, M. Sato, M. Ichioka, and K. Machida, J. Phys. Soc. Jpn. **82**, 113707 (2013).
[118] J. C. Y. Teo and C. L. Kane, Phys. Rev. B **82**, 115120 (2010).
[119] X.-L. Qi, T. L. Hughes, and S.-C. Zhang, Phys. Rev. B **78**, 195424 (2008).
[120] M. Sato, A. Yamakage, and T. Mizushima, Physica E **55**, 20 (2014).