Letter

Emergence of many-body quantum chaos via spontaneous breaking of unitarity

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It is suggested that many-body quantum chaos appears as the spontaneous symmetry breaking of unitarity in interacting quantum many-body systems. It has been shown that many-body level statistics, probed by the spectral form factor (SFF) defined as $K(\eta, t) = \langle|\mathrm{Tr}\exp(-\eta H + itH)|^2\rangle$, is dominated by a diffuson-type mode in a field theory analysis. The key finding of this Letter is that the "unitary" $\eta = 0$ case is different from the $\eta \to 0^{\pm}$ limit, with the latter leading to a finite mass of these modes due to interactions. This mass suppresses a rapid exponential ramp in the SFF, which is responsible for the fast emergence of Poisson statistics in the noninteracting case, and gives rise to a nontrivial random matrix structure of many-body levels. The interaction-induced mass in the SFF shares similarities with the dephasing rate in the theory of weak localization and the Lyapunov exponent of the out-of-time-ordered correlators.

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Both our everyday experience and laboratory experiments indicate that physical systems, initially prepared in a nonequilibrium state, time-evolve into thermal equilibrium. Sets of axioms, such as the eigenstate thermalization hypothesis (ETH) [1–5], have been formulated to justify this generic behavior and the emergence of statistical mechanics. Yet, there is no formal proof of ETH, nor a clear understanding of the fundamental principles underlying the universality of thermalization and ergodicity in a variety of quantum many-body systems. Perhaps the most perplexing is the disconnect between the reversibility of physical laws, governing the unitary evolution of closed quantum systems, and the irreversibility of thermodynamic behavior, which they eventually exhibit. A well-known example of this puzzle is the black hole information paradox [6], but the question itself can be posed for a much wider class of quantum systems [7,8]. How does irreversible dynamics emerge in quantum systems?

Closely related to this line of inquiry is research on many-body quantum chaos, which has attracted much interest recently [9–41]. It can be defined as the presence of Wigner-Dyson level statistics [42–45] of many-body energy levels in an interacting quantum system [46–48]. Chaoticity so defined, ETH, and the thermal behavior in nonequilibrium settings are often assumed to be nearly equivalent notions, although no such equivalence has been proven. Furthermore, no generic derivation of many-body quantum chaos exists, and only a handful of rather fine-tuned models allow a microscopic insight into the fine structure of many-body levels [9–15]. However, given the ubiquity of thermal, "chaotic" behavior, it is natural to ask whether there are generic underlying reasons for its emergence.

This Letter suggests that the spontaneous breaking of unitarity may play a role in the emergence of chaotic behavior of interacting many-body systems. Specifically, we study the distribution of many-body energy levels in the system of N weakly interacting particles, which populate Wigner-Dyson distributed single-particle levels. The choice of the model of interacting fermions "embedded" in a single-particle chaotic background is motivated by the following considerations: First, most actual physical media (e.g., disordered metals) are chaotic from the single-particle perspective (see Ref. [47] for a review). Integrable environments require fine tuning and are not representative of typical physical systems. Second, single-particle quantum chaos gives rise to a universal long-wavelength description (e.g., diffusion in disordered metals) in contrast to a nonuniversal description of systems without intrinsic randomness where ultraviolet physics (e.g., ballistic motion in clean conductors) complicates matters. Third, we notice that in a previous analytical derivation of the many-body level statistics of a Floquet-driven spin chain [10], the many-body random matrix structure emerged after an ensemble averaging. Indeed, there had been arguments that level statistics is not self-averaging and hence introducing an ensemble averaging (or imposing bare single-particle quantum chaos) may be a necessary step to see many-body quantum chaos [49].

One quantity that carries useful statistical information about the spectrum is the two-level correlation function, $R_2(E - E')$. Its Fourier transform can be shown to give rise to the spectral form factor (SFF) [47,48,50–52] $K(t) = \langle \mathrm{Tr} e^{-iHt} \mathrm{Tr} e^{+iHt}\rangle$. This relates the statistical properties of the static spectrum to the SFF, which explicitly involves unitary time-evolution operators. More concrete links between the SFF and actual thermalization dynamics have been considered [53].

This Letter considers the generalized SFF defined as

$$K(\eta, t) = \langle Z(it + \eta) Z(-it + \eta)\rangle = \langle|\mathrm{Tr} e^{-iH(t-i\eta)}|^2\rangle, \quad (1)$$

where $\eta \to 0^{\pm}$ is an infinitesimal. The key observation of the Letter is that

$$\lim_{\eta\to 0}\lim_{N\to\infty} K(\eta, t) \neq \lim_{N\to\infty}\lim_{\eta\to 0} K(\eta, t), \quad (2)$$

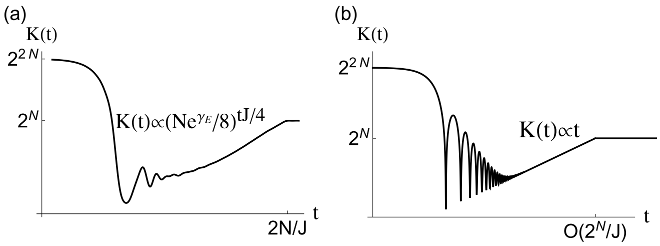

FIG. 1. Schematic log-log plots of the SFF $K(t)$ of the random matrix model given by Eq. (3) for the (a) noninteracting and (b) interacting case. (a) is obtained using the numerical data from our previous study [54], while (b) is the RMT prediction applying the analytical expression for the GUE SFF in Ref. [52]. (a) In the noninteracting case, the SFF consists of an initial slope, an exponential-in-t ramp, and a plateau that starts at $t = 2N/J$. (b) In the presence of interactions, the many-body spectrum exhibits RMT statistics and the exponential ramp is expected to be replaced with a linear one which approaches the plateau at a much larger time $t \sim 2^N/J$. See also Ref. [56] where this SFF transition has been observed numerically in a similar model of Majorana fermions.

and random matrix theory (RMT) statistics appears if the $\eta \to 0$ limit is taken after the $N \to \infty$ limit, suggesting a possible connection between the emergence of quantum chaos and the spontaneous breaking of unitarity. We note that although the calculation is done with $\eta \to 0^+$ being a positive infinitesimal, it can be easily generalized to the case of $\eta \to 0^-$ and the main conclusion still holds.

We consider an interacting random matrix model of fermions populating Wigner-Dyson single-particle levels and generic nonrandom two-body interactions. For simplicity, we restrict ourselves to the case of broken time-reversal symmetry. The Hamiltonian assumes the form

$$H = \sum_{i,j=1}^{N} \psi_i^\dagger h_{ij} \psi_j + \frac{1}{2} \sum_{i,j,k,l=1}^{N} \psi_i^\dagger \psi_j^\dagger V_{ij;kl} \psi_k \psi_l, \tag{3}$$

where h is a $N \times N$ random Hermitian matrix drawn from a Gaussian unitary ensemble (GUE) [42] with the distribution function $P(h) \propto \exp(-\frac{N}{2J^2}\mathrm{Tr}h^2)$. The interaction matrix V is antisymmetric, $V_{ij;kl} = -V_{ji;kl} = -V_{ij;lk} = V^*_{kl;ij}$, and not random. We consider an arbitrary fixed realization of V.

In the absence of interactions, the model is trivially many-body integrable. However, the SFF is still nontrivial showing an initial slope falling off the value of $K(t=0) = 2^{2N}$ and then rapidly increasing via an exponential ramp to a plateau $K(t) = 2^N$ at $t \geqslant 2N/J$ [see Ref. [54] and Fig. 1(a)]. This implies residual correlations in the many-particle spectrum on single-particle energy scales, $\Delta E \sim N^{-1}J$. However, there are negligible correlations on smaller many-body energy scales and the statistics there becomes Poisson—the plateau. Interactions are expected to break integrability and give rise to Wigner-Dyson many-body level statistics, whereas the exponential ramp is replaced with a much slower linear one leading to a plateau at $\Delta E \sim 2^{-N}J$ [see Fig. 1(b)]. The question is, how does this transition occur? It is argued below that this involves two "steps"—spontaneous breaking of unitarity, which selects a unique saddle point out of a manifold of "unitary" saddle points, and gapping out soft modes, which are responsible for the exponential ramp in the noninteracting theory [54,55].

The SFF defined in Eq. (1) for this model can be expressed as the following functional integral,

$$K(\eta, t) = \int \mathcal{D}hP(h) \int \mathcal{D}(\bar{\psi}, \psi) e^{-S[\bar{\psi},\psi]}, \tag{4a}$$

$$S[\bar{\psi}, \psi] = -i \sum_{a=\pm} \int_0^{z_a} dt' \Big[\bar{\psi}_i^a(t')(i\partial_{t'}\delta_{ij} - \zeta_a h_{ij})\psi_j^a(t') - \frac{i}{2}\zeta_a \bar{\psi}_i^a(t')\bar{\psi}_j^a(t') V_{ij;kl} \psi_k^a(t')\psi_l^a(t')\Big] \tag{4b}$$

The Grassmann field ψ_i^a is labeled by a flavor index $i = 1, \dots, N$ along with a replica index $a = \pm$, and is subject to the antiperiodic boundary condition $\psi^a(z_a) = -\psi^a(0)$, with $z_a = t - i\zeta_a\eta$ and $\zeta_a = \pm 1$ for $a = \pm$. The integration over ψ^a yields the partition function $Z(i\zeta_a z_a)$.

Starting from Eq. (4) and following the standard σ-model derivation procedure [57–60], we obtain

$$K(\eta, t) = \frac{1}{Z_\phi Z_Q} \int \mathcal{D}\phi \int \mathcal{D}Q \exp\left(-S[Q, \phi]\right), \tag{5a}$$

$$S[Q, \phi] = -\frac{i}{2} \sum_a \zeta_a z_a \phi_{il}^a\big(-\omega_m^a\big) V_{ij;kl}^{-1} \phi_{jk}^a\big(\omega_m^a\big) + \frac{N}{2J^2}\mathrm{Tr}Q^2 - \mathrm{Tr}\ln[(\mathcal{E}\sigma^3 + iQ) \otimes I_f + \Phi]. \tag{5b}$$

Here, $\mathcal{E}$ and Φ are matrices with elements,

$$\mathcal{E}_{nn'}^{ab} = \delta_{ab}\delta_{nn'}\varepsilon_n^a e^{-i\varepsilon_n^a \delta z_a}, \quad \Phi_{ij;nn'}^{ab} = \delta_{ab}\phi_{ij}^a\big(\omega_{n-n'}^a\big), \tag{6}$$

where $\varepsilon_n^a = 2\pi(n + 1/2)/z_a$ and $\omega_m^a = 2\pi m/z_a$ denote the fermionic and bosonic Matsubara frequencies, respectively. σ^3 indicates the direct product of the third Pauli matrix in the replica space (labeled by a) and the identity matrix in the frequency space (indexed by n), while I_f represents the $N \times N$ identity matrix in the flavor space (labeled by i). The phase factor $e^{-i\varepsilon_n^a\delta z_a}$ in Eq. (6), with δz_a being the time discretization interval for path a, ensures convergence of the integral. We have reduced the problem to a theory of the matrix field Q and bosonic field ϕ. Q is a Hermitian matrix acting in the replica and Matsubara frequency spaces. It decouples the ensemble-average-generated four-fermion term. The bosonic field ϕ is introduced to decouple the interactions [61], and carries replica, frequency, and flavor indices. See Sec. I A of the Supplemental Material [62] for the detailed derivation and the expressions for normalization constants Z_ϕ and Z_Q.

In the large $N \to \infty$ limit, the functional integral Eq. (5) can be evaluated by considering the saddle points and small fluctuations around them. We assume that the decoupling field ϕ does not influence the stationary configuration of the matrix field Q, and obtain from the noninteracting action $S[Q, \phi = 0]$ [Eq. (5b)] the saddle-point equation:

$$Q_{\mathrm{sp}} = J^2(-i\mathcal{E}\sigma^3 + Q_{\mathrm{sp}})^{-1}. \tag{7}$$

This is solved by diagonal matrices Λ with

$$\Lambda_{nm}^{ab} = \pm J\delta_{ab}\delta_{nm}, \quad \big|\varepsilon_n^a\big| \ll J. \tag{8}$$

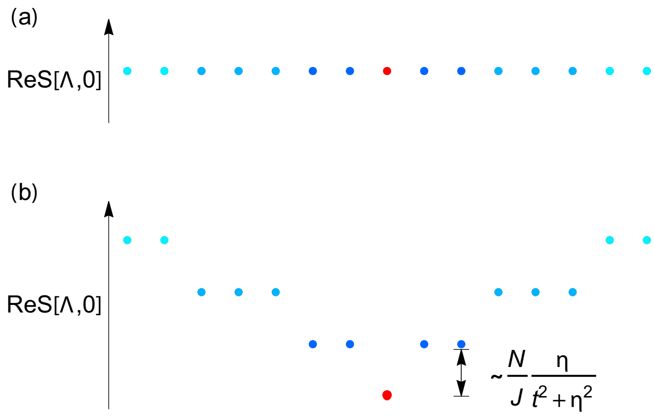

FIG. 2. Schematic plot of the real part of the noninteracting action $\mathrm{Re}\, S[\Lambda, 0]$ (vertical displacement) for various diagonal saddle points (solid circles). (a) For $\eta = 0$, $\mathrm{Re}\, S[\Lambda, 0]$ of all diagonal saddle points are equivalent, and the corresponding contributions to the SFF differ only by phase factors. It is essential to take into account fluctuations around all saddle points for the computation of SFF for zero η. (b) For nonzero η, the differences in $\mathrm{Re}\, S[\Lambda, 0]$ between various saddle points are non-negligible as long as $\eta/(t^2+\eta^2) \gtrsim J/N$. The standard saddle point $\Lambda^{(0)}$ (solid red circle) acquires the minimum $\mathrm{Re}\, S[\Lambda, 0]$ and dominates over all remaining saddle points. In this case, the SFF is dominated by fluctuations around $\Lambda^{(0)}$.

Each diagonal element can take two possible values, resulting in 2^{2N_ε} distinct diagonal saddle points Λ, with N_ε being the total number of Matsubara frequencies considered. We investigate the long-time behavior of the SFF $K(t \gg J^{-1})$, which allows us to focus on the low-energy sector of the theory ($|\varepsilon_n^a| \ll J$) and ignore the correction from nonuniversal single-particle density of states.

In the zero η and δz_a limit, the noninteracting action remains invariant under unitary transformations $Q \to U^\dagger Q U$ satisfying $U\mathcal{E}\sigma^3 U^\dagger = \mathcal{E}\sigma^3$. These unitary transformations U are of the form $U = \prod_{n=1}^{N_\varepsilon} U_n$, where $U_n \in U(2)$ is a rotation acting on the subblock $\left[\begin{smallmatrix} Q^{++}_{nn} & Q^{+-}_{n,-n-1} \\ Q^{-+}_{-n-1,n} & Q^{--}_{-n-1,-n-1} \end{smallmatrix}\right]$. More saddle points can be found by applying the symmetry transformations U to the diagonal ones $Q_{\mathrm{sp}} = U^\dagger \Lambda U$.

The presence of nonzero η breaks this $\prod_{n=1}^{N_\varepsilon} U(2)$ symmetry of the noninteracting theory and allows us to select one dominant saddle point. The noninteracting action of a diagonal saddle point acquires the form

$$S[\Lambda, 0] = i\frac{N}{J^2}\sum_{a,n} \zeta_a \Lambda^{aa}_{nn} \varepsilon^a_n e^{-i\varepsilon^a_n \delta z_a} + \text{const.} \tag{9}$$

The phase factor $e^{-i\varepsilon^a_n \delta z_a}$ is only needed for convergence and can be omitted here. In the case of $\eta/(t^2+\eta^2) \gtrsim J/N$, among various saddle points, $(\Lambda^{(0)})^{ab}_{nm} = J\,\mathrm{sgn}(n+1/2)\delta_{ab}\delta_{nm}$ yields the minimum $\mathrm{Re}\, S[\Lambda, 0]$ and consequently the dominant contribution. By contrast, for $\eta = 0$, the contributions from various saddle points to the SFF differ only by phase factors (see Fig. 2). We call $\Lambda^{(0)}$ the standard saddle point and all remaining diagonal saddle points the nonstandard ones [59,63].

Let us now consider the fluctuations of Q around the diagonal saddle points Λ, which fall into two categories [54,64]: (i) soft modes (or Goldstone modes) generated by unitary rotations of saddle points $Q = R^\dagger \Lambda R$ and associated with the explicitly breakdown of the $U(2N_\varepsilon)$ symmetry of the noninteracting action by the $\mathcal{E}\sigma^3$ term; or (ii) massive modes that cannot be obtained by rotation. In the zero η case, there is a subset of unitary transformations $U \in \prod_{n=1}^{N_\varepsilon} U(2)$ which leaves the noninteracting action invariant. Applying these symmetry transformations to diagonal saddle points $Q = U^\dagger \Lambda U$ generates a special type of soft modes called zero mode with $\delta S[Q, 0] = 0$ [65].

In the noninteracting case, the soft modes are responsible for the exponential-in-t ramp, whereas the massive modes only give rise to a nonessential constant [54]. Therefore, we focus on the interaction effects on the soft modes. If $\eta = 0$, to evaluate the SFF, one must sum over contributions of fluctuations around various saddle points. By contrast, for $\eta \to 0^+$ [or more precisely $\eta/(t^2+\eta^2) \gtrsim J/N$], there is one dominant saddle point $\Lambda^{(0)}$ determined by the η-induced symmetry breaking, and it is sufficient to consider only soft mode fluctuations around it.

In the following, we consider the latter case and explore the influence of interactions on soft mode fluctuations around $\Lambda^{(0)}$ and their contribution to the SFF. In this case, the SFF can be approximated by (see Sec. I D of the Supplemental Material [62])

$$\begin{aligned} K(\eta, t) &= \frac{Z_m e^{-S[\Lambda^{(0)},0]}}{Z_\phi Z_Q}\int \mathcal{D}\phi \int_{\mathcal{M}} \mathcal{D}Q e^{-\delta S[Q,\phi;\Lambda^{(0)}]}, \\ \delta S[Q,\phi;\Lambda^{(0)}] &= i\frac{N}{J^2}\mathrm{Tr}[\sigma^3\mathcal{E}(Q-\Lambda^{(0)})] \\ &\quad + \frac{i}{J^2}\sum_{a,n,n',i} Q^{aa}_{nn'}\phi^a_{ii}(\omega^a_{n'-n}) \\ &\quad - \sum_{a,m,i,j}\frac{i\zeta_a z_a}{2}\phi^a_{il}(-\omega^a_m) \\ &\quad \times \left(V^{-1}_{ij;kl} + i\frac{\zeta_a}{z_a}\tilde{M}^a_m \delta_{ik}\delta_{lj}\right)\phi^a_{jk}(\omega^a_m), \end{aligned} \tag{10}$$

where $\tilde{M}^a_m = -\sum_n (\Lambda^{(0)})^{aa}_{nn}(\Lambda^{(0)})^{aa}_{n+m,n+m}/J^4$, and Z_m denotes the nonessential contribution from massive modes. The integration $\int \mathcal{D}Q$ is over the manifold $\mathcal{M}$ of matrices Q which are generated by unitary rotations of the standard saddle point $\Lambda^{(0)}$:

$$Q = R^{-1}\Lambda^{(0)}R, \quad R \in \frac{U(2N_\varepsilon)}{U(N_\varepsilon)\times U(N_\varepsilon)}. \tag{11}$$

These matrices Q represent soft modes around $\Lambda^{(0)}$ and obey the nonlinear constraints, $\mathrm{Tr}Q = 0$ and $Q^2/J^2 = I$. The aforementioned zero modes are the special soft modes corresponding to

$$Q = U^{-1}\Lambda^{(0)}U, \quad U \in \prod_{n=1}^{N_\varepsilon}\frac{U(2)}{U(1)\times U(1)}. \tag{12}$$

For the matrix field Q governed by the action in Eq. (10), the noninteracting propagator is given by

$$\left(\mathcal{G}^{(0)}_X\right)^{ab;ba}_{nm;mn} = \frac{N}{2J^3}\left\langle Q^{ab}_{nm}Q^{ba}_{mn}\right\rangle_0 = \frac{i}{\zeta_a\varepsilon^a_n - \zeta_b\varepsilon^b_m}, \tag{13}$$

for $n \geqslant 0 > m$. Interactions between Q and ϕ result in a self-energy correction, which gives rise to a mass term λ_{nm}^{ab} in the interaction-dressed inter-replica Q propagator (see Sec. I D 3 of the Supplemental Material [62]),

$$(\mathcal{G}_X)_{nm;mn}^{ab;ba} = \frac{i}{\left(\zeta_a \varepsilon_n^a - \zeta_b \varepsilon_m^b\right)(1+\delta z) + i\lambda_{nm}^{ab}}, \tag{14}$$

for $a \neq b$ and $n \geqslant 0 > m$. Note that the intra-replica propagator $\mathcal{G}_X^{aa;aa}$ contributes to the disconnected SFF $K^{\text{dis}}(\eta, t) = \langle Z(iz_+)\rangle \langle Z(-iz_-)\rangle$ which dominates the early-time slope regime and will not be considered here. Assuming that the main interaction effect on the SFF comes from the mass λ_{nm}^{ab}, we ignore the renormalization effect and set $\delta z = 0$. We obtain a self-consistent equation for the mass λ_{nm}^{ab} [Eq. (S59) in the Supplemental Material [62]], whose explicit form depends on the interaction matrix V and is not important for our purpose. Equation (14) shows that the fluctuations of the bosonic field ϕ lead to decoherence and introduce a cutoff λ in the soft modes.

Carrying out the integration over Q and ϕ in Eq. (10), we find that, up to an irrelevant constant, $K(\eta, t)$ is approximately given by

$$K(\eta, t) \propto \exp\left\{-\sum_{ab}\sum_{n\geqslant 0>m} \ln\left[(\mathcal{G}_X^{-1})_{nm;mn}^{ab;ba}\right] + \frac{1}{2}\sum_{a,m}\left[\operatorname{Tr}\ln \tilde{V}^a(\omega_m^a) - \operatorname{Tr}\ln V\right] - S[\Lambda^{(0)}, 0]\right\}. \tag{15}$$

Here, $\tilde{V}_{ij;kl}^a(\omega_m^a) = -i\zeta_a z_a \langle \phi_{il}^a(\omega_m^a)\phi_{jk}^a(-\omega_m^a)\rangle$ stands for the effective interaction matrix and is proportional to the interaction dressed propagator for the bosonic field ϕ [see Eq. (S49) in the Supplemental Material [62]).

In the exponent in Eq. (15), only the first term with $a \neq b$ contributes to the connected SFF $K^{\text{con}} = K - K^{\text{dis}}$. Substituting the explicit form of the interaction-dressed inter-replica propagator $\mathcal{G}_X^{ab;ba}$ into this term and evaluating the Matsubara frequency summation by the analytical continuation technique, one obtains

$$\ln K^{\text{con}}(\eta, t) = \sum_{a\neq b}\frac{1}{4\pi^2}\int_{\varepsilon,\varepsilon'} \ln\left(1+e^{-i\zeta_a z_a \varepsilon}\right)\ln\left(1+e^{-i\zeta_b z_b \varepsilon'}\right) \times \left(\frac{1}{-i(\varepsilon-\varepsilon') + \lambda^{ab}(\varepsilon+\varepsilon')}\right)^2 \left(\frac{\partial \lambda^{ab}}{\partial \varepsilon} - i\right) \times \left(\frac{\partial \lambda^{ab}}{\partial \varepsilon'} + i\right). \tag{16}$$

Here, $\lambda^{ab}(\varepsilon + \varepsilon')$ now represents the analytic continuation of the inter-replica mass λ_{nm}^{ab} from $\zeta_a \varepsilon_n^a \to \varepsilon$ and $\zeta_b \varepsilon_m^b \to \varepsilon'$, and is assumed to be a function of $\varepsilon + \varepsilon'$ (independent of $\varepsilon - \varepsilon'$). The noninteracting connected SFF can also be obtained from the equation above by setting $\lambda^{ab} = 0$.

Focusing on the difference between the interacting and noninteracting cases, we then replace the factor $\ln(1 + e^{-i\zeta_a z_a \varepsilon})$ in Eq. (16) with $e^{-i\zeta_a z_a \varepsilon}$. We assume that the neglected contribution cancels partially with higher-order fluctuation correction (see Sec. I F of the Supplemental Material [62] and also Ref. [54]) and find

$$\ln K^{\text{con}}(\eta, t) = t\sum_{a\neq b}\int_{-E_{\text{UV}}}^{E_{\text{UV}}} \frac{dE}{2\pi} e^{-2\eta E} e^{-\zeta_a \lambda^{ab}(E)t}\,\Theta(\zeta_a \operatorname{Re}\lambda^{ab}) \times \left[\left(\frac{1}{2}\frac{\partial \lambda^{ab}}{\partial E}\right)^2 + 1\right], \tag{17}$$

where $E_{\text{UV}} \sim J$ denotes the ultraviolet cutoff.

Let us now take the limit $\eta \to 0^+$. In the noninteracting case, after setting $\lambda^{ab} = 0$, it is straightforward to see from the equation above that $\ln K^{\text{con}}(\eta \to 0^+, t) \propto t$, and an infrared divergence originating from the zero modes occurs. This infrared divergence can be resolved by including a higher-order fluctuation correction [54,62] (which is also needed to recover the correct overall coefficient), and is cut off by the mass λ^{ab} in the interacting theory at the quadratic order.

With interactions, fluctuations of decoupling field ϕ lead to the dephasing effect, reflected by the appearance of a mass λ to the soft modes. The mass λ^{ab} results in an exponential factor $e^{-\zeta_a \lambda^{ab}(E)t}\Theta(\zeta_a \operatorname{Re}\lambda^{ab})$ in $\ln K^{\text{con}}(\eta \to 0^+, t)$, and thus suppresses the exponential-in-t growth of the connected SFF. We note that what matters is not the explicit form of the inter-replica mass $\lambda^{ab}(E)$, but its existence.

For the case where $\eta \to 0^-$ is a negative infinitesimal, the dominant saddle point is instead $-\Lambda^{(0)}$ and the soft mode fluctuations around this saddle point are described by a nonlinear σ model which can be obtained by replacing $\Lambda^{(0)}$ in Eqs. (10) and (11) with $-\Lambda^{(0)}$. Through a calculation similar to that of the $\eta \to 0^+$ case, one can show that the inter-replica propagator of the fluctuations around $-\Lambda^{(0)}$ also acquires a mass, which arises from interaction-induced dephasing processes and is responsible for the suppression of the exponential ramp.

The suppression of the exponential ramp is a necessary prerequisite for the emergence of RMT statistics in a many-body spectrum. In particular, for the interacting model (with broken time-reversal symmetry) whose many-body energy levels follow Wigner-Dyson statistics, the connected SFF should grow linearly in t instead of exponentially. However, the derivation of the explicit expression for the SFF requires a consideration of the fluctuation corrections beyond the quadratic order. For the noninteracting case, the many-body SFF can be expressed in terms of the connected n-point single-particle level correlation function $\mathrm{R}_n^{\text{con}}(\varepsilon_1, \ldots, \varepsilon_n)$ as [54,62]

$$\ln K(\eta, t) = \sum_n \frac{N^n}{n!}\int \prod_{k=1}^{n} d\varepsilon_k \left[\sum_{a_k} \ln\left(1+e^{-i\zeta_{a_k} z_{a_k}\varepsilon_k}\right)\right] \times \mathrm{R}_n^{\text{con}}(\varepsilon_1, \ldots, \varepsilon_n). \tag{18}$$

The saddle-point action $S[\Lambda^{(0)}, 0]$ [Eq. (5b)] yields the $n = 1$ term which results in the initial slope, whereas the quadratic fluctuation correction leads to the $n = 2$ term which contributes to the exponential ramp. The contributions from $n > 2$ terms are as important and are necessary to obtain the correct overall coefficient in the exponent of the ramp. In the presence of interactions, Eq. (18) is no longer valid. However, we find that the quadratic fluctuation correction Eq. (16) is analogous to the $n = 2$ term. If the interaction effects on all higher-order fluctuations are similar, their contributions will be suppressed in a similar way.

In Sec. II of the Supplemental Material [62], we also perform an analogous calculation of the SFF for a two-dimensional (2D) disordered system of fermions interacting via density-density interactions. In the ergodic regime $t \gg L^2/D$, with D being the diffusion constant and L the system size, the statistics of noninteracting single-particle energy levels can be universally described by RMT [57,63,64,66–70], and the corresponding field theory becomes effectively the same as the current theory of the random matrix model. In the diffusive regime with $L^2/D \gg t \gg \tau_{\rm el}$, where $\tau_{\rm el}$ denotes the elastic scattering time, the connected SFF is governed by inter-replica diffusons, which acquire a mass from the dephasing processes due to interactions [see Eq. (S132) in the Supplemental Material [62]]. The inter-replica mass gives rise to an exponential decay factor in the exponent of the connected SFF, and is crucial for the emergence of RMT statistics. This mass is similar to the dephasing rate [71,72] that cuts off the quantum interference correction to conductance and the Lyapunov exponent of the out-of-time-ordered correlator (OTOC) [73,74] in interacting disordered metals. All these three quantities are given by the mass of diffusons, and the complex time $z_\pm = t \mp i\eta$ in the SFF now plays the role of inverse temperature in the dephasing of the conductance correction and the Lyapunov exponent of OTOC. If the fluctuations of the interaction decoupling field become ineffective in destroying the coherence, these three types of diffuson mass vanish, Poisson statistics would appear, the quantum conductance correction would diverge [75,76], and the OTOC would no longer grow exponentially in time, suggesting a connection between dephasing failure and many-body localization. However, it is unclear under what condition dephasing failure will occur for these three separate cases, which requires a more detailed analysis of these masses and is a possible direction for future work.

In conclusion, we have shown that the spectral form factor behaves in a drastically different way in the $\eta \to 0^\pm$ limit compared to the $\eta = 0$ case. In the former case, the exponential ramp of the noninteracting theory is suppressed, which leads to the appearance of a plateau at a later time (or equivalently to the emergence of correlations on smaller many-body energy scales). Note that the mathematical problem of calculating level statistics studies properties of the spectrum, has no notion of temperature, and the SFF is described by the correlations of two unitary time-evolution operators. While the $\eta = 0$ case is formally correct, it does not account for the possible external perturbations and noise that the physical system may experience, and which would lead to nonunitary time evolution. Our proposed explanation of the result is that the $\eta \to 0^\pm$ limit (or any other way of imposing a nonunitary structure in the SFF) in effect encodes such tiny nonunitary perturbations, which get magnified in the thermodynamic limit. Mathematically this occurs via a spontaneous symmetry breaking in the σ-model saddle-point manifold and the appearance of a "dephasing" mass of the Goldstone modes, while physically this may imply spontaneous breaking of unitarity triggered by any external perturbations.

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[1] M. Srednicki, Phys. Rev. E. **50**, 888 (1994).
[2] M. Srednicki, J. Phys. A **32**, 1163 (1999).
[3] J. M. Deutsch, Phys. Rev. A **43**, 2046 (1991).
[4] M. Rigol, V. Dunjko, and M. Olshanii, Nature (London) **452**, 854 (2008).
[5] J. M. Deutsch, Rep. Prog. Phys. **81**, 082001 (2018).
[6] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976).
[7] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Adv. Phys. **65**, 239 (2016).
[8] C. Gogolin and J. Eisert, Rep. Prog. Phys. **79**, 056001 (2016).
[9] P. Kos, M. Ljubotina, and T. Prosen, Phys. Rev. X **8**, 021062 (2018).
[10] B. Bertini, P. Kos, and T. Prosen, Phys. Rev. Lett. **121**, 264101 (2018).
[11] B. Bertini, P. Kos, and T. Prosen, Commun. Math. Phys **387**, 597 (2021).
[12] D. Roy and T. Prosen, Phys. Rev. E **102**, 060202(R) (2020).
[13] A. Chan, A. De Luca, and J. T. Chalker, Phys. Rev. X **8**, 041019 (2018).
[14] A. Chan, A. De Luca, and J. T. Chalker, Phys. Rev. Lett. **121**, 060601 (2018).
[15] A. J. Friedman, A. Chan, A. De Luca, and J. T. Chalker, Phys. Rev. Lett. **123**, 210603 (2019).
[16] S. J. Garratt and J. T. Chalker, Phys. Rev. X **11**, 021051 (2021).
[17] S. J. Garratt and J. T. Chalker, Phys. Rev. Lett. **127**, 026802 (2021).
[18] A. Chan, A. De Luca, and J. T. Chalker, Phys. Rev. Research **3**, 023118 (2021).
[19] P. Saad, S. H. Shenker, and D. Stanford, arXiv:1806.06840.
[20] J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, J. High Energy Phys. 05 (2017) 118.
[21] A. Altland and D. Bagrets, Nucl. Phys. B **930**, 45 (2018).
[22] T. Micklitz, F. Monteiro, and A. Altland, Phys. Rev. Lett. **123**, 125701 (2019).
[23] F. Monteiro, T. Micklitz, M. Tezuka, and A. Altland, Phys. Rev. Research **3**, 013023 (2021).
[24] F. Monteiro, M. Tezuka, A. Altland, D. A. Huse, and T. Micklitz, Phys. Rev. Lett. **127**, 030601 (2021).
[25] A. M. García-García, Y. Jia, and J. J. M. Verbaarschot, Phys. Rev. D **97**, 106003 (2018).
[26] A. M. García-García and J. J. M. Verbaarschot, Phys. Rev. D **94**, 126010 (2016).
[27] H. Gharibyan, M. Hanada, S. H. Shenker, and M. Tezuka, J. High Energy Phys. 07 (2018) 124.
[28] Y.-Z. You, A. W. W. Ludwig, and C. Xu, Phys. Rev. B **95**, 115150 (2017).

[29] J. Cotler and N. Hunter-Jones, J. High Energy Phys. 12 (2020) 205.

[30] D. Poilblanc, T. Ziman, J. Bellissard, F. Mila, and G. Montambaux, Europhys. Lett. **22**, 537 (1993).

[31] G. Montambaux, D. Poilblanc, J. Bellissard, and C. Sire, Phys. Rev. Lett. **70**, 497 (1993).

[32] S. Åberg, Phys. Rev. Lett. **64**, 3119 (1990).

[33] P. Jacquod and D. L. Shepelyansky, Phys. Rev. Lett. **79**, 1837 (1997).

[34] A. Hallam, J. G. Morley, and A. G. Green, Nat. Commun. **10**, 2708 (2019).

[35] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. B **93**, 125419 (2016).

[36] A. Altland and J. Sonner, SciPost Phys. **11**, 34 (2021).

[37] A. Altland, D. Bagrets, P. Nayak, J. Sonner, and M. Vielma, Phys. Rev. Research **3**, 033259 (2021).

[38] R. Dubertrand and S. Müller, New J. Phys. **18**, 033009 (2016).

[39] M. Akila, D. Waltner, B. Gutkin, P. Braun, and T. Guhr, Phys. Rev. Lett. **118**, 164101 (2017).

[40] T. Engl, J. D. Urbina, and K. Richter, Phys. Rev. E **92**, 062907 (2015).

[41] T. Engl, J. Dujardin, A. Argüelles, P. Schlagheck, K. Richter, and J. D. Urbina, Phys. Rev. Lett. **112**, 140403 (2014).

[42] M. L. Mehta, *Random Matrices* (Elsevier, Amsterdam, 2004).

[43] F. J. Dyson, J. Math. Phys. **3**, 140 (1962).

[44] E. P. Wigner, Ann. Math. **62**, 548 (1955).

[45] E. Wigner, *Group Theory: and its Application to the Quantum Mechanics of Atomic Spectra*, Vol. 5 (Elsevier, Amsterdam, 2012).

[46] O. Bohigas, M.-J. Giannoni, and C. Schmit, Phys. Rev. Lett. **52**, 1 (1984).

[47] T. Guhr, A. Müller–Groeling, and H. A. Weidenmüller, Phys. Rep. **299**, 189 (1998).

[48] F. Haake, *Quantum Signatures of Chaos* (Springer, Berlin, 2010).

[49] R. E. Prange, Phys. Rev. Lett. **78**, 2280 (1997).

[50] M. V. Berry, Proc. R. Soc. London A **400**, 229 (1985).

[51] J. Cotler, N. Hunter-Jones, J. Liu, and B. Yoshida, J. High Energy Phys. 11 (2017) 48.

[52] J. Liu, Phys. Rev. D **98**, 086026 (2018).

[53] P. Reimann, Nat. Commun. **7**, 10821 (2016).

[54] Y. Liao, A. Vikram, and V. Galitski, Phys. Rev. Lett. **125**, 250601 (2020).

[55] M. Winer, S.-K. Jian, and B. Swingle, Phys. Rev. Lett. **125**, 250602 (2020).

[56] P. H. C. Lau, C.-T. Ma, J. Murugan, and M. Tezuka, J. Phys. A **54**, 095401 (2021).

[57] K. Efetov, Adv. Phys. **32**, 53 (1983).

[58] F. Wegner, Z. Phys. B **35**, 207 (1979).

[59] A. Kamenev and M. Mézard, J. Phys. A **32**, 4373 (1999).

[60] A. Altland and A. Kamenev, Phys. Rev. Lett. **85**, 5615 (2000).

[61] We assume that interaction matrix's inverse V^{-1}, defined by $\sum_{jk} V^{-1}_{i'j;kl'} V_{ji;lk} = \delta_{ii'}\delta_{ll'}$, exists. Otherwise, one can proceed in an analogous manner by decoupling the interactions in a different channel.

[62] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.105.L140202 for the detailed derivation of the spectral form factors for both an interacting random matrix model and a two-dimensional disordered system of fermions with short-range density-density interactions.

[63] A. V. Andreev and B. L. Altshuler, Phys. Rev. Lett. **75**, 902 (1995).

[64] A. Kamenev and M. Mézard, Phys. Rev. B **60**, 3944 (1999).

[65] Note that there are no soft modes around the saddle point $\Lambda = \pm JI$, and no zero modes around the saddle points satisfying $\Lambda^{++}_{nn} = \Lambda^{--}_{-n-1,-n-1}$ for any frequency index n.

[66] L. P. Gorkov and G. M. Eliashberg, Zh. Eksp. Teor. Fiz. **48**, 1407 (1965) [Sov. Phys. JETP **21**, 940 (1965)].

[67] K. B. Efetov, A. I. Larkin, and D. E. Khmelnitskii, Zh. Eksp. Teor. Fiz. **79**, 1120 (1980) [Sov. Phys. JETP **52**, 568 (1980)].

[68] B. L. Altshuler and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **91**, 220 (1986) [Sov. Phys. JETP **64**, 127 (1986)].

[69] J. M. Verbaarschot and M. R. Zirnbauer, J. Phys. A **18**, 1093 (1985).

[70] V. E. Kravtsov and A. D. Mirlin, Pis'ma Zh. Eksp. Teor. Fiz. **60**, 645 (1994) [JETP Lett. **60**, 656 (1994)].

[71] B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky, J. Phys. C: Solid State Phys. **15**, 7367 (1982).

[72] I. L. Aleiner, B. L. Altshuler, and M. E. Gershenson, Waves Random Media **9**, 201 (1999).

[73] A. A. Patel, D. Chowdhury, S. Sachdev, and B. Swingle, Phys. Rev. X **7**, 031047 (2017).

[74] Y. Liao and V. Galitski, Phys. Rev. B **98**, 205124 (2018).

[75] Y. Liao and M. S. Foster, Phys. Rev. Lett. **120**, 236601 (2018).

[76] S. M. Davis and M. S. Foster, Phys. Rev. B **102**, 155101 (2020).