Peak effect in a superconductor/normal-metal strip in a vortex-free state

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We theoretically predict that the critical current I_c and magnetization M of a hybrid superconductor/normal-metal (SN) strip may have nonmonotonous dependence on a perpendicular magnetic field—so-called peak effect. In contrast to familiar peak effect, which is connected either with vortex entry to the superconductor or with peculiarities of vortex pinning, the found phenomenon exists at low fields, in the vortex-free (Meissner) phase. We argue that the effect appears at specific parameters of the studied hybrid structure when its in-plane current-supervelocity relation has two maxima. We expect that the same peak effect may exist in two-band superconductors (like MgB_2) where similar current-supervelocity dependence was predicted at low temperatures.

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The influence of a perpendicular magnetic field H on transport properties of type-II superconductors has been the subject of numerous studies. Usually, the critical current I_c of bulk superconductors is determined mainly by pinning of vortices on defects, and it monotonically decreases with increasing H. However, in conventional low-T_c superconductors, a peak in $I_c(H)$ just below the upper critical field H_{c2} has been observed (see, for example, Refs. [1–3]). The peak in $I_c(H)$ is also accompanied by a peak in the dependence of magnetization M on H, and this phenomenon is called the *peak effect*. The peak effect near H_{c2} is explained by a softening of the vortex lattice [4,5]. Also, the peak was discovered significantly below H_{c2} both in the low-T_c [6,7] and high-T_c superconductors [8,9]. The origin of this type of peak is explained by the transition from a quasi-ordered vortex lattice to an amorphous vortex glass state.

In a homogeneous superconducting strip, the critical current could be determined not by the bulk pinning of vortices but by the edge barrier for their entrance [10–13]. Usually, the effect of the edge barrier is pronounced in a thin strip/bridge with thickness d_S less than the London magnetic field penetration depth λ and at relatively low magnetic fields when there is no dense vortex lattice [13–16]. In a relatively narrow strip (with width $W \ll \Lambda = \lambda^2/d_S$), one may observe a peak in $I_c(H)$ near the field for the first vortex entry [16–18] (the same peak has been observed in thin Pb/In and Nb strips placed in a parallel magnetic field [19,20]). It originates from the entrance of the vortex row at some field which does not exit the strip, and it prevents subsequent vortex entry [21]. It is interesting that competition of the bulk pinning and edge barrier also may lead, at some parameters, to the peak effect at low magnetic fields, as it was predicted in Ref. [22].

Here, we argue that the peak in $I_c(H)$ and $M(H)$ may arise even in the vortex-free state. Below, we show that it could be realized, for example, in a hybrid superconductor/normal-metal (SN) thin strip with a large ratio of resistivities of S and N layers $\rho_S/\rho_N \gg 1$ in the normal state. In Ref. [23], it has been shown that dependence of the superconducting sheet current density J_s (in an ordinary S strip $J_s = j_s d_S$, where j_s is a superconducting current density) on supervelocity v_s or supermomentum $\hbar q = \hbar(\nabla\varphi + 2\pi A/\Phi_0) \sim v_s$ (here, φ is the phase of the superconducting order parameter, and A is the vector potential) may have two maxima at low temperature. The first maximum at small q is connected with suppression of the proximity-induced superconductivity in the N layer, while the second maximum at large q comes from suppression of superconductivity in the S layer. The predicted dependence is rather different from $J_s(q)$ of an ordinary one-band superconductor, which has only one maximum, but it resembles the dependence $J_s(q)$ for two-band superconductors [24,25]. In that case, different maxima correspond to destruction of superconductivity in different bands.

Our model system is shown in Fig. 1. The SN strip with width W has two layers: a superconducting one with thickness d_S and the normal-metal layer with thickness d_N. In calculations, we use one- (1D) and two-dimensional (2D) Usadel equations for normal $g = \cos\Theta$ and anomalous $f = \sin\Theta\exp(i\varphi)$ quasiclassical Green functions, assuming that angle Θ depends only on x and y and the length of the SN strip $L \to \infty$ (equations and details of the model are presented in the Appendix and could be found in Ref. [23]). Our model cannot consider vortex states, so we consider here only the Meissner (vortex-free) state. We consider a narrow strip with width smaller than the magnetic field penetration depth Λ of the single S layer to neglect the contribution of screening currents to the vector potential which we choose as: $\mathbf{A} = (0, 0, Hy)$. In our model, we assume that current reaches the critical value when $q(y = W/2) = q_c$, where $q(y) = \nabla\varphi + 2\pi A(y)/\Phi_0$ ($\nabla\varphi(y) = \text{const.}$), and q_c is the critical value of q corresponding to the reaching depairing current density at the edge. This condition corresponds to instability of the Meissner state with respect to vortex entry [26].

To find $I_c(H)$, we numerically solve either 1D or 2D Usadel equations (see Appendix). In the 1D model, we split the SN strip into filaments with width ξ_c and assume that

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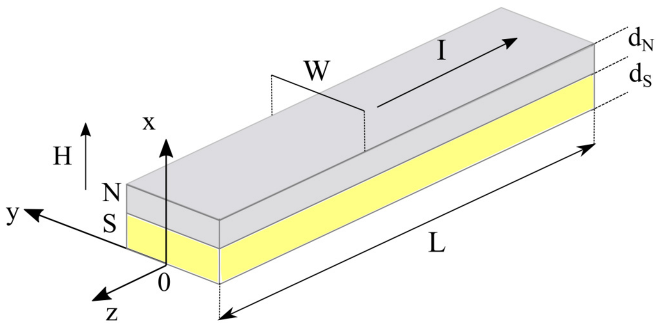

FIG. 1. Sketch of superconductor/normal-metal (SN) strip with transport current I and placed in perpendicular magnetic field H.

$J_s(y) = \int j_s(x, y)dx = \int j_s[x, q(y)]dx = J_s[q(y)]$ (q depends on the y coordinate of the filament) and may be found from the solution of the 1D Usadel equation (in this case, Θ has dependence only on the x coordinate). Then we calculate $I_c = \int J_s[q(y)]dy$. In the 2D model, we solve the 2D Usadel equation with given $q(y)$ and find $I_c = \int j_s(x, y)dxdy$ (Θ depends both on x and y). The difference between these approaches is that, in the 1D model, we neglect the proximity effect between adjacent filaments which brings the difference between $J_s(y)$ and $J_s[q(y)]$. We expect that the filament model gives quantitatively correct results when $W \gg \xi_N = \sqrt{\hbar D_N/k_B T}$ [27], where D_N is a diffusion coefficient in the N layer.

In calculations, we normalize lengths in units of $\xi_c = \sqrt{\hbar D_S/k_B T_{c0}}$, where T_{c0} is the critical temperature, and D_S is the diffusion coefficient of S layer. Sheet current density J_s is normalized in units of depairing sheet current density $J_{dep}(0) = I_{dep}(0)/d_S$ of the S layer at $T = 0$, and the magnetic field is measured in units of $H_s = \Phi_0/2\pi W \xi_c$ (this field is about the first vortex entry field [12,13] to the strip at $I = 0$). We choose a ratio of resistivities (diffusion coefficients) $\rho_S/\rho_N = D_N/D_S = 100$ which corresponds to NbN, NbTiN, MoN, or MoSi as a superconductor and Ag, Cu, or Au as a normal metal.

In Fig. 2(a), we show temperature evolution of $|J_s|(q)$ (it was found from the solution of the 1D Usadel equation) which is used for calculation of the critical current in the filaments model. With decreasing temperature, the dependence $|J_s|(q)$ transforms from the ordinary one (with one maximum) to the dependence with two maxima located at $q = q_{c1}$ and $q = q_{c2}$ (note qualitative similarity with $|J_s|(q)$ for the two-band superconductor MgB_2 [24,25]). The first maximum comes from the suppression of proximity-induced superconductivity in the N layer, where $q_c = q_{c1} \propto \sqrt{1/D_N}$. The second maximum comes from the suppression of superconductivity in the S layer, where $q_c = q_{c2} \propto \sqrt{1/D_S} \gg q_{c1}$. The increase of the amplitude of the first maximum at low temperatures is explained by the enhancement of the proximity-induced superconductivity while the strength of the intrinsic superconductivity in the S layer is already saturated, and the amplitude of the second maximum weakly depends on temperature at low T. As it is discussed in Ref. [23], such an evolution in $J_s(q)$ should lead to the kink on dependence $I_c(T)$ at low T [when $|J_s|(q_{c1})$ becomes larger than $|J_s|(q_{c2})$]. The same kink is also predicted in Refs. [24,25] for MgB_2, and it is caused by the similar change of $J_s(q)$ with temperature.

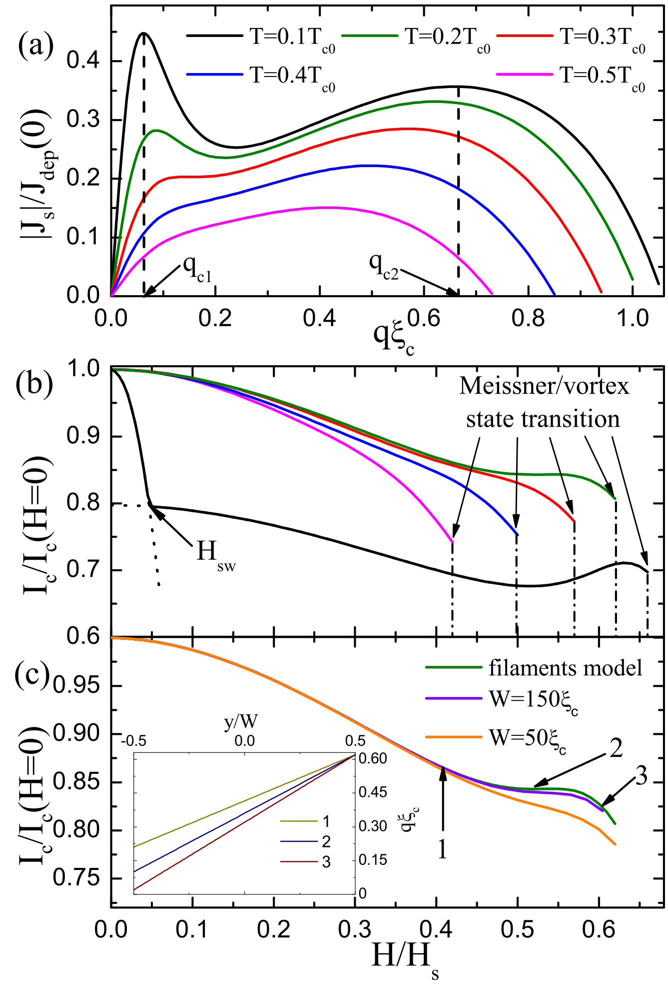

FIG. 2. (a) Dependence of the amplitude of sheet superconducting current density J_s on q in a superconductor/normal-metal (SN) strip calculated at different temperatures T in the one-dimensional (1D) Usadel model. (b) Calculated $I_c(H)$ of a SN strip at different T (filaments model). (c) Calculated $I_c(H)$ in filaments and two-dimensional (2D) Usadel models at $T = 0.2T_{c0}$. In the inset, we show the spatial distribution of $q(y)$ over the SN strip at different fields marked by numbers 1–3. The SN strip has the following parameters: $d_S = 2\xi_c$, $d_N = 4\xi_c$, and $\rho_S/\rho_N = 100$.

The dependence $I_c(H)$ [see Fig. 2(b)] changes with the temperature according to transformation of $J_s(q)$. Indeed, the external magnetic field changes the distribution of q across the strip [see inset in Fig. 2(c)]. When the width of the SN strip is much larger than ξ_N, one may assume that local $J_s(y)$ is determined only by local $q(y)$. With increasing magnetic field, q decreases in the strip (except at the edge $y = W/2$), and it leads to monotonous decrease of $|J_s|$ and critical current $I_c = \int J_s dy$ when dependence $|J_s|(q)$ has only one maximum. However, with decreasing temperature, the additional maximum appears at low q. At first, it leads to flattening of $I_c(H)$ [see Fig. 2(b) at $T = 0.2T_{c0}$] because of flattening of $J_s(q)$. When the height of the first maximum becomes larger than the second one, the dependence $I_c(H)$ changes drastically [see Fig. 2(b) at $T = 0.1\ T_{c0}$]. At low fields, I_c drops fast with increase of H because of much smaller value of $q(W/2) = q_c = q_{c1} \ll q_{c2}$. At some field [marked by H_{sw} in Fig. 2(b)],

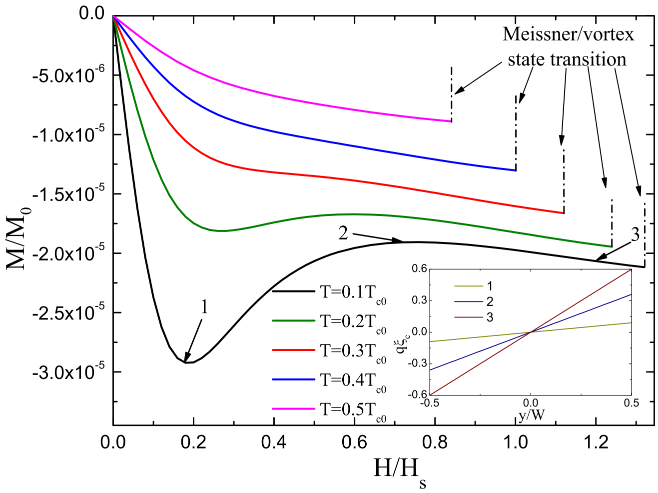

FIG. 3. The magnetization curves of the superconductor/normal-metal (SN) strip calculated in the two-dimensional (2D) model at different temperatures. Magnetization M is measured in units of $M_0 = \Phi_0/2\pi\xi_c^2$. In the inset, we show the distribution $q(y)$ over the SN strip at $T = 0.1T_{c0}$ and different fields marked by numbers 1–3. The SN strip has the width $W = 80\xi_c$, and the other parameters are as in Fig. 2.

the SN strip transit to the state with larger $q(W/2) = q_c = q_{c2}$ because, in this case, the SN strip may carry larger critical current [28]. At $H > H_{sw}$, the peak in $I_c(H)$ appears which is a consequence of the first maximum in $|J_s|(q)$.

At some magnetic field, q and $J_s(q)$ change the sign at $y = -W/2$. It means that vortices which enter at opposite edges ($y = W/2$) cannot exit the SN strip. In an ordinary S strip, it leads to the peak in $I_c(H)$ [21,29]. We expect similar behavior in the SN strip too. Because vortex states cannot be described by the used model, we are bounded by the field $q_{c2}\Phi_0/2\pi W$ at which $q(-W/2) = 0$.

The above discussed features could be seen only for sufficiently wide strips. In a relatively narrow strip with $W \lesssim \xi_N$, the proximity effect from the adjacent regions plays an important role, and nonmonotonous behavior is smeared out [see Fig. 2(c)].

It is known that, in the ordinary superconductors, the peak in $I_c(H)$ is followed by the peak in the magnetization curve $M(H)$ (or vice versa). In the SN strip, we also find the peak in $M(H) = \iint [\mathbf{r} \times \mathbf{j_s}]dxdy/[2c(d_S + d_N)W]$ (see Fig. 3). In contrast to the ordinary peak effect, the peaks are located at different fields for $I_c(H)$ and $M(H)$ dependencies. The reason for this is the following. In absence of the current, the evolution of $q(y)$ with increasing of H is different to the situation with current $I = I_c(H)$ [see inset in Fig. 3—in this case, $q(y) = 2\pi A(y)/\Phi_0$]. It results in larger screening currents at low fields than at high H (at $T = 0.1T_{c0}$), and the peak is located at a lower field. Here, we stop calculations at the magnetic field $H = \Phi_0 q_{c2}/(\pi W)$ when $|q(\pm W/2)| = q_{c2}$, and we expect vortex entrance to the SN strip.

We believe that the same effect should exist in an SS′ bilayer where S′ is a superconductor which has a large diffusion coefficient (low resistivity in the normal state, for example, Al, Pb, or Sn). Due to large $D_{S'}$, the superconductivity in the S′ layer should be destroyed at smaller q, and the current-supervelocity dependence will have two maxima at proper choice of d_S, $d_{S'}$, and temperature. Because qualitatively similar $J_s(q)$ dependence was predicted for the two-band superconductor MgB_2 (see Refs. [24,25]), a peak or plateau in $I_c(H)$ should be observed in a MgB_2 thin strip at fields $\lesssim q_{c2}\Phi_0/2\pi W$. However, an important condition for experimental observation of the predicted effect is approaching of $I_c(H = 0)$ to the depairing current of SN, SS′ or MgB_2. The easiest critical current about I_{dep} could be probably reached in the SN system as it has been demonstrated recently for the MoN/Cu strip [27]. However, in MgB_2 strips/bridges, depairing current has not been reached yet. In Refs. [30–32], $I_c \simeq 15$–30% of the depairing current was claimed, which is not large enough for observation of the predicted peak effect.

To conclude, we hope that the experimental observation of the peak or plateau on $I_c(H)$ and/or $M(H)$ dependencies at low fields would indirectly confirm existence of two peaks in $J_s(q)$ dependence in SN, SS′ hybrid structures or many-band superconducting materials.

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APPENDIX: USADEL MODEL

To calculate the superconducting properties of the SN strip, we use the Usadel model for normal $g = \cos\Theta$ and anomalous $f = \sin\Theta \exp(i\varphi)$ quasiclassical Green functions inside both S and N layers. We neglect the dependence of Θ on the longitudinal coordinate z since the length of the SN strip $L \to \infty$ and the system is uniform in this direction. Therefore, we use the 1D Usadel equation:

$$\frac{\hbar D}{2}\frac{\partial^2\Theta}{\partial x^2} - \left(\hbar\omega_n + \frac{\hbar D}{2}q^2\cos\Theta\right)\sin\Theta + \Delta\cos\Theta = 0, \tag{A1}$$

and the 2D Usadel equation:

$$\frac{\hbar D}{2}\left(\frac{\partial^2\Theta}{\partial x^2} + \frac{\partial^2\Theta}{\partial y^2}\right) - \left(\hbar\omega_n + \frac{\hbar D}{2}q^2\cos\Theta\right)\sin\Theta + \Delta\cos\Theta = 0. \tag{A2}$$

Here, D is a diffusion coefficient ($D = D_S$ and $D = D_N$ in superconducting and normal layers, respectively), $\hbar\omega_n = \pi k_B T(2n + 1)$ are the Matsubara frequencies (n is an integer), and Δ is the superconducting order parameter, which is nonzero only in the S layer. Coordinate axes are presented in Fig. 1. Here, Δ should satisfy the self-consistency equation:

$$\Delta \ln\left(\frac{T}{T_{c0}}\right) = 2\pi k_B T \sum_{\omega_n>0}\left(\sin\Theta_S - \frac{\Delta}{\hbar\omega_n}\right), \tag{A3}$$

where T_{c0} is the critical temperature of a single S layer in the absence of magnetic field. Equations (A1) and (A2) are supplemented by the Kupriyanov-Lukichev boundary conditions between layers [33] with fully transparent interfaces:

$$D_{\rm S}\frac{d\Theta_{\rm S}}{dx}\bigg|_{x=d_{\rm S}-0} = D_{\rm N}\frac{d\Theta_{\rm N}}{dx}\bigg|_{x=d_{\rm S}+0}.$$

On the interfaces between the system and vacuum, we use $d\Theta/dn = 0$.

The superconducting current density is calculated as

$$j_s(x,y) = \frac{2\pi k_{\rm B}T}{e\rho} q \sum_{\omega_n>0} \sin^2\Theta, \tag{A4}$$

where ρ is the resistivity of the corresponding layer. To find $j_s(x, y)$, we numerically solve either Eq. (A1) or (A2) and Eq. (A3). Equations are solved by an iteration procedure using the Newton method combined with a tridiagonal matrix algorithm. Obtained $\Theta(x, y)$ is inserted into Eq. (A3) to find Δ, and then iterations repeat until the self-consistency is achieved.

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