Letter

Suppression of heating by long-range interactions in periodically driven spin chains

Devendra Singh Bhakuni ,[1] Lea F. Santos ,[2] and Yevgeny Bar Lev [1,*]
[1]Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel
[2]Department of Physics, Yeshiva University, New York, New York 10016, USA

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We propose a mechanism to suppress heating in periodically driven many-body quantum systems by employing sufficiently long-range interactions and experimentally relevant initial conditions. The mechanism is robust to local perturbations and does *not* rely on disorder or high driving frequencies. Instead, it makes use of an approximate fragmentation of the many-body spectrum of the nondriven system into bands, with band gaps that grow with the system size. We show that when these systems are driven, there is a regime where *decreasing* the driving frequency *decreases* heating and entanglement buildup. This is demonstrated numerically for a prototypical system of spins in one dimension, but the results can be readily generalized to higher dimensions.

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Periodically driven quantum systems continue to produce fascinating physics and phenomena inaccessible to their static counterparts. Some notable examples include the Kapitza pendulum [1], dynamical localization [2–4], Floquet topological insulators [5–7], dynamical phase transitions [8], induced many-body localization (MBL) [9–13], and Floquet time crystals [14–18]. However, a key obstacle to realizing new phases of matter in driven systems is that typically the drive heats up the system to a featureless infinite-temperature state where all correlations and observables become trivial [19–22].

In one-dimensional systems, heating can be suppressed with the inclusion of sufficiently strong disorder, which leads to the formation of the Floquet-MBL phase [13,20,23–25]. Alternatively, heating can be suppressed at any dimension, whether the system is clean or disordered, by considering driving frequencies greater than the single-particle excitation energy, such that the absorption of a photon from the drive will always result in a multiparticle process [26–32]. Under these conditions the system will spend a significant amount of time in a nontrivial metastable state—a phenomenon called Floquet prethermalization [26–32]. It has been recently demonstrated with nuclear spins using nuclear magnetic resonance techniques [33] and with ultracold atoms in a driven optical lattice [34].

If the driving frequencies are smaller than the single-particle excitation energy, the system can efficiently absorb energy from the drive, which results in fast heating to infinite temperature [35]. But is this the fate of all driven quantum systems? In this Letter, we show that the answer is negative. Heating can actually be suppressed in any dimension and for frequencies smaller than the single-particle excitation if the system has sufficiently long-range interactions.

The physics of nondriven systems with power-law decaying interactions, $r^{-\alpha}$ (where r is the distance between two bodies), has gained considerable attention due to experimental realizations in trapped ions [36–40], where the range of the interactions can be tuned. A particularly intriguing regime is $\alpha < d$ (d being the dimension of the system), where conventional thermodynamics does not apply [41]. Power-law decaying interactions occur in various systems, from spin glasses and magnetically frustrated systems to atomic, molecular, and optical systems [42–46]. They are associated with phenomena that are absent for neighboring interactions [47–54]. They are known to affect transport [55–61], destroy many-body localization [62–68], and facilitate the propagation of correlations [53,54,69–71].

While for $\alpha > d$, the physics is many times only quantitatively different from the physics of systems with local interactions ($\alpha \to \infty$), novel physics often emerges for slowly decaying interactions, $\alpha < d$. An example is the emergence of a Hilbert space fragmentation into weakly connected subspaces. If the dynamics starts in one of these subspaces, it can be effectively described by a local Hamiltonian for a long time [72,73], so despite the presence of long-range interactions, features that are usually associated with short-range interactions may be observed, such as the logarithmic growth of entanglement [52,74], light-cone evolution [72,75], and self-trapping [76]. On the other hand, if the initial state spans multiple subspaces, the dynamics violates the generalized Lieb-Robinson bound and leads to the instantaneous spread of correlations [36–38,54].

The behavior of periodically driven systems with power-law decaying interactions was studied in Refs. [77–80]. For $\alpha > d$ and large driving frequencies, exponentially slow heating and the emergence of Floquet prethermalization were obtained [26–29,78]. In this prethermal regime, a novel nonequilibrium phase of matter dubbed the prethermal time crystal [79], which is similar to the MBL-time crystal [14–16], has been argued to exist. For $\alpha < d$, the general expectation is that to achieve a prethermal plateau, the system needs both to be in the *high-frequency regime* and to be finite. The second condition arises since the single-particle excitation energy

*ybarlev@bgu.ac.il

increases with system size and therefore for fixed frequency, the prethermal plateau shrinks as the system size increases [78].

In this Letter, we show that it is in fact possible to suppress heating in systems with long-range interactions in the *low-frequency regime*, where the driving frequencies are smaller than the single-particle excitation energy. This can be done by taking advantage of the effective fragmentation of the Hilbert space, which is induced by interactions with $\alpha < d$, and by selecting initial states within one of those approximate subspaces, such that the energy absorption from the drive becomes ineffective. In this way, we can achieve prethermal phases whose lifetimes grow as the system size increases and which are viable at any dimension. We demonstrate this behavior by numerically examining the dynamics of the half-chain entanglement entropy and the energy absorption in a spin chain with $\alpha < 1$.

Model. We consider a long-range interacting spin chain of length L described by the Hamiltonian,

$$\hat{H}_0 = J_z\hat{V} + J_x\sum_{\langle i,j\rangle}^{L-1}\hat{\sigma}_i^x\hat{\sigma}_j^x + h_x\sum_{i=1}^{L}\hat{\sigma}_i^x,$$

$$\hat{V} = \sum_{i<j}^{L-1}\frac{1}{|i-j|^\alpha}\hat{\sigma}_i^z\hat{\sigma}_j^z, \tag{1}$$

where $\hat{\sigma}_i^{x,y,z}$ are Pauli operators, J_z is the strength of the long-range term $\hat{V}$ and we set $J_z = 1$, J_x corresponds to the strength of nearest-neighbor interactions in the x direction, and h_x is the amplitude of a transverse magnetic field. The operator norm of the long-range term is $\|\hat{V}\| \sim L^{2-\alpha}$ for $\alpha < 1$, such that it becomes dominant in the thermodynamic limit. Nevertheless, even in this limit, the dynamics is *not* given by $\hat{V}$ for almost all initial states, and is highly nontrivial, since the model stays nonintegrable for any value of α. While one can make $\hat{V}$ extensive by proper rescaling [81–83], this rescaling does not naturally occur in the experiments, where finite systems are studied [17,36,37,84]. We therefore do not consider this rescaling in our work.

The static Hamiltonian $\hat{H}_0$ is periodically driven by the following time-dependent perturbation,

$$\hat{H}_1(t) = \text{sgn}[\sin(\omega t)]\left(h_y\sum_{i=1}^{L}\hat{\sigma}_i^y + h_z\sum_{i=1}^{L}\hat{\sigma}_i^z\right), \tag{2}$$

such that the total Hamiltonian is $\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t)$. Here, $\omega = 2\pi/T$ is the driving frequency, sgn$(\cdot)$ is the sign function, T is the driving period, and h_y and h_z are the magnitudes of the magnetic fields along the y and z directions, respectively. We use a square-wave driving to closely follow the experiment with trapped ions in Ref. [40], and also since it is computationally more efficient than a continuous time-varying drive. However, the results presented here should be insensitive to the choice of the driving protocol. We explore the dynamics of the driven system, $\hat{H}(t)$, with $\alpha < 1$.

To study the heating dynamics, we use the numerically exact Krylov subspace techniques to evolve the system in time [85]. Due to the lack of symmetries, we have to consider the entire Hilbert space of dimension 2^L, so we analyze system

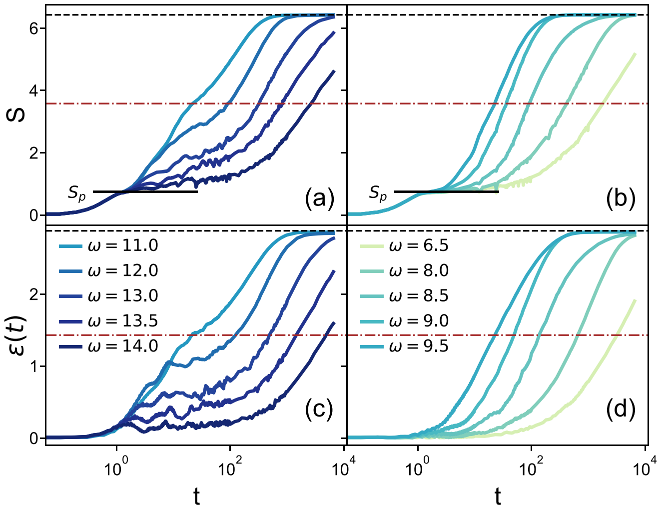

FIG. 1. Dynamics of (a), (b) the half-chain entanglement entropy and (c), (d) the energy absorption, for different ranges of the driving frequencies ω. The infinite-temperature values are marked by horizontal black dashed lines and the prethermal values of $S(t)$ by solid horizontal lines. The dotted-dashed red lines mark the heating time, where the entropy (energy) reaches the halfway mark between its plateau value (initial value) and its infinite-temperature value. The initial state is $|\psi(0)\rangle = |11\cdots11011\cdots11\rangle$, $L = 20$, $\alpha = 0.67$, $J_x = 0.69$, $h_x = 0.23$, $h_y = 0.21$, and $h_z = 0.19$. For these parameters, $J_{\text{eff}} = \Delta_1 = 10.92$.

sizes up to $L = 22$. We investigate the energy density of the static system measured with respect to the initial state,

$$\varepsilon(t) \equiv \frac{1}{L}\text{Tr}\,[[\hat{\rho}(0) - \hat{\rho}(t)]\hat{H}_0], \tag{3}$$

where $\hat{\rho}(t)$ is the density matrix as a function of time, and the half-chain entanglement entropy,

$$S(t) = -\text{Tr}[\hat{\rho}_A(t)\ln\hat{\rho}_A(t)], \tag{4}$$

where $\hat{\rho}_A(t) = \text{Tr}_B\,\hat{\rho}(t)$ is the reduced density matrix of the subsystem A consisting of $L/2$ spins.

Heating suppression. Figure 1 shows the evolution with time of the entanglement entropy and the energy density for $L = 20$, different frequencies, and the initial state $|\psi(0)\rangle = |11\cdots11011\cdots11\rangle$, where all the spins, except the one in the middle, point up. For most frequencies in Fig. 1, the entanglement entropy exhibits three distinct regimes: an initial growth for a short time, which is followed by the emergence of a long-lived prethermal state (Floquet prethermalization), where $S(t)$ saturates to a plateau value S_p (horizontal black solid line), after which the entropy finally reaches an infinite-temperature value (black dashed line) corresponding to the result by Page, $S_{\text{Page}} = (L\ln 2 - 1)/2$ [86]. The dependence of the behavior of the energy density on the frequency is comparable to that for the entropy, so it remains constant during the prethermal phase, and eventually goes to its infinite-temperature value at long times. Those distinct dynamical stages in Fig. 1 were observed before in Ref. [78], where high driving frequencies were considered and the dynamics started with initial product states in the z direction with an equivalent number of spins pointing up and down. But in stark contrast with

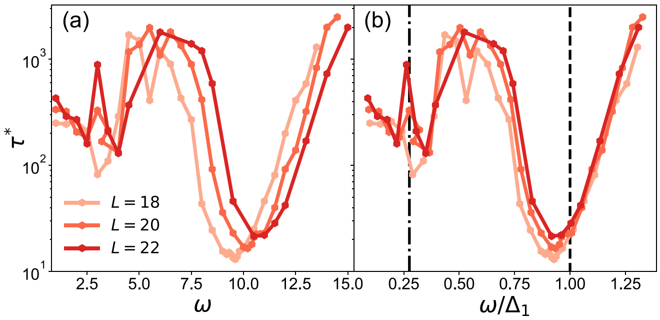

FIG. 2. Heating time τ^* as extracted from the entanglement entropy for different system sizes as a function of (a) frequency and (b) rescaled frequency, ω/Δ_1. The two vertical lines on the x axis in (b) indicate the gaps Δ_2 (dashed-dotted) and Δ_1 (dashed) for $L = 22$. The gap Δ_1 is equal to J_{eff} (see text). As in Fig. 1: initial state $|\psi(0)\rangle = |11\cdots11011\cdots11\rangle$, $\alpha = 0.67$, $J_x = 0.69$, $h_x = 0.23$, $h_y = 0.21$, and $h_z = 0.19$.

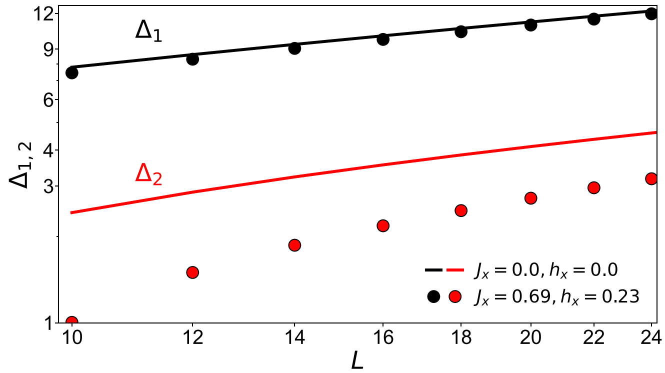

FIG. 3. The lowest energy gaps Δ_1 (black lines and circles) and Δ_2 (red lines and circles) as a function of L. The solid lines indicate the exact calculations for $J_x = h_x = 0$ [Eqs. (6) and (7)] and the circles are the numerical results for $J_x = 0.69$ and $h_x = 0.23$.

previous studies, we find that *below* a certain frequency value, we can extend the prethermal phase and postpone heating by *decreasing* the driving frequency, as shown in Figs. 1(b) and 1(d). Contrary to past studies for which the heating time increases monotonically with the frequency, we have now a nonmonotonic dependence. For frequencies $\omega \gtrsim 11$, the heating time grows as ω increases [Figs. 1(a) and 1(c)], but for a range of frequencies with $\omega < 11$, the heating time shrinks as ω increases [Figs. 1(b) and 1(d)]. For frequencies close to $\omega \sim 11$, the system heats up very quickly, hinting at a resonant behavior.

To show the frequency dependence more explicitly, we define the heating time τ^* as the time when the entanglement entropy reaches a halfway mark between its prethermal plateau and its asymptotic value, $S(\tau^*) \equiv S_p + [S_{\text{Page}} - S_p]/2$, which is indicated with dotted-dashed red lines in Figs. 1(a) and 1(b). We see in Fig. 2(a) that, as expected, for $\omega > 11$ the heating time increases as we increase the driving frequency, however, within a range of values for $\omega < 11$, the heating time increases as we decrease ω and these results further improve as the system size grows. A similar qualitative picture is obtained also for the heating time calculated from the energy density $\varepsilon(t)$. As we explain next, this unusual dependence on the frequency is a consequence of the effective fragmentation of the Hilbert space verified for the nondriven system when $\alpha < 1$ [72].

Energy bands. To better understand the Hilbert space fragmentation of the static system, let us first examine the long-range term $\hat{V}$ of $\hat{H}_0$ [Eq. (1)], which for $\alpha = 0$ can be written in terms of the collective spin operator $\hat{M}_z = \sum_i^L \hat{\sigma}_i^z/2$ as $\hat{V} = 2\hat{M}_z^2 - L/2$. The energy spectrum of $\hat{V}$ consists of degenerate bands with the energies

$$E_b = 2\left(\frac{L}{2} - b\right)^2 - \frac{L}{2}, \quad b = 0, 1, \ldots, \frac{L}{2}, \tag{5}$$

where b indicates the number of spins pointing down in the z direction, and we designate the corresponding energy band as the band b. Since the energy of a product state with b down spins is equal to the energy of a state with $L - b$ down spins, each band is $2\binom{L}{b}$ degenerate for $b < L/2$. For $0 < \alpha < 1$, the degeneracy within each band of the spectrum of $\hat{V}$ is partially lifted, but the different subspaces are still separated in energy. We define the energy gap between two nearby bands as $\Delta_b \equiv E_b - E_{b-1}$, which can be obtained analytically. The gap between the bands $b = 0$ and $b = 1$ can be calculated as

$$\Delta_1 = \sum_{r=1}^{L-1} \frac{2}{r^\alpha} \sim \frac{2}{1-\alpha} L^{1-\alpha}. \tag{6}$$

One sees that the gap increases monotonically with system size for $\alpha < 1$. Similarly, we can obtain Δ_2,

$$\Delta_2 = \left(\sum_{r=2}^{L-1} \frac{2}{r^\alpha} + \sum_{r=1}^{L-2} \frac{2}{r^\alpha}\right) - 2\left[\sum_{r=1}^{L/2-1} \frac{2}{r^\alpha} + \left(\frac{2}{L}\right)^\alpha\right] \sim \frac{2(2-2^\alpha)}{1-\alpha} L^{1-\alpha}, \tag{7}$$

which also increases with the system size, although $\Delta_2 < \Delta_1$.

The other terms of the static Hamiltonian $\hat{H}_0$ couple the states of $\hat{V}$. The J_x term connects states within the same band and states from band b to bands $b \pm 2$, while the h_x term connects states of band b to bands $b \pm 1$. However, if the values of J_x and h_x are smaller than the gap between the bands, they cannot effectively couple them. Furthermore, the numerical calculations for the values of Δ_1 and Δ_2 for $\hat{H}_0$ with $J_x, h_x \neq 0$ approach the gaps between the bands of $\hat{V}$ in the limit $L \to \infty$, as shown in Fig. 3. This implies that the dynamics starting from an initial state within one band gets confined to that approximate subspace for a time that grows with the system size [72].

Resonant transition. The periodic driving of $\hat{H}_0$ tries to establish transitions between the different bands, but for this to happen efficiently it must deposit an amount of energy on par with the gap between the bands, $\omega \approx \Delta_b$. For the initial state considered in Fig. 1, the most relevant bands are $b = 0$, 1, and 2, with the corresponding gaps Δ_1 and Δ_2. To see the dependence of the heating time on the dominant gap more clearly, we rescale the driving frequency by the largest gap, Δ_1, as shown in Fig. 2(b). We see that the heating time τ^* reaches its smallest value when $\omega \approx \Delta_1$, because at this point we hit a resonant transition that leads to fast heating. This can

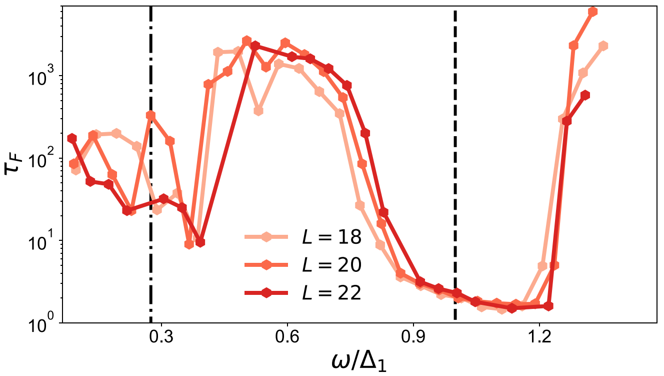

FIG. 4. Half-time decay of the fidelity τ_F as a function of the rescaled frequency ω/Δ_1. The two vertical lines indicate the gaps Δ_2 (dashed-dotted) and Δ_1 (dashed) for $L = 22$. As in Fig. 1: initial state $|\psi(0)\rangle = |11\cdots11011\cdots11\rangle$, $\alpha = 0.67$, $J_x = 0.69$, $h_x = 0.23$, $h_y = 0.21$, and $h_z = 0.19$.

be directly observed also in Fig. 1, where the fastest heating is indeed verified for $\omega \approx \Delta_1 \approx 11$. Another drop in the value of τ^* occurs when $\omega \approx \Delta_2$, which may be due to a multiple photon process. Since a square-wave drive contains multiple harmonics, higher-order transitions might occur. While such processes are suppressed at high enough frequencies, at very low frequencies, they could give rise to a nonmonotonic dependence of the thermalization time on the frequency as is indeed observed in Fig. 2. Next, we explain what causes the suppression of heating as the frequency increases above Δ_1 and, especially, when it decreases within the range $\Delta_2 < \omega < \Delta_1$.

Nonmonotonic frequency dependence. The maximum energy required to flip one spin for *any* initial state, scales as $J_{\rm eff} \equiv \sum_r r^{-\alpha} \sim L^{1-\alpha}$, with the system size. For the initial state considered above, $J_{\rm eff}$ coincides with the largest gap between the energy bands, $J_{\rm eff} = \Delta_1$. In the high-frequency regime, $\omega \gg J_{\rm eff}$, we expect slow heating, as indeed observed in Figs. 1(a) and 1(c). For $\omega < J_{\rm eff}$ one might have expected fast heating to occur, however, because $\Delta_1 = J_{\rm eff}$, one photon from the drive is not sufficient to induce a transition from the band $b = 1$ of the initial state to a neighboring band and the dynamics gets confined to the initial band for a long time, leading to heating *suppression* and the emergence of the prethermal phase in Figs. 1(a) and 1(c). In this case, *increasing* the frequency, $\omega \to \Delta_1$, *induces heating* due to the approach to the resonant condition. Therefore, heating suppression can be achieved by going away from the resonant frequency either by increasing [Figs. 1(a) and 1(c)] or decreasing [Figs. 1(b) and 1(d)] the driving frequency.

To demonstrate that for frequencies off resonance to the gap the dynamics is indeed confined for long times to the band of the initial state, we calculate the fidelity corresponding to the probability to find the evolved state within the initial band,

$$F_b(t) = {\rm Tr}[\hat{\rho}(t)\hat{P}_b], \quad \hat{P}_b = \sum_k |V_k^b\rangle\langle V_k^b|, \tag{8}$$

where $\hat{P}_b$ is the projector to the initial band spanned by the states $|V_k^b\rangle$. In Fig. 4, we plot the time τ_F that it takes for the fidelity to decay to half of its initial value for various frequencies and starting from an initial state in the band $b = 1$. We obtain a behavior very similar to that for the heating time τ^*: The fidelity decays fast for frequencies close to the gap value, $\omega \approx \Delta_1$, and as we move away from it, τ_F increases significantly. This corroborates our claim that the suppression of heating and the emergence of Floquet prethermalization, that we observe, are indeed a result of the confinement of the dynamics to the initial band.

Discussion. We demonstrate that in periodically driven spin systems with long-range interactions, heating can be strongly suppressed not only with driving frequencies larger than the energy it costs to flip a single spin, but also with frequencies smaller than that energy. This is due to the formation of energy bands in the many-body spectrum of the static system, which get further apart as the system size increases. If the system is initialized within one band and the drive is off resonance with the gap between the bands, then heating is significantly suppressed. This results in a nonmonotonic dependence of the heating time on the frequency. For frequencies larger than the gap, increasing the frequency suppresses heating, while for frequencies below the gap, *increasing* the frequencies *enhances* heating.

Our results therefore provide a robust way to suppress heating even for small driving frequencies, which can be tested in experiments with ion traps [36,40]. While in this Letter, due to numerical limitations, we have explored a one-dimensional system, our results should hold for any dimension, provided $\alpha < d$.

In the future, it would be interesting to see if constraining a long-range interacting system to a certain energy band allows us to obtain, at least a transient, time-crystalline behavior, which has been ruled out for $\alpha < d$ [79]. It would be also interesting to study the effect of aperiodic drives on heating in such systems [87,88].

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