Letter

Nonreciprocal Meissner response in parity-mixed superconductors

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Parity breaking gives rise to rich superconducting properties through the admixture of even- and odd-parity Cooper pairs. New light has been shed on parity-breaking superconductors by recent observations of nonreciprocal responses such as nonlinear optical responses and the superconducting diode effect. In this Letter, we demonstrate that nonreciprocal responses are characterized by a unidirectional correction to the superfluid density, which we call nonreciprocal superfluid density. This correction leads to the nonreciprocal Meissner effect, namely, the asymmetric screening of magnetic fields due to the nonreciprocal magnetic penetration depth. Performing a microscopic analysis of an exotic superconductor UTe_2 and examining the temperature dependence and renormalization effect, we show that the nonreciprocal Meissner effect is useful to probe parity-mixing properties and gap structures in superconductors.

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Introduction. Nonlinear and nonreciprocal responses have recently attracted interest in various fields of condensed matter physics. For instance, second harmonic generation and photocurrent creation have been applied to a probe of the symmetry breaking in matter and the topological nature of electrons [1–3].

Recent studies explored the nonreciprocal responses in superconductors. Nonreciprocal electric conductivity [4], a rectified conductivity originating from the parity violation, is strongly enhanced by the superconducting fluctuation [5,6] and by the vortex dynamics [7,8]. Furthermore, recent efforts have clarified the nonreciprocal superconducting phenomena such as the nonreciprocal critical current [9–13] and Josephson current [12,14,15]. The nonreciprocal critical current realizes the superconducting diode effect, indicating that the electrical resistivity is zero in a direction while finite in the opposite direction. Nonreciprocal optical responses have also been observed in superconducting systems whose parity violation stems from the spontaneous order intertwined with superconductivity [16–18] or an injected supercurrent [19–21]. Building on the superconducting properties, nonreciprocal phenomena imply richer functionalities.

There is a symmetry requirement of nonreciprocal responses which is unique to superconductors. In addition to the space-inversion ($\mathcal{P}$) symmetry breaking, its combination with the gauge symmetry has to be broken. In particular, odd-parity superconductivity is insufficient to cause a nonreciprocal response because the above symmetry holds. Superconductors must have no definite parity under the $\mathcal{P}$ operation to host nonreciprocal responses, implying a *parity-mixed superconducting state*. Conversely, the nonreciprocal response may be an indicator of the parity mixing in superconductors as proposed in the prior study of nonreciprocal conductivity in the fluctuation regime [5–7]. This potential indicator of a parity-mixed superconducting state may allow us to identify the relation between the parity violation and superconducting symmetries which has been intensively investigated with noncentrosymmetric superconductors such as $CePt_3Si$ [22].

Considering the high interest in the research community, it is desirable to further explore the nonreciprocal properties of superconductors. While previous theoretical studies have focused on dc or low-frequency charge transport [5–7,10–12], the nonreciprocal nature may appear in the other responses as well. Recently, the authors have identified anomalous contributions to nonlinear optical conductivity in superconductors, which diverges in the low-frequency limit [23], as is the case for linear optical conductivity [24]. This in turn implies an inherent directionality in the Meissner response, i.e., the *nonreciprocal Meissner effect.* Since the Meissner effect plays a central role in superconductivity, clarifying the nonreciprocal Meissner effect may contribute to a deeper understanding of the parity-breaking superconducting states as well as nonreciprocal responses.

This Letter consists of two parts. First, we show that the anomalous nonlinear conductivity arising from parity mixing leads to a unidirectional correction to the rigidity of the superconducting state, which we call *nonreciprocal superfluid density*. Various nonreciprocal responses of superconductors are characterized by the nonreciprocal superfluid density. Second, to demonstrate the exotic phenomena arising from the nonreciprocal superfluid density, we elaborate the nonreciprocal property of the Meissner effect (Fig. 1). With the microscopic analysis implementing the model of a candidate for the parity-mixed superconductor UTe_2, we show that the nonreciprocal Meissner effect is sensitive to the parity violation and hence applicable to the detection of the parity-mixed superconducting state.

Nonreciprocal property of the superfluid density. The nonreciprocal electric conductivity of superconductors has

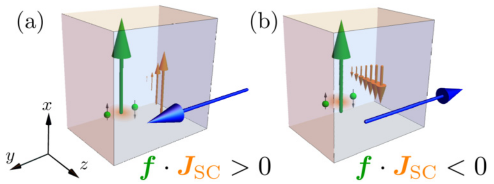

FIG. 1. Nonreciprocal Meissner effect. The nonreciprocal superfluid density $\boldsymbol{f}$ (green arrow) (a) strengthens or (b) diminishes the supercurrent $\boldsymbol{J}_{\mathrm{SC}}$ (orange arrows) shielding the external magnetic field (blue arrow).

recently been formulated [23]. The leading nonreciprocal correction to the electric current is given by the second-order component

$$J_\alpha^{(2)}(\omega) = \int \frac{d\Omega}{2\pi}\sigma_{\alpha;\beta\gamma}(\omega,\Omega)E_\beta(\Omega)E_\gamma(\omega-\Omega). \tag{1}$$

In the low-frequency limit, we obtain the nonlinear optical conductivity

$$\begin{aligned}\sigma_{\alpha;\beta\gamma}(\omega,\Omega) =& \frac{f_{\alpha\beta\gamma}}{2\Omega(\omega-\Omega)} \\ & -\frac{i}{4}\lim_{\boldsymbol{A}\to\boldsymbol{0}}\left(\frac{1}{\Omega}\partial_{A_\beta}\sigma_{\alpha\gamma}^{(\boldsymbol{A})}+\frac{1}{\omega-\Omega}\partial_{A_\gamma}\sigma_{\alpha\beta}^{(\boldsymbol{A})}\right).\end{aligned} \tag{2}$$

The diverging terms proportional to Ω^{-2} or Ω^{-1} are unique to the superconducting state while they are forbidden in the normal state [25]. Thus, we call the terms anomalous contributions. We suppressed $O(\Omega^0)$ terms comprising nondivergent nonlinear optical conductivity since it is negligible in the low-frequency regime.

The anomalous nonreciprocal conductivity in Eq. (2) is determined by the nonreciprocal superfluid density $f_{\alpha\beta\gamma}$ (NRSF) and the conductivity derivative $\partial_{A_\gamma}\sigma_{\alpha\beta}^{(\boldsymbol{A})}$. The NRSF is given by

$$f_{\alpha\beta\gamma} = \lim_{\boldsymbol{A}\to\boldsymbol{0}}\partial_{A_\alpha}\partial_{A_\beta}\partial_{A_\gamma}F_A, \tag{3}$$

where F_A is the free energy obtained from the Hamiltonian containing the vector potential $\boldsymbol{A}$. Since the superfluid density is given by $\rho^{\mathrm{s}}_{\alpha\beta} = \lim_{\boldsymbol{A}\to\boldsymbol{0}}\partial_{A_\alpha}\partial_{A_\beta}F_A$, the NRSF is regarded as a unidirectional correction to ρ^{s}. When the superfluid density is isotropic $\rho^{\mathrm{s}}_{\alpha\beta} = \rho^{\mathrm{s}}\delta_{\alpha\beta}$, the NRSF is recast as the vector $\boldsymbol{f}$ which has the same symmetry as the electric current and the toroidal moment [26–28]. Thus, the NRSF is allowed in the absence of both $\mathcal{P}$ and time-reversal ($\mathcal{T}$) symmetries.

In the second term of Eq. (2), $\sigma_{\alpha\beta}^{(\boldsymbol{A})}$ denotes the regular part of the linear static conductivity calculated with the Bogoliubov–de Gennes Hamiltonian including $\boldsymbol{A}$. Its derivative $\partial_{A_\gamma}\sigma_{\alpha\beta}^{(\boldsymbol{A})}$, which we call a conductivity derivative, is decomposed into symmetric and antisymmetric parts in terms of the permutation of indices (α,β) for $\sigma_{\alpha\beta}^{(\boldsymbol{A})}$. The symmetric part of $\sigma_{\alpha\beta}^{(\boldsymbol{A})}$ corresponds to the Drude contribution, while the antisymmetric part is the Berry curvature term. Thus, we call the corresponding components in $\partial_{A_\gamma}\sigma_{\alpha\beta}^{(\boldsymbol{A})}$ the Drude and Berry curvature derivatives, which vanish in $\mathcal{T}$ and $\mathcal{PT}$ symmetric parity-mixed superconductors, respectively. From Eq. (2), we see that the nonreciprocal optical responses such as photocurrent creation ($\omega = 0$, $\Omega \neq 0$) and second harmonic generation ($\omega = 2\Omega$) show a prominent divergent behavior in the low-frequency regime, which is unique to superconductors [23].

TABLE I. Classification of the nonreciprocal Meissner kernel $K^{(2)}$ based on $\mathcal{T}$ and $\mathcal{PT}$ symmetries. The $O(\Omega^n)$ contributions allowed by the symmetry are summarized.

$O(\Omega^n)$	$\mathcal{T}$	$\mathcal{PT}$
$n=0$	N/A	NRSF
$n=1$	Berry curvature deriv.	Drude deriv.
$(n\geqslant 2)$	(regularized nonlinear conductivity)	

Recalling the minimal coupling between the electrons and electromagnetic field, the vector potential twists the phase of the superfluid and plays the same role as the supercurrent. Thus, when the electromagnetic perturbation is weak, the NRSF determines the nonreciprocal component of the supercurrent induced by a given phase twist. This is confirmed by the argument of the adiabatic process [23]. In the Josephson junction, the phase twist is similarly accumulated through the junction bridging the superconducting leads, and the NRSF also participates in the nonreciprocal Josephson current [12,14].

According to the Ginzburg-Landau analysis, both the NRSF and the nonreciprocal critical current are attributed to the cubic gradient component of the quadratic term as well as the linear gradient component of the quartic term [11,29]. Therefore, the NRSF provides a systematic understanding of various nonreciprocal responses in superconductors, including the optical response, Josephson effect, and critical current. This is similar to the case of conventional superfluid density, which determines the anomalous linear optical conductivity, Meissner effect, and zero-resistance phenomenon [24,30].

Nonreciprocal Meissner effect. We now transform the conductivity into the susceptibility

$$J_\alpha(\omega) = K^{(1)}_{\alpha\beta}A_\beta(\omega) + \int\frac{d\Omega}{2\pi}K^{(2)}_{\alpha;\beta\gamma}(\omega,\Omega)A_\beta(\Omega)A_\gamma(\omega-\Omega). \tag{4}$$

The anomalous nonreciprocal conductivity contributes to the response function $K^{(2)}$ as

$$2K^{(2)}_{\alpha;\beta\gamma}(\omega,\Omega) = -2\Omega(\omega-\Omega)\sigma_{\alpha;\beta\gamma}(\omega,\Omega), \tag{5}$$

$$= -f_{\alpha\beta\gamma} + \frac{i}{2}\lim_{\boldsymbol{A}\to\boldsymbol{0}}\left[(\omega-\Omega)\,\partial_{A_\beta}\sigma^{(\boldsymbol{A})}_{\alpha\gamma} + \Omega\,\partial_{A_\gamma}\sigma^{(\boldsymbol{A})}_{\alpha\beta}\right]. \tag{6}$$

The anomalous conductivity determines the low-frequency behaviors of the nonlinear coupling between the vector potential and electric current. This indicates that the NRSF causes the nonreciprocal property of the Meissner response, that is, the nonreciprocal Meissner effect. Note that the nonreciprocal Meissner effect is unique to the parity-breaking superconductors and distinguished from the *nonlinear* Meissner response [31,32], which is reciprocal in terms of magnetic fields. Although the conductivity derivative may participate in the ac nonreciprocal Meissner response, we hereafter focus on the static response determined by the NRSF. Classification of the nonreciprocal Meissner kernel $K^{(2)}$ based on the $\mathcal{T}$ and $\mathcal{PT}$ symmetries is summarized in Table I.

We phenomenologically introduce a nonlinear correction to the London theory by

$$J_\alpha(\boldsymbol{r}) = -\rho^s_{\alpha\beta}A_\beta(\boldsymbol{r}) - f_{\alpha\beta\gamma}A_\beta(\boldsymbol{r})A_\gamma(\boldsymbol{r}), \tag{7}$$

where the first and second terms are normal and nonreciprocal supercurrents, respectively. Here, we consider a superconductor occupying the spatial region $z \leqslant 0$ and the NRSF vector $\boldsymbol{f} \parallel \hat{x}$ (Fig. 1). When the magnetic field is applied to the y direction and the nonreciprocal effect is assumed to be small, the field-dependent magnetic penetration depth is estimated as $\lambda(B) = \lambda_L(1 + \lambda_L Bf/3\rho^s)$ with the London penetration depth $\lambda_L^{-1} = \sqrt{\mu_0\rho^s}$. Thus, it shows the unidirectional magnetic-field dependence. Intuitively, the magnetic flux is retracted from or drawn into the superconductor when the supercurrent shielding the magnetic flux is parallel or antiparallel to the NRSF vector $\boldsymbol{f}$. Thus, a careful magnetic penetration depth measurement can evaluate the NRSF [33].

The $\mathcal{P}$ and $\mathcal{T}$ symmetries have to be broken in superconductors that host the NRSF. To our best knowledge, three setups are available: (i) systems in which $\mathcal{P}$ and $\mathcal{T}$ symmetries are broken by other spontaneous orders or by the crystal structure and external fields, (ii) superconductors under supercurrent flow, and (iii) exotic superconductors whose order parameter spontaneously breaks the symmetries. Case (i) is realized in various situations, such as noncentrosymmetric superconductors under external magnetic fields [3] and the superconductors undergoing magnetically parity-breaking order [34,35]. Interestingly, case (ii) was recently supported by an experiment where the superconducting NbN thin film was probed under an electric current by second harmonic generation [20]. Case (iii) is further classified into two classes. First, the multiple transitions of even-parity and odd-parity superconductivity make both $\mathcal{P}$ and $\mathcal{T}$ parities ill defined [36]. Second, these symmetries are broken by chiral superconductivity and a noncentrosymmetric crystal structure [37]. Later we will investigate the former class in (iii) by referring to the recent proposal for a heavy fermion superconductor UTe_2 [38].

We discuss the magnitude of the nonreciprocal Meissner response by the ratio

$$\eta_{NR} = \frac{\lambda_L B_{c2} f}{\rho^s}, \tag{8}$$

where B_{c2} is the upper critical magnetic field. First, our analysis based on the Ginzburg-Landau theory shows $\eta_{NR} \propto |T - T_c|^{1/2}$ [29], and thus the nonreciprocal response may be negligible in the vicinity of the transition temperature $T \lesssim T_c$. Next, we estimate the renormalization effect on the ratio η_{NR}. Interestingly, the correlation-induced renormalization effect denoted by the inverse mass-renormalization factor z positively influences the nonreciprocal property of the Meissner response. Since ρ^s, $f \propto z$, $\lambda_L \propto z^{-1/2}$, and $B_{c2} \propto z^{-2}$, the ratio is strongly enhanced by the correlation effect as much as $\eta_{NR} \propto z^{-5/2}$ [39]. Thus, strongly correlated electron systems are potential candidates offering a pronounced nonreciprocal property of the Meissner response. While we will work on a heavy fermion system UTe_2 in the following, another strongly correlated electron systems such as cuprate superconductors and twisted bilayer graphene are also of interest. The cuprates are usually centrosymmetric in the bulk, whereas the parity violation can be evoked by spontaneous ordering [case (i)] and by a supercurrent injection [case (ii)]. As for the former case, loop-current order has been proposed for the pseudogap phase in cuprate superconductors [16,17,40–44]. The NRSF may be a long-sought probe for examining such intertwining order in cuprates.

Microscopic calculations of the UTe_2 *model.* UTe_2 has recently attracted a lot of attention as a candidate material for spin-triplet superconductivity [45,46]. It is argued that the ferromagnetic fluctuation plays a key role in the spin-triplet superconductivity as in ferromagnetic superconductors [45–49], whereas antiferromagnetic fluctuation has also been observed recently [50–54]. Since the antiferromagnetic fluctuation usually stabilizes even-parity spin-singlet superconductivity, multiple magnetic fluctuations are expected to lead to multiple pairing instabilities [38]. Interestingly, UTe_2 shows multiple superconducting transitions [55]. Thus, a coexisting even- and odd-parity pairing state has been proposed for the low-temperature phase [38].

Based on a microscopic model, we investigate the NRSF in the putative parity-mixed phase of UTe_2. The model tight-binding Hamiltonian for the normal state reads

$$H^N_{\boldsymbol{k}} = (\varepsilon_0 - \mu) + V\rho_x + V'\rho_y + \boldsymbol{g}\cdot\boldsymbol{\sigma}\rho_z, \tag{9}$$

which is described by Pauli matrices representing spin (σ_μ) and sublattice (ρ_μ) degrees of freedom. The details are given in the Supplemental Material [29]. The model Hamiltonian reproduces the heavy band mainly consisting of U $5f$ orbitals near the Fermi level, which was obtained in the Density functional theory with a Hubbard-U like correction (DFT+U) calculations [56,57]. We introduce a pair potential for parity-mixed superconductivity

$$\hat{\Delta}_{\boldsymbol{k}} = (\psi_{\boldsymbol{k}} + \boldsymbol{d}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma})i\sigma_y\rho_0, \tag{10}$$

where we consider intrasublattice Cooper pairing. According to theoretical calculations implementing the Eliashberg theory and the DFT+U calculation, even-parity pairing is characterized by the A_g irreducible representation ($\psi_{\boldsymbol{k}} = \Delta_e \cos k_x$), while the odd-parity pairing is either of the A_u or B_{3u} type denoted by $\boldsymbol{d}_{\boldsymbol{k}} = \Delta_o \sin k_y\hat{y}$ or $\Delta_o \sin k_y\hat{z}$, respectively [38]. It is energetically favorable for the relative phase between the pair potentials to be $\pm\pi/2$, when the spin-orbit coupling due to noncentrosymmetric crystal structures is absent or weak. This choice leads to $s + ip$-wave superconductivity preserving the $\mathcal{PT}$ symmetry [28,36]. This case contrasts with the fact that the spin-orbit coupling in a noncentrosymmetric superconductor leads to the zero phase difference indicating a $\mathcal{T}$ symmetric state such as $s + p$-wave superconductivity [22]. Since UTe_2 crystallizes in a centrosymmetric structure, we take the $\mathcal{PT}$ symmetric mean field $\Delta_e^{(0)} = r\Delta_{e+o}^{(0)}$, $\Delta_o^{(0)} = i(1-r)\Delta_{e+o}^{(0)}$ with the parity-mixing ratio r. Here, we denote the pair potentials at zero temperature by those with the superscript "(0)."

Following the symmetry analysis, we obtain the NRSF f_{xyz} for the $A_g + iA_u$ state and f_{xxx}, f_{xyy}, f_{xzz} for the $A_g + iB_{3u}$ state. Here, we investigate the temperature and parity-mixing ratio dependence of the NRSF f_{xxx} in the $A_g + iB_{3u}$ state in detail, while we obtain a similar result for the $A_g + iA_u$ state [29]. The temperature dependence of the pairing potential is

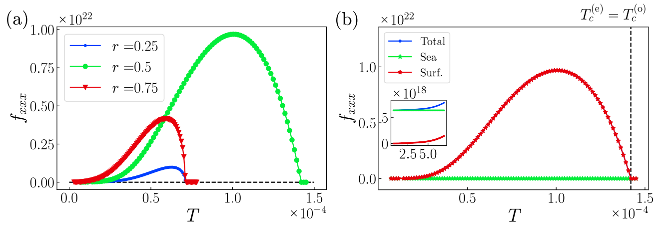

FIG. 2. Temperature dependence of the NRSF f_{xxx} for the $A_g + iB_{3u}$ superconducting state. (a) Plot with several ratios of even- to odd-parity pair potentials, $r = 0.25$, 0.5, and 0.75. (b) Decomposition of the total NRSF into the Fermi-sea and Fermi-surface terms in the case of $r = 0.5$. The dashed line guides the transition temperature $T_c^{(e)} = T_c^{(o)} = 0.5\Delta^{(0)}_{e+o}/1.76$. The inset shows the low-temperature regime with the horizontal axis $T \times 10^6$.

assumed to follow the phenomenological formula

$$\Delta_{e,o}(T) = \Delta^{(0)}_{e,o} \tanh\left(1.74\sqrt{\frac{|\Delta^{(0)}_{e,o}|}{1.76T} - 1}\right). \tag{11}$$

Figure 2 shows the NRSF calculated with several parity-mixing ratios $r = 0.25, 0.5, 0.75$. We do not have any NRSF in the pure spin-singlet or spin-triplet state ($r = 1, 0$) where the $\mathcal{P}$ and $U(1) \times \mathcal{P}$ symmetry respectively forbid the NRSF. In Fig. 2, it is clearly shown that the NRSF arises in the parity-mixed superconducting state. Each plot shows the maximum value at an intermediate temperature. This is because the NRSF is almost determined by the Fermi-surface contribution [Fig. 2(b)] given by

$$-\frac{1}{2}\sum_a \left(J^x_{aa}\right)^3 \frac{\partial^2 f(\varepsilon)}{\partial \varepsilon^2}\bigg|_{\varepsilon=\varepsilon_a}, \tag{12}$$

where $\boldsymbol{J}_{aa}$ is the paramagnetic current density of the Bogoliubov quasiparticle labeled by the quantum number a. On the other hand, the Fermi-surface term gets suppressed at low temperature, and then the Fermi-sea term mainly contributes to the NRSF [inset of Fig. 2(b)]. Decomposition of the NRSF into the Fermi-surface and Fermi-sea terms is formulated in the Supplemental Material [29]. Since the Fermi-surface contribution is much larger than the Fermi-sea contribution, the NRSF shows a nonmonotonic temperature dependence. The sizable Fermi-surface contribution is attributed to the almost nodal superconducting gap. To support this argument, we show that the Fermi-surface term is negligible in a superconducting state with a nearly isotropic gap [29]. Therefore, a significant NRSF and its nonmonotonic temperature dependence are characteristic behaviors of nodal superconductors, and they are useful in identifying the nodal texture in the superconducting gap.

The present study clarified that the Fermi-surface effect is much more significant than the Fermi-sea effect. The behavior is in contrast to the normal superfluid density, which is usually determined by the Fermi-sea effect and detrimentally influenced by the Fermi-surface effect. Since the quasiparticle excitations moderately occur in the intermediate temperature regime, the NRSF $f_{\alpha\beta\gamma}$ as well as the ratio η_{NR} in Eq. (8) are enhanced there.

To estimate the ratio η_{NR} of our model, we first take $\rho^s_{xx} \sim 10^{19}$ A V^{-1} m^{-1} s^{-1} [29], $|f_{xxx}| \sim 10^{22}$ A^2 V^{-2} s^{-2}, and $B_{c2} \sim 1$ T. The superfluid density leads to the penetration depth $\lambda_L \sim 0.3$ μm. Then, we obtain $\eta_{NR} \sim 3 \times 10^{-4}$. The ratio may increase due to the renormalization effect by electron correlations. The adopted model Hamiltonian is based on the DFT+U calculation and does not sufficiently take into account the electron correlation effect of UTe_2. We consider the renormalization factor $z = 0.1$, which enhances the London penetration depth as much as the observed values $\lambda_L \sim 1$ μm [49,58,59]. The adopted renormalization factor is moderate compared to those for prototypical heavy fermion superconductors [60–62]. Then, the ratio η_{NR} increases by ~300 times larger than the above estimation. The enhanced ratio $\eta_{NR} \sim 10^{-1}$ may be within the experimental sensitivity.

Discussion and summary. This work reveals that the NRSF plays an essential role in various nonreciprocal responses of superconductors such as optical responses, supercurrent flow, and Meissner effect. Thus, the NRSF is a potential indicator of nonreciprocal responses and is helpful to probe the parity mixing in a superconducting state, which causes parity breaking required for the nonreciprocal superconducting phenomena. It is noteworthy that the NRSF can be estimated through various experimental techniques implemented in nonlinear optics and magnetic penetration depth measurements. We also note that the NRSF is well defined in the whole temperature region and complementary to the fluctuation-assisted normal nonlinear conductivity, which captures a nonreciprocal response in the vicinity of the transition temperature [5–7]. Furthermore, we clarified that the electron correlation and quasiparticle excitations play essential roles on the NRSF, although these aspects have not been noticed in prior studies of superconducting nonreciprocal responses.

We have mainly discussed the NRSF and resulting nonreciprocal Meissner response in an exotic superconductor UTe_2, where multiple spin fluctuations may lead to spontaneously parity-mixed superconductivity. Although vast experimental works have been devoted, the phase diagram and superconducting symmetry remain unidentified [55,63–67]. The nonreciprocal Meissner response is sensitive to the parity violation and pronounced in the presence of a strong correlation, and therefore it is useful to determine the superconducting symmetry of UTe_2. Notably, $s + ip$-wave superconductivity, which breaks both $\mathcal{P}$ and $\mathcal{T}$ symmetries, can be detected by the nonreciprocal Meissner response, while it is inaccessible by magneto-optical probes such as the polar Kerr rotation measurement [37,64,68,69] due to the $\mathcal{PT}$ symmetry.

Our formulation can apply to the NRSF in a broad class of superconductors with broken $\mathcal{P}$ and $\mathcal{T}$ symmetries. For instance, noncentrosymmetric superconductors under an external magnetic field and some classes of magnetic superconductors satisfy the symmetry condition. As artificially engineered noncentrosymmetric superconductors have realized sizable nonreciprocal transport phenomena in recent experimental works [9,14], it is also expected that the nonreciprocal properties of optical phenomena and the Meissner response due to the NRSF will be observed.

The dynamical nonreciprocal response is also of interest, while we mainly focused on the static nonreciprocal Meissner response in this study. The low-frequency behavior of the nonreciprocal Meissner kernel $K^{(2)}$ is closely related to the space-time symmetry as shown in Table I; $K^{(2)}$ has a static component in the $\mathcal{PT}$ symmetric parity-mixed superconductor, whereas it contains a Ω linear term as a leading order term in the $\mathcal{T}$ symmetric superconductor. Thus, a careful ac experiment on the magnetic penetration depth may distinguish the symmetry of parity-mixed superconductors.

To summarize, we proposed that the NRSF provides a systematic understanding of nonreciprocal responses in superconductors. Accordingly, we clarified the nonreciprocal magnetic penetration phenomenon, which we call the nonreciprocal Meissner effect. According to the calculation for UTe_2, the nonreciprocal Meissner effect mainly arises from the Fermi-surface effect and is sensitive to the superconducting gap structure and electron correlation effect. Therefore, the phenomenon is expected to be a key to identify the symmetry of exotic superconductors such as UTe_2.

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