Letter

Stabilizing volume-law entangled states of fermions and qubits using local dissipation

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(Received 6 August 2021; revised 24 November 2021; accepted 11 April 2022; published 26 April 2022)

We analyze a general method for the dissipative preparation and stabilization of volume-law entangled states of fermionic and qubit lattice systems in one dimension (and higher dimensions for fermions). Our approach requires minimal resources: nearest-neighbor Hamiltonian interactions that obey a suitable chiral symmetry, and the realization of just a single, spatially localized dissipative pairing interaction. In the case of a qubit array, the dissipative model we study maps to an interacting fermionic problem. Nonetheless, we analytically show the existence of a unique pure entangled steady state (a so-called rainbow state). Our ideas are compatible with a number of experimental platforms, including superconducting circuits and trapped ions.

DOI: 10.1103/PhysRevB.105.L140301

Introduction. Quantum reservoir engineering is a powerful tool in quantum information processing. In its simplest form, it involves tailoring dissipative processes to stabilize nonclassical quantum states [1,2]; when generalized to stabilizing a subspace, it can also be used as a route to quantum error correction [3,4]. Many experiments have implemented dissipation engineering in few-body quantum systems comprised of 1–2 qubits or bosonic modes (see, e.g., [5–9]). Theoretical work has also considered extensions to truly many-body systems [10–12], though most proposals are experimentally daunting, as they require engineered dissipation on every site of an extended lattice system. More recent work demonstrated that for noninteracting bosons hopping on a one-dimensional (1D) lattice, a single, local engineered squeezing dissipator can be sufficient to stabilize the entire extended system in a state with long-range entanglement [13,14]; a subtle particle-hole symmetry was shown to be the key ingredient, allowing a generalization to higher dimensions [15]. These protocols are, however, limited to stabilizing Gaussian entangled states, whose use in quantum information is highly constrained [16].

Given this prior work, a natural question is whether a single localized dissipative process can prepare and stabilize more complex many-body entangled states. In particular, can this approach work in systems which have (unlike free bosons) a finite-dimensional local Hilbert space, e.g., lattices of fermions, hard-core bosons or qubits. In this Letter, we show that the answer is, surprisingly, yes. We describe an extremely simple protocol exploiting symmetry and the dissipative analog of Cooper pairing to stabilize highly entangled states in 1D lattices of fermions and qubits, one example being the so-called "rainbow state" (Fig. 1). Such rainbow states feature long-range, volume-law entanglement, and are known to be the ground states of highly structured, spatially nonuniform Hamiltonians [17,18]. Our dissipative approach does not require the realization of such exotic Hamiltonians. Instead, it only uses nearest-neighbor Hamiltonian interactions (which need not be uniform or symmetric) and a *single* localized dissipator; the entangled steady state is the unique steady state irrespective of the size of the lattice. As we discuss, the resources required to implement our protocol already exist in a number of different quantum information processing architectures.

Our results also have interest in the context of general studies of many-body driven dissipative system. The spin version of our problem *cannot be mapped exactly to free fermions*. Nonetheless, we are able to exactly describe the steady state. We discuss how qualitative features of the dynamics can be connected to a model of dissipative fermionic pairing with phase fluctuations. Note that our work is distinct from a recent proposal for using dissipation to generate entangled states in 1D qubit chains [19,20]. These protocols also generated rainbow-like entangled states, but only if the system was initially prepared in a nontrivial, highly nonlocal entangled steady state. In contrast, our approach has, in general, a *unique*

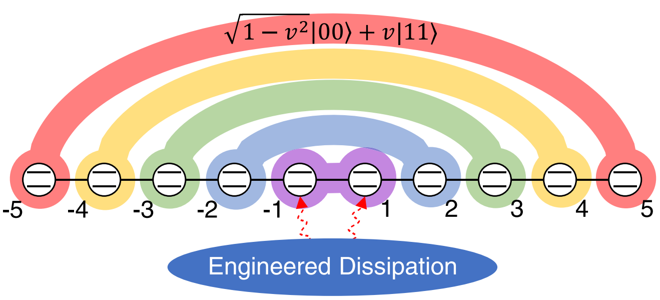

FIG. 1. Schematic of a 1D qubit array with nearest-neighbor XY interactions, where the two central sites are coupled to a common engineered dissipative reservoir with a pairing parameter v [c.f. Eq. (2)]. The dissipative dynamics stabilizes an arbitrary initial state of the qubits into a volume-law entangled rainbow state, where each qubit is entangled with its mirror image qubit (as depicted). The model maps to an interacting fermionic model featuring dissipative pairing with phase fluctuations.

entangled steady state, and hence is completely independent of the initial state: one can start from a trivial product state and still obtain the volume-law entangled rainbow state. Alternate schemes that unconditionally stabilize qubit rainbow states have also been proposed [21,22]. Our scheme is simpler to implement, and is also far more general: it can stabilize a wide class of entangled pure qubit states (many having a correlation structure considerably more complex than a rainbow).

Fermions. We begin by considering noninteracting fermions, using this system to build up the key ideas that will enable our qubit protocol. We consider spinless fermions hopping on a $2N$-site lattice with a tight binding Hamiltonian $\hat{\mathcal{H}}_F = \sum_{i,j} H_{ij}\hat{c}_i^\dagger \hat{c}_j$, where H_{ij} is a Hermitian matrix, and $\hat{c}_i$ annihilates a fermion at lattice site i. $\hat{\mathcal{H}}_F$ is readily diagonalized, with $\hat{d}_\alpha^\dagger = \sum_j \psi_\alpha[j]\hat{c}_j^\dagger$ creating a particle in an energy eigenstate with energy ϵ_α and real-space wave function $\psi_\alpha[j]$. Note that we do not assume translational invariance.

Our goal is to now introduce localized dissipation which stabilizes the entire lattice in a finite-density state with long-range entanglement. For noninteracting bosons, this can be accomplished by coupling a single site to a squeezed Markovian reservoir [13–15]. Such a reservoir attempts to enforce local pairing correlations on the coupled site. For spinless fermions, the Pauli exclusion principle excludes an identical approach. However, one can try the next simplest configuration: introduce a localized Markovian dissipative reservoir that attempts to stabilize fermionic pairing correlations on *two* adjacent sites $j = \bar{0}, \bar{1}$ [i.e., prepare them in the state $(u + ve^{i\phi}\hat{c}_{\bar{0}}^\dagger \hat{c}_{\bar{1}}^\dagger)|00\rangle$]. This corresponds to simply cooling a pair of localized fermionic Bogoliubov modes. As we will see, simply cooling one of these modes generally suffices.

The total system dynamics including the localized dissipative pairing is then described by a Lindblad master equation:

$$\dot{\hat{\rho}} = -i[\hat{\mathcal{H}}, \hat{\rho}] + \Gamma\mathcal{D}[\hat{\beta}_L]\hat{\rho}, \tag{1}$$

Here, $\mathcal{D}[\hat{L}]\hat{\rho} = \hat{L}\hat{\rho}\hat{L}^\dagger - \frac{1}{2}\{\hat{L}^\dagger\hat{L}, \hat{\rho}\}$. For our fermion problem, we have $\hat{\mathcal{H}} = \hat{\mathcal{H}}_F$ and

$$\hat{\beta}_L = u\hat{c}_{\bar{0}} - ve^{i\phi}\hat{c}_{\bar{1}}^\dagger, \tag{2}$$

where $u = \sqrt{1 - v^2}$, with the pairing parameter v real and satisfying $0 \leqslant v \leqslant 1$, and Γ parametrizes the strength of the dissipation, and corresponds to the cooling rate of the localized Bogoliubov mode $\hat{\beta}_L$. The dissipation in Eq. (1) induces an effective non-Hermitian Hamiltonian which includes pairing terms of the form $(iuve^{i\phi}c_{\bar{1}}^\dagger c_{\bar{0}}^\dagger - \text{H.c.})$. Our system thus has the form of an unusual dissipative impurity problem, where the "impurity" corresponds to the local dissipative pairing terms. At a heuristic level, the dissipation injects Cooper pairs on these sites, which can then propagate outward in the lattice. Generically, Eq. (1) will lead to an impure steady state, with fluxes of Cooper pairs both into and out of the lattice. We note that quadratic fermionic models with dissipative pairing have been studied previously in the context of cold atoms [12,23–25], but unlike our work, these assumed pairing on every lattice site.

We next show that if the lattice Hamiltonian obeys a ubiquitous kind of generalized chiral symmetry, then we can ensure the existence of a unique, pure, entangled steady state (see Supplemental Material [26]). This includes, but is certainly not limited to, lattices that can be divided into two equal sized sublattices, denoted A and B, such that $\hat{\mathcal{H}}_F$ only permits hopping from $A \leftrightarrow B$. This structure is found in many lattice systems, including all nearest-neighbor hopping models on square lattices. Observe that this implies that $\hat{\mathcal{H}}_F$ has a chiral symmetry, since the symmetry that sends $\hat{c}_i \to -\hat{c}_i$ if $i \in A$ and $\hat{c}_i \to \hat{c}_i$ if $i \in B$ sends $\hat{\mathcal{H}}_F \to -\hat{\mathcal{H}}_F$. This guarantees we can diagonalize the Hamiltonian such that $\hat{\mathcal{H}}_F = \sum_{\alpha>0} \epsilon_\alpha(\hat{d}_\alpha^\dagger\hat{d}_\alpha - \hat{d}_{-\alpha}^\dagger\hat{d}_{-\alpha})$, where $\hat{d}_{\pm\alpha}^\dagger$ creates an eigenmode with energy $\pm\epsilon_\alpha$.

We can use this to rewrite the jump operator as a sum over nonlocal Bogoliubov modes, which pair positive and negative energy eigenmodes whenever the sites $\bar{0}, \bar{1}$ live on different sublattices [26]:

$$\hat{\beta}_L = \sum_\alpha N_\alpha(\hat{\beta}_\alpha + \hat{\beta}_{-\alpha}), \tag{3}$$

where

$$\hat{\beta}_\alpha = u_\alpha\hat{d}_\alpha - v_\alpha\hat{d}_{-\alpha}^\dagger, \quad \hat{\beta}_{-\alpha} = u_\alpha\hat{d}_{-\alpha} + v_\alpha\hat{d}_\alpha^\dagger. \tag{4}$$

These are an independent set of fermionic annihilation operators obeying canonical anticommutation relations. The constants u_α and v_α encode information about the overlap of the eigenmodes with the dissipation sites $\bar{0}, \bar{1}$ [26]:

$$u_\alpha = \frac{u\psi_\alpha[\bar{0}]}{N_\alpha}, \quad v_\alpha = \frac{ve^{i\phi}\psi_{-\alpha}^*[\bar{1}]}{N_\alpha}, \tag{5}$$

$$N_\alpha = \sqrt{u^2|\psi_\alpha[\bar{0}]|^2 + v^2|\psi_{-\alpha}^*[\bar{1}]|^2}, \tag{6}$$

where N_α fixes normalization. For certain finely tuned parameters, it is possible that some $N_\alpha = 0$, in which case those eigenmodes have no overlap with the dissipation sites, and thus are not cooled. However, for a generic Hamiltonian, $N_\alpha \neq 0$, and so the jump operator is a linear combination of all $2N$ of the Bogoliubov modes; the steady state is *uniquely* their joint vacuum. This state is pure, and has entanglement that grows linearly with system size. We can express the steady state in terms of the eigenmodes as $|\psi\rangle = \prod_{\alpha>0}(u_\alpha - v_\alpha\hat{d}_{-\alpha}^\dagger\hat{d}_\alpha^\dagger)|0\rangle$. The correlators in the steady state are

$$\langle\hat{d}_\alpha^\dagger\hat{d}_\beta\rangle = |v_\alpha|^2\delta_{\alpha\beta}, \quad \langle\hat{d}_\alpha\hat{d}_\beta\rangle = -u_\alpha v_\beta\, \text{sgn}(\alpha)\delta_{\alpha,-\beta}. \tag{7}$$

In real space, the entanglement structure of our dissipatively stabilized steady state is only between the two sublattices A and B, and the amount of entanglement between the two sublattices grows linearly with N [26]. However, despite only being between the sublattices, the spatial pattern can be quite complicated. Given a generic Hamiltonian, any given lattice site will be correlated with the entire sublattice it does not reside on. There are many systems that possess the chiral symmetry required for our scheme. A particularly simple example (that, surprisingly, will generalize to the case of spins) is a 1D lattice with $2N$ sites described by a Hamiltonian with nearest neighbor hopping that possesses an inversion symmetry about its midpoint. Since $\hat{\mathcal{H}}_F$ is symmetric, if the sites $\bar{0}, \bar{1}$ are mapped to each other by the mirror symmetry, then $u_\alpha = u$, $v_\alpha = v\ \forall\alpha$. The correlators defined in Eq. (7) are then also particularly simple in real space, giving the unique steady state $|\psi\rangle = \prod_{i=1}^N(u - v(-1)^i\hat{c}_{-i}^\dagger\hat{c}_i^\dagger)|0\rangle$.

This state exhibits volume-law entanglement (see Fig. 1), and is known as a rainbow state [17,18]. As discussed in [26], many more examples are possible, including the 1D Su-Schrieffer-Heeger model [27,28] and the two-dimensional (2D) Hofstadter model [29].

Qubits. Using the above fermionic setup as inspiration, we now ask whether a similar dissipative preparation scheme is possible for an array of coupled spins or qubits. While any quadratic $A \leftrightarrow B$ hopping Hamiltonian worked for the fermions, it turns out the qubits require the slightly stricter condition of a 1D nearest-neighbor hopping chain. In this case, the sublattices are comprised of every-other lattice sites. Further, while the fermions would take any placement choice of $\overline{0}$, $\overline{1}$ so long as they were on different sublattices, the qubits require they be neighboring [26].

This still leaves all 1D nearest neighbor hopping models, including disordered systems. For simplicity, we will focus below on the case of a lattice with mirror symmetry. It was in this case that the fermions had simplified spatial correlations, giving rise to the rainbow structure. The master equation is then given by Eq. (1) with $\hat{\mathcal{H}} = \hat{\mathcal{H}}_S$ and

$$\hat{\mathcal{H}}_S = -\left[\sum_{i=-N}^{-2} J_i\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \sum_{i=1}^{N-1} J_i\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + J_{-1}\hat{\sigma}_{-1}^+\hat{\sigma}_1^-\right] + \text{H.c.}, \tag{8}$$

$$\hat{\beta}_L = u\hat{\sigma}_{\overline{0}}^- + v\hat{\sigma}_{\overline{1}}^+. \tag{9}$$

Here, $\hat{\sigma}_i^+$ ($\hat{\sigma}_i^-$) is the Pauli raising (lowering) operator on site i. Our lattice has $2N$ sites labeled $(-N, \dots, -1, 1, \dots N)$, i.e., there is no zeroth lattice site. We will constrain the hoppings to obey $J_i = J_{-i-1}$, so $\hat{\mathcal{H}}$ has mirror symmetry. The dissipation-coupled sites $\overline{0}$ and $\overline{1}$ will then be the middle two sites of the lattice, i.e., $\overline{0} = -1$, $\overline{1} = 1$. While the dissipator here may seem exotic, we show below how they can be realized in a number of platforms using existing experimental tools.

The above spin model can be readily mapped to fermions using the Jordan-Wigner (JW) transformation [30]. However, it necessarily maps to an *interacting* fermionic model [in contrast to the quadratic system considered in Eq. (2)]. The most convenient mapping to JW fermions $\hat{c}_j$ is given by the transformation

$$\hat{c}_i = \begin{cases} \left(\prod_{j=1}^{i} \hat{\sigma}_j^z\right)\hat{\sigma}_i^- & 1 \leqslant i \leqslant N, \\ \left(\prod_{j=1}^{N} \hat{\sigma}_j^z\right)\left(\prod_{j=-N}^{i} \hat{\sigma}_j^z\right)\hat{\sigma}_i^- & -N \leqslant i \leqslant -1. \end{cases} \tag{10}$$

This corresponds to using site 1 as the reference for the string operators. Letting $\hat{N}_{\text{tot}}$ be the total fermion number operator, our model can be expressed in terms of these fermionic degrees of freedom as

$$\hat{\mathcal{H}}_S = \sum_{i \neq N,-1} J_i\hat{c}_i^\dagger\hat{c}_{i+1} + J_{-1}(-1)^{\hat{N}_{\text{tot}}}\hat{c}_1^\dagger\hat{c}_{-1} + \text{H.c.}, \tag{11}$$

$$\hat{\beta}_L = u\hat{c}_{-1}(-1)^{\hat{N}_{\text{tot}}} - v\hat{c}_1^\dagger. \tag{12}$$

We see that the presence of the phase operator $(-1)^{\hat{N}_{\text{tot}}}$ in both $\hat{\mathcal{H}}_S$ and the dissipative terms ruins a mapping to free fermions. On a heuristic level, we can interpret this as a modification of Eq. (2) that now describes fluctuations in the phases of the Cooper pairs injected into the system by the reservoir. For the simple fermionic system described by Eq. (2), pairs are always injected with a fixed phase ϕ; in contrast, in Eq. (11), they are injected with a phase ± 1 that depends on the system's parity. We stress that even with other gauge conventions for the JW transformation, it is not possible to eliminate these phase fluctuations (i.e., string operators) from the dissipator.

To better understand our system, we can rewrite the nonlinear dissipation operators in terms of a fixed basis of local Bogoliubov operators $\hat{\tilde{\beta}}_A = u\hat{c}_{-1} - v\hat{c}_1^\dagger$, $\hat{\tilde{\beta}}_B = u\hat{c}_1 + v\hat{c}_{-1}^\dagger$. As was noted in the preceding section, we can write $\hat{\tilde{\beta}}_A$ as a sum over the modes defined in Eq. (4). Further, in the special case of a mirror symmetric Hamiltonian $\hat{\tilde{\beta}}_B$ will also be a sum over these modes, [26]. Defining $\hat{P}_{\text{ev}} = (1 + (-1)^{\hat{N}_{\text{tot}}})/2$ as the projection operator onto even number-parity states, we have

$$\hat{\beta}_L = \hat{\tilde{\beta}}_A\hat{P}_{\text{ev}} - [(u^2 - v^2)\hat{\tilde{\beta}}_A + 2uv\hat{\tilde{\beta}}_B^\dagger](1 - \hat{P}_{\text{ev}}). \tag{13}$$

This provides a simple way to understand the phase fluctuation physics: it is as though the dissipation has a parity-dependent temperature. For even-parity states, the dissipation can only remove Bogoliubov excitations, i.e., it acts like an effective zero temperature bath. In contrast, for odd-parity states and $v \neq 0$, we see that there are amplitudes for the dissipation to either create or destroy excitations (like an effective bath at a non-zero temperature).

Equation (13) also leads to an important conclusion: despite the additional nonlinearity and phase fluctuations in our spin model, the steady state of our simple free fermion model in Eq. (2) is also a steady state of the spin model. This steady state (which here is a rainbow state, given the mirror symmetry of $\hat{\mathcal{H}}$) has a definite even number parity, and hence the Liouvillian acting on this state is identical to the free fermion Liouvillian. At a heuristic level, this state has a definite number parity, and hence phase fluctuations are irrelevant. Returning to our original qubit degrees of freedom, the pure steady state takes the form

$$|\psi_{\text{ss}}\rangle = \prod_{i=1}^{N}(u + (-1)^i v\hat{\sigma}_i^+\hat{\sigma}_{-i}^+)|0\rangle. \tag{14}$$

Equation (13) also lets us show that as long as $v^2 \neq 1/2$, this steady state is unique [26]. Since the Hamiltonian conserves Bogoliubov excitations, moving between manifolds with different Bogoliubov number can only happen dissipatively. The state with no Bogoliubov excitations has even parity, so it can only be cooled by Eq. (13); however, there is no lower number state, and so it is dark to the dissipation. On the other hand, every higher excitation state can be cooled, and so eventually all of the population flows to the Bogoliubov vacuum. Note, this argument also holds for non-rainbow symmetric systems [26]. Thus, we have a central result of this Letter: any initial state of our qubit array (irrespective of its purity or entanglement) will relax into this volume-law entangled pure state. Further, this result holds independently of the magnitude of the hopping parameters J_j/Γ, and even in the presence of additional Hamiltonian terms that preserve the mirror symmetry of $\hat{\mathcal{H}}_S$ [26].

We can now also understand the additional constraints put on the qubits to maintain the purity of the steady state. The fact

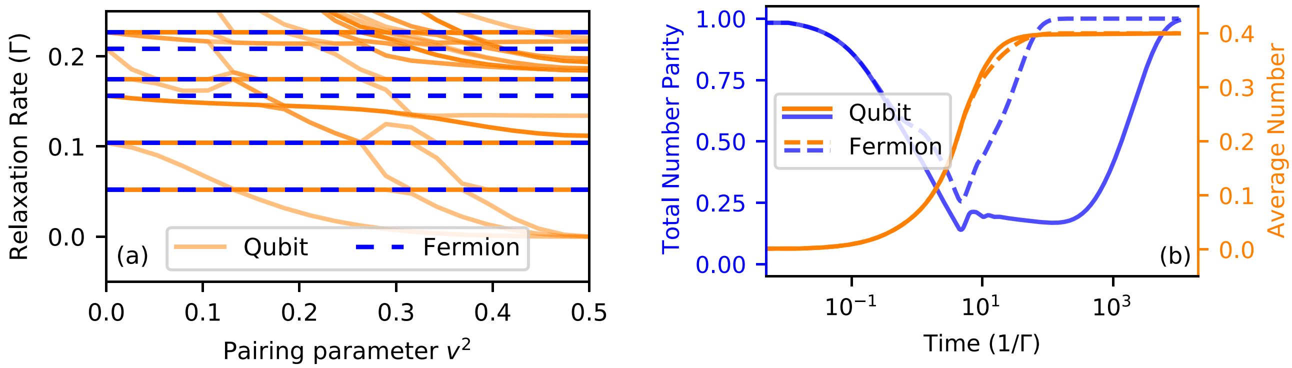

FIG. 2. (a) Low-lying dissipative spectrum of our master equation in Eq. (1) (i.e., real part of Liouvillian eigenvalues) for a six site 1D lattice with $\Gamma = J_j$, plotted as a function of v^2. We take the simple case where the Hamiltonians $\hat{\mathcal{H}}_F, \hat{\mathcal{H}}_S$ have a mirror symmetry. Both the qubit model (orange) and the free-fermion model (blue) are shown. While the spectra coincide for the trivial $v = 0$ case, the fermionic spectrum is independent of v, whereas for qubits, slow modes and complex level structure arise as v is increased. (b) Time evolution of both the average excitation number (orange) and total number parity $(-1)^{\hat{N}}$ (blue) for an eight-site lattice, $v^2 = 0.4$, starting from the vacuum state; other parameters same as (a). The qubit model exhibits an extremely slow relaxation of total number parity, a direct reflection of the pairing phase fluctuations that emerge in its fermionic representation.

that the chain must be 1D nearest neighbor hopping is a result of the nonlocal nature of the Jordan-Wigner transformation. In the same vein, requiring the dissipation coupled sites to be neighboring tells us the only string operator that appears in the master equation is $(-1)^{\hat{N}_{\rm tot}}$. Despite these constraints, we can see that our model is still incredibly robust to disorder. The steady state will still be pure, unique and entangled regardless of the different hopping amplitudes between lattice sites. See [26] for more details.

Dynamics and multistability. While the qubit and free-fermion dissipative arrays share the same pure steady state, the models have strikingly different dynamics. This is a direct consequence of the form of the dissipator given in Eq. (13). The phase fluctuations in the fermionic representation of the qubit model lead to slower overall relaxation, due to the effective nonzero temperature and excitation-creation associated with odd-parity states. As can be seen from Eq. (13), this odd parity heating increases as v is increased from 0, with an amplitude $\propto v\sqrt{1-v^2}$. For free fermions, there is no heating: the dissipative dynamics always corresponds to removing excitations, irrespective of the system state.

Shown in Fig. 2(a) is the numerically calculated dissipative spectrum of the Liouvillians for the free fermion and qubit versions of our model as a function of the pairing parameter v^2. For free fermions, the relaxation rates are independent of v^2; one can show that for large N, the slowest relaxation rate (dissipative gap) scales as $1/N^3$ [26]. In stark contrast, the relaxation rates in the qubit model depend on v^2, with the emergence of an extremely small dissipative gap as $v^2 \to 1/2$. Figure 2(b) demonstrates that this emergent slow timescale manifests itself directly in observable quantities. We see that for both the qubit and free-fermion models, the average particle numbers relax on a similar timescale $\sim 1/\Gamma$. The average parity relaxes on the same timescale for fermions, but for the qubit model, exhibits exponentially slower relaxation. This is a direct manifestation of the effective phase fluctuations encoded in Eq. (11).

The case $u^2 = v^2 = 1/2$ is also of special interest. Eq. (13) indicates that in this case, the dissipation can only remove excitations from even parity states, and can only add excitations to odd parity states. This immediately leads to multistability, as if the system starts in a state with $2m$ Bogoliubov excitations, it will forever be stuck in a manifold of states having either $2m$ or $2m - 1$ excitations. This immediately leads to at least $N + 1$ steady states (see [26] for more details). We stress that there is no multistability in the free-fermion model.

Experimental Implementation. The basic qubit master equation in Eqs. (8) and (9) could be realized in a variety of platforms. Linear arrays of tunnel-coupled qubits have been realized in many systems, including trapped ions [31–33] and superconducting qubits [34–36]. The required dissipation on sites $j = -1, 1$ could be achieved by driving these qubits with two-mode squeezed light via transmission lines or waveguides [37]. In this case, Γ would represent the waveguide coupling rates, and $v/u = \tanh r$ with r the squeezing parameter. While such a scheme could be realized by driving superconducting qubits with two-mode squeezed microwave radiation generated by a Josephson parametric amplifier [38,39], implementation routes that do not require nonclassical microwaves or light are also possible. The required dissipator can be realized by coupling qubits $j = -1, 1$ to a common dissipative bosonic mode (e.g., a lossy microwave cavity), and then either modulating the qubit frequencies, or modulating the qubit-resonator couplings (as was recently achieved [40]). By interfering, e.g., a red sideband process on one qubit with a blue sideband process on the second qubit, the required dissipator can be achieved (with u, v being determined by the modulation amplitudes). Interfering red and blue sideband processes has been used previously in both trapped ion [8] and superconducting qubit [41] experiments for reservoir engineering bosonic modes, but not to control qubit dissipation in the way we suggest. More details on this modulation approach, and on the resilience of our scheme to disorder and unwanted dissipation (i.e., qubit dephasing and relaxation) are presented in [26].

Conclusions. We have demonstrated that the combination of spatially localized pairing dissipation with symmetry constrained Hamiltonian dynamics can be used generically to stabilize entangled states in systems with locally constrained Hilbert spaces. These states can exhibit long range, volume-law entanglement. In the case of a qubit array, our setup corresponds to a dissipative spin chain that is equivalent to an interacting fermionic model, which can be interpreted in terms of dissipative Cooper pairing with phase fluctuations. Our ideas are compatible with a number of different experimental platforms, and could provide an important resource for a variety of quantum information processing protocols.

Acknowledgments. This work is supported by the Air Force Office of Scientific Research MURI program under Grant No. FA9550-19-1-0399.

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