

## Stability of even-denominator fractional quantum Hall states in systems with strong Landau-level mixing

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Mixing of Landau levels has been understood to be essential in governing the nature of the ground state for the even-denominator fractional quantum Hall effect. The incompressibility of the ground state at filling factor  $5/2$  in the strong Landau-level-mixed systems, such as the ZnO quantum well, is not always stable. Here we present an approach to generally deal with this kind of system and satisfactorily explain the recent experiments [Falson *et al.*, *Sci. Adv.* **4**, eaat8742 (2018)] by implementing the screening plus the thickness effect. Further, the phase diagrams of the incompressibility of the ground state indicate that the phase transitions can be explicitly extracted by observing the lowest gap of the collective modes when the magnetic field and the width of the quantum well are tuned. We also predict the incompressibility of the two-dimensional electron gas in higher Landau levels in another strong Landau-level-mixed system, *viz.*, the black phosphorene, by considering the screening effect where the relevant even-denominator fractional quantum Hall effects can possibly be observed.

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For the past four decades, the quantum Hall effects have been the epitome of elegant phenomena in condensed matter physics [1]. Ever since the observation of the even-denominator fractional quantum Hall effect (FQHE) [2], it has been recognized that some difficulties remain in explaining this effect. Experimentally, the spin polarization of the  $5/2$  FQHE state is still debated [3] and very high-quality samples are an absolute requirement to resolve the issue. A couple of trial wave functions have been proposed thus far for this system [4,5]. However, the numerical results are somewhat size dependent so that the trial wave-function candidates are not so satisfactory and show relatively poor performance when compared with the numerical wave functions. It is also possible that both the long-range and the short-range interactions are important but the Haldane's pseudopotentials are limited to a spherical geometry. In a toroidal geometry the numerical strategy, although not entirely trouble free, can be a powerful alternative approach.

In practice, the even-denominator FQHE is proposed to be useful in topological quantum computation [6] since the non-Abelian excitations in the even-denominator FQHE are robust against many environment noises due to the topological protection. Determining the nature of the even-denominator FQHE states is therefore important for the development of quantum computation. The Moore-Read Pfaffian trial wave function (or its particle-hole conjugate anti-Pfaffian state) is

the most plausible candidate for the ground state, albeit the overlap between this wave function and the numerical results is not high. A particle-hole symmetric Pfaffian state for the ground state is proposed [7] that is believed to be stable when the Landau-level (LL) mixing and the influence of disorders in the two-dimensional electron gas (2DEG) are included [8]. Further studies have indicated the existence of a phase transition when the LL mixing increases [9]. The incompressibility of the 2DEG depends on the strength of the LL mixing, which can be effectively considered by screening the Coulomb potential. In the framework of screening, the surprise missing of the  $5/2$  plateau in ZnO quantum well has been explained [10,11]. In a more recent experiment performed in 2018, however, Falson *et al.* reported that the  $5/2$  FQHE is recovered in one sample [12], which could not be explained by our previous theoretical approach. In order to understand the strong LL-mixed 2DEG in general, and why the  $5/2$  FQHE is survived in one sample but is still missing in another sample of the 2018 experiment, we need to consider the screening effect together with the thickness effect associated with the quantum well in our analysis below.

The 2DEG discussed here has a unique property that the LL gap is very small compared to the Coulomb energy gap. The LL mixing is therefore strong and influences the properties of the ground state, and its incompressibility may be absent. A dimensionless quantity is defined to characterize the LL-mixing strength: the ratio of the Coulomb interaction strength  $E_C = e^2/\epsilon\ell$  to the LL gap  $E_{\text{cyc}} = \hbar\omega_c$ ,  $\kappa = E_C/E_{\text{cyc}}$ , where  $\epsilon$  is the dielectric constant of the material,  $\ell = 25.656/\sqrt{B}$  nm is the magnetic length, and  $\omega_c$  is the cyclotron frequency in

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the magnetic field  $B$ . This ratio is typically small in a conventional GaAs quantum well in a strong magnetic field, and is a constant 0.5–0.8 in graphene [13]. In contrast, it is very large in ZnO,  $\kappa > 10$  (the same order as in black phosphorene). The perturbation theory approach, including the three-body interaction which is based on the expansion of  $\kappa$  [14], is not appropriate for these systems. To overcome this difficulty, we make use of the screened Coulomb potential in the relevant LL, which is obtained by integrating out all the other LLs, to effectively describe the LL mixing.

As has been done in previous works [11], we also consider the electron-electron interaction described by the screened Coulomb potential in our present approach. However, if we just consider the pure 2DEG, we only have unstable 5/2 FQHE in both systems considered in the experiment [12,15], which clearly conflicts with the experimental observations. It has been suggested that we need to consider the third dimension, the thickness, of the quantum well in order to get a better understanding of the present situation. For simplicity, the confinement of the quantum well is approximately described either by a parabolic potential [16,17] or an infinite square potential [18]. We will compare these two approximations, of which the results are supposed to be coincident.

The many-body Hamiltonian is given by

$$H = \sum_{n,i,\sigma} E_{n,\sigma} c_{n,i,\sigma}^\dagger c_{n,i,\sigma} + \frac{1}{2} \sum_{\sigma,\sigma'} \sum_{n_1,\dots,n_4} V_{(s),i_1,i_2,i_3,i_4}^{n_1,n_2,n_3,n_4} \times c_{n_1,i_1,\sigma}^\dagger c_{n_2,i_2,\sigma'}^\dagger c_{n_3,i_3,\sigma'} c_{n_4,i_4,\sigma}, \quad (1)$$

where  $c$  is the electron operator,  $E_{n,\sigma}$  is the kinetic energy of the LL  $n$  with spin  $\sigma$ ,  $n_i$  is the LL index, and  $i_{1,\dots,4}$  are the guiding center indices. The Coulomb interaction matrix element  $V_{(s),i_1,i_2,i_3,i_4}^{n_1,n_2,n_3,n_4}$  depends on the confinement potential, which will be explicitly given in the following cases. We perform the exact diagonalization scheme to numerically solve the Hamiltonian with the translational symmetry in the toroidal

geometry [19–21]. The collective modes of the system at zero temperature are obtained, and the incompressibility or instability of the FQHE state is then determined.

The virtual process between these LLs in the density response function should be excluded to avoid double counting of the correlations if more LLs are involved in the Hamiltonian. In the following, we need to analyze the screened dielectric functions in detail for different confinement potentials.

The noninteracting Hamiltonian can be exactly solved in a parabolic potential in a tilted (or perpendicular) magnetic field, and the screened Coulomb potential has been studied [17]. We will numerically evaluate the collective modes in different magnetic fields and different widths of the quantum well based on the formulations of Ref. [17].

For an infinite square potential, the noninteracting Hamiltonian can also be exactly solved when the magnetic field is perpendicular to the  $xy$  plane, in which the  $z$  component of the wave function can be separated,  $\Psi_{m,n,X}(\mathbf{r}, z) = \sqrt{2/L_z} \sin(m\pi z/L_z) \psi_{n,X}(\mathbf{r})$ , where  $L_z$  is the width of the quantum well,  $m$  is the Landau band index,  $n$  is the LL index,  $X$  is the guiding center index, and  $\psi_{n,X}(\mathbf{r})$  is the wave function of a conventional 2DEG in a magnetic field in the Landau gauge.

The screened dielectric function is given by

$$\epsilon_s(q, q_z) = 1 - \frac{4\pi e^2}{(q^2 + q_z^2)\epsilon} \chi_{nm}^0(q, q_z), \quad (2)$$

where the three-dimensional momentum contains the in-plane momentum  $q$  and the  $z$ -component momentum  $q_z$ .  $\chi_{nm}^0(q, q_z)$  is the noninteracting retarded density-density response function computed in the random phase approximation in the static limit [22]. If we consider that the relevant LLs are all in the lowest band, i.e.,  $m_i = 1$ , then the screened Coulomb interaction matrix element is

$$V_{(s),i_1,i_2,i_3,i_4}^{n_1,n_2,n_3,n_4} = \frac{e^2}{\epsilon\ell} \frac{2}{\pi N_s} \sum_{\mathbf{q}} V_{i_1,i_2,i_3,i_4}^{n_1,n_2,n_3,n_4}(\mathbf{q}) \int_0^\infty \frac{dq_z \ell}{\epsilon_s(q, q_z)(q^2 + q_z^2)\ell^2} \frac{32\pi^4 [1 - \cos(q_z L_z)]}{(4\pi^2 q_z L_z - q_z^3 L_z^3)^2}, \quad (3)$$

where  $\sum$  means that the term of  $q = 0$  is excluded in the summation, and  $V_{i_1,i_2,i_3,i_4}^{n_1,n_2,n_3,n_4}(\mathbf{q})$  is the Coulomb interaction matrix element for a 2DEG in a conventional zero-width quantum well without screening [22]. The integral includes the width and screening corrections.

Let us check the theory to see whether it agrees with the experiment [10] by using the following parameters: half width of the wave function ( $W = L_z/1.5$ ) about 5–6 nm, dielectric constant  $\epsilon = 8.5$ , and effective mass of the electron  $m^* = 0.44m_e$  with the electron mass  $m_e$ . We first assume the incompressible phase as the ground state, and evaluate the lowest excitation gap in the collective modes separating the incompressible ground state from the excitations. If the gap is larger than  $10^{-4}e^2/\epsilon\ell$ , then the FQHE is supposed to be stable. The reason for this particular choice of  $10^{-4}$  is because this energy corresponds to about 20 mK, which

guarantees that the thermal fluctuation cannot overwhelm the incompressibility of the system, as otherwise the FQHE cannot be observed in the current experimental condition. It is clearly shown in Fig. 1 that the collective modes (at the crosses) are softened at  $B = 3.75$  T and the translational invariant liquid phase no longer has the lowest energy. The 2DEG is compressible no matter what kind of potential is chosen, which coincides with the experimental observation and the previous theoretic work [11]. Figure 1 also shows the phase diagram of the ground state at  $\nu = 5/2$  for different values of magnetic fields and widths of the wave function, since the qualitative variation of the gap must represent the changes in the ground states. The phase diagrams for the two different potentials are qualitatively similar, i.e., the 5/2 FQHE is only stable when the quantum well is thick and the LL mixing is weak (small  $\kappa$ ). Note that the LLs are crossing and the

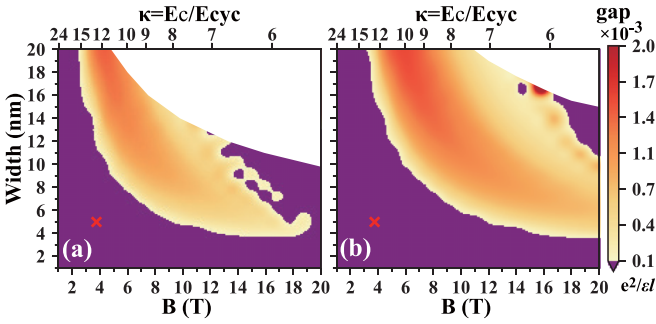


FIG. 1. The lowest energy gaps in the collective modes for different magnetic fields and widths of the wave functions in the ZnO quantum wells. Other parameters are extracted from Ref. [10],  $m^* = 0.44m_e$  and  $\epsilon = 8.5$ , and  $N_e = 11$ . The confinement of the 2DEG in the  $z$  direction is (a) parabolic potential and (b) infinite square well. When the gap is below  $10^{-4}e^2/\epsilon l$  (could be negative), which is shown in the purple region, the 2DEG is not incompressible. The crosses mark the sample in Ref. [10], where 5/2 FQHE is missing.

noninteracting ground states are changed in the white regions in Fig. 1.

Let us now move on to the new experiments [12] where the effective mass is  $m^* = 0.3m_e$  at magnetic fields  $B = 9.6$  T (sample *a*) and 7.2 T (sample *b*), corresponding to  $\kappa = 5.4$  and 6.2, respectively. The perturbation theory is still not very useful. Again, we compute the phase diagrams for this effective mass in parabolic and infinite square potentials, shown in Fig. 2. It is clear that for the infinite square well, we can

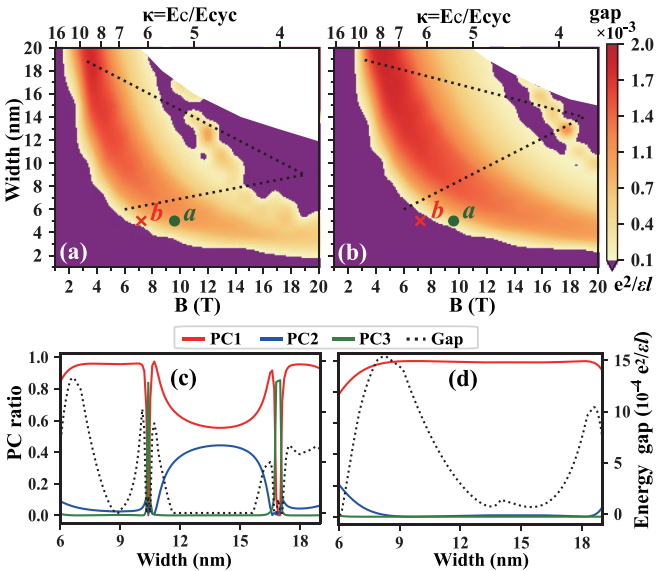


FIG. 2. Lowest gaps in the collective modes for different magnetic fields and the widths of the wave functions in the ZnO quantum well with  $m^* = 0.3m_e$  and  $\epsilon = 8.5$  represent the phase diagram in the strong LL mixed system. The confinement of the 2DEG in the  $z$  direction is (a) a parabolic potential and (b) an infinite square well. The left cross and the right dot mark the phase of samples *b* and *a* in Ref. [12], respectively. The first three principle components of the wave functions, which are extracted from the lines shown in (a) and (b), respectively, are shown in (c) and (d).

quantitatively find that the 5/2 FQHE is stable in sample *a* and unstable in sample *b* (in Ref. [12]), while the incompressibility of the 2DEG can be obtained qualitatively in the parabolic potential. It implies that the infinite square well may be a better approximation to describe the quantum well. Moreover, the second incompressible states found in tilted magnetic field in Ref. [12] can be explained by the phase transition induced by the LLs crossing when the Zeeman coupling is in excess of the LL gap [17].

The phase diagrams are not qualitatively different from those of  $m^* = 0.44m_e$ . Some important features can be found here. First, there is an arclike region isolated around the compressible region where the gaps become smaller or even negative for both types of potentials. It is similar to the case discussed in Refs. [9,23] where a topological phase transition was suggested. Here we also observe that the LL-mixing strength causes the phase transition. In addition, the width of the quantum well is even involved to induce more than one phase transition, which is not expected earlier. Roughly, when  $BW < 30$  T nm where  $W$  is the wave-function width, the 2DEG is compressible. Second, the energy gaps are somewhat size dependent. The gap becomes smaller when the electron number is increased. However, dealing with a larger number of electrons is not currently feasible, and we expect that for larger systems the FQHE in sample *a* still survives. We examine its stability in a three-LL model, i.e., we consider the Hamiltonian with three LLs  $N = 0, 1, 2$  with  $N_e = 15$  and the virtual process in these three LLs are excluded in the screening. The collective mode clearly supports that the ground state is incompressible.

To understand more about the incompressible phase, we perform the principle component (PC) analysis [24] to the ground state at zero momentum, as shown in Figs. 2(c) and 2(d). The wave functions are extracted along the black lines in Figs. 2(a) and 2(b). The first PC is almost the same in the arc region, which means that the ground states in the arc region share the same phase.

We next report on our study of black phosphorene, another strong LL-mixed system. Here we need to consider its bilayer structure in which the interlayer Coulomb potential is different from the intralayer Coulomb potential. We are required to extend the formula of screening to a bilayer system, i.e., the Coulomb potential and the density response function should be replaced by matrices,

$$V^s(q) = \begin{pmatrix} V_{11}^s(q) & V_{12}^s(q) \\ V_{21}^s(q) & V_{22}^s(q) \end{pmatrix}, \quad (4)$$

where  $V_{kl}^s(q)$  is the screened Coulomb potential between layers  $k$  and  $l$  [22].

Black phosphorene has a rectangular crystal lattice. There are four atoms  $A, B, C, D$  in a unit cell in which  $A, B$  are in layer 1 and  $C, D$  are in layer 2. It can be simplified to a two-band model when we only work on the bands near the Fermi surface [25]. The positive and negative filling factors correspond to the conduction and the valance bands, respectively.

The single-particle wave function is written as

$$\psi_{n,X}^{bp} = \frac{1}{\sqrt{2}} \sum_m \begin{pmatrix} u_{n,m} \psi_{m,X}(|B\rangle + |C\rangle) \\ v_{n,m} \psi_{m,X}(|A\rangle + |D\rangle) \end{pmatrix}, \quad (5)$$

where  $u_{n,m}$  and  $v_{n,m}$  can be obtained by diagonalizing the noninteracting Hamiltonian which is given in

$$V_{(s),i_1,i_2,i_3,i_4}^{n_1,n_2,n_3,n_4} = \frac{e^2}{\epsilon_{bp}\ell} \sum_{\mathbf{q}} \frac{V_{I1}^s + V_{I2}^s}{2N_s} \sum_{j_1,\dots,j_4} V_{i_1,i_2,i_3,i_4}^{j_1,j_2,j_3,j_4}(\mathbf{q}) (u_{n_1,j_1}^* u_{n_4,j_4} + v_{n_1,j_1}^* v_{n_4,j_4}) (u_{n_2,j_2}^* u_{n_3,j_3} + v_{n_2,j_2}^* v_{n_3,j_3}).$$

Note that the interlayer distance  $d$  may not be negligible [27], although  $d$  is as small as 2.13 Å.

Black phosphorene is also a large  $\kappa$  system ( $\kappa > 5.6$  when  $B < 10$  T) due to its heavy effective mass. The mobility of this system is sufficiently high so that the  $-4/3$  FQHE has already been observed [28]. We check the stability of this odd denominator FQHE for verification of our present approach. With screening, the  $\nu = \pm 1/3, \pm 4/3$  FQHEs are stable [22], which is not surprising and is compatible with the experiment results. We then explore whether the even denominator FQHE is observable in this material. Interestingly, the even denominator states  $\pm 5/2$  and  $\pm 7/2$  FQHEs are not stable, since the eigen wave function is composed of different  $\psi_{m,i}$ . The weights of  $\psi_{m \neq 1,i}$  play important roles in destabilizing these FQHEs.

For the higher LL  $N = 2$  ( $|\nu| = 9/2, 11/2$ ), the FQHEs survive due to the LL screening. As shown in Fig. 3, the 2DEG may not be incompressible until  $B > 40$  T if the screening is not considered, since the FQHE is not stable for  $N_e = 5$ . However, at  $\nu = 11/2$  the FQHE can be stabilized for a much lower magnetic field ( $B > 26$  T in the conduction band) for  $N_e = 5$ . In addition, the gap does not change too much when  $N_e \geq 7$ . For  $\nu = 9/2$ , although the gap decreases a bit when  $N_e$  is increased from 7 to 11, this gap is still of the order of

Refs. [25,26]. The Coulomb interaction matrix element reads

$10^{-3} e^2 / \epsilon_{bp} \ell$  which is one order larger than that in ZnO. We thus expect that  $11/2$  FQHE can be observed, but the mobility of this material may need to be further increased. We also assume that the  $9/2$  FQHE state is observable (provided the smaller system of  $N_e = 5$  can be ignored). That prediction requires experimental verification.

*Summary and remarks.* From the phase diagrams of the ground state at  $\nu = 5/2$  in the strong LL-mixing region, we find that the two stable FQHE regions (with large energy gap from the incompressible ground state) for both the parabolic and infinite square well potentials roughly coincide: the arc-like region surrounded by low (or negative) energy gap regions. This means that the phase transitions between the incompressible phase and the compressible phase occur more than once when either  $\kappa$  or width is varied. Moreover, even in the incompressible phase region, the topological phase transition between the (anti-)Pfaffian state and the particle-hole symmetric state which is suggested to be stabilized by the LL mixing [8], could also be possible by tuning the magnetic field [9] or the width of the quantum well. The topological features of the ground state should be determined in a thermal transport experiment [29]. We believe that the particle-hole symmetry [30] can be conserved by the extremely strong LL mixing, albeit the topological order of the  $5/2$  FQHE is still puzzling (especially when  $\kappa \sim 1$ ), since the numerical evidence has confirmed that the  $5/2$  FQHE can be stabilized at shift  $-1$  on a sphere when  $\kappa$  is large enough [9]. This argument is also compatible with the screening theory employed here, in which the two-body interaction with renormalized Coulomb interaction does not break the particle-hole symmetry.

By combining the screening and finite width corrections, we have successfully explained the latest experiments in ZnO. In the strong LL mixing limit, this strategy effectively takes the related correlations of other LLs into account and obeys the conservation laws. Moreover, we predict the stability of the  $5/2$  FQHE and present the phase diagrams at this filling factor for different magnetic fields and quantum well widths. The phase diagram should be amenable to verification by the experiments. Another kind of strong LL mixed system, the black phosphorene, is also considered and we have shown that the even denominator FQHEs are stabilized in higher LLs by the screening. This is also expected to be observed experimentally. The screening theory is expected to be applicable for study of the FQHE in black arsenic (in the same nitrogen family as phosphorus) with spin-orbit coupling [31] which also has strong LL mixing, and is probably helpful to understand the even denominator FQHE in monolayer WSe<sub>2</sub> [32]. Indeed, our present approach has laid the foundation for future studies of strongly LL-mixed systems.

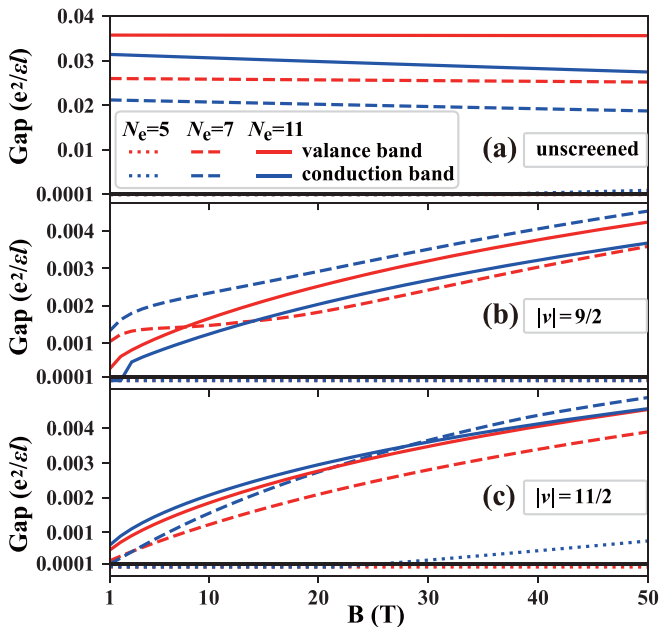


FIG. 3. The lowest gaps in the collective modes of the LL  $N = 2$  black phosphorene states in different magnetic fields: (a) without screening, (b)  $|\nu| = 9/2$  with screening, and (c)  $|\nu| = 11/2$  with screening.

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- [1] K. von Klitzing, T. Chakraborty, P. Kim, V. Madhavan, X. Dai, J. McIver, Y. Tokura, L. Savary, D. Smirnova, A. M. Rey, C. Felser, J. Gooth, and X. Qi, *Nat. Rev. Phys.* **2**, 397 (2020).
- [2] J. P. Eisenstein, in *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley-Interscience, New York, 1996), p. 37; R. Willett, J. P. Eisenstein, H. L. Störmer, D. C. Tsui, A. C. Gossard, and J. H. English, *Phys. Rev. Lett.* **59**, 1776 (1987); M. Greiter, X.-G. Wen, and F. Wilczek, *ibid.* **66**, 3205 (1991); R. L. Willett, *Rep. Prog. Phys.* **76**, 076501 (2013); J. Levallois, M. K. Tran, D. Pouliot, C. N. Presura, L. H. Greene, J. N. Eckstein, J. Uccelli, E. Giannini, G. D. Gu, A. J. Leggett, and D. van der Marel, *Phys. Rev. X* **6**, 031027 (2016).
- [3] M. Stern, P. Plochocka, V. Umansky, D. K. Maude, M. Potemski, and I. Bar-Joseph, *Phys. Rev. Lett.* **105**, 096801 (2010); L. Tiemann, G. Gamez, N. Kumada, and K. Muraki, *Science* **335**, 828 (2012); H. Fu, P. Wang, P. Shan, L. Xiong, L. N. Pfeiffer, K. West, M. A. Kastner, and Xi Lin, *Proc. Natl. Acad. Sci. USA* **113**, 12386 (2016).
- [4] G. Moore and N. Read, *Nucl. Phys. B* **360**, 362 (1991); N. Read and D. Green, *ibid.* **61**, 10267 (2000); N. Read, *Phys. B: Condens. Matter* **298**, 121 (2001).
- [5] B. I. Halperin, *Helv. Phys. Acta* **56**, 75 (1983); N. Read and E. Rezayi, *Phys. Rev. B* **59**, 08084 (1999); Ken K. W. Ma and D. E. Feldman, *ibid.* **100**, 035302 (2019).
- [6] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, *Rev. Mod. Phys.* **80**, 1083 (2008); X. Wan, Z. Wang, and K. Yang, *Physics (Beijing)* **42**, 558 (2013).
- [7] D. T. Son, *Phys. Rev. X* **5**, 031027 (2015).
- [8] P. T. Zucker and D. E. Feldman, *Phys. Rev. Lett.* **117**, 096802 (2016).
- [9] W. Luo and T. Chakraborty, *Phys. Rev. B* **96**, 081108(R) (2017).
- [10] J. Falson, D. Maryenko, B. Friess, D. Zhang, Y. Kozuka, A. Tsukazaki, J. H. Smet, and M. Kawasaki, *Nat. Phys.* **11**, 347 (2015).
- [11] W. Luo and T. Chakraborty, *Phys. Rev. B* **93**, 161103(R) (2016).
- [12] J. Falson, D. Tabrea, D. Zhang, I. Sodemann, Y. Kozuka, A. Tsukazaki, M. Kawasaki, K. von Klitzing, and J. H. Smet, *Sci. Adv.* **4**, eaat8742 (2018).
- [13] J. I. A. Li, C. Tan, S. Chen, Y. Zeng, T. Taniguchi, K. Watanabe, J. Hone, and C. R. Dean, *Science* **358**, 648 (2017); A. A. Zibrov, E. M. Spanton, H. Zhou, C. Kometter, T. Taniguchi, K. Watanabe, and A. F. Young, *Nat. Phys.* **14**, 930 (2018).
- [14] W. Bishara and C. Nayak, *Phys. Rev. B* **80**, 121302(R) (2009); E. H. Rezayi and F. D. M. Haldane, *ibid.* **42**, 4532 (1990); E. H. Rezayi and S. H. Simon, *Phys. Rev. Lett.* **106**, 116801 (2011); I. Sodemann and A. H. MacDonald, *Phys. Rev. B* **87**, 245425 (2013); R. E. Wooten, J. H. Macek, and J. J. Quinn, *ibid.* **88**, 155421 (2013); M. R. Peterson and C. Nayak, *Phys. Rev. Lett.* **113**, 086401 (2014); E. H. Rezayi, *ibid.* **119**, 026801 (2017).
- [15] J. Falson, *Physica E* **110**, 49 (2019); J. Falson and M. Kawasaki, *Rep. Prog. Phys.* **81**, 056501 (2018).
- [16] T. Chakraborty and P. Pietiläinen, *Phys. Rev. B* **39**, 7971 (1989); V. Halonen, P. Pietiläinen, and T. Chakraborty, *ibid.* **41**, 10202 (1990); V. Halonen, *ibid.* **47**, 4003 (1993); **47**, 10001(R) (1993).
- [17] Wenchen Luo and Tapash Chakraborty, *Phys. Rev. B* **94**, 161101(R) (2016).
- [18] M. R. Peterson, Th. Jolicoeur, and S. Das Sarma, *Phys. Rev. Lett.* **101**, 016807 (2008).
- [19] F. D. M. Haldane, *Phys. Rev. Lett.* **55**, 2095 (1985).
- [20] T. Chakraborty and P. Pietiläinen, *The Quantum Hall Effects* (Springer, New York, 1995); *The Fractional Quantum Hall Effect* (Springer, New York, 1988).
- [21] T. Chakraborty and P. Pietiläinen, *Phys. Rev. B* **38**, 10097(R) (1988).
- [22] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevB.104.L161302> for detailed formula on screening and more numerical data.
- [23] N. Samkharadze, D. Ro, L. N. Pfeiffer, K. W. West, and G. A. Csáthy, *Phys. Rev. B* **96**, 085105 (2017).
- [24] N. Jiang, S. Ke, H. Ji, H. Wang, Z.-X. Hu, and X. Wan, *Phys. Rev. B* **102**, 115140 (2020); N. Jiang and M. Lu, *Chin. Phys. Lett.* **37**, 117302 (2020).
- [25] J. M. Pereira, Jr. and M. I. Katsnelson, *Phys. Rev. B* **92**, 075437 (2015).
- [26] A. Ghazaryan and T. Chakraborty, *Phys. Rev. B* **92**, 165409 (2015).
- [27] W. Luo and R. Côté, *Phys. Rev. B* **90**, 245410 (2014); W. Luo and T. Chakraborty, *ibid.* **92**, 155123 (2015).
- [28] Jiawei Yang, Son Tran, Jason Wu, Shi Che, Petr Stepanov, Takashi Taniguchi, Kenji Watanabe, Hongwoo Baek, Dmitry Smirnov, Ruoyu Chen, and Chun Ning Lau, *Nano Lett.* **18**, 229 (2018).
- [29] M. Banerjee, M. Heiblum, V. Umansky, D. E. Feldman, Y. Oreg, and A. Stern, *Nature (London)* **559**, 205 (2018).
- [30] E. H. Rezayi and F. D. M. Haldane, *Phys. Rev. Lett.* **84**, 4685 (2000).
- [31] Feng Sheng, Chenqiang Hua, Man Cheng, Jie Hu, Xikang Sun, Qian Tao, Hengzhe Lu, Yunhao Lu, Mianzeng Zhong, Kenji Watanabe, Takashi Taniguchi, Qinglin Xia, Zhu-An Xu, and Yi Zheng, *Nature (London)* **593**, 56 (2021).
- [32] Qianhui Shi, En-Min Shih, M. V. Gustafsson, D. A. Rhodes, Bumho Kim, Kenji Watanabe, Takashi Taniguchi, Zlatko Papić, James Hone, and Cory R. Dean, *Nat. Nanotechnol.* **15**, 569 (2020).