First-principles prediction of the Landau parameter for Fermi liquids near the unitarity limit

Shashin Pavaskar [*] and Ira Z. Rothstein
Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA

(Received 15 May 2021; accepted 25 May 2022; published 9 June 2022)

This paper explores the behavior of systems of cold fermions as they approach unitary above the critical temperature. Symmetry arguments indicate that at unitarity the Fermi-liquid picture breaks down. As we move away from unitarity, by decreasing the scattering length, the dilaton, the Goldstone boson resulting from the spontaneous breaking of Schrodinger symmetry by the Fermi sea, becomes gapped. At energies below this gap, the interaction between quasiparticles will be dominated by dilaton exchange. The dilaton mass can, in turn, be related via anomaly matching to the scattering length and contact parameter within the confines of a systematic expansion. We use this relation to predict that the quasiparticle width is given by the expression $\Gamma(E,T) = \frac{8m}{9\pi\tilde{C}^2}(\sqrt{\frac{m}{m^\star}}\frac{a\mu_F^2}{4\hbar E_F^2})^2(E^2+\pi^2k_B^2T^2)$ where a is the the scattering length, $m_\star$ is the effective mass, and $\tilde{C}$ is the dimensionless contact parameter. This prediction is valid for $(\frac{E_F}{E})^2 \gg ak_f \gg 1$.

DOI: 10.1103/PhysRevB.105.235107

I. INTRODUCTION

First-principles analytic predictions for strongly correlated systems are typically obstructed by the lack of an expansion parameter. Such predictions are possible when symmetry sufficiently constrains the dynamics. The set of such systems is, of course, extremely limited. In this paper we will show that we can make analytic predictions for a continuum of strongly correlated systems, within a systematic expansion, by expanding around the conformally symmetric limit of a gas of strongly interacting fermions in a cold atomic gas. In this limit the system behaves as a "non-Fermi liquid" which has no well-defined notion of a quasiparticle and is an a open puzzle in the field of strongly correlated systems. To best attack this problem it is wise to study non-Fermi-liquid behavior in as simple a system as possible. A particularly prudent choice of systems is cold atoms, since one may control the coupling strength and all the complications of an underlying lattice are absent. Recent experimental progress in producing such "uniform quantum gases" [1–7] via boxed traps has opened the door to the study of such ideal systems.

Fermions in the unitary limit cannot be described by the canonical Fermi-liquid EFT (as described, e.g., in Refs. [8–10]) because there is no way to nonlinearly realize the spontaneously broken conformal and boost invariance and maintain Fermi-liquid behavior, as shown in Refs. [11,12]. At present we do not know how to calculate in a systematic expansion in the unitary limit. Here we will instead calculate far enough away from unitarity that we can treat it as a Fermi liquid but close enough to keep some approximate symmetries. By doing so we will able to predict the aforementioned Landau parameters.

To understand how to calculate near unitarity we must first understand why Fermi-liquid theory breaks down at unitarity. At unitarity the atomic underlying theory is invariant under the full nonrelativistic conformal (Schrodinger) group. The existence of the Fermi sea breaks a subset of symmetries: Three boosts, dilatations, and special conformal transformations. While the breaking of global internal symmetries leads to gapless Goldstone modes, one per broken generator, when space-time symmetries are broken, this is no longer true [13,14]. In such a case, the Ward identities can be saturated by excitations which can be arbitrarily wide, i.e., they need not be quasiparticles [15]. At the level of the action, invariance may be maintained despite the dearth of Goldstones. The modes for which the corresponding broken generators' commutator with unbroken translations yields another broken generator (not in the same multiplet) can be eliminated from the action. This is called the Inverse Higgs mechanism (IHM) and one can use the space-time coset construction [13,14] to determine invariant constraints which eliminate the extra Goldstones. However, there are cases where there are no IHMs at play and yet the Goldstones, which seemingly should be in the spectrum, are not. The classic example of this is ^{3}He where only boosts are broken, and there are no corresponding Goldstones. In such systems, dubbed "framids" [16], the symmetry is realized by constraining the form of the interactions [12]. In fact, the famous Landau condition on Fermi liquids is the constraint that must be imposed on the action to ensure boost invariance. Such a condition can be considered a "Dynamical Inverse Higgs Constraint" (DIHC) [12]. In Ref. [11] it was shown that in the unitary limit, in three spatial dimensions, the symmetries can be realized either by imposing another DIHC or by the inclusion of a dilaton. In either case the system cannot behave like a canonical Fermi liquid above T_c.

Fermi-liquid theory starts with the assumption that quasiparticles (in our case fermionic) exist in the spectrum with widths that scale as $\Gamma \sim E^2$ due to Pauli blocking. Such systems will have two marginal couplings, the "BCS" and the forward scattering channels, with the former growing strong in the IR leading to breaking of the particle number $U(1)$

[*]Corresponding author: spavaska@andrew.cmu.edu.

symmetry. However, when interactions in the UV become strong, the Fermi-liquid description can break down at which point there may no longer be any stable quasiparticles, leading to non-Fermi-liquid behavior. Such is the case for fermion in the unitary limit.

In this paper we explore the approach to this non-Fermi-liquid behavior by calculating how the quasiparticle width begins to deviate from Fermi-liquid behavior as the scattering length (a) is increased. The starting point is the effective field theory of Fermi liquids [8] where we consider small fluctuations around the Fermi surface. We are interested in studying the normal phase of the theory where $T > T_c$. Furthermore, as will be explained below, to maintain calculational control we will keep the scattering length finite.

Our approach begins by utilizing the pattern of spontaneous breaking of space-time symmetries. In Ref. [11] it was shown that at unitarity, non-Fermi-liquid behavior emerges due to the presence of a nonderivatively coupled gapless Goldstone (the dilaton) that arises as a consequence of the symmetry-breaking pattern. Typically Goldstones are derivatively coupled and therefore decouple in the far IR; however, for spontaneously broken space-time symmetries, for certain symmetry-breaking patterns, Goldstone bosons, such as the dilaton, couple nonderivatively [12,17] leading to a strong coupling in the infrared.

When we perturb away from unitarity, the dilaton gets gapped, with its mass acting as a control parameter which can be used to study the crossover behavior. When the mass is nonvanishing but sufficiently small, Fermi-liquid behavior is expected and dilaton exchange will dominate the fermion-fermion interaction. Moreover, the dilaton mass can be determined by matching the conformal anomaly between the UV theory (where it is exactly known) and the IR theory. Using this result, along with the fact that the dilaton coupling is fixed by symmetry, allows us to to predict the s-wave Landau parameter in terms of the scattering length, the effective mass of the fermion, and the contact parameter. With this result in hand we then predict the value of the compressibility and the quasiparticle lifetime.

II. THE EFT

In the normal phase of a gas of cold atoms the only spontaneously broken symmetries are boosts. Despite this fact, the spectrum has no Goldstone bosons and the broken boosts are still nonlinearly realized via the nontrivial (Landau) relation between the effective mass and the p-wave Landau parameter.

The unitary limit in the trivial vacuum is a point of enhanced symmetry realizing the full 13-parameter Schrodinger group. The Fermi surface spontaneously breaks boosts (K), dilatations (D), and special conformal transformations(C). The way these broken symmetries can be realized was discussed in Refs. [11,12] which for completeness we summarize here. In the case at hand, the Goldstone associated with the breaking of conformal symmetry can be eliminated using the IHC arising from the relation

$$[H, C] = iD, \tag{1}$$

leaving only the dilaton, the Goldstone associated with the broken scale invariance. The boost Goldstone called the framon is necessary to write down a Galilean invariant action for the dilaton. However, it was shown in Ref. [12] that one can eliminate the framon using an operator constraint called the DIHC. In the Fermi-liquid theory, the DIHC is nothing but the aforementioned Landau relation. The logical possibility remains that the action obeys further constraints, such that there is no dilaton in the action. However, as shown in Ref. [12], without a dilaton in the action the quasiparticle would have to obey a quadratic dispersion relation and the coupling would have to undergo power-law running. Moreover, independent of the choice of field variables, at unitarity there still must be a gapless singularity in the stress-energy correlation function, though it should be expected to be highly damped. Moving away from unitarity toward a quasiparticle description, this gapped channel will be nothing but the massive dilaton.

Let us explore the consequences of the existence of a light ($m_\phi \ll E_F$) dilaton in the spectrum. We will treat the dilaton mass as the leading-order perturbation in the conformal symmetry breaking, with higher-order corrections being down by powers of m_ϕ/E_F. We begin by first writing down the action in the conformal/unitary limit. Since the scattering length diverges in this limit, the only scale in the theory is the Fermi energy E_F. To write down the action for quasiparticles and the dilaton, we utilize the technique of space-time coset constructions [13,14] which is a systematic way of nonlinearly realizing the symmetries. We present here the results given in Ref. [11] and refer the reader to that paper for details.

At the unitary point, the coset element can be written as

$$U = e^{iHt}e^{-i\vec{P}.\vec{x}}e^{-i\vec{K}.\vec{\eta}}e^{-iD\phi}e^{-iC\xi}, \tag{2}$$

where $\vec{\eta}(x,t)$, $\phi(x,t)$ and $\xi(x,t)$ are the framon, dilaton, and Goldstone of the conformal transformation, respectively. In the remainder of the work, we will explicitly drop any x and t dependence from the fields. Using the Maurer-Cartan form, one can extract the covariant derivatives for the Goldstones which transform linearly under the broken group. The coupling of the dilaton ϕ in the quasiparticle action is given by

$$\begin{aligned} S_\psi = \int dt d^3x e^{-\frac{5\phi}{\Lambda}}[\tilde{\psi}^\dagger(ie^{\frac{2\phi}{\Lambda}}\partial_t\tilde{\psi} \\ - (\varepsilon(e^{\frac{\phi}{\Lambda}}i\vec{\partial}) - \mu_F)\tilde{\psi}) \\ + f_0(\tilde{\psi}^\dagger\tilde{\psi})^2 + f_1(\tilde{\psi}^\dagger\sigma_i\tilde{\psi})^2], \end{aligned} \tag{3}$$

where $\tilde{\psi} = e^{\frac{3\phi}{2\Lambda}}\psi$. We have kept only the $l = 0$ Landau parameter. The addition of higher l's will not change our predictions as we shall see.

We have introduced a scale Λ to normalize the dilaton field in the exponential. Under dilatations, the dilaton shifts by a constant $\phi \longrightarrow \phi + c\Lambda$ whereas the coordinates transform as $t \longrightarrow e^{2c}t$ and $x \longrightarrow e^c x$. The quasiparticle fields and their covariant derivatives have to transform as a linear representation of the unbroken group $\psi(x,t) \rightarrow e^{-3/2c}\psi(x,t)$. One is free to add an invariant term of the form $V_{dil} = Ce^{-5\phi/\Lambda}$ to the dilaton Lagrangian. Thus maintaining a light dilaton implies C must be fine-tuned to be small, as its natural value is of order of the cutoff. This is analogous to the cosmological constant problem, the most egregious fine-tuning in nature. However, in the context of fermions at unitarity, the appropriate fine-tuning is achieved by choosing the magnetic field such that the atomic system is sitting near the Feshbach resonance.

Expanding to leading order in the dilaton field ϕ in the quasiparticle action,

$$S_\psi = \int d^3x\, dt\; i\psi^\dagger \partial_t \psi + \psi^\dagger \vec{v}_F \cdot \vec{\partial}\psi - \frac{2\mu_F}{\Lambda}\phi\psi^\dagger\psi + \cdots, \tag{4}$$

where $\vec{v}_F$ is the Fermi velocity. Power counting dictates that the dilaton momenta must scale homogeneously under an RG transformation in all directions ($\vec{p} \to \lambda\vec{p}$) and thus will only scatter nearby points on the Fermi surface. Any other choice of scalings would lead to a power suppression. The quasiparticle and the dilaton energies scale in the same way as we move toward the Fermi surface ($\omega \sim \lambda\omega$). From the kinetic terms in the dilaton and quasiparticle actions, we can read off the scaling of the momentum space dilaton and quasiparticle fields

$$\psi(p,t) \sim \lambda^{-1/2} \quad \phi(p,t) \sim \lambda^{-2}. \tag{5}$$

The scaling of the dilaton-quasiparticle interaction is marginal as can be seen by going to momentum space and noting that, as in the four-point quasiparticle interaction, the delta function enforcing the three-momentum conservation scales as $1/\lambda$ while the momentum space measure will scale as

$$d^3p_1 d^3p_2 d^3k \sim \lambda^5, \tag{6}$$

as all three-momentum components of the dilaton, as well as the quasiparticle momenta along the direction normal to the Fermi surface, scale as λ.

III. THE APPROACH TO NON-FERMI-LIQUID BEHAVIOR

As we move away from the unitary point, the scattering length becomes finite and scale invariance becomes an approximate symmetry of the effective theory. Hence the dilaton becomes a gapped pseudo-Goldstone. As we will see, we can determine the mass of dilaton in terms of the scattering length and the contact parameter. We are working in the units where the fermion mass is one and $\hbar = 1$, and the length dimensions will be

$$[t] = 2 \quad [\phi] = -\tfrac{1}{2} \quad [\psi] = -\tfrac{3}{2}. \tag{7}$$

Away from unitarity, the conformal symmetry is explicitly broken; however if we keep the scale of explicit symmetry breaking (the inverse scattering length) small compared with the scale of spontaneous symmetry breaking (the Fermi wave number) we may still treat the dilaton as a pseudo-Goldstone boson. The smallness of the dilaton mass follows from the fact that the scattering length is tuned to be large. The mass of the dilaton is treated as a spurion such that the action is invariant if we scale it according to its dimensions,

$$\delta L = \tfrac{1}{2} m_\phi^2 \phi^2. \tag{8}$$

We now use a matching procedure to calculate m_ϕ. In the effective theory away from unitarity, the scale current is not conserved,

$$\partial_\mu s^\mu = m_\phi^2 \Lambda\, \phi. \tag{9}$$

We will use current algebra to extract the mass by matching it onto the full theory result. From the Noether construction the dilatation charge is given by

$$D^0(0) = \Lambda \int d^3x\, \pi(\vec{x},0), \tag{10}$$

where $\pi(x)$ is the conjugate momentum to ϕ. Hence using Eq. (9) we have

$$\int_x [D^0(0), \partial_\mu s^\mu(\vec{x},0)] = \int d^3x\, m_\phi^2 \Lambda^2. \tag{11}$$

We match this commutator to the full theory, which is a microscopic description of the theory, in terms of fermions with action

$$S = \int dt \int d^3x\; i\chi^\dagger \partial_t \chi + \frac{1}{2}\chi^\dagger \nabla^2 \chi + g(\mu)(\chi^\dagger\chi)^2, \tag{12}$$

where χ is two-spinor. The van der Waals scale(Λ_{VDW}) provides the upper cutoff in the theory that suppresses higher dimensional operators. The renormalized coupling can be written in terms of the scattering length as [18]

$$g(\mu) = \frac{4\pi}{-\frac{2}{\pi}\mu + \frac{1}{a}}. \tag{13}$$

The four-fermion interaction defined in (12) explicitly breaks scale invariance. One can verify that the dilatation charge, the divergence of the scale current, and their commutators are given, respectively, by

$$D^0(0) = \int d^3x \left(\frac{3}{2}\chi^\dagger(\vec{x},0)\chi(\vec{x},0) + \chi^\dagger(\vec{x},0)\vec{x}\cdot\vec{\partial}\chi(\vec{x},0) \right) \tag{14}$$

$$\partial_\mu s^\mu = (g(\mu) + \beta(g))(\chi^\dagger\chi)^2 \tag{15}$$

$$\int d^3x\, m_\phi^2 \Lambda^2 = 3 \int d^3x\, (g(\mu) + \beta(g))\, (\chi^\dagger\chi)^2, \tag{16}$$

where in (16) we have matched the commutators in the full and the effective theory using (11). $\beta(g)$ is the beta function associated with the coupling. Note that the RHS of (16) is an RG invariant, and the dilaton mass is independent of the scale μ. The coupling and the four-fermion operator both depend on the scale μ but the dependence cancels exactly in (16) to give a scale-independent mass as required. Evaluating the beta function and taking the expectation value, we have

$$m_\phi^2 \Lambda^2 = \frac{3}{4\pi a}\langle g^2\, \chi_\uparrow^\dagger \chi_\uparrow \chi_\downarrow^\dagger \chi_\downarrow \rangle \equiv \frac{3}{4\pi a}\,\mathcal{C}, \tag{17}$$

where we have now made the spin state explicit and $\mathcal{C}$ is the contact parameter [19] whose vacuum expectation value is a measure of the local pair density of the fermions and is independent of the RG scale μ. For any system consisting of fermions with two spin states and large scattering length, one can define universal relations which depend on the contact. Note that Λ is still an undetermined free parameter. However, we will see that it will cancel in the calculation of the Landau parameter f_0.

If the dilaton mass is sufficiently small it will dominate the quasiparticles interactions, as other contributions to the interaction, arising from integrating out other modes, will be parametrically suppressed by powers of m_ϕ/E_F.

IV. QUASIPARTICLE WIDTH

The self-energy only gets contributions from the forward scattering coupling, as the other marginal coupling (BCS) is restricted to back-to-back interactions. The imaginary part of the self-energy of a Fermi liquid due to singlet interaction is $\Gamma_{FL}(E,T) = 4f_0^2 I(E,T)$, where $I(E,T) = \frac{m_\star^3}{2(2\pi)^3}(E^2 + \pi^2 k_B^2 T^2)$ is the imaginary part of the two-loop self-energy diagram at finite temperature. We integrate out the dilaton to generate net interaction

$$L_{\text{int}} = (f_0 + \frac{8\pi a\mu_F^2}{3\mathcal{C}})(\psi^\dagger\psi)^2 + \cdots, \qquad (18)$$

where the corrections are suppressed by powers of $E^2/m_\phi^2 = k_F a \frac{E^2}{E_F^2}$ where we have taken $\Lambda \sim k_F^{1/2}$ as the symmetry-breaking scale. Thus if we are in the regime,

$$\left(\frac{E_F}{E}\right)^2 \gg ak_F \gg 1, \qquad (19)$$

then the dilaton exchange dominates so that the effective coupling, after repristinating factors of $\hbar$ and the atomic mass m, is given by

$$f_D \equiv f_0 = \frac{8\pi a\mu_F^2 m}{3\hbar^4 k_F^4 \tilde{\mathcal{C}}}. \qquad (20)$$

$\tilde{\mathcal{C}} = \frac{\mathcal{C}}{k_F^4}$ is the dimensionless contact parameter which has been measured to be of order one (when $k_F a > 1$) in the cases of a trapped system [20]. Note that since the coupling to the dilaton is scalar in nature, the spin-triplet channel (f_1) as well as higher angular momentum interactions will be subleading in our expansion. Using our result (20) we can then calculate the quasiparticle width

$$\Gamma(E,T) = 4f_D^2 I(E,T), \qquad (21)$$

such that

$$\Gamma(E,T) = \frac{8m}{9\pi\tilde{\mathcal{C}}^2}\left(\sqrt{\frac{m}{m^\star}}\frac{a\mu_F^2}{4\hbar E_F^2}\right)^2(E^2 + \pi^2 k_B^2 T^2) \qquad (22)$$

and $E_F = \frac{k_F^2}{2m_\star}$. The theoretical errors in this predictions are of order

$$\frac{\Delta\Gamma_T}{\Gamma} \sim O\left(\frac{1}{k_F a}\right) + O\left(k_F a\left(\frac{E^2}{E_F^2}\right)\right). \qquad (23)$$

We may also utilize this result to calculate the compressibility, which is given by

$$\kappa = \frac{1}{n^2}\frac{N_F}{1 - \hbar^2 N_F f_D} = \frac{1}{n^2}\frac{N_F}{1 - \frac{2}{3\pi}\frac{m}{m^\star}\frac{k_F a\mu_F^2}{\tilde{\mathcal{C}}E_F^2}}, \qquad (24)$$

where $N_F = \frac{m^\star p_F}{\hbar^3\pi^2}$ is the density of states at the Fermi surface and n is the Fermion number density. As indicated by the theory error, these results are not valid in the region where the scattering lengths diverges, since in this region one cannot integrate out the dilaton. Also in the limit ($a \to 0^-$), this prediction is not applicable since the symmetry-breaking parameter ($1/a$) diverges and hence the dilaton is not a pseudo-Goldstone anymore. Finally, note that the spin susceptibility is not predictable because it depends on the $S = 1$ interaction which is not mediated by the dilaton.

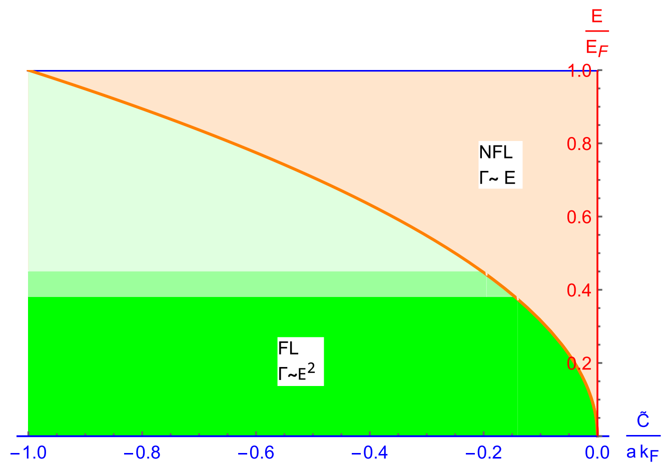

FIG. 1. The plot shows the phase diagram as a function of the energy and the scattering length. Our prediction for the self-energy Γ is valid below the green line ($E \sim 0.4E_F$) in the Fermi-liquid (FL) region. The dilaton mass curve (orange) separates this region from the non-Fermi-liquid (NFL) region. The region above $E \sim 0.4E_F$ (green line) (this line is a rough guess of the region where the EFT breaks down) is where the effective theory begins to break down.

V. CONCLUSIONS

It is known that degenerate fermionic systems cross over from Fermi to non-Fermi liquids as unitarity is approached. Symmetry requires that Fermi gases at unitarity manifest a gapless excitation in response to external stress. This “dilaton” mode will look like an overdamped sound mode, but since we are working in the attractive regime where there is no zero sound, this mode can be isolated. Furthermore, by working below the hydrodynamic limit, there will be no contamination from the second sound.

Perturbing away from the unitary limit gaps this mode. For energy scales large compared with the gap, the quasiparticle excitations are expected to behave as in a non-Fermi liquid with a width that scales linearly with the energy. However, as the energy of the quasiparticle drops below the gap, the dilaton-mediated interaction localizes and Fermi-liquid behavior with the width scaling quadratically with energy is expected. The behavior of the system as a function of energy and scattering length is depicted in Fig. 1.

The key insight noted here is that the mass of the dilaton can be fixed by matching the effective theory current algebra to that of the full theory, the result of which leads to a prediction for the mass in terms of the scattering length and contact parameter, which in turn allows us to make a prediction for the quasiparticle lifetime including the normalization. The width is predicted to scale quadratically with the ratio of scattering length to the contact parameter. Note also that the dilaton, because it is not derivatively coupled, will only generate the $l = 0$ Landau parameter. Thus we have the additional prediction that the $l = 0$ Landau parameter will dominate all other channels. We also calculate the compressibility of the Fermi liquid as a function of the scattering length. These predictions have a limited range of validity. The energy must be small enough that the dilaton exchange can still be treated as a local interaction. This limitation also implies our EFT breaks down when the scattering length, which is inversely proportional to the dilaton mass, becomes large, i.e., in the NFL region. The range of validity of the theory is shown in Fig. 1 as the region bounded by the green line from above.

ACKNOWLEDGMENTS

We thank Shauna Kravec, John McGreevy, Cliff Cheung, and Nir Navon for helpful comments on an earlier version of this paper. This work was partially supported by the U.S. Department of Energy under Grants No. DE-FG02-04ER41338 and No. FG02-06ER41449.

[1] A. L. Gaunt, T. F. Schmidutz, I. Gotlibovych, R. P. Smith, and Z. Hadzibabic, Bose-Einstein Condensation of Atoms in a Uniform Potential, Phys. Rev. Lett. **110**, 200406 (2013).

[2] B. Mukherjee, Z. Yan, P. B. Patel, Z. Hadzibabic, T. Yefsah, J. Struck, and M. W. Zwierlein, Homogeneous Atomic Fermi Gases, Phys. Rev. Lett. **118**, 123401 (2017).

[3] K. Hueck, N. Luick, L. Sobirey, J. Siegl, T. Lompe and H. Moritz, Two-Dimensional Homogeneous Fermi Gases, Phys. Rev. Lett. **120**, 060402 (2018).

[4] L. Baird, X. Wang, S. Roof and J. E. Thomas, Measuring the Hydrodynamic Linear Response of a Unitary Fermi Gas, Phys. Rev. Lett. **123**, 160402 (2019).

[5] M. Horikoshi, M. Koashi, H. Tajima, Y. Ohashi and M. Kuwata-Gonokami, Ground-State Thermodynamic Quantities of Homogeneous Spin-1/2 Fermions from the BCS Region to the Unitarity Limit, Phys. Rev. X **7**, 041004 (2017).

[6] H. Biss, L. Sobirey, N. Luick, M. Bohlen, J. J. Kinnunen, G. M. Bruun, T. Lompe, and H. Moritz, Excitation Spectrum and 4 Superfluid Gap of an Ultracold Fermi Gas, Phys. Rev. Lett. **128**, 100401 (2022).

[7] N. Navon, R. P. Smith, and Z. Hadzibabic, Quantum Gases in Optical Boxes, Nat. Phys. **17**, 1334 (2021).

[8] G. Benfatto and G. Gallavotti, Renormalization-group approach to the theory of the Fermi surface, Phys. Rev. B **42**, 9967 (1990).

[9] R. Shankar, Renormalization group approach to interacting fermions, Rev. Mod. Phys. **66**, 129 (1994).

[10] J. Polchinski, Effective field theory and the Fermi surface, In Boulder 1992, Proceedings, Recent directions in particle theory, 235–274. arXiv:hep-th/9210046.

[11] I. Z. Rothstein and P. Shrivastava, Symmetry Obstruction to Fermi Liquid Behavior in the Unitary Limit, Phys. Rev. B **99**, 035101 (2019).

[12] I. Z. Rothstein and P. Shrivastava, Symmetry Realization via a Dynamical Inverse Higgs Mechanism, J. High Energy Phys. 05 (2018) 014.

[13] E. A. Ivanov and V. I. Ogievetsky, The Inverse Higgs Phenomenon in Nonlinear Realizations, Theor Math Phys **25**, 1050 (1975).

[14] D. V. Volkov, Phenomenological Lagrangians, Fiz. Elem. Chast. Atom. Yadra **4**, 3 (1973).

[15] For a recent discussion of this issue see L. Alberte and A. Nicolis, Spontaneously broken boosts and the Goldstone continuum, J. High Energy Phys. 07 (2020) 076.

[16] A. Nicolis, R. Penco, F. Piazza, and R. Rattazzi, Zoology of condensed matter: Framids, ordinary stuff, extra-ordinary stuff, J. High Energy Phys. 06 (2015) 155.

[17] H. Watanabe and A. Vishwanath, Criterion for stability of Goldstone Modes and Fermi Liquid behavior in a metal with broken symmetry, Proc. Natl. Acad. Sci. USA **111**, 16314 (2014).

[18] For a review see E. Braaten and H. W. Hammer, Universality in few-body systems with large scattering length, Phys. Rep. **428**, 259 (2006).

[19] S. Tan, Energetics of a strongly correlated Fermi gas, Ann. Phys. **323**, 2952 (2008).

[20] J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Verification of Universal Relations in a Strongly Interacting Fermi Gas, Phys. Rev. Lett. **104**, 235301 (2010).