Letter Editors' Suggestion

Quantized phase-coherent heat transport of counterpropagating Majorana modes

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We demonstrate that phase-coherent heat transport constitutes a powerful tool to probe Majorana physics in topological Josephson junctions. We predict that the thermal conductance transverse to the direction of the superconducting phase bias is universally quantized by half the thermal conductance quantum at phase difference $\phi = \pi$. This is a direct consequence of the parity-protected counterpropagating Majorana modes which are hosted at the superconducting interfaces. Away from $\phi = \pi$, we find a strong suppression of the thermal conductance due to the opening of a gap in the Andreev spectrum. This behavior is very robust with respect to the presence of magnetic fields. It is in direct contrast to the thermal conductance of a trivial Josephson junction which is suppressed at any phase difference ϕ. Thus, thermal transport can provide strong evidence for the existence of Majorana modes in topological Josephson junctions.

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I. INTRODUCTION

Topological insulators (TIs) have received considerable interest during the past decade [1–3]. They are characterized by an insulating bulk and topologically protected surface states. Two-dimensional TIs that can be realized in HgTe/CdTe quantum wells manifest themselves in the quantum spin Hall effect [4,5] where two helical edge states give rise to a quantized electrical conductance of $2e^2/h$ [6]. In three-dimensional TIs, such as strained HgTe [7,8], two-dimensional Dirac surface states with linear dispersion and spin-momentum locking arise due to relativistic corrections.

Similarly, topological superconductors have a gapped bulk and topologically protected surface states. These surfaces states, at zero energy, are Majorana modes which are their own particle-hole conjugate [9,10]. Majorana modes have received attention in the past few years [11–13] because they can form nonlocal qubits for fault-tolerant quantum computing. The manipulation of quantum information in such qubits becomes possible via braiding of Majorana fermions due to their non-Abelian braiding statistics [14]. Majorana modes occur in the form of localized zero modes whose braiding is, however, challenging and requires a sophisticated three-turn protocol [15]. One-dimensional Majorana modes provide a promising alternative for easier braiding [16]. They have been predicted to occur in superconductor-quantum anomalous Hall systems where they give rise to a half-integer quantization of the electrical two-terminal conductance [17,18]. However, the same signature occurs when the superconductor shorts opposite Hall edges. Indeed the robust appearance of half-integer quantized electrical conductances in recent experiments [19] indicates that its observation does not provide any evidence of chiral Majorana modes in contrast to earlier claims [20].

In this Letter, we propose phase-coherent heat transport as an alternative method to probe the presence of one-dimensional Majorana modes. It can overcome the issues of electrical detection schemes because superconductors are perfect electrical conductors but thermal insulators. Phase-dependent, longitudinal thermal transport of a temperature-biased Josephson junction was studied theoretically [21–26] and observed in conventional Josephson junctions [27]. Phase-dependent thermal transport arises due to thermally excited quasiparticles above the superconducting gap. It is sensitive to Andreev bound states in the superconducting gap due to a direct connection of the thermal conductance to the local density of states. In particular, for Josephson junctions based on TIs, the longitudinal thermal conductance exhibits a robust minimum at a phase difference of π which is linked to the presence of localized, zero-dimensional Majorana modes in the junction [28–30].

Contrary to previous studies, in this Letter, we consider four-terminal devices where the gradient of temperature is applied perpendicular to the phase-dependent Josephson junction. We dub transport in these devices transverse phase-dependent thermal conductance, see Fig. 1(a). To get signatures of Majorana states, we analyze the transverse thermal conductance due to transport below the gap in phase-biased Josephson junctions based on surface states of three-dimensional TIs proximitized by s-wave superconductors (S). We predict that the transverse thermal conductance is quantized as a half of the thermal conductance quantum when the junction hosts one-dimensional helical Majorana modes, while it is exponentially suppressed otherwise. The thermal conductance quantization is very robust and persists in the presence of magnetic fields as long as transport is dominated by counterpropagating Majorana modes. For

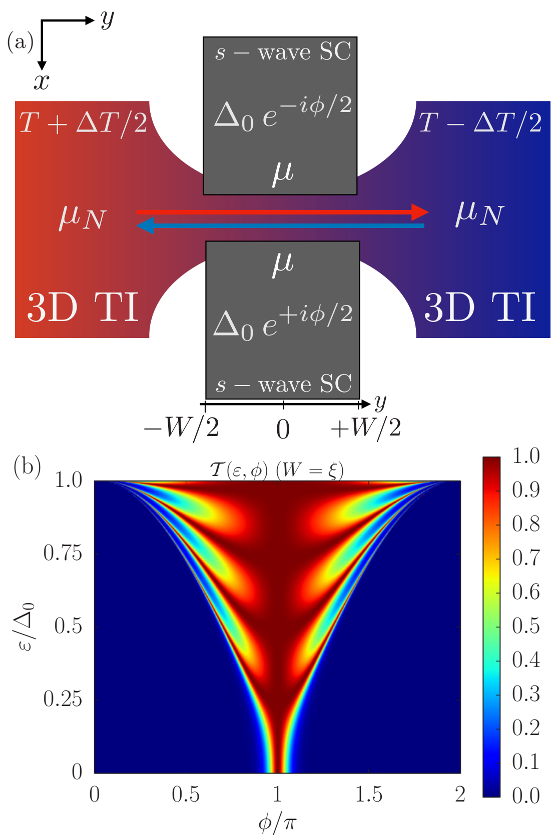

FIG. 1. (a) Short S-TI-S Josephson junction of width W embedded in a point contact geometry to measure heat transport perpendicular to the direction of phase bias ϕ. Thermal transport is driven by a temperature difference ΔT between the two TI leads and occurs via two counterpropagating modes (red and blue arrow) in the leads and in the superconducting region. (b) Quasiparticle transmission $\mathcal{T}$ given by Eq. (9) as a function of energy ε and phase difference ϕ for $\mu = 10\Delta_0$ and $Z_0 = \pi/4$. For $\phi = \pi$, we find perfect transmission while for $\phi \neq \pi$, the transmission is exponentially suppressed at low energies.

trivial junctions, the phase dependence of thermal transport is markedly different. Therefore, we conclude that transverse heat transport offers a compelling tool to distinguish between trivial and nontrivial junctions. Moreover, our setup is very different from that of Refs. [31,32] where intrinsic p-wave superconductors are considered and where chiral Majorana modes give rise to a quantized thermal Hall effect.

II. MODEL

We consider a Josephson junction based on the surface states of a three-dimensional topological insulator located in the $x-y$ plane, see Fig. 1. The junction has a width W in the y direction. Its length L in the x direction is assumed to be short compared to the superconducting coherence length ξ. We consider the limit $L/\xi \to 0$ for convenience but our results also apply to short junctions of finite length as demonstrated in the Supplemental Material [33]. The central part of the junction contains a potential barrier of strength V_0 and a Zeeman field $\mathbf{B}$ pointing in an arbitrary direction. A phase bias of ϕ is applied between the two superconductors which both have chemical potential μ. The junction is embedded into a point-contact geometry in the y direction. Thermal transport along the y direction is driven by a temperature bias ΔT between the left and the right lead which are formed by the normal surface states of the topological insulator.

We describe transport in the y direction in terms of an effective one-dimensional Hamiltonian

$$H(\hat{p}_y) = \begin{cases} H_S(\hat{p}_y) & |y| < W/2 \\ H_N(\hat{p}_y) & |y| > W/2 \end{cases}, \tag{1}$$

where the central part of the junction is described by $H_S(\hat{p}_y)$ while the two leads are described by $H_N(\hat{p}_y)$ with $\hat{p}_y = -i\hbar\partial_y$ denoting the transverse momentum. The Hamiltonian $H_S(\hat{p}_y)$ is obtained by projecting the transverse modes of the setup onto the two-level system formed by the two topological Andreev bound states (ABS) which emerge in the short S-TI-S junction at energies below the superconducting gap Δ_0. This is a reasonable approximation as long as the base temperature satisfies $k_B T \ll \Delta_0$. The modes are localized in the x direction but propagate along the y direction and can be described by

$$H_S(\hat{p}_y) = E_0(\phi)\sigma_z + v_M(\phi)\hat{p}_y\sigma_y + v_J(\phi)\hat{p}_y\sigma_0. \tag{2}$$

The first term $E_0(\phi)\sigma_z$ provides the 4π-periodic spectrum of the two topological ABS branches for zero transverse momentum. Explicitly,

$$E_0(\phi) = \frac{\Delta_0 \cos(\phi/2 + Z_y)}{\sqrt{\cos^2(Z) + Z_0^2 \sin^2(Z)/Z^2}}, \tag{3}$$

with $Z^2 = Z_0^2 - Z_x^2 - Z_z^2$, $Z_0 = V_0/(\hbar v_F)$ and $Z_{x,y,z} = B_{x,y,z}/(\hbar v_F)$. In the absence of any Zeeman field, $E_0(\phi)$ reduces to the well-known energy spectrum $E_0(\phi) = \Delta_0 \cos(\phi/2)$ [34]. The second term $v_M(\phi)\hat{p}_y\sigma_y$ describes the two counterpropagating modes while the third term $v_J(\phi)\hat{p}_y\sigma_0$ emerges only in the presence of a Zeeman field along the x direction and describes a tilting of the dispersion. Just like $E_0(\phi)$, the two velocities $v_M(\phi)$ and $v_J(\phi)$ depend on the Zeeman field $\mathbf{B}$ and the barrier strength V_0. The explicit dependence on these parameters as well as a detailed derivation of H_S are given in the Supplemental Material [33].

We assume that the leads support two propagating modes which are not subject to superconductivity or any Zeeman field. Therefore, we can model the TI leads by

$$H_N(\hat{p}_y) = v_N\hat{p}_y\sigma_y - \mu_N, \tag{4}$$

where $v_N > 0$ denotes the Fermi velocity in the leads and μ_N takes into account a mismatch of the chemical potential with respect to the superconducting region. As we will see below, the thermal conductance is actually independent of μ_N and v_N. Furthermore, we demonstrate in the Supplemental Material that the choice of Pauli matrices in H_N does not have any qualitative effect on our results because backscattering at the junction is forbidden due to spin-momentum locking and Majorana-parity conservation.

In order to calculate the thermal conductance in the y direction, we determine the transmission $\mathcal{T}$ from a scattering approach. An incoming quasiparticle from the left lead can either be reflected with reflection amplitude r or transmitted to the right lead via the superconducting region with transmission amplitude t. This leads to the plane-wave ansatz

$$\Psi(y) = \begin{cases} \psi_+ e^{i k_+ y} + r\psi_- e^{ik_- y} & y < -W/2 \\ a\tilde{\psi}_+ e^{i \tilde{k}_+ y} + b\tilde{\psi}_- e^{i\tilde{k}_- y} & |y| < W/2 \; , \\ t\psi_+ e^{ik_+ y} & y > +W/2 \end{cases} \tag{5}$$

where $\psi_\pm$ and $\tilde{\psi}_\pm$ are the right- and left-moving scattering states of H_N and H_S, respectively, for a given energy ε. They are given by

$$\psi_\pm = 1/\sqrt{2}\,(1, \pm i)^T,$$
$$\tilde{\psi}_\pm \propto (\pm v_J E_0 + \gamma, \pm i v_M(\varepsilon - E_0))^T, \tag{6}$$

where we dropped the ϕ dependence of E_0 for brevity and defined $\gamma = \sqrt{(v_J^2 - v_M^2)E_0^2 + v_M^2\varepsilon^2}$. The corresponding wave vectors read $k_\pm = \pm(\varepsilon + \mu_L)/(\hbar v_L)$ and $\tilde{k}_\pm = [v_J\varepsilon \mp \gamma]/[\hbar(v_J^2 - v_M^2)]$. The conservation of quasiparticle currents at the interfaces $y = \pm W/2$ is ensured by the boundary conditions

$$\hat{J}_N^{+1/2}\Psi(\pm(W/2 + 0^+)) = \hat{J}_S^{+1/2}\Psi(\pm(W/2 - 0^+)), \tag{7}$$

where $\hat{J}_{N,S} = \partial_{\hat{p}_y} H_{N,S}$ are the quasiparticle current operators of the leads and the superconducting region. In the absence of a Zeeman field in the x direction ($v_J = 0$), Eq. (7) reduces to the standard continuity of the wave function. From the wave-function matching procedure, we find t and subsequently the transmission function $\mathcal{T}(\varepsilon, \phi) = |t|^2$.

If the temperature bias ΔT is small compared to the base temperature T, thermal transport is fully characterized by the linear thermal conductance which is given by

$$\kappa(\phi) = \frac{1}{h}\int_0^\infty d\varepsilon\, \varepsilon\mathcal{T}(\varepsilon, \phi)\frac{df}{dT}. \tag{8}$$

Here, $f = [\exp(\varepsilon/(k_B T)) + 1]^{-1}$ denotes the equilibrium Fermi function.

III. QUANTIZED THERMAL CONDUCTANCE

First, we focus on the setup without Zeeman field. Then, $v_J = 0$ in Eq. (2) and we find

$$\mathcal{T}(\varepsilon, \phi) = \frac{\varepsilon^2 - E_0^2(\phi)}{\varepsilon^2 - E_0^2(\phi)\cos^2\left(\frac{W\sqrt{\varepsilon^2 - E_0^2(\phi)}}{\hbar v_M(\phi)}\right)}, \tag{9}$$

which is plotted in Fig. 1(b). At phase difference $\phi = \pi$, we find that the system is universally transparent $\mathcal{T} = 1$ independent of all other system parameters because of the presence of gapless, counterpropagating Majorana modes with $E_0(\phi = \pi) = 0$. The unitary transmission can be understood directly from a symmetry argument. For $\phi = \pi$, the mass term $E_0\sigma_z$ in H_S drops out and the full Hamiltonian (1) represents a chain of gapless helical systems. Consequently, backscattering processes are forbidden and the junction becomes fully transparent. In contrast, the energy spectrum of H_S is gapped if we tune ϕ away from $\phi = \pi$ where a finite mass term $E_0\sigma_z$ appears. On the one hand, this implies that quasiparticles can be backscattered for $\varepsilon > E_0(\phi)$. On the other hand, transport is additionally suppressed by tunnel processes for $\varepsilon < E_0(\phi)$. The combination of both effects leads to a

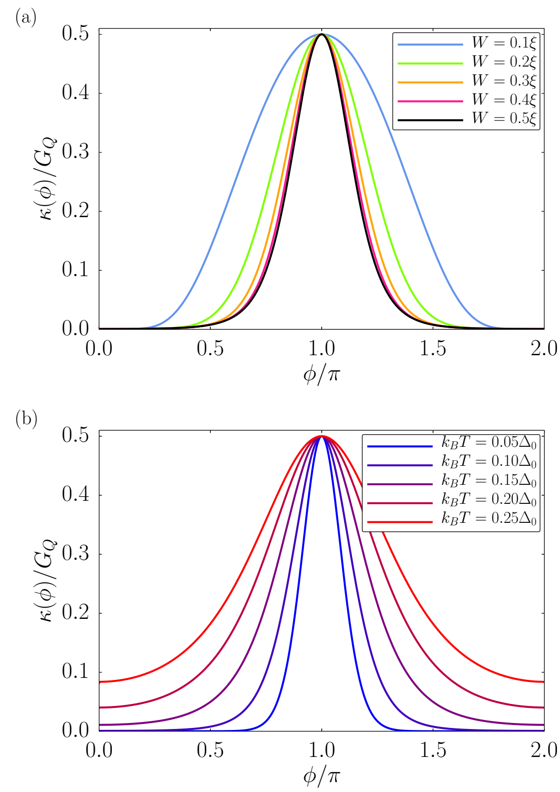

FIG. 2. Heat conductance $\kappa(\phi)$ in units of the thermal conductance quantum $G_Q = \pi^2 k_B^2 T/(3h)$ in the absence of any Zeeman field. (a) Width dependence at fixed base temperature $k_B T = 0.1\Delta_0$. (b) Temperature dependence at fixed width $W = \xi/2$. The thermal conductance $\kappa(\phi = \pi) = G_Q/2$ is quantized independently of the chosen parameters. Other parameters as in Fig. 1.

strong suppression of the transmission for $\phi \neq \pi$. This behavior of the quasiparticle transmission translates directly into the thermal conductance $\kappa(\phi)$ given by Eq. (8). As shown in Fig. 2(a), κ is perfectly quantized by half the thermal conductance quantum $G_Q = \pi 2 k_{\mathrm{B}}^2 T/(3h)$ for $\phi = \pi$ and decreases rapidly for $\phi \neq \pi$. The suppression of the thermal conductance arises in particular at low energies $\varepsilon < E_0(\phi)$ where transport is dominated by tunneling processes. In consequence, the conductance peak at $\phi = \pi$ becomes more pronounced as the width W of the junction in the y direction is increased, cf. Fig. 2(a). Furthermore, Figure 2(b) illustrates that a lower base temperature $k_B T$ reduces thermal broadening and consequently also leads to a more pronounced peak in the phase-dependent thermal conductance.

IV. MAGNETIC MANIPULATION

From an experimental point of view, it is highly desirable to probe and manipulate the quantized thermal conductance by additional parameters. Therefore, we now discuss the effect of a Zeeman term in the center of the S-TI-S junction. Due to the parity protection of the Majorana modes, this term cannot remove the protected zero-energy crossing. At most, the crossing is shifted in ϕ space [34,35].

The effects of a finite Zeeman field are the following: (i) an in-plane field B_y in y direction only shifts the spectrum and

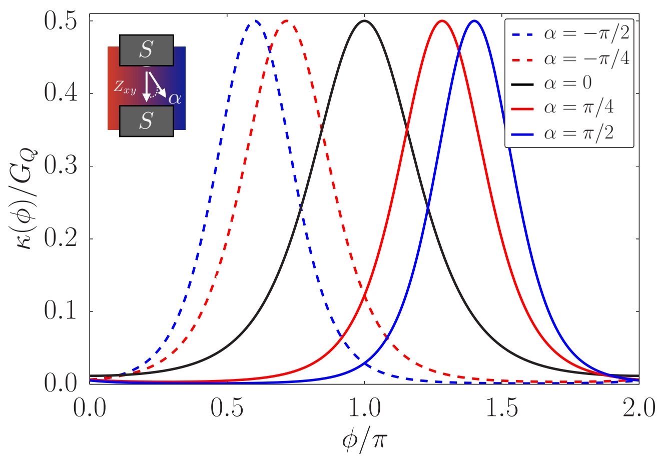

FIG. 3. (a) Heat conductance $\kappa(\phi)$ in units of the thermal conductance quantum G_Q for an in-plane Zeeman field $Z_{xy} = \pi/5$. While a finite Z_x broadens the conductance peak, a finite Z_y leads to a phase shift. The thermal conductance is quantized even in the presence of a finite Zeeman field. The remaining parameters are chosen as in Fig. 2 with $W = \xi/2$ and $k_B T = 0.1\Delta_0$.

thus the transmission,

$$\mathcal{T}(\varepsilon, \phi) \to \mathcal{T}(\varepsilon, \phi + 2Z_y); \tag{10}$$

(ii) field components in x or z direction lead to a renormalization of $E_0(\phi)$ and $v_M(\phi)$; (iii) in addition, a finite x component of the Zeeman field also gives rise to a finite $v_J(\phi)$ in Eq. (2). Even though $v_J(\phi)$ leads to an asymmetry of the group velocities $v \propto \partial_{k_y}\varepsilon$ for the left- and right-moving modes, one finds from Eq. (7) that quasiparticle current conservation still ensures a transparent junction at $\phi + 2Z_y = \pi$ as long as transport occurs via two counterpropagating modes which is the case for $|v_M| > |v_J|$ [33]. Consequently, $\kappa(\phi)$ remains quantized even in the presence of a Zeeman field, independent of its orientation. For illustration, Fig. 3(a) shows the effect of an in-plane Zeeman field Z_{xy} on the thermal conductance peak. While the x component $Z_x = Z_{xy}\cos(\alpha)$ leads to a broadening of the thermal conductance peak, the y component $Z_y = Z_{xy}\sin(\alpha)$ shifts the peak position in ϕ space according to Eq. (10).

V. COMPARISON WITH TRIVIAL JUNCTIONS

We will now show that the quantization of $\kappa(\phi)$ at $\phi = \pi$ allows us to distinguish topological from trivial junctions. To this end, we compare our results for the S-TI-S system to the phase-dependent transverse thermal conductance of a short junction based on a trivial S-N-S system without Zeeman field. Similar to the topological case, one can derive an effective two-level Hamiltonian corresponding to a projection onto the two ABS forming in short trivial junctions. For $\phi = \pi$, we find that the modes which propagate parallel to the SN interfaces can be described by

$$\widetilde{H}_S(k_y) = \left[\widetilde{E}_0(\phi = \pi) + \frac{\hbar^2 k_y^2}{2m_E}\right]\sigma_z. \tag{11}$$

Here, $\widetilde{E}_0(\phi) = \Delta_0\sqrt{1 - \sin^2(\phi/2)/(1 + Z_0^2)}$ denotes the spectrum of trivial ABS while m_E is a renormalized effective mass [33]. A key observation here is that a simple potential barrier in the N region of the SNS junction (giving rise to a finite Z_0) leads to a gapped system. Therefore, quasiparticle transport is exponentially suppressed as $\exp[-W/(\hbar v_F/\widetilde{E}_0(\phi = \pi))]$ at $\phi = \pi$ in this case. Consequently, the junction is no longer transparent ($\mathcal{T} \ll 1$ for $Z_0 \gg 1$) and the thermal conductance of the SNS system is *not* quantized ($\kappa/G_q \ll 1/2$) at $\phi = \pi$. In contrast, the S-TI-S setup is always gapless at $\phi = \pi$ due to the topological protection of the Majorana modes. The clear distinction between topologically trivial and nontrivial junctions is a direct consequence of our geometry where the finite width W of the Josephson junction in y direction suppresses tunnel processes which result from the gapped SNS spectrum.

VI. EXPERIMENTAL FEASIBILITY

We now comment on the experimental feasibility of our proposal. Thermal conductances have by now been measured in various nanosystems ranging from Josephson junctions [27] over quantum Hall setups [36] to quantum dots [37]. Topological Josephson junctions have been realized in a number of recent experiments. The induced superconducting gaps are of the order of $\Delta_0 \approx 100\,\mu$eV [38]. At a base temperature of $T \approx 30$ mK [38] our assumption $\Delta_0 \gg k_{\mathrm{B}}T$ is thus well fulfilled. For a Fermi velocity $\hbar v_F = 250$ meVnm [39], the superconducting coherence length is $\xi \approx 2.5\,\mu$m such that junctions with widths $W \sim \xi$ are experimentally accessible. A Zeeman term can be proximity-induced by a ferromagnetic insulator [40–42] or by an in-plane magnetic field such that orbital effects of the magnetic field can be neglected. Using the above parameters and a g factor of 20, we estimate that the helical Majorana modes exist up to fields of about 1.5 T. For large magnetic fields, the probability density of the Majorana modes is no longer constant across the junction; instead they become localized at opposite junction edges with a localization length $\propto (B_x^2 + B_z^2 - \mu^2)^{-1/2}$.

VII. CONCLUSION

We have demonstrated that phase-coherent heat transport provides a powerful method to probe the properties of helical Majorana modes in topological Josephson junctions. In particular, we have shown that the thermal conductance perpendicular to the direction of the phase bias is quantized for a phase difference of $\phi = \pi$. This is a direct consequence of the presence of gapless Majorana modes propagating parallely to the S-TI interface. Away from $\phi = \pi$, the Majorana mode is destroyed by broken time-reversal symmetry, a gap opens in the spectrum and leads to an exponential suppression of thermal conductance. The quantization of thermal conductance persists even in the presence of magnetic fields and is in stark contrast to the thermal transport properties of conventional Josephson junctions where the conductance is suppressed at any phase difference. The predicted effect is within the reach of current experimental technology, paving the way for future phase-coherent caloritronics with Majorana modes.

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