Erratum: Spin and orbital angular momenta of acoustic beams [Phys. Rev. B 99, 174310 (2019)]

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In the original paper, we introduced the canonical momentum and angular momentum (AM) densities of monochromatic sound waves in fluids or gases using quadratic forms involving the vector velocity field $\mathbf{v}(\mathbf{r})$ and the scalar pressure field $P(\mathbf{r})$. This approach is based on the analogy with electromagnetic dual-symmetric canonical quantities, involving the electric- and magnetic-fields $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$, treated on equal footing [1–4]. It is also supported by the quadratic energy-density form for sound waves including the kinetic (velocity) and potential (pressure) terms,

$$W = \frac{1}{4}(\rho|\mathbf{v}|^2 + \beta|P|^2) \equiv T + U. \tag{1}$$

However, recent studies [5–7] revealed that the proper definition of the canonical momentum and angular momentum densities in acoustic waves should involve only the velocity field $\mathbf{v}$, in agreement with the original proposition in Ref. [8]. This follows from the mechanical origin of the acoustic canonical momentum and spin, which are associated with the motion of the medium particles: the Stokes drift [6,7,9] and microscopic elliptical motion of the particles [8,10], respectively.

With these definitions, Eq. (6) in our paper should read

$$\mathbf{p} = \frac{\rho}{2\omega}\mathrm{Im}[\mathbf{v}^* \cdot (\boldsymbol{\nabla})\mathbf{v}]. \tag{2}$$

This restores the correct form of the Belinfante-Rosenfeld relation between the canonical momentum, spin, and kinetic momentum (energy flux divided by c^2) densities [1,3,4,6,11] so that Eq. (7) becomes

$$\frac{\boldsymbol{\Pi}}{c^2} = \mathbf{p} + \frac{1}{2}\boldsymbol{\nabla} \times \mathbf{S}, \quad \mathbf{S} = \frac{\rho}{2\omega}\mathrm{Im}(\mathbf{v}^* \times \mathbf{v}), \tag{3}$$

as in Eq. (12). The general relations (8) for the orbital and total angular momentum densities remain unchanged.

The corrected definitions affect calculations of the momentum and angular momenta of acoustic Bessel beams in Sec. III. Since the new definitions involve only the velocity field, it is convenient to normalize all the quantities by the kinetic-energy density T rather than the total energy density W. In doing so, the modified first Eq. (18) and Eqs. (19)–(21) can be written as

$$p_z = k_z\frac{2T}{\omega}, \tag{4}$$

$$L_z = p_\varphi r = \ell\frac{2T}{\omega} - \frac{\rho|A'|^2}{4\omega}\kappa^2\big[J_{\ell-1}^2(\kappa r) - J_{\ell+1}^2(\kappa r)\big], \tag{5}$$

$$S_z = \frac{\rho|A'|^2}{4\omega}\kappa^2\big[J_{\ell-1}^2(\kappa r) - J_{\ell+1}^2(\kappa r)\big], \tag{6}$$

$$S_\varphi = -\frac{\rho|A'|^2}{4\omega}\frac{k_z r}{\ell}\kappa^2\big[J_{\ell-1}^2(\kappa r) - J_{\ell+1}^2(\kappa r)\big], \tag{7}$$

where

$$T = \frac{\rho|A'|^2}{4}\left[k_z^2 J_\ell^2(\kappa r) + \frac{\kappa^2}{2}\big[J_{\ell-1}^2(\kappa r) - J_{\ell+1}^2(\kappa r)\big]\right].$$

These expressions immediately improve Eq. (22) for the total angular momentum density, which now acquires a more natural form similar to that for optical cylindrical modes [12–14],

$$J_z = L_z + S_z = \ell\frac{2T}{\omega}. \tag{8}$$

TABLE I. Comparison of acoustic and electromagnetic quantities and properties.

	Acoustics	Electromagnetism
Fields	Velocity $\mathbf{v}$, pressure P	Electric $\mathbf{E}$, magnetic $\mathbf{H}$
Constraints	$\boldsymbol{\nabla} \times \mathbf{v} = \mathbf{0}$	$\boldsymbol{\nabla} \cdot \mathbf{E} = \boldsymbol{\nabla} \cdot \mathbf{H} = 0$
Energy density	$\frac{1}{4}(\rho\lvert\mathbf{v}\rvert^2 + \beta\lvert P\rvert^2)$	$\frac{1}{4}(\varepsilon\lvert\mathbf{E}\rvert^2 + \mu\lvert\mathbf{H}\rvert^2)$
Canonical momentum density	$\frac{\rho}{2\omega}\mathrm{Im}[\mathbf{v}^* \cdot (\boldsymbol{\nabla})\mathbf{v}]$	$\frac{1}{4\omega}\mathrm{Im}[\varepsilon\mathbf{E}^* \cdot (\boldsymbol{\nabla})\mathbf{E} + \mu\mathbf{H}^* \cdot (\boldsymbol{\nabla})\mathbf{H}]$
Kinetic momentum density	$\frac{1}{2c^2}\mathrm{Re}(P^*\mathbf{v}) = \mathbf{p} + \frac{1}{2}\boldsymbol{\nabla} \times \mathbf{S}$	$\frac{1}{2c^2}\mathrm{Re}(\mathbf{E}^* \times \mathbf{H}) = \mathbf{p} + \frac{1}{2}\boldsymbol{\nabla} \times \mathbf{S}$
Spin AM density	$\frac{\rho}{2\omega}\mathrm{Im}(\mathbf{v}^* \times \mathbf{v})$	$\frac{1}{4\omega}\mathrm{Im}(\varepsilon\mathbf{E}^* \times \mathbf{E} + \mu\mathbf{H}^* \times \mathbf{H})$
Orbital AM density	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$
Integral AM values	$\langle\mathbf{M}\rangle = \langle\mathbf{L}\rangle, \langle\mathbf{S}\rangle = \mathbf{0}$	$\langle\mathbf{M}\rangle = \langle\mathbf{L}\rangle + \langle\mathbf{S}\rangle, \langle\mathbf{S}\rangle \neq \mathbf{0}$
Helicity	$\mathfrak{S} \equiv 0$	$\mathfrak{S} \not\equiv 0$

Notably, for propagating acoustic modes, the integral values of the kinetic and potential energies are equal to each other: $\langle T\rangle = \langle U\rangle = \langle W\rangle/2$ (see Ref. [15], Secs. 5 and 10-O). Therefore, the second Eq. (18) and Eq. (23) for the integral values remain valid: The normalizations by $2\langle T\rangle$ and $\langle W\rangle$ are equivalent.

Finally, Table I in the original, paper, summarizing the main properties of acoustic and electromagnetic waves, should have the form shown below. The above corrections do not affect the conclusions of our paper.

[1] M. V. Berry, Optical currents, J. Opt. A: Pure Appl. Opt. **11**, 094001 (2009).

[2] S. M. Barnett, Rotation of electromagnetic fields and the nature of optical angular momentum, J. Mod. Opt. **57**, 1339 (2010).

[3] K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, Dual electromagnetism: helicity, spin, momentum and angular momentum, New J. Phys. **15**, 033026 (2013).

[4] K. Y. Bliokh and F. Nori, Transverse and longitudinal angular momenta of light, Phys. Rep. **592**, 1 (2015).

[5] L. Burns, K. Y. Bliokh, F. Nori, and J. Dressel, Acoustic versus electromagnetic field theory: Scalar, vector, spinor representations and the emergence of acoustic spin, New J. Phys. **22**, 053050 (2020).

[6] K. Y. Bliokh, H. Punzmann, H. Xia, F. Nori, and M. Shats, Field theory spin and momentum in water waves, Sci. Adv. **8**, eabm1295 (2022).

[7] K. Y. Bliokh, Y. P. Bliokh, and F. Nori, Ponderomotive forces, stokes drift, and momentum in acoustic and electromagnetic waves, arXiv:2204.07035.

[8] C. Shi, R. Zhao, Y. Long, S. Yang, Y. Wang, H. Chen, J. Ren, and X. Zhang, Observation of acoustic spin, Natl. Sci. Rev. **6**, 707 (2019).

[9] T. S. van den Bremer and Ø. Breivik, Stokes drift, Philos. Trans. R. Soc. A **376**, 20170104 (2018).

[10] W. L. Jones, Asymmetric wave-stress tensors and wave spin, J. Fluid Mech. **58**, 737 (1973).

[11] D. E. Soper, *Classical Field Theory* (Wiley, New York, 1976).

[12] S. J. van Enk and G. Nienhuis, Commutation rules and eigenvalues of spin and orbital angular momentum of radiation fields, J. Mod. Opt. **41**, 963 (1994).

[13] K. Y. Bliokh, M. A. Alonso, E. A. Ostrovskaya, and A. Aiello, Angular momenta and spin-orbit interaction of nonparaxial light in free space, Phys. Rev. A **82**, 063825 (2010).

[14] M. F. Picardi, K. Y. Bliokh, F. J. Rodríguez-Fortuño, F. Alpeggiani, and F. Nori, Angular momenta, helicity, and other properties of dielectric-fiber and metallic-wire modes, Optica **5**, 1016 (2018).

[15] B. A. Auld, *Acoustic Fields and Waves in Solids* (Wiley, New York, 1973).