Letter

Linear magnetoconductivity in magnetic metals

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We theoretically describe a mechanism of low-field linear magnetoconductivity in helical magnetic metals. Two ingredients for the mechanism in three-dimensional metals are identified to be the spin-orbit coupling and momentum-dependent ferromagnetic exchange interaction. We propose and study a number of minimal theoretical models which have linear magnetoconductivity and discuss their implications for recent experiments.

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Onsager's relations [1,2] dictate that the low-field electric conductivity of the system in the applied magnetic field must be even under the reversal of the magnetic field when the time-reversal symmetry is not violated in the system. However, when the time-reversal symmetry is broken in the system by, for example, spontaneous ferromagnetic order, the Onsager relations allow for the low-field linear magnetoconductivity in the system.

There is a number of recent experiments [3–5] which observe linear magnetoconductivity in ferromagnetic metals. Indeed, based on Onsager's relation argument, one would expect that, when spontaneous magnetization $\mathbf{M}$ is present in the system, there might be terms in the electric current which will depend on the magnetization and result in linear magnetoconductivity. Three such possible terms with a pronounced angle dependence between the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields and magnetization are proportional to $(\mathbf{E}\cdot\mathbf{B})\mathbf{M}$, $(\mathbf{E}\cdot\mathbf{M})\mathbf{B}$, and $(\mathbf{M}\cdot\mathbf{B})\mathbf{E}$ combinations, namely,

$$\delta\mathbf{j} = \alpha_1(\mathbf{E}\cdot\mathbf{B})\mathbf{M} + \alpha_2(\mathbf{E}\cdot\mathbf{M})\mathbf{B} + \alpha_3(\mathbf{M}\cdot\mathbf{B})\mathbf{E}, \quad (1)$$

where $\alpha_{1,2,3}$ are material dependent coefficients. Thus, varying the direction of either magnetic field, magnetization, or the current, one can identify the presence of each term in the system; see Fig. 1. However, besides the knowledge of the Onsager relation, the microscopic mechanism behind these three terms is still not fully understood. The aim of the present Letter is to introduce a number of theoretical models which provide a possible mechanism of linear magnetoconductivity in magnetic metals.

We assume that the spontaneous magnetization in the metals is due to the localized fermions, while the conduction fermions are responsible for the transport in these metals. The localized fermions interact with the conducting fermions via the ferromagnetic exchange interaction, which is proportional to the magnetization. In order to couple the magnetization with the momentum of conducting fermions we propose that the metals are helical, meaning that there is a spin-orbit coupling [6–9] which leads to the momentum-spin locking of conducting fermions. In the case of pure linear in momentum three-dimensional spin-orbit coupling, the ferromagnetic exchange interaction acting on the spin of conducting fermions just like the regular Zeeman magnetic field cannot affect the velocity of fermions unless there is spin-orbit coupling affecting motion of the conducting fermions. Indeed, ferromagnetic exchange interaction acting on conducting fermions can be gauged away by simply shifting the momentum of fermions. However, we show that the momentum-dependent ferromagnetic exchange interaction [10,11] does affect the velocity of conduction fermions, and leads to linear magnetoconductivity with all terms present in Eq. (1). The effect of momentum dependent ferromagnetic exchange on the magnetoconductivity has already been theoretically recognized in [12–14]. In the case of two-dimensional spin-orbit coupling, the Zeeman-like ferromagnetic exchange interaction can affect the velocity of fermions, but only when it has a component parallel to the spin-orbit coupling vector. We discuss such a scenario in our second example of the theoretical models. We show that the current Eq. (1) will depend only on one particular component of the magnetization.

The mechanism of linear magnetoconductivity proposed in this Letter is due to the effects of Berry curvature and orbital magnetization [15,16]. The Lorentz force in all of the presented cases does not result in linear magnetoconductivity.

Three-dimensional spin-orbit coupling. As a model of a three-dimensional metal with spin-orbit coupling (helical metal) we pick the Weyl semimetal [17] with two chiralities each described by a linear spectrum. We also include three possible momentum-dependent terms to the Hamiltonian, which might be present due to the finite magnetization $\mathbf{M}$ in the system. Our model Hamiltonian for the $s = \pm$ chiralities is

$$\hat{H}_s = sv(\boldsymbol{\sigma}\cdot\mathbf{k}) + (\boldsymbol{\sigma}\cdot\mathbf{M}) - \mu + sa_{\mathrm{A}}(\mathbf{M}\cdot\mathbf{k}) + a_{\mathrm{B}}\sum_n M_n k_n^2\sigma_n + a_{\mathrm{C}}(\mathbf{M}\cdot\mathbf{k})(\boldsymbol{\sigma}\cdot\mathbf{k}), \quad (2)$$

where v is the velocity of conducting fermions, μ is the chemical potential, $\boldsymbol{\sigma}$ are the Pauli matrices describing spin of fermions, and $\mathbf{k} = (k_x, k_y, k_z)$ is the three-dimensional momentum. The term with $a_{\rm A}$ is a tilt of the Dirac cones. The tilt breaks the time-reversal symmetry. The other two terms in the second line of Eq. (2), with $a_{\rm B}$ and $a_{\rm C}$, are momentum dependent ferromagnetic exchange interaction, which break the time-reversal symmetry as well. They were considered in [10,11] in studies of fermion's g-factor anisotropy in quantum wells. We note that the $a_{\rm A,B,C}$ terms can be understood as next order expansion in momentum of the fermion Hamiltonian in the vicinity of the $s = \pm$ Weyl nodes. The second term in the first line of Eq. (2) is the regular ferromagnetic exchange interaction, analogous to the usual Zeeman magnetic field. This term simply splits the two chiralities $s = \pm$ in momentum and can be shifted away from the Hamiltonian of a given chirality. Since we are interested in effects linear in $\mathbf{M}$ the shift will not affect the terms in the second line of Eq. (2). In all of the models symmetry between the chiralities is broken by the terms in the second line of Eq. (2).

Our three models, which we will be calling A, B, and C in accord with $a_{\rm A}$, $a_{\rm B}$, and $a_{\rm C}$ terms in Eq. (2) correspondingly, are three dimensional metals with spin-orbit coupling. This implies the presence of the Berry curvature and orbital magnetization in the description of the fermions [15,16]. To study the electric current, we employ the method of the kinetic equation,

$$\frac{\partial n_{\mathbf{k}}^{(s)}}{\partial t} + \dot{\mathbf{k}}^{(s)} \frac{\partial n_{\mathbf{k}}^{(s)}}{\partial \mathbf{k}} + \dot{\mathbf{r}}^{(s)} \frac{\partial n_{\mathbf{k}}^{(s)}}{\partial \mathbf{r}} = I_{\rm coll}\big[n_{\mathbf{k}}^{(s)}\big], \tag{3}$$

with equations of motion updated in the presence of the Berry curvature [16], $\dot{\mathbf{r}}^{(s)} = \frac{\partial \epsilon_{\mathbf{k}}^{(s)}}{\partial \mathbf{k}} + \dot{\mathbf{k}}^{(s)} \times \boldsymbol{\Omega}_{\mathbf{k}\eta}^{(s)}$, and $\dot{\mathbf{k}}^{(s)} = e\mathbf{E} + \frac{e}{c}\dot{\mathbf{r}}^{(s)} \times \mathbf{B}$, where $\boldsymbol{\Omega}_{\mathbf{k}}^{(s)}$ is the Berry curvature. The current is given by $\mathbf{j} = \sum_{s=\pm}\int_{\mathbf{k}} e\dot{\mathbf{r}}_{\mathbf{k}}^{(s)} n_{\mathbf{k}}^{(s)}$. In the fermion collision integral $I_{\rm coll}[n_{\mathbf{k}}^{(s)}]$ we consider two scattering processes described by different lifetimes. The first one is scattering within the chiralities denoted by τ and the other between the chiralities denoted by $\tau_{\rm V}$, namely $I_{\rm coll}[n_{\mathbf{k}}^{(\pm)}] = (\bar{n}^{(\pm)} - n_{\mathbf{k}}^{(\pm)})\tau^{-1} + (\bar{n}^{(\mp)} - n_{\mathbf{k}}^{(\pm)})\tau_{\rm V}^{-1}$, where $\bar{n}^{(s)} = (4\pi)^{-1}\int \sin(\theta) d\theta\, d\phi [1 + \frac{e}{c}(\mathbf{B}\cdot\boldsymbol{\Omega}_{\mathbf{k}}^{(s)})] n_{\mathbf{k}}^{(s)}$ is the distribution function averaged over the angles. To analyze the electric current we follow approximations used in [13].

In [13] the electric current for a system with $a_{\rm B} = a_{\rm C} = 0$ was studied, and it was shown that there is indeed linear magnetoconductivity due to an interplay of the chiral anomaly and the tilt of the Dirac cones. Based on the findings of [13] (also see the Supplemental Material [18]), we here distinguish three contributions to the current. All contributions are due to the asymmetry of velocities of fermions of opposite chiralities. The asymmetry is either due to the $a_{\rm A,B,C}$ terms or due to the Berry curvature. The first one is due to the chiral anomaly [19]—in other words, when a difference of charges at different chiralities builds up in the presence of electric and magnetic fields, namely $N_+ - N_- \propto \tau_{\rm V}(\mathbf{E}\cdot\mathbf{B})$. This contribution results in the $\propto\tau_{\rm V}(\mathbf{E}\cdot\mathbf{B})\mathbf{M}$ term in the current. The second contribution to the current is similar in nature to the first one, with the only difference being that a buildup of a nonzero chiral charge in the two valleys happens for a given absolute value of the momentum when only the electric field is present. Namely, $\bar{n}_k^{(+)} - \bar{n}_k^{(-)} \propto \tau_{\rm V}(\mathbf{E}\cdot\mathbf{M})$ and recall that $N_s = \int \frac{k^2 dk}{2\pi^2}\bar{n}_k^{(s)}$. We show that there is no chiral anomaly due to this contribution in all the three models. This contribution results in $\propto\tau_{\rm V}(\mathbf{E}\cdot\mathbf{M})\mathbf{B}$ to the current. Note that the first two contributions are defined by interchirality relaxation processes and are proportional to $\tau_{\rm V}$. The third contribution to the current is due to the Berry curvature and orbital magnetization corrections to the fermion velocity. This contribution is primarily defined by relaxation processes within the chirality and thus defined by time τ. All three terms present in Eq. (1) can be derived for the electric current from the third contribution. However, we are assuming that $\tau_{\rm V} \gg \tau$, which allows us to select from them only the unique term of the $\propto\tau(\mathbf{M}\cdot\mathbf{B})\mathbf{E}$ type. This assumption is legitimate given that the splitting between the chiralities defined by $\mathbf{M}$ is large. Details of the derivations are given in the Supplemental Material [18]. Here we list calculated expressions for the linear magnetotransport for the three models,

$$\delta\mathbf{j}_{\rm A} \approx \frac{e^3 a_{\rm A}}{\pi^2 c}\left\{-\tau_{\rm V}\left[\frac{1}{4}(\mathbf{E}\cdot\mathbf{B})\mathbf{M} + \frac{1}{6}(\mathbf{E}\cdot\mathbf{M})\mathbf{B}\right] + \frac{2\tau}{15}(\mathbf{M}\cdot\mathbf{B})\mathbf{E}\right\}, \tag{4}$$

$$\delta\mathbf{j}_{\rm B} \approx \frac{e^3 \mu a_{\rm B}}{\pi^2 c v}\left\{-\frac{\tau_{\rm V}}{15}[4(\mathbf{E}\cdot\mathbf{B})\mathbf{M} + (\mathbf{E}\cdot\mathbf{M})\mathbf{B}] + \frac{\tau}{7}(\mathbf{M}\cdot\mathbf{B})\mathbf{E}\right\}, \tag{5}$$

$$\delta\mathbf{j}_{\rm C} \approx -\frac{e^3 \mu a_{\rm C}}{\pi^2 c v}\left\{\frac{\tau_{\rm V}}{9}[(\mathbf{E}\cdot\mathbf{B})\mathbf{M} + (\mathbf{E}\cdot\mathbf{M})\mathbf{B}] + \frac{5\tau}{3}(\mathbf{M}\cdot\mathbf{B})\mathbf{E}\right\}. \tag{6}$$

In all three models all terms listed in Eq. (1) are present. The signs and numerical coefficients are model dependent. We also find in our calculations a $\propto \frac{1}{|\mathbf{M}|^2}(\mathbf{E}\cdot\mathbf{M})(\mathbf{M}\cdot\mathbf{B})\mathbf{M}$ term in the current for the model B.

Quasi-two-dimensional systems. Here we consider a quasi-two-dimensional system with two-dimensional Rashba spin-orbit coupling in the x-y plane, i.e., with Rashba spin-orbit coupling vector in z direction, and magnetization $\mathbf{M}$ pointing in z direction. This is a model of a hypothetical BiTeI type material with spontaneous magnetization pointing in z direction. The Hamiltonian of the system is

$$\hat{H}_{\rm D} = \frac{\mathbf{k}^2}{2m} + \lambda(k_x\sigma_y - k_y\sigma_x) + M_z\sigma_z - \mu, \tag{7}$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the three-dimensional momentum. The spectrum consists of two branches $\epsilon_{\mathbf{k};\pm} = \frac{k^2}{2m} \pm \sqrt{M_z^2 + (\lambda k_\parallel)^2}$ and we assume that the chemical potential μ is such that both branches are occupied. The Berry curvature can only point in the z direction. Moreover, integrating the kinetic equation over the angles, one can check that there is no chiral anomaly in the system, meaning that the $N_+ - N_- = 0$ in applied electric and magnetic fields. We approximate $\tau = \tau_{\rm V}$ as the two chiralities are close to each other in momentum and energy space. We calculate linear magnetoconductivity following the same steps outlined above, and we get

$$\delta\mathbf{j}_{\rm D} = \frac{e^3\lambda^2}{2m^2 c}\tau I_1[(E_z M_z)\mathbf{B} + (\mathbf{E}\cdot\mathbf{B})M_z\mathbf{e}_z] + \frac{e^3\lambda^2}{8m^2 c}\tau(6I_2 - I_3)(M_z B_z)\mathbf{E}, \tag{8}$$

where I_1, I_2, and I_3 are defined in the SM. Again, all terms listed in Eq. (1) are present in Eq. (8), but the current depends only on M_z. In addition to Eq. (8) we also find a $\propto M_z B_z E_z \mathbf{e}_z$ term in the current (see the Supplemental Material [18] for details). We cannot generalize the obtained expression Eq. (8) to any direction of the magnetization, because M_x and M_y can be shifted away from the Hamiltonian Eq. (7). One might wonder what nonlinear time-reversal symmetry breaking due to $\mathbf{M} = (M_x, M_y, M_z)$ corrections to the Hamiltonian, similar to those with a_B and a_C in Eq. (2), will do to the linear magnetoconductivity. According to [20], all such possible corrections which would enter the Hamiltonian Eq. (8) with σ_x and σ_y Pauli matrices will not affect the Berry curvature and orbital magnetization to linear order in magnetization $\mathbf{M}$. The result of those entering with σ_z in the case when $M_z = 0$ and $M_x \neq 0$ and $M_y \neq 0$ can be traced from the following argument. According to [20], one can think of a system which will have a nontrivial Berry curvature and orbital magnetization when $M_z = 0$, $M_x \neq 0$, and $M_y \neq 0$ in Eq. (7). In such a case one will then need to add the spin-orbit coupling term obeying, for example, a C_{3v} symmetry to Eq. (7). Such a spin-orbit coupling (which can be thought of having a vector in the y direction) reads as $H_{\rm SOC} \propto v_{\rm D} k_x \sigma_z + \alpha k_x (k_x^2 - 3k_y^2)\sigma_z$, where $v_{\rm D}$ and α are coefficients. Then, even in this case, the linear magnetoconductivity will be of the Eq. (8) form with the only difference of M_z being replaced by M_y with an appropriate coefficient.

Finally, note that the term with $\propto(\mathbf{E} \cdot \mathbf{B})$ in Eq. (8) recalls the chiral anomaly contribution; however, it is of different origin. In other words, there is no difference in chemical potentials of the two chiralities when electric and magnetic fields are applied, and as already mentioned $N_+ - N_- = 0$.

Discussion. Typically, low-field linear magnetoconductivity has a small magnitude, and at some point it gets overshadowed by quadratic magnetoconductivity as the magnetic field is increased. Despite that, linear magnetoconductivity has a rich anistropic structure, which can be tested in the experiment (see Fig. 1).

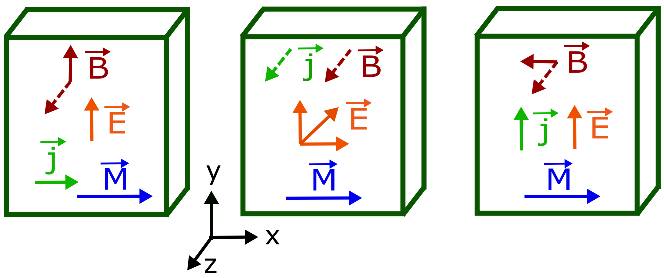

FIG. 1. Schematics of the proposed experimental setup to measure three unique angular dependences of the linear magnetoconductivity. Dashed lines are pointing in z direction. Left: the current is passed in x direction, the magnetic field is varied in z-y plane, and magnetization is in x direction, while the electric field is measured in the y direction. Hence only the $(\mathbf{E} \cdot \mathbf{B})\mathbf{M}$ component of the current is active. Center: the current is passed in z direction, magnetic field is in z direction, and magnetization is in x direction, while the electric field is measured in the x-y plane. In this case only the $(\mathbf{E} \cdot \mathbf{M})\mathbf{B}$ component of the current is active. Right: the current is passed in y direction, magnetic field is varied in z-x plane, and magnetization is in x direction, while the electric field is measured in the y direction. Thus only the $(\mathbf{M} \cdot \mathbf{B})\mathbf{E}$ component of the current is active.

We think that the low-energy description of the conduction fermions in ferromagnets, which experimentally show linear magnetoconductivity, fall into the classes of the theoretical models presented above. Or, it might as well be, into some other models with the same ingredients, namely, the spin-orbit coupling and momentum dependent ferromagnetic exchange interaction. As a result, if either of the α_1, α_2, or α_3 components of the current Eq. (1) is observed in the experiment, the other two must also be present. Based on our findings, below we comment on the two recent experiments.

In the experiment Ref. [4] linear magnetoconductivity was observed in ferromagnetic metal $SmCo_5$ and in ferromagnetic domains of the $Cd_2Os_2O_7$ antiferromagnet. Terms with $(\mathbf{E} \cdot \mathbf{B})\mathbf{M}$ and $(\mathbf{M} \cdot \mathbf{B})\mathbf{E}$ in Eq. (1) were observed in the experiment. Based on our findings, we think that a $(\mathbf{E} \cdot \mathbf{M})\mathbf{B}$ component of the current was overlooked [21]. We hope further experiments will identify this missing term, thus confirming our theoretical models and discussed above mechanism of linear magnetoconductivity. Moreover, we think that the $(\mathbf{E} \cdot \mathbf{B})\mathbf{M}$ term in the current observed in Ref. [4] might be due to the chiral anomaly. However, further analysis should be made to eliminate possible quasi-two-dimensional properties, where, as is shown for model D, Eq. (8), such term is present but is not due to the chiral anomaly.

In another experiment in Ref. [5] linear magnetoconductivity was observed in the magnetic Weyl semimetal $Co_3Sn_2S_2$ [22,23]. There it was claimed that the effect might be due to the tilt of the Weyl cones, namely due to the $a_{\rm A}$ term in Eq. (2)—a mechanism first proposed in [13]. Our findings introduced above suggest that the mechanism due to the tilt might not be the only one. Below we will make one more comment on the experiment in Ref. [5].

Below are four comments on the model Hamiltonians, Eqs. (2) and (7). First, any three-dimensional linear in momentum spin-orbit coupling will be described by the physics of Weyl semimetals. Hence the choice of the model Hamiltonian Eq. (2)—Weyl semimetal with two chiralities. However, one can reengineer the Hamiltonian, for example, by adding a regular, $\propto\frac{k^2}{2m}$ type, term. Then, in models B and C such a term will allow us to simplify the spectrum by reducing it to only one valley. Note that the problem of chiral anomaly would not be faced in this case, and the overall charge will be conserved. This is because there will still be two Fermi surfaces with opposite chiralities. Model A, on the other hand, cannot be reduced to only one valley.

Second, we saw that two main ingredients for the linear magnetoconductivity are the linear in momentum spin-orbit coupling and momentum-dependent ferromagnetic exchange interaction. However, this is not a unique combination, and one can achieve the same effect of momentum-dependent exchange interaction by introducing, in addition to linear spin-orbit coupling, next in expansion, if symmetry allows, cubic in momentum term. Then, ferromagnetic exchange interaction can be kept to zeroth order in momentum (just like a regular Zeeman term). The two schemes are similar to each other and should result in similar linear magnetoconductivity.

Third, in the realistic bulk systems the spin-orbit might not be pure, meaning that some Pauli matrix in the Hamiltonian is not, or only partly, describing the spin of the electrons. Instead, it might be describing pseudospin or some mixture of spin and, for example, the unit cell's degree of freedom. In this case **M** in the components in the current will be anisotropic, and, in the most severe example, the current might only depend on one projection of the magnetization **M**. For example, in model D, Eq. (7), we saw that it is only on M_z projection that the linear magnetoconductivity depends. However, all three terms in the linear magnetoconductivity are present in the model. When model D is reduced to two dimensions, only the $M_zB_z\mathbf{E}$ out of the three terms will survive.

Fourth, depending on the symmetries of the crystal structure of the experimental magnetic system, the magnetization **M** entering the current Eq. (1) and the second line in the Hamiltonian Eq. (2) might be replaced with its rotated direction, for example, $\mathbf{M} \to \mathbf{M} \times \mathbf{e}_{x,y,z}$ or even with other configurations [for example, see Eq. (3) in Ref. [20]]. Note that we have already discussed a possibility of such a replacement after Eq. (8). Quite likely, this situation was observed in the experiment of Ref. [5]. Namely, Ref. [5] observed two linear magnetoconductivity terms, $j_x \propto E_xM_zB_y$ and $j_y \propto E_xM_zB_x$. In terms of Eq. (1), the two can be understood as

$$\delta j_x = \alpha_3([M_z\mathbf{e}_z \times \mathbf{e}_x]\cdot\mathbf{B})E_x,$$
$$\delta j_y = \alpha_1(\mathbf{E}\cdot\mathbf{B})[M_z\mathbf{e}_z \times \mathbf{e}_x]_y. \quad (9)$$

Therefore, we predict that the $\delta\mathbf{j} = \alpha_2(\mathbf{E}\cdot[M_z\mathbf{e}_z \times \mathbf{e}_x])\mathbf{B} = \alpha_2(E_yM_z)\mathbf{B}$ term should also be observed in the experiment of Ref. [5].

Essentially, the theory presented in this Letter is based on the effect of the Berry curvature on the fermion properties. We note that there are other known scattering processes which contribute to the anomalous velocity of fermions. These are the skew-scattering and side-jump processes which are known, for example, to contribute to the anomalous Hall effect [24–26]. As is discussed in [27] these processes might contribute to the linear magnetoconductivity as well. Whether they will result in current of the Eq. (1) type is a question for future research.

Since the theory of the linear magnetoconductivity presented in this Letter stems from the Berry curvature of fermions, and the anomalous Hall effect does too [28], we think that both effects, linear magnetoconductivity and the anomalous Hall effect, should be experimentally looked for in the same material. For example, the model A is known [29] to show anomalous Hall effect as a function of the tilt; here and in [13] we concluded that it shows linear magnetoconductivity due to the same tilt as well. The model D [28] has the same feature. It can be checked that the remaining B and C models have the same property.

In passing, more magnetic Weyl and topological semimetals have been recently experimentally identified [30–32], and based on our findings here, we anticipate that linear magnetoconductivity, just like in experiments [3–5], should be observed in these systems. Moreover, we believe that linear magnetoconductivity should be added to a plethora of effects and properties such as the Fermi arcs [33], chiral anomaly driven positive longitudinal magnetoconductivity [19,34], and symmetric in magnetic field so-called planar Hall effect [both are the components of the $\delta\mathbf{j} \propto (\mathbf{E}\cdot\mathbf{B})\mathbf{B}$ current [13], and see comment [30] in Ref. [20]], anomalous Hall effect (due to the chirality splitting [35] and due to the tilt a_A alone [29]), chiral collective modes [36,37], and others, which make Weyl semimetals unique physical systems [38].

Conclusions. In this Letter we theoretically discussed the mechanism of linear magnetoconductivity in magnetic metals. We identified two necessary ingredients for the minimal model of the Hamiltonian of conducting fermions and three-dimensional spin-orbit coupling and momentum dependent coupling to the magnetization. If the spin-orbit coupling is two dimensional, the coupling to the magnetization is of regular exchange interaction. We proposed and studied four models, Eqs. (2) and (7), of such two scenarios. In all of the models linear magnetoconductivity contains three unique terms outlined in Eq. (1), with the model dependent coefficients; see Eqs. (4), (5), (6), and (8).

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current, as the work only focused on the terms due to the chiral anomaly, while [12] misses $(\mathbf{E}\cdot\mathbf{M})\mathbf{B}$ and $(\mathbf{E}\cdot\mathbf{B})\mathbf{M}$ terms in the current.

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