Letter Editors' Suggestion

Strong-coupling emergence of dark states in XX central spin models

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It was recently shown that the XX central spin model is integrable in the presence of a magnetic field perpendicular to the plane in which the coupling exists. A large number of its eigenstates are such that the central spin is not correlated to the environmental spins it is coupled to. In this work, we first demonstrate that the XX-central spin model remains integrable in the presence of an arbitrarily oriented magnetic field. We then show that, provided the coupling is strong enough, dark states can actually be found even in the presence of an in-plane magnetic field. We finally provide a simple explanation of this result and demonstrate its universality for a variety of distinct distributions of the coupling of the central spin to the various bath spins.

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I. INTRODUCTION

The integrability of the XX central spin model in the presence of a z-axis magnetic field,

$$\hat{H} = B_0^z\hat{S}_0^z + \sum_{k=1}^{N-1}\Gamma_k\left(\hat{S}_0^x\hat{S}_k^x + \hat{S}_0^y\hat{S}_k^y\right), \tag{1}$$

has recently been demonstrated [1]. Moreover, it was shown that a fraction of its eigenstates can be characterized as dark states for which the central spin $\hat{S}_0$ remains, independently of the magnitude of the coupling to the $k = 1\dots N-1$ bath spins, in an eigenstate $|\uparrow\rangle$ or $|\downarrow\rangle$ of the magnetic field part of the Hamiltonian, $B_0^z\hat{S}_0^z$, as if completely decoupled from the bath spins [1]. The idea was further studied by explicitly showing the existence of these dark states through a Bethe ansatz approach [2]. The dark state structure was also shown to be robust against certain perturbations [3].

In spin qubits, based on the spin of single electrons trapped in quantum dots [4], the coupling of the qubit to the bath of environmental spins is detrimental in that it ultimately leads to decoherence of the central spin and to the loss of the quantum information it should store [5–7]. Dark states (and dark subspaces of the Hilbert space) then become remarkably desirable since they can provide protection against these bath-induced negative effects leading to long-lived quantum states of the qubit [8,9] both in nitrogen-vacancy centers in diamond [10–12] and in semiconductor quantum dots [13–15]. This extreme anisotropy of the XX model, characterized by its lack of coupling along the z axis, can, when placing oneself in a properly chosen rotating frame, be used to describe nitrogen-vacancy centers [12] or in the context of more generic quantum chips [16].

II. INTEGRABILITY OF XX MODELS IN A GENERIC MAGNETIC FIELD

In this work we study the fate of the dark states in XX central spin models submitted to an arbitrarily oriented magnetic field. Adding XY-plane components to the magnetic field breaks the rotational U(1)-symmetry of the model (1) and therefore, the total z-axis magnetization of the system is no longer conserved and the eigenstates no longer have a fixed value of $\sum_{i=0}^{N-1}\hat{S}_i^z$. Nonetheless, they remain integrable, a fact that we now demonstrate by taking an appropriate limit of the integrable N spin-1/2 nonskew symmetric elliptic Richardson-Gaudin (RG) models. The latter are defined by a set of N operators [17,18]:

$$\hat{R}_j = \sum_{k=0(\neq j)}^{N-1}\left(\Gamma_{jk}^x\hat{S}_j^x\hat{S}_k^x + \Gamma_{jk}^y\hat{S}_j^y\hat{S}_k^y + \Gamma_{jk}^z\hat{S}_j^z\hat{S}_k^z\right) + B_j^x\hat{S}_j^x + B_j^y\hat{S}_j^y + B_j^z\hat{S}_j^z, \tag{2}$$

where, in any given direction $\alpha \in \{x, y, z\}$, the coupling constants and magnetic field components, are given by

$$\Gamma_{jk}^\alpha = g\frac{\sqrt{(\epsilon_j + j_\alpha)(\epsilon_k + j_\beta)(\epsilon_k + j_\gamma)}}{\epsilon_k - \epsilon_j},\quad B_j^\alpha = \frac{B^\alpha}{\sqrt{\epsilon_j + j_\alpha}} \tag{3}$$

with β and γ the two directions orthogonal to α. These models are known to be integrable in that, for arbitrary values of the parameters ϵ_a, j_α, and B^α, the N operators (2) commute with one another and consequently share a common set of eigenstates.

In order to reach the XX model limit, we first choose $j_y = j_x = j_\perp$ which leads to an XXZ model where the couplings along both the x and y directions are equal. Taking now the specific value $\epsilon_0 = -j_z + \Delta$, choosing $B^z \equiv B_0^z\sqrt{\Delta}$ and, finally, taking the $\Delta \to 0$ limit of the operator $\hat{R}_0$, one then finds that the XX model, in an arbitrarily oriented magnetic field

$$\hat{H} \equiv \lim_{\Delta\to 0} \hat{R}_0 = B_0^z \hat{S}_0^z + B_0^x \hat{S}_0^x + B_0^y \hat{S}_0^y + \sum_{k=1}^{N-1} \Gamma_k \left(\hat{S}_0^x \hat{S}_k^x + \hat{S}_0^y \hat{S}_k^y\right) \tag{4}$$

is also integrable. It remains so for arbitrary components of the magnetic field since B_0^z, $B_0^x \equiv \frac{B^x}{\sqrt{j_\perp - j_z}}$, and $B_0^y \equiv \frac{B^y}{\sqrt{j_\perp - j_z}}$ can all be chosen freely, independently of one another. The $N-1$ values of the resulting couplings in $\hat{H}$ are given by $\Gamma_k \equiv g\frac{\sqrt{(j_\perp - j_z)(\epsilon_k + j_\perp)(\epsilon_k + j_z)}}{\epsilon_k + j_z}$ and can therefore all be arbitrarily chosen, while maintaining integrability, using the $N-1$ free parameters ϵ_k. By taking the same limit for the other $\hat{R}_{j>0}$ operators, one directly shows that the Hamiltonian (4) commutes with the $N-1$ following conserved charges:

$$\hat{R}_j = \frac{B^x}{\sqrt{\epsilon_j + j_\perp}} \hat{S}_j^x + \frac{B^y}{\sqrt{\epsilon_j + j_\perp}} \hat{S}_j^y + \sum_{\alpha}^{x,y,z} \sum_{k\neq 0,j}^{N-1} \Gamma_{jk}^\alpha \hat{S}_j^\alpha \hat{S}_k^\alpha - g\frac{\sqrt{(j_\perp - j_z)^2(\epsilon_j + j_z)}}{\epsilon_j + j_z} \hat{S}_0^z \hat{S}_j^z, \tag{5}$$

with the values of Γ_{jk}^α given by Eq. (3) with $j_y = j_x = j_\perp$.

The RG conserved charges (2) obey simple quadratic relations [19,20] which can provide access to the spectrum and ultimately to the expectation values of local spin operators. Indeed, for an eigenstate labeled $|\psi_n\rangle$, the set of N eigenvalues $(r_0^n \cdots r_{N-1}^n)$ such that $\hat{R}_j|\psi_n\rangle = r_j^n|\psi_n\rangle$ corresponds to one of the solutions of the system:

$$[r_j^n]^2 = -\frac{1}{2}\sum_{k\neq j} C_{jk} r_k^n + \sum_{\alpha}\sum_{k\neq j}\left[\frac{\Gamma_{jk}^\alpha}{4}\right]^2 + \sum_{\alpha}\left[\frac{B_j^\alpha}{2}\right]^2, \tag{6}$$

with $C_{jk} = -g\frac{\sqrt{(\epsilon_k + j_\perp)^2(\epsilon_k + j_z)}}{\epsilon_k - \epsilon_j}$, and $\epsilon_0 = -j_z$ so that $C_{j0} = 0$.

Specific solutions to this system can be found numerically by smoothly deforming a given $g = 0$ solution to the desired coupling amplitude g [21]. Those $g = 0$ solutions are simply the various tensor products $\bigotimes_{i=0}^{N-1} |\pm_i\rangle$ built from the eigenstates $|\pm_i\rangle$ of $\hat{R}_i^{g=0} = \vec{B}_i \cdot \vec{\hat{S}}_i$. This allows us to index individual eigenstates, at finite coupling, simply by specifying the $g = 0$ parent state which is deformed into it. These will be denoted by the sequence of $g = 0$ eigenstates, ordered by spin index from left to right. For example, $|+--+\dots\rangle$ means the central spin S_0 is in the eigenstate of $\vec{B}_0 \cdot \vec{\hat{S}}_0$ with eigenvalue $+\frac{1}{2}$, spin S_1 in the eigenstate of $\vec{B}_1 \cdot \vec{\hat{S}}_1$ with eigenvalue $-\frac{1}{2}$, and so on.

To access the expectation values $\langle\psi_n|\hat{S}_j^\alpha|\psi_n\rangle$, one can use the Hellmann-Feynman theorem which expresses them in terms of derivatives of the eigenvalues with respect to the Hamiltonian's parameter. In the specific problems of interest here, one has

$$\langle \hat{S}_j^{x,y}\rangle = \sqrt{\epsilon_j + j_\perp}\,\frac{\partial r_j^n}{\partial B^{x,y}} \quad \text{with } \epsilon_0 = -j_z,$$
$$\langle \hat{S}_0^z\rangle = \frac{\partial r_0^n}{\partial B_0^z};\quad \langle \hat{S}_{j\neq 0}^z\rangle = \sqrt{\epsilon_j + j_z}\,\frac{\partial}{\partial B_0^z}\left[\frac{r_j^n}{\sqrt{\Delta}}\right], \tag{7}$$

where the expression for $\langle \hat{S}_{j\neq 0}^z\rangle$ requires taking the derivative of r_j^n with respect to B^z, before taking the $\Delta \to 0$ limit. Once the values of r_j^n, defining an eigenstate, have been found, these derivatives are accessible by solving the linear system of equations obtained after taking the derivatives of (6) with respect to the parameter of interest [18].

III. DARK STATES

As mentioned in the Introduction, dark states are known to occur in the XX central spin model subjected to a z-oriented magnetic field. They are characterized by the central spin being, for any coupling amplitude, exactly in one of the two eigenstates of the $B_z\hat{S}_0^z$ operator [1]. They can, therefore, in general, be written as a tensor product $|\uparrow_0\rangle \otimes |\psi_n^{\text{bath}}\rangle$ (or$|\downarrow_0\rangle \otimes |\psi_m^{\text{bath}}\rangle$) where the various possible states of the bath spins can be found as a solution to a set of reduced Bethe equations [2].

If this product state structure can be maintained in the presence of in-plane components of the magnetic field, it could possibly lead to a dark state in which the central spin would be in a well defined eigenstate of the arbitrarily oriented $\vec{B}_0 \cdot \vec{\hat{S}}_0$. On the other hand, the mechanism through which dark states arise could require that the magnetic field be orthogonal to the XY plane in which the coupling exists. In this case, an XY plane component of the magnetic field would be sufficient to correlate the central spin and the bath preventing the appearance of dark states. In this work, we will show that dark states can actually arise in the presence of an in-plane magnetic field, but that they will require a strong enough coupling to do so.

Here we chose to work with a definition of a dark state which would be valid for an arbitrarily oriented central spin. We simply require a tensor product structure making the reduced density matrix of the central spin describe an arbitrary pure state. On the other hand, a generic (bright) eigenstate of this coupled system would, typically, lead to a reduced density matrix for which the central spin is in a statistically mixed state. Differentiating between dark and bright states is then possible by computing the quantity $\gamma_0 = \langle\hat{S}_0^x\rangle^2 + \langle\hat{S}_0^y\rangle^2 + \langle\hat{S}_0^z\rangle^2$, we will dub the purity factor. Indeed, $\gamma_0 = \frac{1}{4}$ for any central spin pure state, $\alpha|\uparrow_0\rangle + \beta|\downarrow_0\rangle$, and systematically gets reduced ($\gamma_0 < \frac{1}{4}$) when the central spin is entangled with the bath leading to a mixed state. This generalizes the simple definition $|\langle S_0^z\rangle| = \frac{1}{2}$ used, for example, in [3] and provides an easy-to-compute alternative to the entanglement entropy.

The Hamiltonian (4) can, of course, be divided by any constant without affecting its eigenstates which will only depend on the ratio between the coupling and the magnetic field. We will therefore rely on the rescaled coupling $\tilde{g} \equiv \frac{1}{|B|}\sum_{j=1}^{N-1}\Gamma_j$ to characterize the system. Having zero coupling is therefore equivalent to having an infinite magnetic field and vice versa. In the zero-coupling limit (or infinite field limit), the various spins are independent of one another and, for every eigenstate, they all are found in a pure state. In this particular limit we will therefore trivially always find $\gamma_0 = \frac{1}{4}$. Alternatively, when the coupling is strong enough for $|B|$ to be negligible, we find ourselves in the zero-field limit. The $\gamma_0 = \frac{1}{4}$ dark states found in the absence of in-plane field still exist when $B_z = 0$ and should therefore be found, in the current problem,

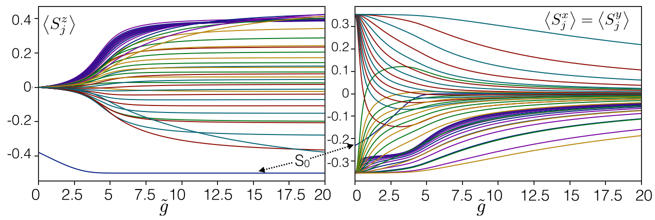

FIG. 1. Expectation values of the $N = 42$ spins in the dark state resulting from the deformation of the $g = 0$ state $|--+--+\ldots\rangle$. The coupling constants are given by $\Gamma_j = 7.254\sqrt{N-j}$ for $j = 1\ldots N-1$, and the magnetic field is oriented at azimuthal angle $\theta \approx \frac{\pi}{4}$ ($B_0^x = B_0^y = 2.23$; $B_0^z = 3.162$).

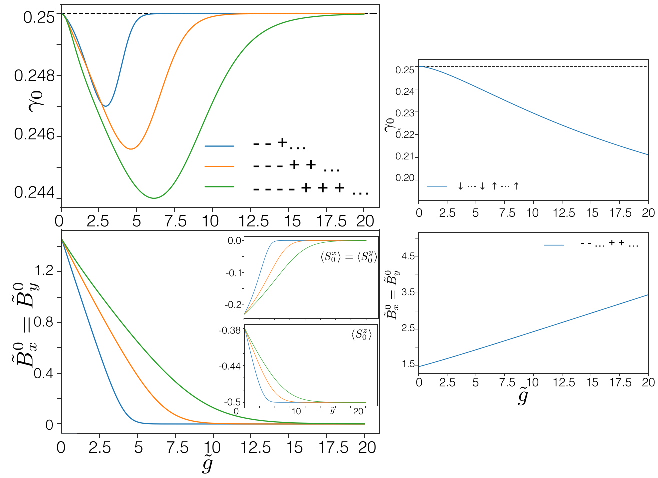

FIG. 2. The purity factor γ_0 of the central spin (top left panel) and the effective in-plane magnetic field acting on the central spin (bottom left panel) in three dark states ($g = 0$ parent states given by: $|--+--+--+\ldots\rangle$, $|---++----++\ldots\rangle$, and $|-----+++------+++\ldots\rangle$). Insets: Expectation values of the central spin components. Smaller figures on the right show a bright state

when $|B| = 0$. In light of these two particular limiting points, a complete working definition of a dark state therefore requires the existence of an extended range of finite rescaled coupling $\tilde{g}$ for which an eigenstate has $\gamma_0 = \frac{1}{4}$. Here, finite rescaled coupling means that neither the magnetic field term nor the coupling to the bath are perturbatively small in front of one another.

In Fig. 1 we first present the expectation values of the z and the in-plane x and y (both identical by symmetry since the external field is chosen with $B_x = B_y$) in the eigenstate whose $g = 0$ parent state is given by an alternance of two negative and one positive eigenvalue: $|--+--+--+\ldots\rangle$. They are presented, for every spin in the system, as a function of the rescaled coupling $\tilde{g}$.

At $\tilde{g} = 0$, the central spin S_0 is naturally in the pure state which is the eigenstate of $\vec{B}_0 \cdot \hat{\vec{S}}_0$ with eigenvalue $-\frac{1}{2}$. As the coupling is increased, we see that it finally reaches the down pointing state (eigenstate of $B_z S_0^z$ with eigenvalue $-\frac{1}{2}$) at large enough coupling. This resulting strong coupling state is therefore also a pure state of the central spin, oriented just like the dark states found for the U(1)-symmetric case (1) with a z-oriented magnetic field.

The bath spins, at $\tilde{g} = 0$, all start as $\pm$ eigenstates of the $\vec{B}_j \cdot \hat{\vec{S}}_j$ and are therefore polarized in the XY plane since the magnetic fields $\vec{B}_j$ have $B_j^z = 0$ [see Eq. (5)]. As the coupling gets stronger, we see that they get tilted out of that plane and gain an important z-component expectation value. The central spin has reached its strong coupling z-polarized state at $\tilde{g} \approx 5$ for the parameters used in this calculation. However, the bath spins evolve much more slowly with coupling since, at the same value of $\tilde{g}$, they are still significantly changing with increasing coupling. In the end, at strong enough coupling, their XY-plane component finally decreases as $\frac{1}{\tilde{g}}$ as will be shown when discussing Fig. 2. One should notice that, at $\tilde{g} \approx 5$, the B_0^x, B_0^y components of the magnetic field are, in no way, negligible when compared to the total coupling to the bath. Nonetheless, the central spin appears to already reproduce the dark state behavior expected in the absence of those in-plane components of the external field.

In order to better understand the physics at play we present, in Fig. 2, the γ_0 purity factor for four different eigenstates using the same set of parameters as for Fig. 1. Three of them (including the state detailed in Fig. 1) are presented in the top left panel and form a dark state at strong coupling. The fourth state, shown in the smaller figures on the right, remains a bright state for all finite coupling as demonstrated by the fact that $\gamma_0 < \frac{1}{4}$.

One notices that the states which ultimately become dark states at strong coupling, see their "darkness" reduced in the weak to intermediate coupling regime. The central spin is then, as soon as $\tilde{g} \neq 0$, in a statistically mixed state, i.e., not in a pure state on the Bloch sphere. Adding an in-plane magnetic field to (1) suppresses the dark state structure over a range of relatively weak couplings. Nonetheless, dark states reemerge at strong enough coupling and, as noted before, they do so even when the in-plane field is not negligible in front of the coupling term. This seems to point to the existence of a mechanism which allows the central spin not to feel the in-plane magnetic field. A clear understanding of the process involved in this reemergence of dark states can actually be obtained, as we show next.

For the three dark states, we plot in the bottom left panel the effective mean-field in-plane magnetic field felt by the central spin when considering the Overhauser field contribution of the bath spins. By replacing, in a typical mean-field approach, the bath spins by their expectation values, one can indeed build an effective Hamiltonian for the central spin which is simply given by

$$H_{\text{eff}} = \vec{B}_0 \cdot \hat{\vec{S}}_0 + g\sum_{k=1}^{N-1} \tilde{\Gamma}_k \big[\hat{S}_0^x \langle \hat{S}_k^x \rangle + \hat{S}_0^y \langle \hat{S}_k^y \rangle\big]$$
$$= \tilde{\vec{B}} \cdot \hat{\vec{S}}_0, \tag{8}$$

where the various effective magnetic field components are given by

$$\tilde{B}^{x,y} = B_0^{\alpha} + g\sum_{k=1}^{N-1} \tilde{\Gamma}_k \langle \hat{S}_k^{x,y} \rangle; \quad \tilde{B}^z = B_0^z, \tag{9}$$

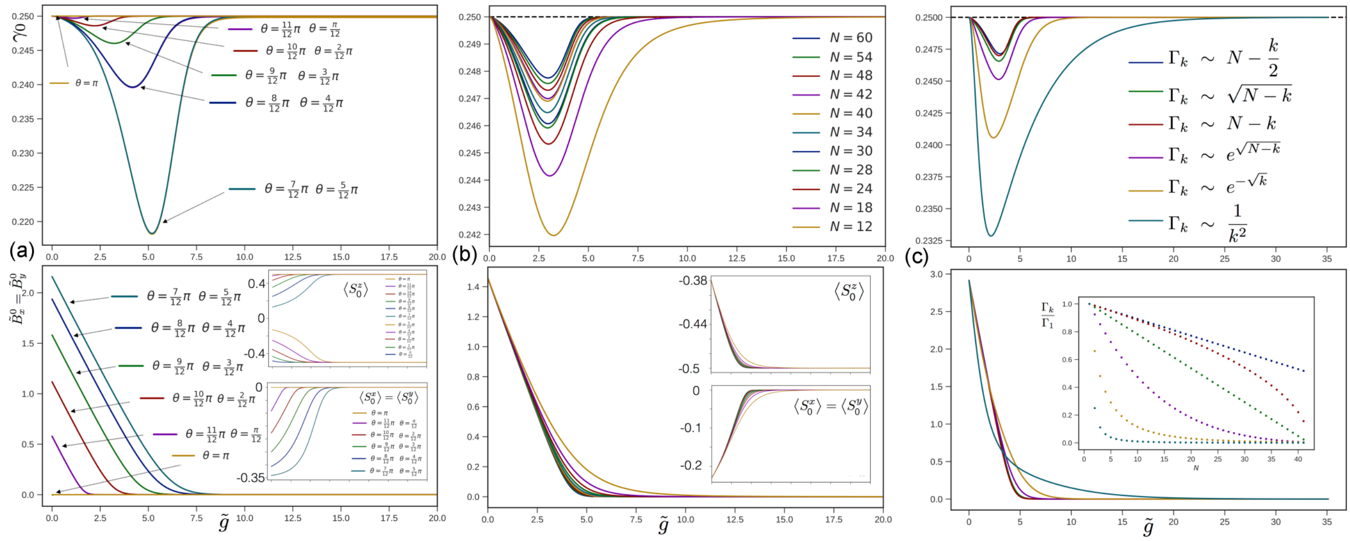

FIG. 3. Purity factors γ_0 of the central spin (top) and effective in-plane field (bottom) as a function of the rescaled coupling strength. Panel (a): For a variety of magnetic field orientations $B_0^z = |B|\cos(\theta)$, $B_0^x = B_0^y = \frac{|B|}{\sqrt{2}}\sin(\theta)$, keeping a fixed norm $|B|$. Panel (b): For a variety of system sizes. Panel (c): For a variety of coupling distributions always with $N = 42$ spins and the same total coupling $\sum_{k=1}^{N-1}\Gamma_k$. Inset: Distribution of coupling constants.

having defined $\Gamma_k \equiv g\tilde{\Gamma}_k$. Judging from these figures, one can clearly see that the return to a pure state ($\gamma_0 = 0.25$) is perfectly correlated with the coupling strength at which the effective in-plane components of the field reach $\tilde{B}_x = \tilde{B}_y = 0$. The cancellation of the effective in-plane magnetic field (9) is then maintained over the whole range of large couplings. This indicates that, as was mentioned previously, the expectation value of the in-plane components of the bath spins becomes, at large coupling, linear in $\frac{1}{g}$ (and therefore in $\frac{1}{\tilde{g}}$ as well).

This cancellation can only occur when the coupling is strong enough for the in-plane magnetic field: $|B|\sin(\theta)$ to be lower than or equal to $\frac{1}{2}|\sum_{k=1}^{N-1}\Gamma_k|$. This value corresponds to the maximal possible in-plane Overhauser field which can be found when all of the bath spins are perfectly aligned and completely in the XY plane. In terms of the rescaled coupling, this corresponds to $\tilde{g} = 2\sin(\theta)$ which is then the minimal coupling required for the effective in-plane field to possibly cancel. This is a much less stringent requirement on the coupling strength than $\tilde{g} \ll 1$, which would be required for the magnetic field to be negligible. It is therefore only through this effective cancellation of the in-plane field by the Overhauser contribution that we can explain why dark states are found, even at a coupling strength too low for the field to be negligible.

In order to confirm this picture, we present in Fig. 3(a) the purity factor and effective magnetic field for a fixed magnitude of the external field $\vec{B}_0$ but for different values of its azimuthal angle θ. One sees that the range of coupling strength over which the purity factor of the central spin state is lower than $\frac{1}{4}$ becomes wider with increasing angle. These results are completely consistent with the physical picture proposed in this work, namely, that dark states reemerge, at large enough coupling, when the Overhauser field cancels the in-plane applied field which gets larger with large θ. In the weak coupling regime where dark states are suppressed, the deeper dip in the purity factor of the central spin indicates that the entanglement between the central spin and the bath gets larger for more strongly titled magnetic fields. Since the total magnetic field has the same magnitude for each of the curves, one clearly understands that the underlying physics is exclusively controlled by the magnitude of its in-plane components. While the in-plane field component could, in principle, be canceled at the minimal coupling strength $\tilde{g} = 2\sin(\theta)$, doing so would require that the bath spins be all completely aligned in the XY plane. Since, in reality, the bath spins retain a nonzero expectation value $\langle S_k^z\rangle$ along the z axis, the cancellation happens at slightly stronger coupling than this minimal value. Nonetheless, as we saw previously, dark states still emerge at couplings considerably lower than what would be required for the in-plane field to become negligible.

In Fig. 3(b) the eigenstate which was presented in Fig. 1 is now shown for a variety of bath sizes. The coupling constants are given by $\Gamma_k = C_N\sqrt{N-k}$ where C_N are chosen in such a way that the total coupling to the bath $\sum_{k=1}^{N-1}\Gamma_k$ stays the same for every system size N. One finds that the dip in purity is systematically limited to a similar range of rescaled couplings. However, the depth of this dip is reduced by increasing the number of bath spins. This allows us to infer that, in the thermodynamic limit $N \to \infty$, for this particular distribution of couplings, the dark state structure should (as is the case in the absence of in-plane magnetic field) be maintained for arbitrary values of the coupling strength $\tilde{g}$. This demonstrates that the factor controlling the purity dip is not, in itself, the total coupling to the bath but actually the way this coupling is spread out over the different spins in the bath.

We therefore present, in Fig. 3(c) results for different coupling constant distributions, all characterized by the same total coupling and the same number of spins. These results first demonstrate that the reemergence of dark states, at strong enough coupling, is a universal feature of the model;

it happens for every distribution of couplings. However, the distribution of couplings does have an impact on the range of coupling strengths over which we deviate from a perfect dark state and the importance of this deviation. The results are highly reminiscent of the finite size scaling shown in the middle panel (b) and one can conclude that the purity dip is in fact controlled by an effective number of active bath spins. Spins which are too weakly coupled to the central spin do not participate significantly to the Overhauser field and to the cancellation of the in-plane magnetic field. The thermodynamic limit discussed before will therefore depend on the nature of the coupling distribution. When the couplings decrease faster than $\frac{1}{k}$, the sum $\lim_{N\to\infty}\sum_{k=1}^{N-1}\frac{\Gamma_k}{\Gamma_1}$ does not diverge at large N. In these cases, going to large N only adds more weakly coupled spins which will not contribute appreciably. As $N\to\infty$, the system will still behave as a smaller finite sized system and there will remain, at weak coupling, a range over which the central spin will deviate from a pure state. Dark states will then only appear at a larger value of $\tilde{g}$. This fact was numerically verified explicitly for the $\frac{1}{k^2}$ distribution, for which the results stop changing significantly for $N > 14$. On the other hand, if, as in panel (b), the couplings decrease slower than $\frac{1}{k}$, the dip will resorb in the thermodynamic limit since any additional spin will have a significant contribution to the cancellation of the applied field. Dark states should then exist for any coupling strength.

In this work we have first shown that the XX central spin model remains integrable in the presence of XY-plane components of the magnetic field. Dark states, which are found for any coupling strength in the absence of in-plane field were then shown to get suppressed at weak couplings. However, at strong enough coupling their reemergence appears to be a universal feature of the model. It can be understood as a consequence of the restructuration of the bath spins such that the resulting mean-field Overhauser field exactly cancels out the in-plane magnetic field. The resulting dark states are such that the central spin is aligned with the z direction just like in the absence of in-plane field. Due to the underlying restructuration of the bath required here, it remains an open question whether dark states will remain stable against perturbatively small zz couplings, as is the case in the absence of in-plane field [3]. What we can globally conclude from this work is that, in what seems like a paradoxical statement, in the XX central spin model with in-plane field, one must increase the coupling to decouple the central spin from its environment.

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