Letter

Electronic pumping of heat without charge transfer

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(Received 19 October 2021; accepted 8 February 2022; published 22 February 2022)

A mechanism of electron-mediated pumping of heat in the absence of net charge transfer is proposed. It may be realized in charge-neutral electron systems, such as graphene, coupled to an external electric potential. The flow of heat in this pumping cycle is not accompanied by a buildup of voltage along the system, which offers advantages over traditional thermoelectric cooling setups. The efficiency of heat pumping and the magnitude of heat flux are studied in the hydrodynamic regime for weak disorder. It is shown that the cycle efficiency may approach the Carnot limit. In a pristine system, even for an infinitesimal pumping potential, the heat flux remains finite. In particular, for a potential in the form of a traveling wave moving with velocity c, the pumping is perfect; the entire heat content of the electron liquid is advected with velocity c. For a general pumping cycle the heat flux is determined by the cycle geometry and disorder strength.

DOI: 10.1103/PhysRevB.105.L081410

In typical heat pumping cycles the heat is transferred between hot and cold reservoirs convectively by some physical substance—the working body, or coolant. Convection enables the rapid transfer of heat between spatially separated reservoirs. In contrast, nonconvective spreading of heat relative to physical substances typically proceeds via a much slower diffusion process.

However, the diffusive character of the spreading of heat through matter is not universal; under some conditions the propagation of heat through physical substances may become ballistic. The well-known examples are second sound waves [1] in superfluid ^{4}He,[1] and pure crystals in the regime of phonon hydrodynamics [3]. In contrast to the usual adiabatic sound, in which entropy and matter move together, the temperature oscillations in the second sound wave are practically decoupled from the density oscillations. They correspond to temperature/entropy waves that move ballistically through matter. Crystallization waves in ^{4}He [4] represent another example of decoupling between the flows of order and matter. In particular, a fall of a ^{4}He crystallite in superfluid ^{4}He proceeds partly via melting at the top and crystallization at the bottom of the crystallite [5]. As a result the crystalline order propagates at a faster speed than the matter itself.

The ballistic propagation of heat can also be realized in electronic systems [6] in the regime of electron hydrodynamics [7–9]. In systems with equal densities of electrons and holes (e.g., in graphene at charge neutrality) the hydrodynamic flow of the electron liquid is decoupled from charge flow and corresponds to the flow of heat (see Ref. [10] for a recent review). A hydrodynamic regime of charge-neutral electron liquid has been realized recently in monolayer and bilayer graphene [11,12], and may be realized in clean semimetals.

The ballistic spreading of heat through matter presents an enticing possibility to design novel heat pumps, in which the pumping of heat is accomplished in the absence of matter flow. Such a pumping cycle requires an ability to induce and guide the flow of heat through the system by coupling it to external perturbations. In this respect electronic systems seem particularly promising because the flow of electrons can be readily controlled by time-dependent gate voltages, thus obviating the need for mechanically moving parts.

The pumping of charge in microelectronic devices has been extensively studied for more than four decades. This work was largely stimulated by Thouless' prediction of the quantization of pumped charge in adiabatic quantum pumps [13]. The quantization of pumped charge in Coulomb blockade devices [14,15] is used in metrological applications, e.g., the modern realization of a capacitance standard [16,17]. The pumping of charge in open mesoscopic systems [18–21], and its accuracy [22–25] were also extensively studied.

The linear coupling between the heat and charge current, i.e., the Peltier effect [26], enables using electron pumps for heat pumping. The thermoelectric effects in microelectronic devices in the Coulomb blockade regime [27–29] and in the regime of electron hydrodynamics [9,10,30–32] have been studied. Traditional thermoelectric cooling corresponds to essentially convective heat transfer in which the electron liquid is used as a coolant.

The goal of this Letter is to show that the electronic pumping of heat of a principally different kind can be realized, in which the heat transfer proceeds in the absence of net charge flow. This pumping mechanism utilizes the decoupling of

[1]Recently, the ballistic spreading of heat was also predicted [2] to occur in quantum one-dimensional liquids at low temperature.

heat and charge flows in electron liquids at charge neutrality. As shown below, the coupling between these flows caused by deviations from charge neutrality can be used to pump the heat by electrostatic modulation of the system. In this pumping cycle the charge redistribution does not involve a net transfer of charge between the reservoirs, and serves only as an intermediary agent for electric coupling to heat and guiding its flow.

Let us consider a pumping setup, in which heat transfer is mediated by an electron liquid in a charge-neutral system, such as monolayer or bilayer graphene, subjected to an external electric pumping potential $U(\mathbf{r},t)$. The latter may be generated by applying a time-dependent voltage to a series of gates, or, for example, using the method developed in Ref. [33] to achieve quantized charge pumping in carbon nanotubes—by placing the system on a piezoelectric substrate driven by a surface acoustic wave (SAW).

The essential features of the pumping mechanism can be understood most easily for clean systems in the regime of electron hydrodynamics. Consider a unidirectional geometry with a periodic pumping potential of the form of a traveling wave, $U(x-ct)$. At zero temperature difference between the reservoirs, and for slow pumping velocities c, the electron liquid will remain in local thermal equilibrium corresponding to the instantaneous realization of the pumping potential $U(x-ct)$. Therefore the densities of electrons, $n(x,t)$, and entropy, $s(x,t)$, will be given by the equilibrium values corresponding to the local value of U. Since the latter moves with velocity c, the electron liquid will also move with the hydrodynamic velocity $u=c$. The local densities of charge and entropy of the electron liquid will propagate with the same velocity. Therefore, the entire heat content of the electron liquid will be entrained by this flow, producing a net heat flux density $T\bar{s}c$, where $\overline{\cdots}$ denotes the spatial average. In contrast, the net charge pumping current will vanish because of the vanishing average electron density at charge neutrality, $\bar{n}=0$.

In the presence of disorder and temperature gradient the entrainment of the electron liquid by the pumping potential will no longer be prefect; the pressure gradient proportional to the temperature gradient and the disorder-induced friction force will cause the electron liquid to lag behind the pumping potential. Evaluating the heat flux and pumping cycle efficiency in this case requires a quantitative theory.

Below, a theory of heat pumping at charge neutrality is developed in the regime of electron hydrodynamics. The hydrodynamic description applies provided the rate of momentum-conserving electron-electron collisions exceeds the momentum relaxation rate and the pumping frequency ω. Let us assume that the spatial scale of the pumping potential exceeds the correlation radius ξ of the disorder potential. In this case pumping may be described by averaging the flow of the electron liquid over length scales of order ξ [9,10,32]. For slow pumping the corresponding macroscopic hydrodynamic equations may be written in the form

$$\partial_t n + \nabla\cdot\mathbf{j} = 0, \tag{1a}$$

$$\partial_t \mathbf{p} + k\mathbf{u} + \nabla P + n\nabla(U+e\phi) = \nabla\cdot\hat{\sigma}', \tag{1b}$$

$$\partial_t s + \nabla\cdot\mathbf{j}_s = \dot{s}. \tag{1c}$$

Here, P is the pressure, $\mathbf{u}$ is the hydrodynamic velocity, $\hat{\sigma}'$ denotes the viscous stress tensor, and k denotes the disorder-induced "friction" coefficient. The densities of particles, entropy, and momentum are denoted by n, s, and $\mathbf{p}$, respectively. The electric potential ϕ is related to the electron charge density en by the Poisson equation. The current densities of particles $\mathbf{j}$ and entropy $\mathbf{j}_s$ may be expressed as

$$\begin{pmatrix}\mathbf{j}\\ \mathbf{j}_s\end{pmatrix} = \begin{pmatrix}n\\ s\end{pmatrix}\mathbf{u} - \begin{pmatrix}\sigma/e^2 & \gamma/T\\ \gamma/T & \kappa/T\end{pmatrix}\begin{pmatrix}-e\boldsymbol{\mathcal{E}}\\ \nabla T\end{pmatrix}, \tag{2}$$

where the first term on the right-hand side (rhs) represents the equilibrium components of the currents, while the second represents the dissipative components. The latter are linear in the temperature gradient ∇T and the electromotive force (emf) $e\boldsymbol{\mathcal{E}} = -\nabla(\mu+U+e\phi)$ (with μ being the chemical potential). The elements of the Onsager matrix of the intrinsic kinetic coefficients of the electron liquid are the electrical and thermal conductivities σ and κ, and thermoelectric coefficient γ. Finally, $\dot{s}$ is the entropy production rate per unit area, caused by dissipative processes in the electron liquid and loss of heat to the lattice.

Adiabatic pumping. Let us focus on the regime of slow pumping, where the heat flux is linear in the rate of change $\partial_t U$ of the pumping potential, and further assume that the temperature difference ΔT between the hot and cold reservoirs is small. In this case we may work within linear order accuracy in $\partial_t U$ and ΔT. Note that momentum density $\mathbf{p}$ is linear in these variables. Since its time derivative is further proportional to the pumping rate, $\partial_t\mathbf{p}$ is quadratic in $\partial_t U$ and ΔT. Similarly, the entropy production rate $\dot{s}$ is also quadratic in these variables. Therefore, in our approximation we may neglect the force density $\partial_t\mathbf{p}$ and entropy production rate $\dot{s}$ in Eq. (1), thereby reducing the entropy evolution equation (1c) to a continuity relation for the entropy current.

Let us consider a unidirectional geometry, in which a two-dimensional system of length L (in the x direction) and width w (in the y direction) is subjected to a periodic in space and time pumping potential of the form $U(x,t)=U(x+\lambda,t)=U(x,t+\tau)$. Being interested in the bulk effects we will evaluate the pumping heat flux per unit width of the system for $L\gg\lambda$. In this case without loss of generality we will set L/λ to be an integer, and impose at the reservoirs periodic boundary conditions on the system variables $e\mathcal{E}$, $\partial_x T$, and u.

Using the thermodynamic identity $dP = nd\mu + sdT$ and expressing the relevant component of the viscous stress tensor as $\sigma'_{xx} = (\eta+\zeta)\partial_x u$, where η and ζ are the shear and bulk viscosities, we obtain from Eq. (1b) the following force balance relation,

$$ne\mathcal{E} - s\partial_x T - [k - \partial_x(\eta+\zeta)\partial_x]u = 0. \tag{3}$$

The remaining two relations between u, $e\mathcal{E}$, and $\partial_x T$ are obtained by integrating the continuity equations (1a) and (1c) over x, and using Eq. (2):

$$nu + \sigma_0 e\mathcal{E} - \frac{\gamma\partial_x T}{T} = j(t) - \int_0^x d\tilde{x}\,\partial_t n(\tilde{x},t), \tag{4a}$$

$$su - \frac{\kappa\partial_x T}{T} + \frac{\gamma e\mathcal{E}}{T} = j_s(t) - \int_0^x d\tilde{x}\,\partial_t s(\tilde{x},t). \tag{4b}$$

Here, the integration constants $j(t)$ and $j_s(t)$ represent, respectively, the current densities of particles and entropy evaluated at the reservoirs, $x = 0$ and $x = L$ (due to the periodic boundary conditions the two are equal).

Equations (3) and (4) determine the hydrodynamic velocity u, temperature gradient $\partial_x T$, and emf $e\mathcal{E}$, which arise in the presence of pumping. They do not assume that the pumping potential is weak, only that the pumping, described by the integrals on the rhs of Eq. (4), is adiabatically slow. In the general case of a strong pumping potential not only the densities of particles and entropy, but also the kinetic coefficients depend on the pumping potential, and are given by their locally equilibrium values [9]. These equations must be supplemented by the boundary conditions for the temperature and electrochemical potential at the reservoirs. At charge neutrality they are given by $\overline{e\mathcal{E}} = 0$ (vanishing voltage bias), and $\overline{\partial_x T} = \Delta T/L$.

For simplicity, let us assume that both pumping potential U and disorder are small in comparison to T. We will work to lowest-order accuracy in $U/T \ll 1$. To this end we will approximate all quantities to leading order in U/T. Since the local electron density n is linear in the pumping potential, $n/s \propto U/T \ll 1$. The thermoelectric coefficient γ, being odd in n, is also linear in U. In contrast, deviations of the entropy density from its value at charge neutrality s_0 are quadratic in U/T and may be neglected. Similarly, the intrinsic electrical and thermal conductivities σ and κ may be replaced by their values at charge neutrality. Then it follows from Eq. (4b) that to within linear order accuracy in U/T the entropy flux j_s is spatially uniform, $j_s = j_s(t) = s_0 u - \kappa \partial_x T/T + \gamma e\mathcal{E}/T$. Excluding $\partial_x T$ from Eqs. (3) and (4b) we get

$$\left[\frac{Ts_0^2}{\kappa} + k - \partial_x(\eta + \zeta)\partial_x\right]u + \left[\frac{\gamma s_0}{\kappa} - n\right]e\mathcal{E} = \frac{Ts_0 j_s(t)}{\kappa}. \tag{5}$$

At charge neutrality the spatially uniform component of the emf vanishes, $\overline{e\mathcal{E}} = 0$, whereas the inhomogeneous part of $e\mathcal{E}$ is $\propto U/T$. Therefore, it follows from the above equation that to within linear order accuracy in U/T the hydrodynamic velocity is spatially uniform. Furthermore, the force balance equation (3) shows that to linear order in U/T the temperature gradient is also spatially uniform, $\partial_x T = \Delta T/L$, and is related to the hydrodynamic velocity $u(t)$ by

$$ku(t) = \overline{ne\mathcal{E}} - s_0\frac{\Delta T}{L}. \tag{6}$$

The second term on the rhs describes the force density caused by a thermally induced pressure gradient. This force is balanced by the disorder-induced friction force on the left-hand side (lhs) and the force exerted by the pumping potential (first term on the rhs). This force may be evaluated by finding the local emf from Eq. (4a). Isolating the inhomogeneous part of Eq. (4a) we obtain within our accuracy

$$e\mathcal{E}(x,t) = -\frac{e^2}{\sigma_0}\left[n(x,t)u(t) + \int_0^x d\tilde{x}\,\partial_t n(\tilde{x},t)\right]. \tag{7}$$

Substituting Eq. (7) into Eq. (6) and expressing the friction coefficient in terms of the variance $\langle(\delta n)^2\rangle$ of disorder-induced density modulations, $k = \frac{\sigma_0}{2e^2}\langle(\delta n)^2\rangle$ [11,32], we obtain the hydrodynamic velocity in the form

$$u(t) = -\frac{\frac{\sigma_0}{e^2}\frac{s_0\Delta T}{L} + \overline{n(x,t)\int_0^x d\tilde{x}\,\partial_t n(\tilde{x},t)}}{\frac{\langle(\delta n)^2\rangle}{2} + \overline{n^2(x,t)}}. \tag{8}$$

The entropy current may be expressed in terms $u(t)$ using Eq. (5). To leading (zeroth) order in U/T one obtains

$$j_s(t) = s_0 u(t), \tag{9}$$

which corresponds to the flow of entropy density s_0 of the electron liquid with velocity $u(t)$.

Equations (8) and (9) describe the flow velocity $u(t)$ of the electron liquid and entropy flux in the presence of pumping and temperature difference between the reservoirs. The flow is caused by the thermally induced pressure gradient $s_0\Delta T/L$ [first term in the numerator in Eq. (8)] and the force exerted on the liquid by the pumping potential (second term).

At $\Delta T = 0$ the flow velocity $u(t)$ is determined only by the pumping cycle parameters and disorder strength. This pumping contribution is given by

$$u_{\rm P}(t) = -\frac{\overline{n(x,t)\int_0^x d\tilde{x}\,\partial_t n(\tilde{x},t)}}{\frac{\langle(\delta n)^2\rangle}{2} + \overline{n^2(x,t)}}. \tag{10}$$

In particular, for a potential in the form of a traveling wave generated by SAW, $U(x,t) = U_0(x - ct)$, we get $u_{\rm P}^{\rm SAW} = c[1 + \langle(\delta n)^2\rangle/2\overline{n^2}]^{-1}$. At vanishing disorder this expression reproduces the expected result of perfect pumping, $u_{\rm P}^{\rm SAW} = c$.

Geometric pumping. At weak disorder, $\langle(\delta n)^2\rangle \ll \overline{n^2(x,t)}$, the pumping velocity in Eq. (10) becomes independent of the amplitude of the pumping potential for a general pumping cycle. In other words, it depends only on the shape of that pumping cycle $U(x,t)$. Let us denote this value by $u_{\rm P}^{(0)}(t)$. Using the Fourier series representation for the electron density, $n(x,t) = \sum_n c_n(t)e^{i\frac{2\pi n}{\lambda}x}$, the pumping velocity $u_{\rm P}^{(0)}(t)$ can be expressed as

$$u_{\rm P}^{(0)}(t) = \frac{-i\sum_n \frac{\lambda}{2\pi n}c_n^*(t)\partial_t c_n(t)}{\sum_n |c_n(t)|^2}, \tag{11}$$

and may be interpreted as an adiabatic connection [34] in the space of periodic functions with zero mean.

Average heat flux. Averaging the $u(t)$ in Eq. (8) over time we can write the average pumping heat flux in Eq. (9) in the form

$$T\,j_s = Ts_0\bar{\bar{u}}_{\rm P} - \kappa_{\rm eff}\frac{\Delta T}{L}, \tag{12}$$

where $\bar{\bar{u}}_{\rm P} = \frac{1}{\tau}\int_0^\tau dt\,u_{\rm P}(t)$ is the time-averaged velocity, and $\kappa_{\rm eff}$ represents the effective thermal conductivity of the system,

$$\kappa_{\rm eff} = \frac{T\sigma_0 s_0^2}{e^2}\frac{1}{\tau}\int_0^\tau \frac{dt}{\frac{\langle(\delta n)^2\rangle}{2} + \overline{n^2(x,t)}}. \tag{13}$$

It is inversely proportional to the variance of deviations of electron density from charge neutrality, which are caused by both disorder and the pumping potential. The factor 1/2 arises because disorder-induced density variations $\langle(\delta n)^2\rangle$ are statistically isotropic, whereas the density variations induced by the pumping potential depend only on the x coordinate. At

vanishing pumping potential Eq. (13) reproduces the result of Ref. [11].

Equations (10), (12), and (13) describe the average heat flux across the system. At $\Delta T = 0$ the pumping heat current is given by the first term on the rhs of Eq. (12). The pumping velocity of the electron liquid depends only on the pumping potential and disorder strength. At weak disorder, $\langle(\delta n)^2\rangle \ll \overline{n^2}$, the pumping velocity in Eq. (10) becomes independent of the amplitude of pumping potential, and is determined only by the pumping cycle geometry [see Eq. (11)]. The flow of heat due to $\Delta T \neq 0$ is described by the second term on the rhs of Eq. (12) and is proportional to the effective thermal conductivity of the system in Eq. (13). The latter also depends on the pumping potential. Note that, similar to peristaltic pumps, the pumping potential is distributed throughout the system. Therefore, the pumped heat flux [first term on the rhs of Eq. (12)] does not decrease with the system length L. In contrast, the flow of heat caused by the temperature difference ΔT [second term on the rhs of Eq. (12)] is inversely proportional to L. Therefore, for a fixed ΔT, cooling may be achieved even at low pumping strengths for sufficiently long systems.

Cycle efficiency. Let us now discuss the heat pump efficiency. To keep the expressions simpler we consider the traveling wave potential of the form $U_0(x - ct)$. The efficiency of the heat pumps is characterized by the coefficient of performance (COP), defined as the ratio of useful power to the power $\dot{W}$ consumed by the pump. For a cooling/heating cycle the useful power (per unit width of the system) is given by $T j_s = T s_0 u$, where T is the temperature of the cold/hot reservoir. The power $\dot{W}$ consumed by the pump may be obtained from a mechanical consideration. Using Eq. (7) it is easy to see that for a traveling wave potential $U_0(x - ct)$, the force density exerted by the pumping potential on the electron liquid in Eq. (6) has the form of a friction force, $\overline{ne\mathcal{E}} = \frac{e^2}{\sigma_0}\overline{n^2(x,t)}(u - c)$. Multiplying it by the velocity c and integrating over the system length, one finds

$$\dot{W} = L\frac{e^2}{\sigma_0}\overline{n^2}\, c(c - u). \tag{14}$$

Dividing this by heat flux $T s_0 u$, one gets

$$\mathrm{COP} = \mathrm{COP_C}\frac{s_0 u \Delta T}{L\frac{e^2}{\sigma_0}\overline{n^2}c(c-u)},$$

where $\mathrm{COP_C} = \frac{T}{\Delta T}$ is the Carnot efficiency. Expressing ΔT in terms of u using Eq. (8) we get

$$\frac{\mathrm{COP}}{\mathrm{COP_C}} = \frac{u}{c} - \alpha\frac{u^2}{c(c-u)}, \tag{15}$$

where we introduced the dimensionless disorder strength[2]

$$\alpha = \frac{\langle(\delta n)^2\rangle}{2\,\overline{n^2}}. \tag{16}$$

[2]If in addition to long-range disorder other sources of friction are present, the parameter α may be expressed in terms of the friction coefficient k and the pumping potential in the form $\alpha = \frac{e^2 k}{\sigma_0 \overline{n^2}}$.

At a fixed temperature difference, the maximal COP in Eq. (15) is achieved at

$$\frac{u}{c} = 1 - \sqrt{\frac{\alpha}{1+\alpha}}. \tag{17}$$

From Eq. (8) it is easy to see that this occurs at

$$\Delta T = \frac{e^2}{\sigma_0}\frac{cL\overline{n^2}}{s_0}[\sqrt{\alpha(1+\alpha)} - \alpha]. \tag{18}$$

The maximal value of COP is given by

$$\frac{\mathrm{COP_m}}{\mathrm{COP_C}} = (\sqrt{1+\alpha} - \sqrt{\alpha})^2. \tag{19}$$

For weak disorder, $\alpha \ll 1$, it nearly reaches the Carnot limit. In this case $u/c \approx 1$ in Eq. (17), and heat pumping is almost perfect.

Equation (14) and subsequent results for the heat pump efficiency may be also obtained by considering energy dissipation in the pump. The considerations above Eq. (7) show that within second-order accuracy in U/T the contributions of temperature gradients and viscous stresses to energy dissipation may be neglected. Thus the rate of energy dissipation per unit area is given by $T\dot{s} = ku^2 + \sigma_0\mathcal{E}^2$. Then using Eq. (7) we obtain

$$T\dot{s} = \frac{e^2}{\sigma_0}\left[\frac{\langle(\delta n)^2\rangle}{2}u^2 + n^2(u-c)^2\right]. \tag{20}$$

Note that the energy dissipation rate in the system is characterized by the intrinsic conductivity. This reflects the fact that at small deviations from charge neutrality dissipation is caused by the electric fields arising in the electron liquid. The power consumed by the pump is given by sum of dissipated energy $TL\dot{s}$ and heat flux TJ_s. This reproduces Eq. (14).

It is important to note the pumping mechanism discussed above is not limited to the hydrodynamic regime. This is especially clear for pristine systems in a traveling wave setup. In this case, as long as the system size L exceeds the mean free path due to electron-electron scattering l_{ee}, the electron liquid will equilibrate in the reference frame moving with the pumping potential, regardless if the relation between l_{ee} exceeds the spatial scale λ.

To summarize, a mechanism of electronic pumping of heat at charge neutrality was considered. Since the system is, on average, charge neutral, the heat transfer proceeds at zero net charge current, and is not accompanied by voltage buildup along the system. This may prove advantageous for potential cooling applications (e.g., of microelectronic devices) where voltage buildup or the presence of mechanically moving parts is undesirable. Consideration focused on slow pumping in the regime of electron hydrodynamics. The key parameter of the pumping cycle is the dimensionless ratio of disorder to pumping strength [α in Eq. (16) in a traveling wave setup]. At $\alpha \ll 1$ the heat flux is determined by the geometry of the pumping potential. For potentials in the form of a traveling wave the optimal efficiency of the pump, Eq. (19), may come

close to the Carnot limit. Equation (18) shows that optimal efficiency may be reached for a wide range of temperature differences by adjusting the pumping parameters.

The author is grateful to V. Antonov, A. Barnard, D. Cobden, M. Feigel'man, P. Goldbart, K. Matveev, M. Rudner, and B. Spivak for useful discussions.

[1] I. Khalatnikov, *An Introduction to the Theory of Superfluidity* (Avalon Publishing, New York, 1989).
[2] K. A. Matveev and A. V. Andreev, Second Sound in Systems of One-Dimensional Fermions, Phys. Rev. Lett. **119**, 266801 (2017).
[3] V. L. Gurevich, *Transport in Phonon Systems*, Modern Problems in Condensed Matter Sciences (North-Holland, Amsterdam, 1986).
[4] A. F. Andreev and A. Y. Parshin, Equilibrium shape and oscillations of the surface of quantum crystals, Sov. Phys. JETP **48**, 763 (1978).
[5] V. Tsymbalenko, The effect of the interphase kinetics on the motion of a quantum crystal in superfluid liquid, J. Low Temp. Phys. **195**, 153 (2019).
[6] T. V. Phan, J. C. Song, and L. S. Levitov, Ballistic heat transfer and energy waves in an electron system, arXiv:1306.4972.
[7] R. N. Gurzhi, Hydrodynamic effects in solids at low temperature, Phys. Usp. **11**, 255 (1968).
[8] M. J. M. de Jong and L. W. Molenkamp, Hydrodynamic electron flow in high-mobility wires, Phys. Rev. B **51**, 13389 (1995).
[9] A. V. Andreev, S. A. Kivelson, and B. Spivak, Hydrodynamic Description of Transport in Strongly Correlated Electron Systems, Phys. Rev. Lett. **106**, 256804 (2011).
[10] A. Lucas and K. C. Fong, Hydrodynamics of electrons in graphene, J. Phys.: Condens. Matter **30**, 053001 (2018).
[11] J. Crossno, J. K. Shi, K. Wang, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki, and K. C. Fong, Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene, Science **351**, 1058 (2016).
[12] C. Tan, D. Y. Ho, L. Wang, J. Li, I. Yudhistira, D. A. Rhodes, T. Taniguchi, K. Watanabe, K. Shepard, P. L. McEuen *et al.*, Realization of a universal hydrodynamic semiconductor in ultra-clean dual-gated bilayer graphene, arXiv:1908.10921.
[13] D. J. Thouless, Quantization of particle transport, Phys. Rev. B **27**, 6083 (1983).
[14] L. P. Kouwenhoven, A. T. Johnson, N. C. van der Vaart, C. J. P. M. Harmans, and C. T. Foxon, Quantized Current in a Quantum-Dot Turnstile using Oscillating Tunnel Barriers, Phys. Rev. Lett. **67**, 1626 (1991).
[15] H. Pothier, P. Lafarge, C. Urbina, D. Esteve, and M. H. Devoret, Single-electron pump based on charging effects, Europhys. Lett. **17**, 249 (1992).
[16] M. W. Keller, J. M. Martinis, and R. L. Kautz, Rare Errors in a Well-Characterized Electron Pump: Comparison of Experiment and Theory, Phys. Rev. Lett. **80**, 4530 (1998).
[17] B. Camarota, H. Scherer, M. W. Keller, S. V. Lotkhov, G.-D. Willenberg, and F. J. Ahlers, Electron counting capacitance standard with an improved five-junction *R*-pump, Metrologia **49**, 8 (2011).
[18] P. W. Brouwer, Scattering approach to parametric pumping, Phys. Rev. B **58**, R10135 (1998).
[19] I. L. Aleiner and A. V. Andreev, Adiabatic Charge Pumping in Almost Open Dots, Phys. Rev. Lett. **81**, 1286 (1998).
[20] F. Zhou, B. Spivak, and B. Altshuler, Mesoscopic Mechanism of Adiabatic Charge Transport, Phys. Rev. Lett. **82**, 608 (1999).
[21] M. Switkes, C. M. Marcus, K. Campman, and A. C. Gossard, An adiabatic quantum electron pump, Science **283**, 1905 (1999).
[22] A. Andreev and A. Kamenev, Counting Statistics of an Adiabatic Pump, Phys. Rev. Lett. **85**, 1294 (2000).
[23] Y. Makhlin and A. D. Mirlin, Counting Statistics for Arbitrary Cycles in Quantum Pumps, Phys. Rev. Lett. **87**, 276803 (2001).
[24] A. V. Andreev and E. G. Mishchenko, Full counting statistics of a charge pump in the Coulomb blockade regime, Phys. Rev. B **64**, 233316 (2001).
[25] D. A. Bagrets and Y. V. Nazarov, Full counting statistics of charge transfer in Coulomb blockade systems, Phys. Rev. B **67**, 085316 (2003).
[26] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (Courier Dover, New York, 2017).
[27] C. W. J. Beenakker and A. A. M. Staring, Theory of the thermopower of a quantum dot, Phys. Rev. B **46**, 9667 (1992).
[28] A. A. M. Staring, L. W. Molenkamp, B. W. Alphenaar, H. van Houten, O. J. A. Buyk, M. A. A. Mabesoone, C. W. J. Beenakker, and C. T. Foxon, Coulomb-blockade oscillations in the thermopower of a quantum dot, Europhys. Lett. **22**, 57 (1993).
[29] K. A. Matveev and A. V. Andreev, Thermopower of a single-electron transistor in the regime of strong inelastic cotunneling, Phys. Rev. B **66**, 045301 (2002).
[30] A. Levchenko, T. Micklitz, J. Rech, and K. A. Matveev, Transport in partially equilibrated inhomogeneous quantum wires, Phys. Rev. B **82**, 115413 (2010).
[31] W. DeGottardi and K. A. Matveev, Electrical and Thermal Transport in Inhomogeneous Luttinger Liquids, Phys. Rev. Lett. **114**, 236405 (2015).
[32] S. Li, A. Levchenko, and A. V. Andreev, Hydrodynamic electron transport near charge neutrality, Phys. Rev. B **102**, 075305 (2020).
[33] P. J. Leek, M. R. Buitelaar, V. I. Talyanskii, C. G. Smith, D. Anderson, G. A. C. Jones, J. Wei, and D. H. Cobden, Charge Pumping in Carbon Nanotubes, Phys. Rev. Lett. **95**, 256802 (2005).
[34] A. Shapere and F. Wilczek, *Geometric Phases in Physics*, Vol. 5 (World Scientific, Singapore, 1989).