

## Thermoelectrics of a two-channel charge Kondo circuit: Role of electron-electron interactions in a quantum point contact

A. V. Parafilo <sup>1,\*</sup>, T. K. T. Nguyen <sup>2</sup>, and M. N. Kiselev <sup>3</sup>

<sup>1</sup>Center for Theoretical Physics of Complex Systems, Institute for Basic Science, Expo-ro, 55, Yuseong-gu, Daejeon 34126, Republic of Korea

<sup>2</sup>Institute of Physics, Vietnam Academy of Science and Technology, 10 Dao Tan, Hanoi, Vietnam

<sup>3</sup>The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, I-34151 Trieste, Italy



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In this Letter we investigate the properties of a quantum impurity model in the presence of additional many-body interactions between mobile carriers. The fundamental question which is addressed here is how the interactions in the charge and spin sectors of an itinerant system affect the quantum impurity physics in the vicinity of the intermediate coupling fixed point. To illustrate the general answer to this question we discuss a two-channel charge Kondo circuit model. We show that the electron-electron interactions resulting in the formation of a massive spin mode in an itinerant electron subset drive the system away from the unstable non-Fermi-liquid (NFL) fixed point to the stable Fermi-liquid (FL) regime. We discuss the thermoelectric response as a benchmark for the NFL-FL crossover.

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**Introduction.** The quantum thermoelectricity of low-dimensional systems is a rapidly developing direction of modern condensed matter physics. Owing to the incredible development of the nanotechnology, the fabrication of highly controllable and fine-tunable nanodevices gives access to a broad variety of charge, spin, and heat quantum transport phenomena. Since the early 1990s [1–5], thermoelectric efficiency has been predicted to be enhanced in low-dimensional systems in comparison with bulk materials. Moreover, heat quantization [6–8], heat Coulomb blockades [9], and the universality of thermoconductance fluctuations [10] have been investigated in different nanostructures.

One of the most prominent nanodevices is a *single-electron transistor* (SET) [11], whose transport properties are fully governed by the Coulomb blockade (CB) phenomenon [12–14]. The SET usually consists of a small island [a so-called quantum dot (QD)], connected to electron reservoirs [15] by tunnel barriers or by quantum point contacts (QPCs) [16]. With its small size, electrostatically tunable properties, and sensitive thermoelectric response, SET provides important information about strong electron-electron interactions, interference effects, and resonance scattering on the quantum transport [17,18]. Recent experiments on the thermopower (TP) in QD systems quantified the role of sequential tunneling and cotunneling on thermoelectric transport through the SET [19] and also demonstrated pronounced nonlinear thermoelectric effects [20]. Moreover, fine tuning the coupling between the QD and leads allowed access to thermoelectric transport through the Kondo-quantum impurity [19]. These promoted studies in both experiment and theory of thermoelectric transport in QDs in the Kondo regime.

The Kondo effect [21], which shows both resonance scattering and strong interactions [22], has been detected in SETs fine tuned by the gate to odd CB valleys [23]. The QD behaves as a quantum spin-1/2 impurity [24] since the strong correlations between it and the conduction electrons in the reservoirs lead to the removal of the Coulomb blockade, and result in a nonmonotonic temperature dependence of the conductance at low temperatures [25–27].

While the conventional Kondo phenomenon is attributed to a spin degree of freedom of the quantum impurity, the charge Kondo effect deals with an isospin implementation of the charge quantization. The latter occurs when a large metallic QD in the Coulomb blockade regime is strongly coupled to one (or several) lead(s) through a (or several) almost fully transmitting single-mode QPC(s) [28–30]. This setup is described by the Flensberg-Matveev-Furusaki model (FMF). In the absence of a magnetic field, the FMF setup is mapped into a two-channel Kondo (2CK) model: The left- and right-moving modes are treated as isospin variables, whereas the spin projection quantum numbers of electrons serve as different channels [29–31]. Very recently, the FMF model has been achieved in breakthrough experiments [32,33]. These experiments mark an important step in the study of multichannel Kondo (MCK) problems in which the universality class known as non-Fermi-liquid (NFL) behavior dominates [34]. The NFL picture in the FMF model, however, is extremely sensitive to variations of external parameters. Since the intermediate coupling NFL fixed point is unstable, the Fermi-liquid (FL) ground state [35,36] is achieved by applying relevant small perturbations. For instance, any small but finite external magnetic field applied to the SET results in channel asymmetry and thus changes the universality class from the two-channel Kondo to the single-channel Kondo (1CK) regime [35,36].

\*aparafil@ibs.re.kr

The significant difference of the FMF model compared to previous theoretical models that have been used to explain the Kondo effect is that the transmission of electrons through QPCs happens in one dimension (1D), therefore, the Abelian bosonization technique [37–40] is applied to solve the problems. Previous works studying the FMF model [28–31,35,36] disregarded the effects of an electron-electron interaction in the QPC(s). One thus raises an important question regarding whether or not the NFL property can be broken spontaneously (without any variation of any external parameter) and how the electron-electron interactions affect the NFL picture in the FMF model.

In an interacting 1D system, the state resulting from the addition or removal of an electron may decay quickly into collective charge and spin excitations which propagate with different velocities (spin-charge separation) [41]. A theoretical model describing a 1D interacting electron system which is predicted to behave quite differently from the FL, is called the Luttinger-liquid (LL) model [42,43]. The advantage of the LL model is that most of the interaction processes (namely, the forward  $g_{4,\parallel}, g_{4,\perp}, g_{2,\parallel}, g_{2,\perp}$  and the backward  $g_{1,\parallel}$  scatterings) [44] can be described by the quadratic terms of the bosonic fields. The LL Hamiltonian in the bosonic representation is thus modified from the FL one through the effective Fermi velocities  $v_{F\rho}, v_{F\sigma}$  (the indices  $\rho, \sigma$  stand for the charge and spin modes, respectively) and additional dimensionless parameters  $g_\rho, g_\sigma$  (so-called Luttinger parameters) [45]. However, the term describing the backscattering  $g_{1,\perp}$  process between electrons with opposite spin projection values cannot be expressed quadratically in bosonic representation, and it must be written explicitly [40]. As mentioned in Ref. [40], the effects of the  $g_{1,\perp}$  term are quite drastic, and it is important to consider them.

In this Letter we investigate the effects of the electron-electron interactions in the LL, especially the role of spin-dependent backward scattering, on the thermoelectric coefficients in the FMF setup. In the absence of a  $g_{1,\perp}$  process or in the case when it is irrelevant ( $g_\sigma \geq 1$ ), the low-temperature scaling behavior of the TP is  $S \propto T^{g_\sigma-1} \log T$ . This scaling is paradigmatic for the two-channel Kondo (2CK) model. In addition, the reflection (transmission) coefficient at the QPC is renormalized due to the electron interactions in the LL [as known in the Kane-Fisher phenomenon (KFP) [46]]. Any relevant  $g_{1,\perp}$  process appears when  $g_\sigma < 1$  opens a gap in the spin mode (or one says that the spin field is massive) [47], and the TP is proportional to the temperature  $S \propto TM^{g_\sigma/2}$  with  $M$  the spin field's mass. We predict that the backscattering process between electrons with opposite spin  $g_{1,\perp}$  in the LL can destroy the local NFL-2CK state and drive the system to the FL-1CK regime. In other words, our results show evidence of the existence of a  $g_{1,\perp}$  process if experimentalists find the FL behavior of TP in the FMF setup.

*Model.* We consider a nanodevice as shown in Fig. 1 consisting of a large metallic QD in the weak Coulomb blockade regime weakly coupled to the left electrode (the source) via a tunnel barrier and strongly coupled to the right one through a single-mode QPC. The QD-QPC structure (the drain) is built of a two-dimensional electron gas (2DEG) and assumed to be in thermal equilibrium at temperature  $T$ . By applying

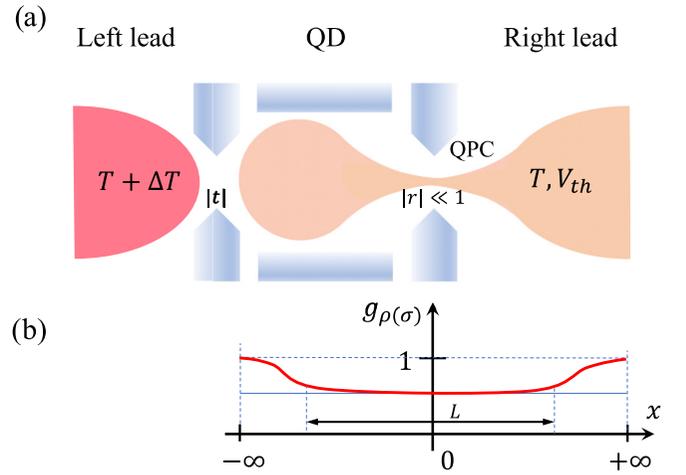


FIG. 1. (a) Schematic of a single-electron transistor device: A large metallic quantum dot (QD) is weakly coupled to the left electrode through a weak barrier and strongly coupled to the right electrode through a single-mode quantum point contact (QPC). The QD and electrodes are formed in a two-dimensional electron gas (2DEG). The tunnel barrier is characterized by a small transparency  $|t| \ll 1$ , while the electron scattering in the QPC is determined by a reflection amplitude  $|r| \ll 1$ . The QD and the right electrode (the drain, marked by the orange color) are at the reference temperature  $T$  while the left electrode (the source, marked by the red color) is at higher temperature  $T + \Delta T$ . A thermovoltage  $V_{th}$  is applied to the drain to compensate the charge flow induced by the temperature drop  $\Delta T$ . The effects of electron-electron interactions in the source are accounted by the Fermi-liquid theory. The role of the electron-electron interaction in the drain, in particular, in the narrow constriction of the size  $L$  (the QPC) is the main subject discussed in this Letter. (b) An example showing the evolution of the charge and spin Luttinger parameters  $g_\rho$  and  $g_\sigma$ : The interactions asymptotically vanish both at the position of the tunnel barrier ( $x = -\infty$ ) and away from the QPC ( $x = +\infty$ ).

an external gate, one can control the electron interactions in the vicinity of the QPC [48–50]. The electrons in this 1D constriction are thus described by the LL model [40]. The size  $L$  characterizing the typical length scale for the area where the electron-electron interactions are appreciable is assumed to satisfy the condition that the energy  $v_F/L$  is low enough, especially  $v_F/L \ll g_\rho E_C$ . The source separated from the QD by a tunnel contact and considered at higher temperature  $T + \Delta T$  is also formed by 2DEG and can be described without any loss of generality by conventional Fermi-liquid theory. The temperature drop  $\Delta T$  is controlled by using a current heating technique [19]. The  $\Delta T$  across the tunnel barrier is assumed to be small compared to the reference temperature  $T$  to guarantee the linear response regime. Applying thermovoltage  $V_{th}$  to implement a zero-current condition for the electric current between the source and drain allows us to compute the thermopower (TP) as  $S = G_T/G|_{I=0} = -V_{th}/\Delta T$ , where  $G_T = I/\Delta T$  is the thermoelectric coefficient, and  $G = I/V_{th}$  is the electric conductance.

The weak coupling between the left electrode and the QD is described by a tunnel Hamiltonian  $H_t = \sum_{k\alpha} (t c_{k\alpha}^\dagger d_\alpha + \text{H.c.})$ , where  $|t| \ll 1$  is the hopping amplitude, and operators  $c_{k\alpha}$  and  $d_\alpha$  account for electrons with spin ( $\alpha = \uparrow, \downarrow$ ) in the

noninteracting left electrode and in the QD at the point of tunnel junction ( $x = -\infty$ ), respectively.

At the lowest order of perturbation theory over a small transparency  $|t| \ll 1$  the transport coefficients can be computed through the energy-dependent tunneling density of states (DOS) related to the Matsubara Green's function. Here, we assume that the DOS of electrons in the source  $\nu_L$  is a constant, and electrons in the QD at the weak link ( $x = -\infty$ ) are noninteracting. We can therefore apply the Fermi golden rule at the weak link with  $\Delta T/T \ll 1$ .

At the end, the conductance  $G$  and the thermoelectric coefficient  $G_T$  are defined through a correlation function  $K(\tau)$  of the interacting electrons in the drain as follows [30,31],

$$G = G_L \frac{\pi T}{2} \int \frac{1}{\cosh^2(\pi T t)} K\left(\frac{1}{2T} + it\right) dt, \quad (1)$$

$$G_T = -\frac{i\pi^2 G_L T}{2e} \int \frac{\sinh(\pi T t)}{\cosh^3(\pi T t)} K\left(\frac{1}{2T} + it\right) dt, \quad (2)$$

where  $G_L \ll e^2/h$  denotes the conductance of the left tunnel barrier without the influence of the dot.

It is convenient to describe the interacting electrons in the QD-QPC in the bosonized representation [28–31,35,36]. In the spirit of Matveev-Andreev theory [31], the time-ordered correlation function  $K(\tau)$  is computed through the functional integration over the bosonic fields  $\phi_{\uparrow(\downarrow)}(x, t)$ ,

$$K(\tau) = Z(\tau)/Z(0), \quad (3)$$

$$Z(\tau) = \prod_{\alpha=\uparrow,\downarrow} \int \mathcal{D}\phi_\alpha \exp[-S_0 - S_C(\tau) - S'], \quad (4)$$

where  $S_0$ ,  $S_C$ , and  $S'$  are Euclidean actions describing the free Luttinger liquid, the Coulomb blockade in the QD, and the backscattering at the QPC, respectively. The action  $S_0$  is presented as a sum of two independent actions [37,38,40]  $S_0 = S_0^{(\rho)} + S_0^{(\sigma)}$ , where

$$S_0^{(\rho)} = \frac{v_{F\rho}}{2\pi g_\rho} \int dx \int_0^\beta dt \left[ \frac{(\partial_t \phi_\rho)^2}{v_{F\rho}^2} + (\partial_x \phi_\rho)^2 \right], \quad (5)$$

$$S_0^{(\sigma)} = \int dx \int_0^\beta dt \left\{ \frac{v_{F\sigma}}{2\pi g_\sigma} \left[ \frac{(\partial_t \phi_\sigma)^2}{v_{F\sigma}^2} + (\partial_x \phi_\sigma)^2 \right] + \frac{2g_{1\perp} D^2}{(2\pi v_F)^2} \cos[\sqrt{8}\phi_\sigma(x, t)] \right\}. \quad (6)$$

Here, the charge  $\phi_\rho = (\phi_\uparrow + \phi_\downarrow)/\sqrt{2}$  and spin  $\phi_\sigma = (\phi_\uparrow - \phi_\downarrow)/\sqrt{2}$  degrees of freedom are separated.  $v_F$  is the Fermi velocity in the noninteracting system, while  $v_{F\rho}$  and  $v_{F\sigma}$  are the interaction renormalized Fermi velocities of charge and spin modes [45]. The dimensionless charge and spin Luttinger parameters  $g_\rho$  and  $g_\sigma$  characterize the  $g_{4,\parallel}$ ,  $g_{4,\perp}$ ,  $g_{2,\parallel}$ ,  $g_{2,\perp}$ ,  $g_{1,\parallel}$  electron interaction processes [37,38,40]. From the theory of LL,  $0 < g_{\rho(\sigma)} < 1$  ( $g_{\rho(\sigma)} > 1$ ) describes 1D electrons with a repulsive (attractive) Coulomb interaction, and  $g_{\rho(\sigma)} = 1$  corresponds to the noninteracting situation. The prefactor  $g_{1\perp}$  in formula (6) characterizes the  $2k_F$  spin-flip backscattering in which the fermion fields with opposite spin projection values are coupled and they exchange sides of the Fermi surface after the interaction. Due to the fact that the  $g_{1\perp}$  process is not

quadratic in a bosonic representation, its effect is not included in the Luttinger parameters [see the third (cosine) term in Eq. (6)]. Therefore, the free action of the spin mode contains a massive term, in contrast to the massless charge excitation as shown in Eq. (5). The relevance of the mass can be studied through a *renormalization group* (RG) analysis of the sine-Gordon model (see, e.g., Refs. [39,40] for the details). The mass term of the spin mode is irrelevant if  $g_\sigma \geq 1$ , while it is relevant for  $g_\sigma < 1$ . In the above equations,  $D$  is an ultraviolet cutoff, which is related to the length parameter  $a = v_F/D$  in the LL-related literature [39,40], and  $\beta = 1/T$  (here we adopt units  $\hbar = k_B = 1$ ).

The Coulomb interaction in the QD is described by the Hamiltonian  $H_C = E_C[\hat{n} - N]^2$ , where  $E_C = e^2/2C$  is the charging energy ( $C$  is the QD capacitance), and  $\hat{n} = \hat{n}_L + \sum_{\alpha=\uparrow,\downarrow} \phi_\alpha(0, t)/\pi$  is the operator of the number of electrons entered through the tunnel barrier and the QPC, respectively [51];  $N$  is a dimensionless parameter proportional to the gate voltage  $V_g$ . Without loss of generality, the number of electrons entering the dot from the left electrode can be replaced by a time-dependent function  $n_\tau = \theta(t)\theta(\tau - t)$ , where  $\theta(t)$  is the Heaviside step function. Therefore, the Coulomb blockade action  $S_C$  in bosonic representation reads [28–31,35,36,51]

$$S_C = E_C \int_0^\beta dt \left[ n_\tau(t) + \frac{\sqrt{2}}{\pi} \phi_\rho(0, t) - N \right]^2. \quad (7)$$

Finally, the contribution  $S'$  in the action of the QD-QPC structure characterizes the weak backscattering at the QPC,

$$S' = -\frac{2D}{\pi} |r| \int_0^\beta dt \cos[\sqrt{2}\phi_\rho(0, t)] \cos[\sqrt{2}\phi_\sigma(0, t)], \quad (8)$$

where  $|r| \ll 1$  is a small reflection amplitude. Interestingly, one notices that both the  $g_{1\perp}$  interaction process in the LL and the backscattering (8) happen simultaneously at the QPC.

*Massless spin field.* We first study the situation in which the spin field  $\phi_\sigma$  is massless. In accordance with the RG analysis [40], it occurs when  $g_\sigma \geq 1$ .

In the absence of backscattering  $r = 0$ , the functional integral Eq. (4) is Gaussian. The correlator  $K_0(\tau) \equiv K(\tau)|_{r=0}$  is computed at low temperature  $T \ll E_C$  and at  $\tau \gg E_C^{-1}$ . The main contribution to the electric conductance is the zero-order term of the perturbation expression over the reflection amplitude  $|r|$  with the condition we will mention later. Therefore,

$$G = G_L C(g_\rho) \left( \frac{T}{g_\rho E_C} \right)^{\frac{1}{g_\rho}}, \quad (9)$$

with

$$C(g_\rho) = \frac{\sqrt{\pi}}{2} \left( \frac{\pi^2}{2\gamma} \right)^{1/g_\rho} \frac{\Gamma(1 + g_\rho/2)}{\Gamma(3/2 + g_\rho/2)}, \quad (10)$$

depends only on the value of the charge Luttinger parameter  $g_\rho$ ,  $\Gamma(y)$  is the gamma function,  $\gamma = e^C$ , and  $C \approx 0.577$  is Euler's constant. The electron interactions in the LL renormalize both the scaling of the conductance ( $G \propto T^{1/g_\rho}$ ) and the charging energy ( $g_\rho E_C$ ). Note that at  $r = 0$  the conductance depends only on the interaction in the charge mode through the parameter  $g_\rho$ . The integrals over the spin field  $\phi_\sigma$  are unaffected by  $n_\tau(t)$ , and the correlator  $K_0(\tau)$  is thus

independent from the free spin mode action in Eq. (6). If  $g_\rho = 1$ , we restore the result for the noninteracting case  $G = (\pi^3 G_L / 8\gamma)(T/E_C)$  as shown in Ref. [30]. The temperature scaling of the conductance in Eq. (9) is relevant with the results explained in Refs. [28,29,52–54]:  $G \propto T^{2/\mathcal{M}}$ , where  $\mathcal{M}$  is the number of channels in the charge Kondo effect (it is two, the number of electron's spin projection in FMF model or the number of the QPCs in the experimental integer quantum Hall setup [32,33,52]). In addition, the  $G \propto T^{1/g_\rho}$  scaling also represents the fact that there are no stable electronlike quasiparticles in the LL. As a consequence, the quasiparticle residue vanishes and the power-law behavior appears in many observables [46,55–57].

The thermoelectric coefficient  $G_T$  vanishes at the  $|r| = 0$  limit due to the electron-hole symmetry. A finite contribution in  $G_T$  is computed in perturbation theory over a small reflection coefficient  $|r| \ll 1$ . Expanding the partition function Eq. (4) over  $S'$ , we obtain  $K(\tau) = K_0(\tau)[1 + (\langle S'^2 \rangle_\tau - \langle S'^2 \rangle_0)/2]$ . One should notice that the fluctuations of the massless spin mode are not suppressed at low energies, and the average  $\langle S' \rangle$  vanishes. Thus, a nonvanishing backscattering correction to the correlation function appears only in the second order in  $|r|$ . The thermoelectric coefficient  $G_T$  is computed with logarithmic accuracy as

$$G_T = -\frac{G_L |r^*|^2}{e} A(g_\rho, g_\sigma) C_T(g_\rho, g_\sigma) \sin(2\pi N) \times \log\left(\frac{E_C}{T}\right) \left(\frac{T}{g_\rho E_C}\right)^{\frac{1}{g_\rho} + g_\sigma - 1}, \quad (11)$$

where  $|r^*| = |r|(g_\rho E_C / D)^{(g_\rho + g_\sigma)/2 - 1}$  is the interaction renormalized reflection amplitude [58], and the interaction-dependent prefactors,  $A(g_\rho, g_\sigma) = (2\gamma)^{g_\rho - \frac{1}{g_\rho}} \pi^{\frac{2}{g_\rho} - g_\rho + g_\sigma - 1}$ , and

$$C_T(g_\rho, g_\sigma) = \int_{-\infty}^{\infty} dz \frac{\sinh(z)}{[\cosh(z)]^{3 + \frac{1}{g_\rho}}} \{\tilde{F}(z_-) - \tilde{F}(z_+)\}. \quad (12)$$

Here, we define  $\tilde{F}(z) = e^{ig_\sigma(z - \pi/2 - i \ln 2)} \times {}_2F_1[g_\sigma/2, g_\sigma, (2 + g_\sigma)/2, e^{2iz}]/g_\sigma$ , where  ${}_2F_1(a, b, c, d)$  is the hypergeometric function, and  $z_\pm = \pi/2 \pm iz$ . The thermoelectric coefficient  $G_T$  shows the temperature-dependent  $T^{1/g_\rho + g_\sigma - 1} \log T$  scaling. In the noninteracting regime,  $g_\rho = g_\sigma = 1$ , it recalls the result  $G_T \propto T \log(T)$  in Ref. [31].

The effect of the electron interaction on  $G_T$  is threefold: (i) the power-law temperature dependence  $G_T \propto (T)^{1/g_\rho + g_\sigma - 1}$ ; (ii) the renormalization of the charging energy  $E_C \rightarrow g_\rho E_C$ ; and (iii) the renormalization of the weak scattering potential at the QPC  $r \rightarrow r^*$ . The last effect reveals the KFP [46]. The interaction renormalized reflection amplitude is consistent with the corresponding RG analysis showing that if the interaction in the LL ( $g_\rho, g_\sigma < 1$ ) is repulsive, the effective reflection amplitude increases, achieving the weak coupling limit ( $r^* \rightarrow 1$ ). On the contrary, the scattering at the QPC becomes irrelevant ( $r^* \rightarrow 0$ ) for the attractive interactions ( $g_\rho, g_\sigma > 1$ ).

Plugging Eqs. (9) and (11) into the definition formula of TP,  $S = G_T/G$ , we obtain

$$S = -\frac{|r^*|^2}{e} C_S(g_\rho, g_\sigma) \sin(2\pi N) \log\left(\frac{E_C}{T}\right) \left(\frac{T}{g_\rho E_C}\right)^{g_\sigma - 1}, \quad (13)$$

where  $C_S(g_\rho, g_\sigma) = A(g_\rho, g_\sigma) C_T(g_\rho, g_\sigma) / C(g_\rho)$ . The temperature scaling of TP,  $T^{g_\sigma - 1} \log T$ , in Eq. (13) vanishes when  $T \rightarrow 0$  for  $g_\sigma > 1$ . This zero-temperature vanishing characteristic is consistent with the corresponding nonperturbative scaling of the TP maximum for 2CK in Ref. [31]. In the charge Kondo effect the charge mode is always blockaded locally, while the spin mode usually fluctuates freely. The gapless spin mode is responsible for the NFL behavior. Therefore, only the spin Luttinger parameter appears in the  $T^{g_\sigma - 1} \log T$  scaling. The effect of the interaction in the spin mode becomes more dominant in the case when the spin mode is massive. In addition, the TP  $S$  in Eq. (13) diverges at zero temperature in the noninteracting spin field case  $g_\sigma = 1$ , showing the breakdown of the perturbation theory at sufficiently low temperature. Thus, the validity the perturbation theory is justified by the condition for temperature:  $|r^*|^2 g_\rho E_C \ll T \ll g_\rho E_C$  [59] if  $|r^*|^2 g_\rho E_C > v_F/L$ . Another dramatic manifestation of the LL properties in the behavior of TP is the appearance of the KFP through the renormalized reflection amplitude at the QPC  $|r^*|$ , which is completely different from the results shown in Refs. [60–63].

Note that Eqs. (9) and (11) can be obtained by applying the spatially inhomogeneous Green's function method for the finite LL wire in the so-called “high-temperature” regime  $v_{F\rho(\sigma)}/L \ll T \ll g_\rho E_C$  [64–66]. In fact, the theory of quantum transport in a 2CK-FMF model with a finite LL wire demonstrating the QPC vicinity is studied [67]. We find that the temperature scalings of the thermoelectric coefficients are independent of the LL length  $L$ . For the purpose of investigating the electron interaction effects, we focus on a discussion of the limit  $L \rightarrow \infty$ . The finite- $L$  effects will be considered elsewhere [67].

*Massive spin field.* In this section, we address the question of how the pinning potential of the spin mode [cosine term in Eq.(6)] affects the thermoelectric properties of the spinful LL-based QD-QPC structure in the case  $g_\sigma < 1$ . The saddle-point solution of the spin mode is  $\phi_{\sigma, \text{sp}} = 2\pi n / \sqrt{8}$  for  $g_{1\perp} < 0$ , while  $\phi_{\sigma, \text{sp}} = \pi / \sqrt{8} + 2\pi n / \sqrt{8}$  for  $g_{1\perp} > 0$  [40] ( $n$  is an integer number).

The free action of the spin fluctuations  $\varphi_\sigma = \phi_\sigma - 2\pi n / \sqrt{8}$  around the saddle-point solution reads as

$$S_0^{(\sigma)} = \int dx \int_0^\beta dt \left\{ \frac{v_{F\sigma}}{2\pi g_\sigma} \left[ \frac{(\partial_t \varphi_\sigma)^2}{v_F^2} + (\partial_x \varphi_\sigma)^2 \right] + \frac{2g_{1\perp} D^2}{\pi^2 v_F^2} \varphi_\sigma^2 \right\}. \quad (14)$$

From Eq. (14), the mass of the spin mode can be defined as  $M = 2D(v_{F\sigma}/v_F) \sqrt{g_\sigma |g_{1\perp}| / \pi v_{F\sigma}}$ . However, the correct value of  $M$  should be found self-consistently by applying Feynman's variational principle [40] or a more strict RG analysis of the sine-Gordon model [39,40]. Both methods give a more complicated dependence of the mass on the interaction constant and Luttinger parameter,  $M = D(v_{F\sigma}/v_F) (|g_{1\perp}| / \pi v_{F\sigma})^{1/(2-2g_\sigma)}$ , in comparison with the perturbative analysis. We will use the latter expression of  $M$  in this Letter.

The spin mode of the LL is now pinned at low energies, and the nonvanishing backscattering correction to the cor-

relation function  $K(\tau)$  is thus obtained in the first order of the perturbation theory over the small reflection amplitude ( $|r| \ll 1$ ):  $K(\tau) = K_0(\tau)[1 - \langle S' \rangle_\tau + \langle S' \rangle_0]$ . Straightforward calculations give the expression of the TP at low temperature  $T \ll M, g_\rho E_C$  as

$$S = -\frac{1}{e} |r^*| C_S^*(g_\rho) \sin(2\pi N) \frac{T}{g_\rho E_C} \left( \frac{M}{2\sqrt{2}g_\rho E_C} \right)^{\frac{g_\sigma}{2}}, \quad (15)$$

in which the interaction-dependent prefactor is

$$C_S^*(g_\rho) = \frac{\xi \pi^{4+g_\rho}}{C(g_\rho)} \left( \frac{2\gamma}{\pi^2} \right)^{\frac{g_\rho}{2} - \frac{1}{g_\rho}} \int_{-\infty}^{\infty} dy \frac{\sinh^2(y)}{\cosh^{4+\frac{1}{g_\rho}}(y)}, \quad (16)$$

where  $\xi \approx 1.59$  [31,53]. Similar to the way that the Coulomb blockade acts on the charge fluctuations, the existence of the finite mass in the spin mode suppresses its fluctuations around the saddle point at low energies, and the TP Eq. (15) is thus proportional to  $|r^*|$  and temperature. The ratio of two “masses”  $M/g_\rho E_C$  determines the strength of TP at a given spin mode Luttinger parameter  $g_\sigma$ . In fact, the backscattering at the QPC, which determines the efficiency of the thermoelectric transport, is similar to the backward interaction  $g_{1\perp}$  process of the LL. Therefore, the influence of the  $g_{1\perp}$  process on the TP is dominant.

Equations (13) and (15) represent the central results of this Letter. Interestingly, the TP for the massless spin mode case depends on temperature nonmonotonically [ $S \propto T^{g_\sigma-1} \log T$ , with  $g_\sigma \geq 1$ , as shown in Eq. (13)] while the TP for the massive spin mode case with  $g_\sigma < 1$  is proportional to the temperature [ $S \propto T$ , as shown in Eq. (15)]. The former shows the NFL property characterizing the 2CK, while the latter shows the FL picture of the 1CK. What physical quantities control this crossover from 2CK to 1CK? Notably different from the fact that a finite *external* magnetic field breaks the symmetry of the up-spin and down-spin as explained in Ref. [36], our current results show that the relevant backward  $g_{1\perp}$  scattering process in the LL (at  $g_\sigma < 1$ ) induces the instant asymmetry of these two Kondo channels at the QPC. This Kondo channel symmetry breaking induces the crossover from 2CK to 1CK. An alternative point of view

for the 2CK-1CK crossover in this work can be represented as follows: In the charge Kondo effect the charge mode is always blocked (locally) while the spin mode is usually unblocked. It refers to the gapless spin mode. This gapless mode results in the local NFL property of 2CK, in contrast to the local FL appearing in the 1CK regime. If the spin mode is additionally gapped (e.g., either by a “trivial” Zeeman effect or by “nontrivial” many-body effects in the LL), the local NFL state is destroyed.

It was argued (see, e.g., Ref. [68]) that interactions in a QPC are the source of the so-called “0.7 anomaly” and Kondo-like effects are invoked for those explanations [68], but recent studies show no link between the Kondo effect and the 0.7 anomaly [69]. In this work, we deal with the mesoscopic Coulomb blockade which assumes a weak charge quantization and does not pronounce conductance steps. However, reducing interactions in the QPC is helpful for the enhancement of TP and the protection of the NFL properties. Therefore, it is necessary to fabricate a clean ballistic QPC [70].

*Conclusions.* In this Letter, we have investigated theoretically the influence of the electron interactions in the LL-based QD-QPC structure on the two-channel charge Kondo problem. Using the Abelian bosonization technique and calculating the thermoelectric coefficients perturbatively with respect to the reflection amplitude at the QPC, we predict the low-temperature scaling behavior of the Seebeck coefficient as  $S \propto T^{g_\sigma-1} \log T$  for the massless spin mode case and  $S \propto T$  for the massive one. We predict that the relevance of the backscattering  $g_{1\perp}$  process induces a universal crossover from NFL-2CK to FL-1CK. It opens an interesting possibility for investigating the crossover between multi- and single-channel Kondo regimes in experiments.

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- [45] The effective Fermi velocity  $v_{F\alpha}$  and the dimensionless Luttinger-liquid parameter  $g_\alpha$  for the charge ( $\alpha = \rho$ ) and spin ( $\alpha = \sigma$ ) modes are defined as  $v_{F\alpha} = \sqrt{(2\pi v_F + g_{4\alpha})^2 - g_{12\alpha}^2}/2\pi$ ,  $g_\alpha = \sqrt{(2\pi v_F + g_{4\alpha} + g_{12\alpha})/(2\pi v_F + g_{4\alpha} - g_{12\alpha})}$ , where  $v_F$  is the Fermi velocity,  $g_{4,\alpha} = g_{4,\parallel} \pm g_{4,\perp}$ , and  $g_{12\alpha} = g_{1,\parallel} - g_{2,\parallel} \mp g_{2,\perp}$  [40]. Here, the upper and lower signs stand for  $\alpha = \rho$  and  $\sigma$ , respectively.  $v_{F,\alpha} = v_F$  and  $g_\alpha = 1$  in the absence of electron-electron interactions.
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- the interaction is assumed to be pointlike. However, taking into account the finite size of the Coulomb interaction  $\tilde{a}$  will result in replacing  $D/v_F \rightarrow \tilde{a}^{-1}$ . Thus, the value of the renormalized reflection coefficient remains as the fitting parameter.
- [59] The validity of the perturbation theory for the  $G_T$  in the massless spin field case should be determined from Eq. (11). Thus, the lower bound value for the Luttinger parameter  $g_\rho^{\min}$  is found using the following condition:  $|r^*|^2 A(g_\rho^{\min}, g_\sigma) C_T(g_\rho^{\min}, g_\sigma) \sim 1$ . Assuming  $|r| = 0.2$  and  $E_C/D = 0.1$  we estimate numerically  $g_\rho^{\min} \approx 0.51$ . An analogous calculation for the massive spin mode case gives  $g_\rho^{\min} \approx 0.63$ .
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