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Letter

Reservoir-assisted energy migration through multiple spin domains

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The transfer of energy through a network of nodes is fundamental to how both nature and current technology operate. Traditionally, we think of the nodes in a network being coupled to channels that connect them, in which energy is passed from node to channel to node until it reaches its targeted site. Here we introduce an alternate approach to this, where our channels are replaced by collective environments (or, actually, reservoirs) which interact with pairs of nodes. We show how energy initially located at a specific node can arrive at a target node—even though that environment may be at zero temperature. Further, we show that such a migration occurs on much faster timescales than the damping rate associated with a single spin coupled to the reservoir. Our approach shows the power of being able to tailor both the system and environment and the symmetries associated with them to provide new directions for future quantum technologies.

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Introduction. Nature has developed many methods for the transport of energy on length scales ranging from the atomic to cosmological [1–4]. Photosynthesis is one extremely well-known example where pigment cofactors absorb the light and transfer it to antennae pigments where it is converted to chemical energy [5–13]. Such energy transport is not restricted to natural processes, but is central to how our modern society and current technologies operate. We are always looking at new approaches to achieve this, but one needs to keep the possible applications in mind and the properties they require. In general, both classical and quantum systems are affected by the environment [14]. The natural question here is the following: Does noise help or hinder this transport process? Actually (and counterintuitively), it was found that energy transport can be enhanced by adding environmental noise [15–20]. Further, quantum mechanics provides unique opportunities in how energy transport could be enhanced using the principles of superpositions and entanglement [21–23], and establishes tight bounds on how fast such energy transport processes can be [24–26].

The recent developments in quantum technology have given us excellent design options to tailor both our system and environmental properties to the tasks we want to achieve [27–35]. It has been shown that a hybrid quantum system composed of an ensemble of negatively charged nitrogen-âvacancy (NV^-) centers in diamond coupled to a resonator [36] exhibits superradiant decay [37,38]—a collective effect where radiation is amplified by the coherence of multiple emitters. In fact, they showed collective decay that is 12 orders of magnitude faster than the decay of a single NV^- center [36]. Interestingly, the reverse process "superabsorption" also exists—when radiation is absorbed much faster into the ensemble [39], which has been experimentally realized by implementing a time-reversal process of superradiance [39]. From an energy transport point of view, combining the two phenomena would allow extremely fast energy transfer.

In our work, we show an unexplored mechanism to migrate energy via dissipation only, from an initial node to a spatially separated target node. We consider a network where the ith node is an ensemble with N_i spins. The energy is not being transferred into the reservoir from one node and then released to the next node. Instead, energy becoming present in the later nodes arises from symmetries associated with the collective decay. This is a very different way for energy to migrate through a chain of nodes. Additionally, due to superradiant decay, it is possible to achieve this migration extremely fast (much faster than the relaxation rate). Actually, at site i, it occurs $\sim N_i/\ln N_i$ times faster.

Generally, one would consider each ensemble coupling to its own environment. Hama *et al.* [40,41] recently considered collective coupling of two ensembles to a reservoir and noted an unusual observation. They investigated what occurs if the first much larger ensemble was initially fully excited while the second is in its ground state and found that the first ensemble "superradiantly" decays while the second ensemble undergoes "superabsorption" (and can become fully populated). However, the process is not that simple due to the nature of the coupling to the environment [40,42], which can induce coherent coupling [43,44]. The key to explain the observed behavior lies in the symmetries of the system, which can be seen from this very simple example. Consider two spins A and

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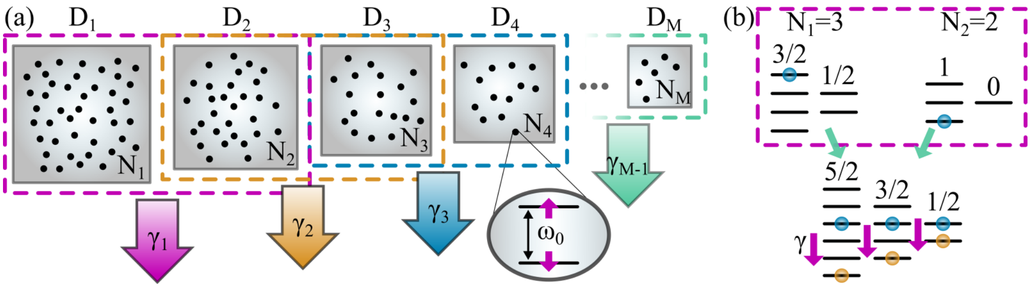

FIG. 1. (a) Illustration of a dissipatively coupled chain of spin domains. Each domain D_i contains N_i identical spin-1/2 particles. All spins have an energy $\hbar\omega_0$ with associated frequency $\omega_0/2\pi$. Two neighboring domains D_i and D_{i+1} are interacting via a common (zero-temperature) reservoir mediated by the dissipation rate γ_i. (b) Schematic representation of the *effective* excitation transfer from the first domain (with $N_1 = 3$ spins and total angular momentum $j_1 = 3/2$ and 1/2) to the second domain (with $N_2 = 2$ spins and total angular momentum $j_2 = 1$ and 0): initially, D_1 is fully excited while D_2 is in the ground state. This corresponds to the (partial) excitation of states with different total angular momentum (bottom row). Due to the collective decay, which preserves the total angular momentum, the system relaxes down the ladders and reaches a steady state where D_1 and D_2 are locally not in their respective ground states. This results in excitations arising in the second domain—even though it was initially in its ground state.

B initially in a state $|\psi\rangle = |1\rangle_A|0\rangle_B = |1\rangle|0\rangle$, which can also be expressed as

$$|\psi\rangle = \tfrac{1}{2}[|1\rangle|0\rangle + |0\rangle|1\rangle] + \tfrac{1}{2}[|1\rangle|0\rangle - |0\rangle|1\rangle]. \qquad (1)$$

Under collective decay to a zero-temperature bath, the first Bell state (a triplet state) decays to $|0\rangle|0\rangle$, while the second term (a dark state) remains unchanged. This means the mean population of spin B has increased from zero to $\bar{n}_B = 1/4$ via that collective coupling to the reservoir. We must emphasize that there is no direct coupling between the spins, meaning we are not seeing simple energy transfer. Further, those spins only collectively couple to a zero-temperature reservoir, meaning energy is not being given to the second spin from it. This is a quantum process associated with the collective decay breaking a symmetry in the system and the symmetries of the initial state. The triplet (or symmetric) part of the initial state can decay but the dark (antisymmetric) part cannot. While this behavior can be seen in two spins [45], similar behavior can be seen with two ensembles collectively coupled to the environment [40,41,46]. We would like to highlight that this process is different from energy transfer in the traditional sense. Instead, we call this energy migration to distinguish it.

In this Letter, we will build upon Refs. [40,41] and extend these results to multiple nodes. Additionally, by utilizing the well-known phenomena of superradiance and superabsorption [38], we show that our system can facilitate energy migration as fast as needed between the multiple nodes. Further, we will show that this energy migration can proceed even if the reservoirs are at zero temperature. We will discuss how such a technique can be used to migrate energy around small networks where each node is an ensemble of spins that are collectively coupled to an environment.

Our model. Let us begin with a simple mathematical model of our system which extends a double domain system [40,41] to the multiple spin domain regime. Our system depicted in Fig. 1(a) consists of M different noninteracting spin domains D_i, each containing N_i identical spin-1/2 particles (with frequency $\omega_0/2\pi$). Pairwise, these domains are collectively coupled to a zero-temperature reservoir. These reservoirs are modeled as a collection of bosonic modes with frequencies $\omega_{k_i}/2\pi$ and bosonic creation (annihilation) operators $a_{k_i}^\dagger$ (a_{k_i}). Importantly, our system is symmetric under the exchange of any two spins within each domain, but not within the overall system. Therefore, it is useful to define collective spin operators for the ith domain $J_i^\alpha = \sum_{n_i=1}^{N_i} S_{n_i}^\alpha$, with $\alpha = x, y, z$ and where $S_{n_i}^\alpha$ are the n_ith spin operators. Further the ith domain raising and lower operators are given by $J_i^\pm = J_i^x \pm iJ_i^y$. The Hamiltonian of the total system with M ensembles and $M - 1$ reservoirs is

$$H = \hbar\omega_0 \sum_i^M J_i^z + \sum_i^{M-1} \sum_{k_i} \hbar\omega_{k_i} a_{k_i}^\dagger a_{k_i} + \sum_i^{M-1} \sum_{k_i} [t_{k_i}(J_i^+ + J_{i+1}^+) a_{k_i} + t_{k_i}^* a_{k_i}^\dagger (J_i^- + J_{i+1}^-)], \qquad (2)$$

where the first and second term represents the Hamiltonian of the spin ensembles and the bosonic reservoirs, respectively. The third term is the interaction of the spin ensemble i and $i + 1$ with their common reservoir (labeled as i), where t_{k_i}, $t_{k_i}^*$ represent emission (absorption) amplitudes that fix the spectral density of the reservoirs, $\Gamma_i(\omega) = 2\pi \sum_{k_i} |t_{k_i}|^2 \delta(\omega - \omega_{k_i})$. Within the standard weak-coupling approach (Born-Markov approximation) and assuming zero-temperature reservoirs, the Lindblad master equation of the system can be written in the rotating frame as [14,47]

$$\dot{\rho}_s = \sum_i^{M-1} \frac{\gamma_i}{2} \mathcal{D}[J_i^- + J_{i+1}^-]\rho_s, \qquad (3)$$

where the Lindblad term is $\mathcal{D}[O]\rho = 2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O$ for any operator O. Additionally, the weak-coupling regime necessarily assumes ω_0/γ is very large. The dissipative coupling between the different spin ensembles is mediated via the rates $\gamma_i = \Gamma_i(\omega_0) = \alpha_i\omega_0$ in the wide band limit, where α_i is constant. Previous works have used dissipative coupling to induce frustration [48] and quantum synchronization of oscillators [49] and atomic ensembles [50]. The work in Ref. [43] experimentally demonstrated a double spin domain system implemented via two distinct nitrogen-vacancy ensembles in diamonds collectively coupled

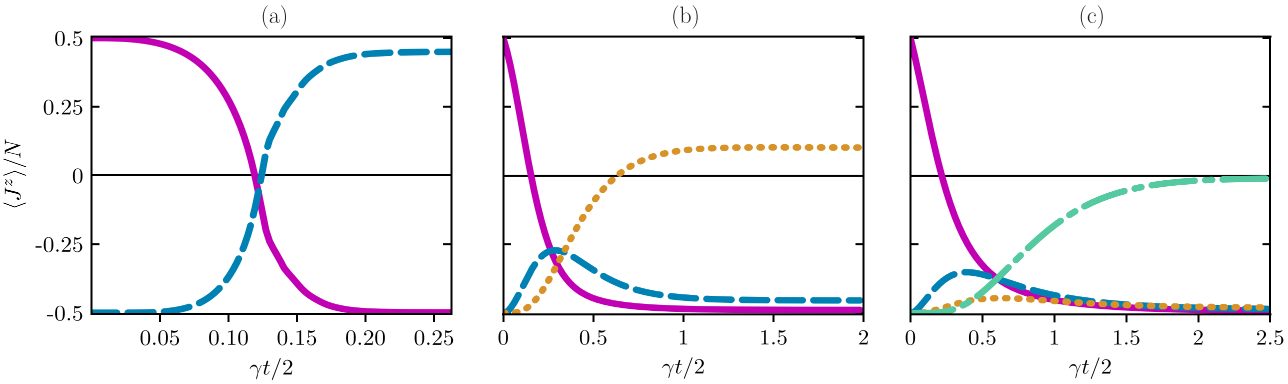

FIG. 2. Normalized collective spin relaxation $\langle J_i^z \rangle / N_i$ with dynamics governed by the master equation (3). (a) Two spin domains with $N_1 = 40$ (solid magenta), $N_2 = 2$ (dashed blue); (b) three domains with $N_1 = 12$ (solid magenta), $N_2 = 6$ (dashed blue), $N_3 = 2$ (dotted gold); (c) four domains with $N_1 = 8$ (solid magenta), $N_2 = 6$ (dashed blue), $N_3 = 5$ (dotted gold), and $N_4 = 1$ (dot-dashed mint green).

to a resonator; potentially an extension of this work could see the realization of the multiple spin domain regime.

System dynamics. As we are mostly interested in excitation migration through the different spin domains, our results will focus on the situations where the first domain is initialized with all spins in their excited states while all spins in subsequent domains begin in the ground state. Our initial state can be expressed as

$$|\psi_{\text{in}}\rangle = |\uparrow \ldots \uparrow\rangle_1 \otimes |\downarrow \ldots \downarrow\rangle_2 \otimes \cdots \otimes |\downarrow \ldots \downarrow\rangle_M . \quad (4)$$

As observed in [41] for the two-domain case, this initial state (4) is not symmetric under the exchange of the ensembles 1 and 2. It is worth noting that the dissipative terms in the master equation (3) induce correlations between neighboring domains i and $i+1$ because it describes their collective decay. The initial state can be decomposed as a superposition of symmetric and antisymmetric states. As a consequence of the decay of the symmetric subspace components, the average number of excitations stays finite at the steady state despite the presence of the zero-temperature reservoir [see Fig. 1(b)]. Furthermore, the system may relax into a steady state, where the second domain population of spins in the excited state is greater than 50%. However, an unbalanced configuration of domain sizes—specifically, $N_1 \gg N_2$—is necessary for this situation to occur. For this reason, we restrict ourselves to the unbalanced configurations where $N_1 > N_2 > \cdots > N_M$ and explore the dynamics of excitation migration from the first to the last domain.

We are now in the position to explore the dynamics of the dissipatively coupled spin ensembles. The master equation (3) represents a series of coupled differential equations which can be solved in conjunction with the initial conditions (4). The only parameters governing the dynamics of our collective spin relaxation are the domain size N_i and the reservoir coupling γ_i. For simplicity, we will set all reservoir couplings to be equal and all results presented in this work use a time axis that is rescaled with $\gamma/2$. In Figs. 2–4, we show the collective spin relaxation of a system with two, three, and four domains with initial state $|\psi_{\text{in}}\rangle$ given by (4). For the two-domain system [Fig. 2], we set $N_1 = 40$ (magenta) and $N_2 = 2$ (blue), while for the three-domain system [Fig. 3], we have $N_1 = 12$ (magenta), $N_2 = 6$ (blue), and $N_3 = 2$ (gold). Similarly, for the four-domain system [Fig. 4], we have $N_1 = 8$ (magenta), $N_2 = 6$ (blue), $N_3 = 5$ (gold), and $N_4 = 1$ (mint green). Since we solve Eq. (3) numerically, we are restricted to rather small ensemble sizes, especially as the number of spin domains increases. Nevertheless, small systems provide valuable insights into the general dynamics and allow conclusions to be drawn.

It can be seen in Fig. 2 that the decay of the first domain first leads to the excitation of the second domain as a result of the reservoir-mediated interaction between the two domains. In the case of only two domains [Fig. 2], the dynamics comes to a halt and the system reaches a steady state. Due to the large imbalance $N_1 > N_2$, the first domain (magenta) is (almost) completely deexcited and the second domain (blue) gets close to the fully excited state [41]. For more than two domains, the second domain (at a slower rate) also decays due to the additional dissipation channel and the ensemble excitation is transferred to the third domain [Fig. 3]. This process will continue until the last (smallest) domain absorbs the excitation and the system finally reaches its steady-state solution

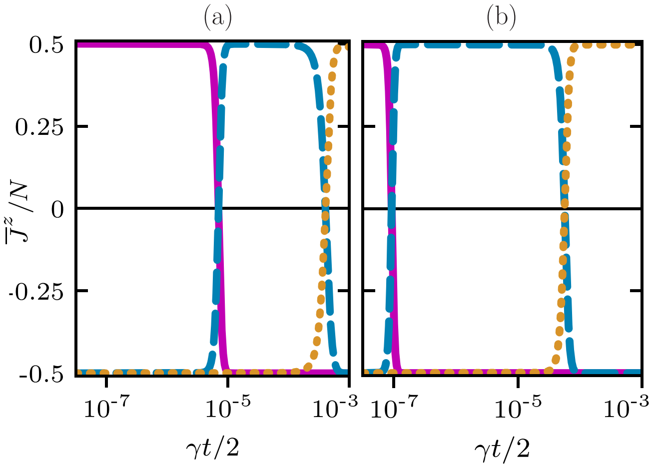

FIG. 3. Normalized collective spin relaxation $\langle J_i^z \rangle / N_i$ for three domains using mean-field dynamics. (a) $N_1 = 10^6$ (solid magenta), $N_2 = 10^4$ (dashed blue), and $N_3 = 10^2$ (dotted gold). (b) $N_1 = 10^8$ (solid magenta), $N_2 = 10^5$ (dashed blue), and $N_3 = 10^2$ (dotted gold).

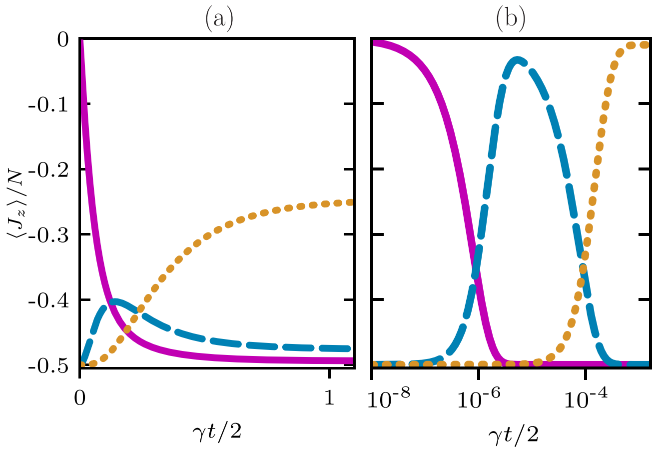

FIG. 4. Collective spin relaxation $\langle J_i^z \rangle / N_i$ for an initial state of the first domain with $\langle J_1^z \rangle$ $(t = 0) = 0$ showing that the relative population of the later domains never exceeds $\langle J_{2,3}^z \rangle = 0$. (a) The master equation approach with $N_1 = 12$ (solid magenta), $N_2 = 6$ (dashed blue), and $N_3 = 2$ (dotted gold). (b) The mean-field approach with $N_1 = 10^6$ (solid magenta), $N_2 = 10^4$ (dashed blue), and $N_3 = 10^2$ (dotted gold).

where the smallest ensemble is excited. For the system sizes considered here, the intermediate domain population of spins in the excited state stays below 50% and the average number of excitations in the last domain is considerably less than the ensemble size. However, the results of the two-domain system depicted in Fig. 2 indicate that larger ensemble sizes may allow the fully excited initial state to dissipatively migrate from the first domain along the chain to the last domain.

The dynamics that we are able to access from numerically solving Eq. (3) are intriguing; however, due to the scaling of the Hilbert space with system size, we are limited in the number of spins. In order to investigate larger domain sizes, we perform a mean-field (MF) approximation by factorizing different moments (see Supplemental Material [51]). In the following, we consider the dynamics of three dissipatively coupled spin domains which is described by a closed set of 13 coupled differential equations for the expectation values of the collective spin operators, which we denote by an overbar to emphasize their MF character.

In Fig. 3, we show the collective spin relaxation according to the MF description for two different ensemble sizes of the first and second domain. In Fig. 3(a), we set $N_1 = 10^6$ (magenta), $N_2 = 10^4$ (blue), and $N_3 = 10^2$ (gold), and in Fig. 3(b), $N_1 = 10^8$ (magenta), $N_2 = 10^5$ (blue), and $N_3 = 10^2$ (gold). Unlike the smaller system sizes shown in Fig. 2, each domain differs substantially in the number of spins from the previous domain. Consequently, we are able to witness (almost) full excitation of the second and third domains, with D_3 relaxing to a steady state of $\bar{J}_3^z / N_3 \approx 0.49$. This is in contrast to the results of Fig. 3, where $\langle J_2^z \rangle / N_2$ and $\langle J_3^z \rangle / N_3$ clearly do not reach the maximum of $\langle J_1^z \rangle / N_1 (t = 0) = 1/2$.

Additionally, the discrete migration of excitation from one spin domain to the next can clearly be witnessed (notice the logarithmic timescale), in contrast to the collective spin relaxations of Fig. 2 where all domains absorb and decay on the same timescale. This is a clear signature of the superradiant decay and thus superradiant excitation transfer, which scales as $1/N_i$ for large system sizes. From the dynamics shown in Fig. 3, we note that the superradiant decay time of D_1 as well as the superradiant absorption time of D_2 are mostly governed by the size of D_1, and thus occur at a time that is orders of magnitude before the superradiant decay of D_2 and absorption of D_3. Therefore, efficient migration in this dissipatively coupled system occurs when the spin population of the initial excited domain (N_1) is sufficiently larger than the final domain (in this case, N_3).

So far, we have explored the superradiant migration of excitations when the first domain is initially fully excited and all subsequent domains are in their respective ground state. However, one may assume that a partially excited initial state is sufficient to fully excite the last domain as the number of spins within each domain decreases along the chain. In the following, we show that this is, in fact, not the case and, moreover, that the maximum relative excitation transferable from one domain to the other is bounded by the initial relative excitation.

Let us start by numerically exploring the three-domain system, where small and large system sizes can be investigated. We first study the effect of initial conditions on the excitation transfer described by the master equation (3) for domain sizes $N_1 = 12, N_2 = 6$, and $N_3 = 2$. As $N_2 = N_1/2$, we choose as the initial state the first domain to be only half excited ($\langle J_1^z \rangle (t = 0) = 0$); however, the second and third domains are to be in their respective ground state. In Fig. 4, one sees that for this initial configuration, the second (blue) and third (gold) domains are both less excited compared to the initial state $|\text{is}\rangle$ [cf. Fig. 3], and, especially, the third domain is considerably below half excited. In contrast, in Fig. 4, the spin relaxation dynamics is shown for $N_1 = 10^6$, $N_2 = 10^4$, and $N_3 = 10^2$ [same system sizes as Fig. 3(a)] with half of the spins in D_1 initialized in the excited state and half in the ground state. Here, we make use of the MF equations to solve the dynamics. Even though the number of spins in the second and third domains is significantly less than the number of initially excited spins in the first domain, both $\bar{J}_2^z / N_2$ and $\bar{J}_3^z / N_3$ always remain below the value of $\bar{J}_1^z / N_1 (t = 0)$. Interestingly, this occurs for any value of N_1 and any proportion of excited spins in the initial state. That is, we observe $\bar{J}_1^z / N_1 (t = 0) \geqslant \max(\bar{J}_2^z / N_2)$ and $\bar{J}_1^z / N_1 (t = 0) \geqslant \max(\bar{J}_3^z / N_3)$. This already suggests that the initial population of spins in the excited state limits the transferable amount of excitations.

The results we have observed so far for the three-domain case also hold for the case of only two domains. That is, the maximum relative excitation of the second domain is bounded by the initial relative excitation of the first domain. In fact, as we show in the Supplemental Material [51], for the two-domain case with $N_1 \gg N_2$, the steady state of the second domain is given by

$$\frac{\bar{J}_2^z}{N_2}(t \to \infty) \approx \frac{\bar{J}_1^z}{N_1}(t = 0). \tag{5}$$

As we discussed in the previous section, for superradiant excitation transfer to occur, we need large differences in the number of spins within each domain. This results in a timescale separation of transfer between the first and second domains, and transfer between the second and third domains. Because of this timescale separation, the second domain reaches its maximum relative excitation before transport to the third domain takes place. We thus conclude from Eq. (5) that the maximum relative excitation of the last domain is bounded by the initial state, i.e., $\bar{J}^z_M/N_M(t \to \infty) \approx \bar{J}^z_1/N_1(t = 0)$. This has implications for quantum thermodynamics and especially the charging of quantum batteries [52].

Discussion. It is well established that movement or transfer of energy around physical system is a primitive operation with applications in many diverse fields. We are always looking for new ways to achieve this in faster and more efficient ways. In this Letter, we have shown an energy migration approach in a small scale quantum network based on collective coupling to a reservoir. Energy is not flowing from node to node. Instead, our initial state is not symmetric with respect to the collective coupling to the reservoirs and so different parts of the quantum wave function decay at different rates (or not at all). This results in populations arising in nodes which were initially unoccupied. Combining this behavior with superradiant decay and absorption, we show the apparent flow of energy from node to node in the network. In exploring the dynamics of energy migration in the network, we were able to find the conditions which facilitate the fastest and most efficient energy transfer. By tailoring the system and environment, and symmetries associated with them, our approach can illustrate different directions for the future of quantum technologies.

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