Letter

Quantum criticality and confinement in weak Mott insulators

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Electrons undergoing a Mott transition may shed their charge but persist as neutral excitations of a quantum spin liquid (QSL). We introduce concrete two-dimensional models exhibiting this exotic behavior as they transition from superconducting or topological phases into fully charge-localized insulators. We study these Mott transitions and the confinement of neutral fermions at a second transition into a symmetry-broken phase. In the process, we also derive coupled-wire parent Hamiltonians for a non-Abelian QSL and a $\mathbb{Z}_4$ QSL.

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Introduction. The Mott transition [1] of spinful fermions is central to numerous compelling phenomena in quantum many-body physics. High-temperature superconductivity in the cuprates, for example, arises near the transition between a nonmagnetic metal and an antiferromagnetic insulator [2]. In addition, "weak Mott insulators," i.e., systems just barely on the insulating side of the quantum phase transition (QPT), provide fertile ground for exotic forms of quantum magnetism, as observed in various organic compounds [3,4]. A small charge gap promotes multispin interactions, stabilizing QSL ground states with fractional "spinon" excitations [5–10].

Many properties of QSLs, such as deconfined spin-$\frac{1}{2}$ excitations, are naturally present in weakly interacting metals or superconductors. There, they are the electronic or Bogoliubov quasiparticles, respectively. In the weak Mott insulator, spinon excitations may fruitfully be viewed as an inheritance of the nearby itinerant phase. Within this framework, QSLs arise when electrons discard their charge but evolve otherwise smoothly across the QPT. Similarly, a singlet Cooper pair relates to a dimer (valence bond) in a spin model. The natural fate of a superconductor undergoing a Mott transition is thus either a valence bond solid (VBS) with frozen dimers or a QSL with fluctuating dimers [11].

We focus on two-dimensional systems of spin-$\frac{1}{2}$ fermions or bosons at an average filling of one particle per unit cell. When such systems undergo a Mott transition, one of two things must happen concomitantly with the localization of unit charge to each site: Either some symmetry breaks spontaneously or a QSL forms. Experimentally, Mott transitions are typically first order. Theoretical studies of these QPTs are challenging due to the lack of any small parameter—the transition occurs when the interaction strength and bandwidth are comparable. Still, field-theoretical analyses have shown that second-order transitions are also possible in both cases [12–16].

A prototypical phase diagram of electrons undergoing a Mott transition is illustrated in Fig. 1 for the example of a superconductor. When both sides of the QPT are conventional phases, it is either first order or exhibits "deconfined" criticality [17]. By contrast, the Mott transition into a QSL may be governed by the critical point of the classical three-dimensional (3D) XY model [18]. Finally, QSLs and topologically trivial Mott insulators are separated by a confinement transition.

To describe the Mott transition, we begin deep within a superconductor or topological insulator. We locally deform its Hamiltonian to write it as a sum of two parts, H^{charge} and H^{spin}, which commute up to irrelevant contributions. The "spin" part does not involve charge transfer between different sites. Consequently, the system remains in its ground state as charge carriers localize due to strong on-site repulsion. The competition between the latter and H^{charge}, projected onto the ground state of H^{spin}, then characterizes the Mott transition. In one dimension, this property is quite generic; Luttinger liquids factorize into charge and spin sectors, and phase transitions in the former do not affect the latter.

To carry out these steps in two dimensions we build on the coupled-wire framework [19]. In particular, the Mott transition and subsequent confinement transitions are accessible in a well-controlled way. Known properties of QPTs out of Abelian and non-Abelian QSLs are reproduced in an almost

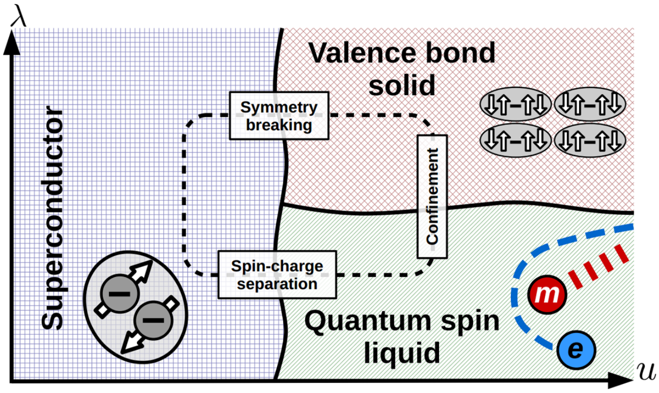

FIG. 1. Phase diagram of a superconductor that undergoes a Mott transition upon increasing the on-site repulsion u. The parameter λ modifies the spin correlations within the superconductor. Depending on its value, the insulating phase may be a valence bond solid or a $\mathbb{Z}_2$ spin liquid. The coupled-wire formalism affords us theoretical control along the dashed line, passing through all phases and phase transitions.

pedestrian manner, without reference to gauge theories [20]. Additionally, we derive the critical properties of two QPTs that were not previously discussed: (i) between an s-wave superconductor and VBS, and (ii) between a bosonic Laughlin state and chiral QSL. Crucially, for each QPT, we construct a physically sensible microscopic model, i.e., one that is local and only involves two-body interactions.

Superconductor and VBS. The conceptually clearest example is the Mott transition out of an s-wave superconductor. To describe it, we begin with an array of one-dimensional quantum wires in the Luttinger liquid phase. At low energies, spin-σ electrons near the right (R) or left (L) Fermi points are annihilated by $\psi_{y,\sigma,R/L}$. The singlet Cooper-pair operator is $\hat{\Delta}_y = \psi_{y,\uparrow,R}\psi_{y,\downarrow,L} - \psi_{y,\downarrow,R}\psi_{y,\uparrow,L}$. We couple neighboring wires via pair hopping, i.e., $H_{\rm SC} = g\hat{\Delta}^\dagger_{y+1}\hat{\Delta}_y + \text{H.c.}$ (Here and throughout, we lighten the notation by leaving the summation over wires and integration along the wire implicit. The wire index y will also be suppressed unless needed.) When $H_{\rm SC}$ is relevant in the renormalization group sense or has a large coefficient, a superconductor with $\langle\hat{\Delta}\rangle \neq 0$ and a hard spin gap arises.

Two-particle intrawire umklapp processes described by $H_{\rm Mott} = u\psi^\dagger_{\uparrow,R}\psi^\dagger_{\downarrow,R}\psi_{\uparrow,L}\psi_{\downarrow,L} + \text{H.c.}$ induce the Mott transition. Microscopically, they arise from density-density interactions between electrons, e.g., on-site repulsion in a Hubbard model. At half filling, repulsion favors localizing the electrons on each wire and thus competes against pair hopping. When the former prevails, it suppresses the latter, but the spin gap may persist. Indeed, at second order in g, $H_{\rm SC}$ generates $H^{\rm spin}_{\rm SC} \sim \hat{\Delta}^\dagger_y\hat{\Delta}_y = \psi^\dagger_{\uparrow,R}\psi_{\uparrow,L}\psi^\dagger_{\downarrow,L}\psi_{\downarrow,R} + \text{H.c.}$ This intrawire interaction opens a spin gap in Luttinger liquids and survives the Mott transition by not transferring charge.

To characterize the insulator and the QPT, we employ Abelian bosonization [21,22]. The charge and spin degrees of freedom are encoded by canonically conjugate long-wavelength operators $\theta^{c/s}_y$ and $\varphi^{c/s}_y$; we use the convention where their densities are $\rho_{c/s} = \frac{1}{\pi}\nabla\theta^{c/s}$. The Cooper-pair operator is then $\hat{\Delta} = e^{i\varphi^c}\cos[2\theta^s]$. The intrawire interactions introduced above are $H_{\rm Mott} \sim \cos[4\theta^c]$ and $H^{\rm spin}_{\rm SC} \sim \cos[4\theta^s]$. According to these two terms alone, each wire breaks translation symmetry independently, and the ground state is macroscopically degenerate. Residual interwire couplings, the leading of which is $\delta H^{\rm spin}_{\rm VBS} = u'\cos[2\theta^c_{y+1} + 2\theta^c_y]\cos[2\theta^s_{y+1} + 2\theta^s_y]$, will lock the order parameters of individual wires into a global symmetry-breaking pattern. The resulting VBS ground state does not exhibit topological order, i.e., fractional excitations are confined.

By construction, $H^{\rm spin}_{\rm SC}$ does not experience competition from $H_{\rm Mott}$, $\delta H^{\rm spin}_{\rm VBS}$, or $H_{\rm SC}$, i.e., the latter is the "charge" Hamiltonian. The transition is thus governed by $H_{\text{SC-VBS}} \equiv \langle H_{\rm Mott} + \delta H^{\rm spin}_{\rm VBS} + H_{\rm SC}\rangle_{H^{\rm spin}_{\rm SC}}$. Here, projection onto the "spin" ground state amounts to replacing all instances of the pinned operators θ^s by c numbers; we find

$$H_{\text{SC-VBS}} = u'\cos\left[2\theta^c_y + 2\theta^c_{y+1}\right] + u\cos\left[4\theta^c_y\right] + \cos\left[\varphi^c_{y+1} - \varphi^c_y\right]. \tag{1}$$

This Hamiltonian also describes the transition between an easy-plane antiferromagnet and a spatially anisotropic VBS [17]. Its ultimate fate is believed to be a first-order transition [23–27]. Explicitly breaking the wire translations permits the lower-order term $\cos[2\theta^c_y]$, placing the transition into the universality class of the 3D XY model.

Superconductor and $\mathbb{Z}_2$ QSL. We now modify the parent superconductor to obtain a Mott transition into a deconfined phase. As we will see, a QSL arises from a phase with the order parameter $\hat{\Delta}'_y = \psi_{y,\uparrow,R}\psi_{y-1,\downarrow,L}$ for odd y and with $R \leftrightarrow L$ for even y. Pairing is induced by Cooper-pair hopping, i.e., $H_{\Delta'} = g_y\hat{\Delta}'^\dagger_{y+1}\hat{\Delta}'_y + \text{H.c.}$ Only half of the low-energy electrons participate in this interaction. We gap out the others with $H_m = \hat{m}_y + \text{H.c.}$, where $\hat{m}_y = \psi^\dagger_{y-1,\uparrow,L}\psi_{y,\uparrow,R}$ for even y and $\hat{m}_y = \psi^\dagger_{y-1,\downarrow,R}\psi_{y,\downarrow,L}$ for odd y [28]. The ground state of $H_{\rm SC'} = H_{\Delta'} + H_m$ features a spin gap and spontaneously breaks charge conservation. It also exhibits accidental edge states that depend on the termination. These do not play an important role here and are addressed below.

Both terms in $H_{\rm SC'}$ involve electrons hopping across wires and are suppressed once charges localize. Still, terms generated from these two interactions may persist across the Mott transition. Consider specifically $H^{\rm spin}_{\rm SC'} \equiv H_{\Delta'}|_{g\to\hat{g}}$ with $\hat{g}_y = \hat{m}_{y+1}\hat{m}_y$. Inside the superconductor, $\langle\hat{g}_y\rangle$ is of order unity, and $H_{\Delta'}$, $H^{\rm spin}_{\rm SC'}$ are interchangeable. Crucially, the interactions in the latter, $e^{i4\tilde{\theta}^s_{2y+1}} \equiv \hat{\Delta}'^\dagger_{2y+2}\hat{\Delta}'_{2y+1}\hat{g}_{2y+1}$ and $e^{i2\tilde{\varphi}^s_{2y}} \equiv \hat{\Delta}'^\dagger_{2y+1}\hat{\Delta}'_{2y}\hat{g}_{2y}$, do not involve charge transfer between wires. Their phases, expressed through operators that satisfy canonical commutations with φ^s_{2y+1} and θ^s_{2y}, thus remain pinned across the Mott transition. All the charge transfer is contained in H_m, which takes on the role of $H^{\rm charge}_{\rm SC'}$.

In the Mott insulator, $H^{\rm spin}_{\rm SC'}$ realizes precisely the $\mathbb{Z}_2$ QSL described in Ref. [29]. The QPT is described by $H_{\text{SC}'\text{-}\mathbb{Z}_2} \equiv \langle H^{\rm charge}_{\rm SC'} + H_{\rm Mott}\rangle_{H^{\rm spin}_{\rm SC'}}$. We find

$$H_{\text{SC}'\text{-}\mathbb{Z}_2} = \cos\left[\tfrac{1}{2}\left(\tilde{\varphi}^c_{y+1} - \tilde{\varphi}^c_y\right)\right] + u\cos\left[4\theta^c_y\right], \tag{2}$$

with dressed charge operators $\tilde{\varphi}^c_y$ that are canonical conjugates to θ^c_y and avoid competition with $H^{\rm spin}_{\rm SC'}$. The same Hamiltonian describes the Mott transition of bosons $e^{i\tilde{\varphi}^c/2}$ at integer filling, which is in the universality class of the 3D XY model. In the present case, there is no local boson with unit charge, and the transition is refined to the XY* type [12]. The slowest fluctuating observable is the Cooper pair with anomalous exponent $\eta \approx 1.47$ [30,31]. It is encoded as $e^{i\tilde{\varphi}^c_y} = \hat{\Delta}'_y\hat{m}_y e^{i2\tilde{\theta}^s_y}$ for odd y and $e^{i\tilde{\varphi}^c_y} = \hat{\Delta}'_y\hat{m}_y e^{i\tilde{\varphi}^s_y}$ for even y.

To complete the analysis of the Mott transition, we trace the evolution of individual electrons into spinons. Consider the operator $\mathcal{O}_{2y'+1,2y} = \psi^\dagger_{2y'+1,\uparrow,R}\psi_{2y,\uparrow,L}$. We obtain $\mathcal{O}^{\rm spin}$ by dressing it with the unique product of $\hat{m}$ that compensates for all interwire charge transfer. Inside the superconductor SC′, the bare and dressed operators are interchangeable. By construction, the latter evolves smoothly across the QPT. Finally, $\langle\mathcal{O}^{\rm spin}\rangle_{H_{\rm Mott}}$ yields the creation operator for a spinon particle-hole pair (see also Supplemental Material [32]) [29].

Confinement transition. The QPT between the $\mathbb{Z}_2$ QSL and the VBS is described by $H_{\mathbb{Z}_2\text{-VBS}} \equiv \langle\lambda H^{\rm spin}_{\rm SC'} + H^{\rm spin}_{\rm SC} +$

$\delta H^{\rm spin}_{\rm VBS}\rangle_{H_{\rm Mott}}$. Notice that $\tilde{\theta}^s$ are pinned in both the VBS and QSL. Replacing them with c numbers, we find

$$H_{\mathbb{Z}_2\text{-VBS}} = u' \cos\left[\theta^s_{2y+2} - \theta^s_{2y}\right] + \lambda \cos\left[2\tilde{\varphi}^s_{2y}\right] + \cos\left[4\theta^s_{2y}\right]. \tag{3}$$

The first two terms are equivalent to Eq. (2). The final term introduces a fourfold anisotropy, which is (dangerously) irrelevant at the 3D XY transition [33]. Breaking wire-translation symmetry explicitly permits a strongly relevant twofold anisotropy. Then, the confinement transition occurs independently of spatial symmetries and is described by a (dual) Ising model.

The models above realize all the phases in Fig. 1. To complete the phase diagram, we show that SC and SC′ are in the same phase. Recall that the latter exhibits accidental edge modes. Specifically, when terminating on an odd wire, the electron modes $\psi_{R,\downarrow}$, $\psi_{L,\uparrow}$ there are decoupled. These modes become gapped when coupled to a nearby region described by $H_{\rm SC}$, suggesting no phase transition occurs. To verify that SC and SC′ are smoothly connected, we use that $\hat{\Delta}$ and $\hat{\Delta}'$ have nonzero expectation values in their respective superconductors. We thus describe SC and SC′ by free fermion models $H^{\rm MF}_{\rm SC} = \hat{\Delta} + {\rm H.c.}$ and $H^{\rm MF}_{\rm SC'} = H_m + (\hat{\Delta}' + {\rm H.c.})$ and find no gap closure when tuning between them (see Supplemental Material [32]).

Topological superconductor and non-Abelian QSL. The simplest topological superconductor is comprised of spinless fermions [34]. To realize this phase, we thus begin by trivially gapping the $\downarrow$ electrons using H_m and $H_\downarrow = \psi^\dagger_{2y+1,\downarrow,R}\psi_{2y,\downarrow,L} + {\rm H.c.}$ The remaining electrons are effectively spinless. A topological superconductor can be obtained from them by diligently constructing interwire interactions such that a chiral Majorana fermion at the boundary remains uncoupled [35]. Alternatively, one may use the fact that a topological superconductor arises upon inducing pairing to Dirac electrons [36]. A single Dirac cone for part of the $\uparrow$ electrons is realized by

$$H_{\uparrow,\rm Dirac} = \psi^\dagger_{4y-2,\uparrow,L}[\psi_{4y+1,\uparrow,R} - \psi_{4y-3,\uparrow,R}] + {\rm H.c.} \tag{4}$$

To induce pairing without explicitly violating charge conservation, we first let the remaining electrons form a trivial (strongly paired) superconductor. Specifically, we take $H_{\uparrow,\rm SP} = \hat{\Delta}^\dagger_{\uparrow,4y+3}\hat{\Delta}_{\uparrow,4y-1} + {\rm H.c.}$, with the Cooper-pair operator $\hat{\Delta}_{\uparrow,y} = \psi_{y+1,\uparrow,L}\psi_{y,\uparrow,R}$. Finally, the "proximity" coupling $H_{\uparrow,\Delta} = g\hat{\Delta}^\dagger_{\uparrow,2y+1}\hat{\Delta}_{\uparrow,2y-1} + {\rm H.c.}$ yields a topological superconductor with a single chiral Majorana fermion at the edge.

All terms in this model involve charge transfer between wires and are suppressed upon undergoing the Mott transition. However, as before, there are vestigial terms. To obtain the "spin" part of all couplings described above, we multiply them by the unique product of $\hat{m}$ operators that compensates for all interwire charge transfer. By construction, the resulting terms do not compete with the opening of the Mott gap.

To identify the insulating phase, we note that $H^{\rm spin}_\downarrow$, $H^{\rm spin}_{\uparrow,\rm SP}$ do not face competition and pin three operators per four-wire unit cell. The final mode plays an entirely different role. To reveal it, we define neutral fermions $f_\chi = e^{i\tilde{\phi}^\chi}$, with

$$\tilde{\phi}^\chi_{4y} \equiv \varphi^s_{4y+1} - \varphi^s_{4y} + \theta^c_{4y} - \theta^c_{4y-1} - \theta^s_{4y} - \theta^s_{4y-1} + 2\begin{cases}\theta^c_{4y+1}, & \chi = R,\\ -\theta^c_{4y+2} - \theta^s_{4y+2} - \theta^s_{4y+1}, & \chi = L.\end{cases} \tag{5}$$

Notice that interwire hopping of these fermions is not a local process. Still, they constitute deconfined excitations on top of the topologically nontrivial background formed by the pinned operators. Their effective Hamiltonian is $H_f \equiv \langle H^{\rm spin}_{\uparrow,\rm Dirac} + H^{\rm spin}_{\uparrow,\Delta}\rangle_{H^{\rm spin}_\downarrow + H^{\rm spin}_{\uparrow,\rm SP}}$. We find

$$H_f = f^\dagger_{4y,L}[f_{4y+4,R} - f_{4y,R}] + g f^\dagger_{4y,R} f^\dagger_{4y,L} + {\rm H.c.}, \tag{6}$$

which describes a neutral version of the electronic model discussed above. In particular, the f fermions form a topological superconductor with a single chiral Majorana fermion at each edge. Consequently, the physical spin system realizes a non-Abelian QSL. The Mott transition is described by Eq. (2) with modified microscopic expressions for $\tilde{\varphi}^c$ [32]. Still, the Cooper-pair operator $\hat{\Delta}_{4y-1}\hat{m}_{4y}$ exhibits critical correlations with anomalous exponent $\eta \approx 1.47$.

Before we conclude this example, we note that a lattice model realizing the electronic band structure, $H_\downarrow + H_m + H_{\uparrow,\rm Dirac}$, is readily constructed by engineering a suitable flux background [32]. Adding the pairing terms described above and an on-site repulsion then results in a lattice model for this non-Abelian QSL.

Quantum Hall insulators and chiral QSLs. We turn to the Mott transition out of topological insulators. Here, the appealing perspective of Cooper pairs evolving into dimers is not applicable. Still, the techniques introduced above apply, and their implementation is even more straightforward. As a prototypical example of quantum Hall insulators, we consider a bilayer Laughlin state of bosons at filling factor $\nu_\sigma = \frac{1}{2}$. A parent Hamiltonian for this phase is the sum of [19,35]

$$H^{\rm charge}_{220} = \cos\left[2\Theta^c_y\right]\cos\left[2\Theta^s_y\right], \quad H^{\rm spin}_{220} = \cos\left[4\Theta^s_y\right], \tag{7}$$

with canonical variables

$$\Phi^{c/s}_y \equiv \tfrac{1}{4}\left(\varphi^{c/s}_y + \varphi^{c/s}_{y-1}\right) + \theta^{c/s}_y - \theta^{c/s}_{y-1}, \tag{8a}$$

$$\Theta^{c/s}_y \equiv \tfrac{1}{4}\left(\varphi^{c/s}_y - \varphi^{c/s}_{y-1}\right) + \theta^{c/s}_y + \theta^{c/s}_{y-1}. \tag{8b}$$

The insulator described by $\langle H^{\rm spin}_{220}\rangle_{H_{\rm Mott}}$ is a variant of the Kalmeyer-Laughlin chiral QSL [37], discussed in wire models by Refs. [38,39].

The nature of the QPT becomes apparent upon introducing fermions $\Psi^c_{R(L)} \equiv e^{i(\Phi^c \pm \Theta^c)}$. We find that $H_{220\text{-CSL}} \equiv \langle H^{\rm charge}_{220} + H_{\rm Mott}\rangle_{H^{\rm spin}_{220}}$ reads

$$H_{220\text{-CSL}} = \Psi^{c,\dagger}_{L,y}\left[u\Psi^c_{R,y+1} - \Psi^c_{R,y}\right] + {\rm H.c.}, \tag{9}$$

which describes a single-Dirac-cone band structure with mass $|1-u|$. Short-range interactions between Dirac fermions are irrelevant in two dimensions. Consequently, the transition is described by a single Dirac cone of free fermions that are spinless but carry unit electric charge. Individual fermions are nonlocal, but Cooper pairs represent pairs of physical bosons with opposite spins. At the QPT, this charge-2e operator exhibits power-law correlations with scaling dimension 2. By contrast, single-particle excitations remain gapped across the transition.

Other topological phases of bosons or fermions with 2×2 K matrices can be analyzed analogously. The resulting Mott insulators are chiral QSLs unless the Hall conductance of the parent topological phase vanishes. For an example of the latter, consider a quantum spin Hall state described by $H_{\rm QSH} = \sum_\sigma \psi^\dagger_{y,\sigma,R}\psi_{y+\sigma,\sigma,L} + {\rm H.c.}$ Upon undergoing a Mott transition, only the correlated process of electrons swapping wires survives. In the insulating phase, where θ^c are replaced by c numbers, we find this term to be $H^{\rm spin}_{\rm QSH} = \cos[\varphi^s_{y+1} - \varphi^s_y]$. It describes an easy-plane antiferromagnet, whose ground state spontaneously breaks U(1) spin-rotation symmetry. The Mott transition is described by Eq. (2) with $\tilde{\theta}^c$, $\tilde{\varphi}^c$ replaced by

$$\Theta_y \equiv \tfrac{1}{8}\big(2\theta^s_{y+1} + 2\theta^s_y - \varphi^c_{y+1} + \varphi^c_y\big), \qquad (10a)$$

$$\Phi_y \equiv \varphi^s_{y+1} + \varphi^s_y - 2\theta^c_{y+1} + 2\theta^c_y, \qquad (10b)$$

respectively, and $u \to u^{-1}$. Therefore, the transition is in the universality class of the 3D XY model, agreeing with previous findings for the Kane-Mele-Hubbard model [40–44]. We note that interchanging the spin and charge modes yields a transition from the quantum spin Hall state to the superconductor of the first example. [Reference [45] studied a related transition for SU(2) symmetric models, which results in a different universality].

Parton approach. An alternative route for describing Mott transitions is based on the parton mean-field approach [46,47]. Specifically, a microscopic electron (boson) ψ_σ is expressed as $\psi_\sigma = c f_\sigma$, where f_σ are fermionic (bosonic) "spinons" and c is a bosonic "chargon." The latter is at unit filling and, within a mean-field treatment, may undergo a conventional boson-Mott transition without changing the phase of the former. Fluctuations of the mean-field parameters take the form of a compact U(1) gauge field that couples to chargons and spinons.

When f_σ form a superconductor, this emergent photon acquires a Higgs mass and does not modify the critical behavior. When f_σ form a chiral phase, the emergent photon is rendered massive by a Chern-Simons term. Here, the transition is modified, as we found in Eq. (9). Mean-field states where f_σ form nonchiral insulators are unstable to monopole proliferation and require a different analysis. By contrast, our approach treats all cases on equal footing and allows us to derive their critical theories. We demonstrated this for VBS and QSH, confining phases within the parton approach.

The superconductor-VBS transition described by Eq. (1) does not permit a straightforward description in terms of c and f_σ. Fortunately, an alternative is suggested by the wire model. This QPT is related to the antiferromagnet-VBS QPT [29] by interchanging spin and charge variables. Consequently, we propose the decomposition $\psi_\uparrow = s h_+$, $\psi_\downarrow = s^\dagger h_-^\dagger$. Specifically, the bosonic spinon s is gapped on either side of the QPT, while the fermionic chargons $h_\pm$ transition from a trivial to a QSH insulator.

Discussion. We adapted the coupled-wire approach for describing weak Mott insulators and the nearby itinerant phases. We used concrete models to understand what conditions favor topologically ordered Mott insulators over trivial VBS phases. In the superconductor denoted by SC, neighboring wires interact only through the fluctuations of $\hat{\Delta}$ around its expectation value, i.e., via the Goldstone mode. In particular, interwire spin correlations are strictly zero. Consequently, an intrawire VBS phase is its natural fate after undergoing a Mott transition. Deforming the superconductor away from this limit into the one denoted by SC′ creates more nontrivial spin correlations, permitting a $\mathbb{Z}_2$ QSL to form.

Beyond the Mott transition itself, the weak-Mott-insulator lens has been conceptually useful for understanding exotic insulators. This perspective becomes practically useful for generating parent Hamiltonians of these phases. First, conventional coupled-wire constructions for spin-chain arrays involve carefully tuned and seemingly unnatural many-spin interactions. By contrast, the ones derived here originate in electron models with local hopping and two-body interactions only. Second, the latters' simplicity may make them more appealing than effective spin models for numerical techniques such as the density matrix renormalization group.

Finally, the coupled-wire framework readily captures intrinsically non-mean-field states and even certain gapless QSLs [48]. Examples of the former are charge-$4e$ superconductors [49,50]. There is no conceptual difference compared to the charge-$2e$ superconducting case, and there are no surprises [32]. The insulating phase is a $\mathbb{Z}_4$ QSL, and the transition is governed by the 3D XY** universality class, i.e., the critical correlations of the charge-$4e$ order parameter are determined by the fourth power of the classical XY order parameter. Extensions to gapless states would be an exciting direction for future studies.

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[1] M. Imada, A. Fujimori, and Y. Tokura, Metal-insulator transitions, Rev. Mod. Phys. **70**, 1039 (1998).

[2] P. A. Lee, N. Nagaosa, and X.-G. Wen, Doping a Mott insulator: Physics of high-temperature superconductivity, Rev. Mod. Phys. **78**, 17 (2006).

[3] B. J. Powell and R. H. McKenzie, Quantum frustration in organic Mott insulators: from spin liquids to unconventional superconductors, Rep. Prog. Phys. **74**, 056501 (2011).

[4] T. Furukawa, K. Miyagawa, H. Taniguchi, R. Kato, and K. Kanoda, Quantum criticality of Mott transition in organic materials, Nat. Phys. **11**, 221 (2015).

[5] L. Balents, Spin liquids in frustrated magnets, Nature (London) **464**, 199 (2010).

[6] L. Savary and L. Balents, Quantum spin liquids: a review, Rep. Prog. Phys. **80**, 016502 (2017).

[7] X.-G. Wen, Colloquium: Zoo of quantum-topological phases of matter, Rev. Mod. Phys. **89**, 041004 (2017).

[8] Y. Zhou, K. Kanoda, and T.-K. Ng, Quantum spin liquid states, Rev. Mod. Phys. **89**, 025003 (2017).

[9] J. Knolle and R. Moessner, A field guide to spin liquids, Annu. Rev. Condens. Matter Phys. **10**, 451 (2019).

[10] C. Broholm, R. J. Cava, S. A. Kivelson, D. G. Nocera, M. R. Norman, and T. Senthil, Quantum spin liquids, Science **367**, eaay0668 (2020).

[11] P. W. Anderson, Resonating valence bonds: A new kind of insulator?, Mater. Res. Bull. **8**, 153 (1973).

[12] T. Senthil and M. P. A. Fisher, Z_2 gauge theory of electron fractionalization in strongly correlated systems, Phys. Rev. B **62**, 7850 (2000).

[13] C. Lannert, M. P. A. Fisher, and T. Senthil, Quantum confinement transition in a d-wave superconductor, Phys. Rev. B **63**, 134510 (2001).

[14] L. Balents, L. Bartosch, A. Burkov, S. Sachdev, and K. Sengupta, Putting competing orders in their place near the Mott transition, Phys. Rev. B **71**, 144508 (2005).

[15] L. Balents, L. Bartosch, A. Burkov, S. Sachdev, and K. Sengupta, Putting competing orders in their place near the Mott transition. II. The doped quantum dimer model, Phys. Rev. B **71**, 144509 (2005).

[16] T. Senthil, Theory of a continuous Mott transition in two dimensions, Phys. Rev. B **78**, 045109 (2008).

[17] T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Deconfined quantum critical points, Science **303**, 1490 (2004).

[18] The universality class is known as XY^*, but the critical theory is the same as for XY [12,30,31].

[19] C. L. Kane, R. Mukhopadhyay, and T. C. Lubensky, Fractional Quantum Hall Effect in an Array of Quantum Wires, Phys. Rev. Lett. **88**, 036401 (2002).

[20] C. Xu and L. Balents, Quantum phase transitions around the staggered valence-bond solid, Phys. Rev. B **84**, 014402 (2011).

[21] T. Giamarchi, *Quantum Physics in One Dimension* (Clarendon Press, Oxford, U.K., 2004).

[22] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, U.K., 2004).

[23] A. B. Kuklov, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer, Deconfined criticality, runaway flow in the two-component scalar electrodynamics and weak first-order superfluid-solid transitions, Ann. Phys. **321**, 1602 (2006).

[24] S. Kragset, E. Smorgrav, J. Hove, F. S. Nogueira, and A. Sudbø, First-Order Phase Transition in Easy-Plane Quantum Antiferromagnets, Phys. Rev. Lett. **97**, 247201 (2006).

[25] G. Chen, J. Gukelberger, S. Trebst, F. Alet, and L. Balents, Coulomb gas transitions in three-dimensional classical dimer models, Phys. Rev. B **80**, 045112 (2009).

[26] J. D'Emidio and R. K. Kaul, First-order superfluid to valence-bond solid phase transitions in easy-plane SU(n) magnets for small n, Phys. Rev. B **93**, 054406 (2016).

[27] J. D'Emidio and R. K. Kaul, New Easy-Plane $\mathbb{CP}^{N-1}$ Fixed Points, Phys. Rev. Lett. **118**, 187202 (2017).

[28] With unbroken x-translation symmetry, such terms require a staggered magnetic field and spin-orbit coupling.

[29] E. Leviatan and D. F. Mross, Unification of parton and coupled-wire approaches to quantum magnetism in two dimensions, Phys. Rev. Research **2**, 043437 (2020).

[30] T. Grover and T. Senthil, Quantum phase transition from an antiferromagnet to a spin liquid in a metal, Phys. Rev. B **81**, 205102 (2010).

[31] S. V. Isakov, R. G. Melko, and M. B. Hastings, Universal signatures of fractionalized quantum critical points, Science **335**, 193 (2012).

[32] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.105.L201110 for explicit bosonized expressions, the adiabatic connection between mean-field superconductors, the charge-$4e$ superconductor-$\mathbb{Z}_4$ QSL transition, and an electronic lattice model for the non-Abelian QSL.

[33] J. V. José, L. P. Kadanoff, S. Kirkpatrick, and D. R. Nelson, Renormalization, vortices, and symmetry-breaking perturbations in the two-dimensional planar model, Phys. Rev. B **16**, 1217 (1977).

[34] N. Read and D. Green, Paired states of fermions in two dimensions with breaking of parity and time-reversal symmetries and the fractional quantum Hall effect, Phys. Rev. B **61**, 10267 (2000).

[35] J. C. Y. Teo and C. L. Kane, From Luttinger liquid to non-Abelian quantum Hall states, Phys. Rev. B **89**, 085101 (2014).

[36] L. Fu and C. L. Kane, Superconducting Proximity Effect and Majorana Fermions at the Surface of a Topological Insulator, Phys. Rev. Lett. **100**, 096407 (2008).

[37] V. Kalmeyer and R. B. Laughlin, Theory of the spin liquid state of the Heisenberg antiferromagnet, Phys. Rev. B **39**, 11879 (1989).

[38] G. Gorohovsky, R. G. Pereira, and E. Sela, Chiral spin liquids in arrays of spin chains, Phys. Rev. B **91**, 245139 (2015).

[39] T. Meng, T. Neupert, M. Greiter, and R. Thomale, Coupled-wire construction of chiral spin liquids, Phys. Rev. B **91**, 241106(R) (2015).

[40] S. Rachel and K. Le Hur, Topological insulators and Mott physics from the Hubbard interaction, Phys. Rev. B **82**, 075106 (2010).

[41] D.-H. Lee, Effects of Interaction on Quantum Spin Hall Insulators, Phys. Rev. Lett. **107**, 166806 (2011).

[42] D. Zheng, G.-M. Zhang, and C. Wu, Particle-hole symmetry and interaction effects in the Kane-Mele-Hubbard model, Phys. Rev. B **84**, 205121 (2011).

[43] C. Griset and C. Xu, Phase diagram of the Kane-Mele-Hubbard model, Phys. Rev. B **85**, 045123 (2012).

[44] M. Hohenadler, T. C. Lang, and F. F. Assaad, Correlation Effects in Quantum Spin-Hall Insulators: A Quantum Monte Carlo Study, Phys. Rev. Lett. **106**, 100403 (2011).

[45] T. Grover and T. Senthil, Topological Spin Hall States, Charged Skyrmions, and Superconductivity in Two Dimensions, Phys. Rev. Lett. **100**, 156804 (2008).

[46] G. Kotliar and A. E. Ruckenstein, New Functional Integral Approach to Strongly Correlated Fermi Systems: The Gutzwiller Approximation as a Saddle Point, Phys. Rev. Lett. **57**, 1362 (1986).

[47] S. Florens and A. Georges, Quantum impurity solvers using a slave rotor representation, Phys. Rev. B **66**, 165111 (2002).

[48] R. G. Pereira and S. Bieri, Gapless chiral spin liquid from coupled chains on the kagome lattice, SciPost Phys. **4**, 004 (2018).

[49] S. A. Kivelson, V. J. Emery, and H. Q. Lin, Doped antiferromagnets in the weak-hopping limit, Phys. Rev. B **42**, 6523 (1990).

[50] C. Wu, Competing Orders in One-Dimensional Spin-3/2 Fermionic Systems, Phys. Rev. Lett. **95**, 266404 (2005).