Letter

Vacuum anomalous Hall effect in gyrotropic cavity

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We consider the ground state of an electron gas embedded in a quantum gyrotropic cavity. We show that the light-matter interaction leads to a nontrivial topology of the many-body electron-photon wave function characterized by a nonzero Berry curvature. Physically, this manifests as the anomalous Hall effect, appearance of equilibrium edge/surface currents, and orbital magnetization induced by vacuum fluctuations. Remarkably, closed analytical expressions for the anomalous Hall conductivity and macroscopic magnetization are obtained for the interacting many-body case.

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Recent advances in nanotechnology allowed for the increase in the effective light-matter coupling to the limit when the border between the condensed matter theory and quantum optics becomes completely blurred. Since the pioneering work of Hopfield in 1958 [1], it has been anticipated that the light-matter interaction would lead to the emergence of hybrid quasiparticles—polaritons, which inherit properties of both photonic and matter excitations. Since then, polaritons were studied in a vast majority of systems ranging from the single atoms and molecules to superconductors [2] and the emergent fields of cavity (CQED) and waveguide (WQED) quantum electrodynamics [3,4] explore the fundamental quantum properties of the polaritons as well as the novel quantum information processing protocols exploiting these structures.

Until recently, most of the WQED and CQED setups could be adequately described within the rotating wave approximation (RWA). Under RWA the light-matter coupling Hamiltonian contains only terms preserving the total number of excitations. As a consequence, the ground state of the system is comprised of a photonic vacuum and the material ground state and remains unaffected by the light-matter interaction. The RWA is applicable whenever the ratio of the characteristic energy of the light-matter coupling g and the photonic excitation energy Ω is negligible, $g/\Omega \ll 1$, and remains an absolutely adequate approximation in most of conventional cavity systems with the photonic excitations in the optical range $\Omega \sim 1\,\text{eV}$. However, for the last decade, a plethora of cavity designs where the ratio g/Ω could reach and even exceed 0.1 have been demonstrated for the optical [5], terahertz [6], and microwave [7]. The pioneering experiments boosted the interest in the so-called ultrastrong coupling (USC) regime of the light-matter interaction [8].

In the USC regime, the terms in the light-matter coupling Hamiltonian which do not preserve the total number of excitations can no longer be neglected. The ground state then becomes a mixture of the matter and photonic degrees of freedom and is characterized by nonzero values of the matter and photon occupation numbers. This may lead to substantial modifications of the material ground state and opens a new route to the versatile control over the material properties via the ultrastrong coupling with the cavity electromagnetic field vacuum fluctuations, which has been recently termed *cavity QED materials engineering* [9]. The emergent effects range from the modification of chemical reactions [10–16] to cavity-mediated superconductivity [17–20] and other cavity-mediated phase transitions [21–25]. The case of the spatially uniform vacuum field is particularly attractive since it allows for the exact analytical solutions for a wide class of problems. On the other hand, some cavity-mediated transitions, e.g., superradiance, are forbidden for the spatially uniform field [26] (while allowed for spatially varying cavity modes [27]). Moreover, there is still no definite answer as to whether cavity-mediated corrections to the ground state of a material are extensive quantities and thus can affect macroscopic observables. Recently [28], it was showed that this is not the case for a collection of N two-level atoms inside a single-mode cavity: the cavity-mediated corrections to macroscopic observables depend only on the effective coupling of a single two-level system to the cavity and thus vanish in the thermodynamic limit.

Here, we study an electron gas confined inside a gyrotropic (or chiral) cavity. The gyrotropy results in the energy splitting of the right- and left-circularly polarized modes. We show that vacuum fluctuations of an electromagnetic field in such a cavity induce the ground-state orbital magnetization of the electron gas, and derive the corresponding anomalous Hall conductivity of the system.

While our results are quite general, for definiteness, we consider a two-dimensional electron gas (2DEG) inside a Fabry-Perot cavity of width D as shown in Fig. 1. The interior of the cavity is vacuum $\varepsilon_0 = 1$ and the mirrors are modeled by half spaces of a ferromagnetic metal characterized the

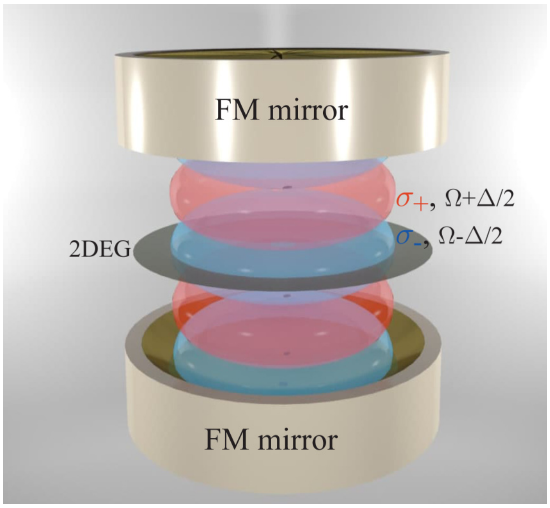

FIG. 1. Geometry of the structure. Two-dimensional electron gas is placed inside a Fabry-Perot cavity with ferromagnetic mirrors. The magnetization of the mirrors splits the energies of the circularly polarized cavity modes, which splitting induces the anomalous magnetization of 2DEG.

effective permittivity tensor $\hat{\varepsilon}$,

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & 4i\pi\sigma_H/\omega & 0 \\ -4i\pi\sigma_H/\omega & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}, \quad (1)$$

where $\varepsilon < 0$ is the diagonal permittivity. The off-diagonal conductivity σ_H is proportional to magnetization and responsible for the gyrotropy. We omit the frequency dispersion of ε and σ_H as well as material losses inside the cavity mirrors. The equation for photon eigenmodes in the cavity can be obtained using standard transfer matrix method [29] and reads

$$\tan(\omega D/2c) = \kappa_\pm, \quad (2)$$

where $\kappa_\pm = \sqrt{|\varepsilon| \pm \varepsilon_H}$ and $\varepsilon_H = 4\pi\sigma_H/\omega$. The modes corresponding to plus and minus sign are right- and left-circularly polarized modes, respectively. Equation (2) has an infinite number of solutions which correspond to the eigenfrequencies in the cavity. In the physically relevant case of $|\varepsilon| \gg 1$, the lowest eigenfrequencies are given by $\omega_\pm = \Omega_0 \pm 4\sigma_H/|\varepsilon|^{3/2}$, where $\Omega_0 = \frac{c}{D}(\pi - 2/\sqrt{|\varepsilon|})$.

The permittivity of Eq. (1) translates to the following expression for the energy of electro-magnetic field in the gyrotropic cavity: $E_{EM} = E_{EM}^{(0)} + \frac{1}{2c^2}\int d^3\mathbf{r}\sigma_H(z)[\dot{\mathbf{A}} \times \mathbf{A}]$, where $E_{EM}^{(0)}$ corresponds to the cavity with $\sigma_H = 0$. We then quantize the system in the basis of eigenstates related to $E_{EM}^{(0)}$, and truncate the basis to the two lowest-energy states, which are degenerate and characterized by orthogonal linear polarizations. The x- and y-components of the vector potential operator then read

$$\hat{A}_{x,y}(z) = \sqrt{\frac{\hbar c^2}{2V\Omega_0}}(\hat{a}_{x,y} + \hat{a}_{x,y}^\dagger)\phi(z) = \sqrt{\frac{\hbar c^2}{V\Omega_0}}\hat{q}_{x,y}\phi(z), \quad (3)$$

where $V = SD$ is the mode volume, S is the cavity area, and $\phi(z)$ is the normalized mode profile. Operators $\hat{a}_{x,y}$ are the conventional bosonic annihilation operators. By noticing that the time derivative of the vector potential $\dot{A}_{x,y}$ is proportional to the canonical momentum $\hat{\pi}_{x,y}$, which satisfies $[\hat{q}_i, \hat{\pi}_j] = i\delta_{ij}$, we obtain the following Hamiltonian of electromagnetic field in the gyrotropic cavity:

$$\begin{aligned} H_{\rm EM} &= \hbar\Omega_0(\hat{a}_x^\dagger\hat{a}_x + \hat{a}_y^\dagger\hat{a}_y + 1) + i\hbar\Delta(\hat{a}_x^\dagger\hat{a}_y - \hat{a}_y^\dagger a_x) \\ &= \frac{\hbar\Omega_0}{2}\hat{\boldsymbol{\pi}}^2 + \hbar\Delta\hat{\boldsymbol{\pi}}(\boldsymbol{e}_z \times \mathbf{q}) + \frac{\hbar\Omega_0}{2}\mathbf{q}^2, \end{aligned} \quad (4)$$

where in the limit of $|\varepsilon| \gg 1$ the gyration parameter $\Delta = 4\sigma_H/|\varepsilon|^{3/2}$. This limit corresponds to the small skin depth of the EM field in the mirrors, and since Δ is proportional to skin depth, in this limit $\Delta \ll \Omega_0$. This Hamiltonian yields two circularly polarized modes with energies reproducing the eigenfrequencies of the classical problem. Equation (4) shows that in a gyrotropic cavity the photons are mapped to the excitations of a harmonic oscillator rotating at the frequency Δ or, equivalently, of an oscillator subjected to an effective magnetic field $B_{\rm eff} = 2\Delta/\Omega_0$.

The Hamiltonian of the electron gas coupled to the cavity photons is given by

$$H_e = \sum_{i=1}^{N}\left[\frac{\left(\hat{\mathbf{p}}_i - \frac{e}{c}\hat{\mathbf{A}}\right)^2}{2m} + U(\mathbf{r}_i) + \frac{1}{2}\sum_{j\neq i} V_{\mathbf{r}_i - \mathbf{r}_j}\right], \quad (5)$$

where N is the number of electrons, U is the external confining potential, $V_{\mathbf{r}_i - \mathbf{r}_j}$ is the direct electron-electron interaction, and the light-matter interaction is described by a minimal coupling to $\hat{\mathbf{A}}$ of Eq. (3). The total electron-photon Hamiltonian is given by the sum $H = H_{EM} + H_e$.

In the following we concentrate on the ground state of a homogeneous 2DEG with a large area S, such that the density $n = N/S$ remains finite even in the limit $S, N \to \infty$. Bulk properties of such systems are customarily addressed by setting $U(\mathbf{r}) = 0$ and imposing periodic boundary conditions, thus making the problem formally translation invariant. Further, we assume, as usual, that the spatial dependence of the relevant electromagnetic mode functions can be omitted, which corresponds to the dipole approximation. After these simplifications it can be immediately noticed that the cavity field couples only to the center-of-mass (COM) degree of freedom of the electron gas. Moreover, the COM and the relative motions become separable. In is convenient to perform this separation in terms of the scaled COM coordinate $\mathbf{R} = \frac{1}{\sqrt{N}}\sum_i \mathbf{r_i}$ and its conjugate momentum, $\hat{\mathbf{P}} = \frac{1}{\sqrt{N}}\sum_i \hat{\mathbf{p}}_i$. The total Hamiltonian then reads

$$H = H_{\rm rel} + H_{\rm EM} + \frac{1}{2m}(\hat{\mathbf{P}} - g_0\sqrt{N}\hat{\mathbf{q}})^2, \quad (6)$$

where $g_0 = [\hbar e^2/(SD\Omega_0)]^{1/2}$ and $H_{\rm rel}$ describes the relative motion of electrons not affected by the electromagnetic (EM) field.

Let us study a parametric dependence of the many-body eigenstates on the COM momentum. This dependence can be formally introduced via the following unitary transformation, which is equivalent to the so-called twisted boundary conditions trick [30]:

$$H_{\mathbf{k}} = e^{-i\mathbf{kR}}He^{i\mathbf{kR}}. \quad (7)$$

Standard arguments [30,31] relate the Berry curvature $\mathcal{F}_{xy}$, associated to the ground state $|\Psi_{0,\mathbf{k}}\rangle$ of $H_{\mathbf{k}}$, to the anomalous

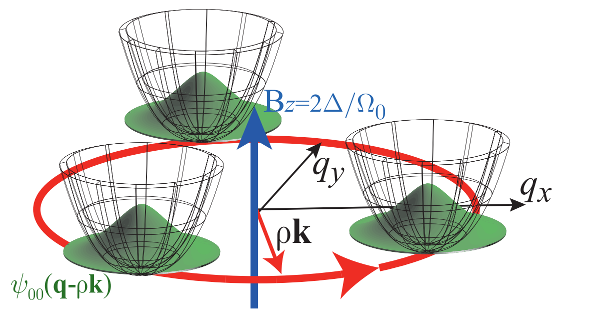

FIG. 2. The schematic image of the polaritonic states which are eigenfunctions of a shifted harmonic oscillator in the presence of the an effective magnetic field $B_{\rm eff} = \frac{2\Delta}{\Omega_0}$. When transported along a closed contour in **k**-space they accumulate a nontrivial Berry phase.

Hall conductivity σ_{xy} of 2DEG,

$$\sigma_{xy} = \frac{e^2}{\hbar} n \mathcal{F}_{xy}|_{\mathbf{k}=\mathbf{0}}, \tag{8}$$

where $\mathcal{F}_{xy} = 2{\rm Im}\langle \partial_{k_x}\Psi_{0,\mathbf{k}}|\partial_{k_y}\Psi_{0,\mathbf{k}}\rangle$. To proceed further we represent $H_\mathbf{k}$ of Eq. (7) explicitly as follows:

$$H_\mathbf{k} = H_{\rm rel} + H_{\rm EM} + \frac{1}{2m}(\hbar\mathbf{k} - g_0\sqrt{N}\mathbf{q})^2 = H_{\rm rel} + \frac{\hbar\Omega_0}{2}\left[\hat{\boldsymbol{\pi}} + \frac{\Delta}{\Omega_0}(\boldsymbol{e}_z \times \mathbf{q})\right]^2 + \frac{\hbar\Omega}{2}(\mathbf{q} - \rho\mathbf{k})^2 + \frac{\hbar^2\mathbf{k}^2}{2m^*}. \tag{9}$$

Here $\Omega = (\tilde{\Omega}_0^2 + \gamma^2)/\Omega_0$ with $\tilde{\Omega}_0^2 = \Omega_0^2 - \Delta^2$ and $\gamma^2 = \frac{g_0^2 N\Omega_0}{\hbar m}$, $m^* = m(1 + \gamma^2/\tilde{\Omega}_0^2)$ is the electron mass renormalized due to the electron-photon interaction, and ρ is given by

$$\rho = \frac{g_0\sqrt{N}}{m\Omega} = \sqrt{\frac{\hbar\Omega_0}{m}}\frac{\gamma}{\tilde{\Omega}_0^2 + \gamma^2}. \tag{10}$$

From Eq. (9) we see that $H_\mathbf{k}$ is nothing but the shifted Fock-Darwin Hamiltonian with a shift proportional to the COM momentum of electrons. This explains the origin of the Berry phase of the many-body polaritonic state. When **k** is moving along a contour enclosing a unit area in the **k**-space, the wave function is transported in the **q**-space along a contour enclosing the area ρ^2. The flux $\rho^2 B_{\rm eff}$ of the effective magnetic field $B_{\rm eff} = \frac{2\Delta}{\Omega_0}$ gives the Berry phase accumulated in this process, see Fig. 2.

More formally, eigenfunctions of $H_\mathbf{k}$ in Eq. (9) are the products $\Psi_\mathbf{k} = \Psi_{\rm rel}\Phi_\mathbf{k}(\mathbf{q})$ of the wave functions $\Psi_{\rm rel}$ for the relative motion, and the polaritonic states

$$\Phi_k(\mathbf{q}) = e^{-i\frac{\Delta}{\Omega_0}\rho[\mathbf{e}_z\times\mathbf{k}]\mathbf{q}}\psi_{n,l}(\mathbf{q} - \rho\mathbf{k}), \tag{11}$$

where $\psi_{n,l}(\mathbf{q})$ are eigenfunction of a two-dimensional (2D) harmonic oscillator (the ground state corresponds to $\{n = l = 0\}$). As the **k**-dependence of the wave function is known explicitly we can directly compute the Berry connection $\boldsymbol{\mathcal{A}}_\mathbf{k}$ as

$$\boldsymbol{\mathcal{A}}_\mathbf{k} = -i\int d\mathbf{q}\,\Phi_k^*(\mathbf{q})\nabla_\mathbf{k}\Phi_k(\mathbf{q}) = \frac{\Delta}{\Omega_0}\rho^2\,\mathbf{e}_z \times \mathbf{k}. \tag{12}$$

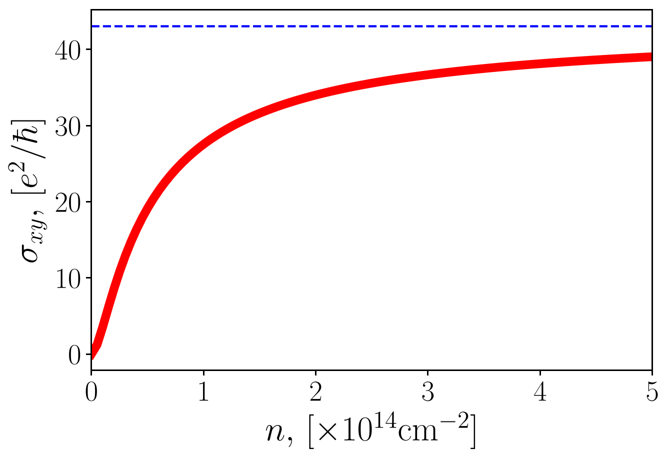

FIG. 3. Hall conductivity as a function of electronic concentration n. Blue dashed line shows asymptotics for large n

The associated Berry curvature $\mathcal{F}_{xy} = (\nabla_\mathbf{k} \times \boldsymbol{\mathcal{A}}_\mathbf{k})_z$ reads

$$\mathcal{F}_{xy} = \frac{2\Delta}{\Omega_0}\rho^2 = \frac{\hbar}{m}\frac{2\Delta\gamma^2}{\left(\tilde{\Omega}_0^2 + \gamma^2\right)^2}, \tag{13}$$

as expected. We note that in the thermodynamic limit $N, S \to \infty$ at $N/S \to n$, the coupling $\gamma \to \sqrt{e^2 n/mD}$ is the effective plasma frequency of electrons in the cavity. Since the Berry curvature does not depend on **k** the corresponding Hall conductivity of Eq. (8) is just

$$\sigma_{xy} = \frac{e^2 n}{m}\frac{2\gamma^2\Delta}{\left(\tilde{\Omega}_0^2 + \gamma^2\right)^2}. \tag{14}$$

In Fig. 3 we plot Eq. (14) as a function of electron concentration n for the specific case of $\Omega_0/2\pi = 1$THz, and $\Delta/\Omega_0 = 0.05$. For small n the conductivity grows quadratically and then approaches asymptotic value of $c(2\pi\Delta)/\Omega_0$ at large electron density. We note that the electron concentrations $\sim 10^{13}\,{\rm cm}^{-2}$ correspond to the Fermi energies of several tens of meV and can be routinely achieved in the state-of-the-art doped semiconducting quantum wells. As can be seen the value of the σ_{xy} depends strongly on the cavity gyrotropy parameter Δ/Ω_0.

We stress here that the electron Berry curvature emerges solely due to the electron-photon coupling. This effect is thus essentially different to the recently proposed quantum Hall effect for the graphene sheet in a magnetic cavity [32,33]. In the case of graphene, the electronic bands are initially characterized by the nonvanishing curvature and the role of the cavity is limited to the opening of the gap in the Dirac point.

The analysis is straightforwardly extended to the case of the infinite number of the cavity modes spatially uniform in the 2DEG plane [the details can be found in the Supplementary Material (SM)]. If the mode index is labeled by α with the corresponding γ_α, Δ_α, and Ω_α the expression for the conductivity reads [34]

$$\sigma_{xy} = \frac{e^2 n}{m}\sum_\alpha \frac{2\gamma_\alpha^2\Delta_\alpha}{\tilde{\Omega}_\alpha^4\left(1 + \sum_\beta \gamma_\beta^2/\tilde{\Omega}_\beta^2\right)^2}. \tag{15}$$

For a Fabry-Perot cavity, this expression results in

$$\sigma_{xy} = \frac{e^2 n}{m}\frac{\pi^4}{90}\frac{2\gamma_1^2\Delta_1}{\left(\tilde{\Omega}_1^2 + \frac{\pi^2}{6}\gamma_1^2\right)^2}, \tag{16}$$

where index 1 corresponds to the fundamental Fabry-Perot mode. As can be seen, inclusion of the multiple modes only weakly renormalizes the Hall conductivity. The Hall conductivity of 2DEG induced by the cavity modes can be rewritten as

$$\sigma_{xy}/c \approx \frac{\varepsilon_H}{|\varepsilon|^{\frac{3}{2}}}\frac{1}{\left(1 + \frac{\tilde{\Omega}^2}{\gamma^2}\right)^2}, \tag{17}$$

which directly relates it to the Hall permittivity of the mirrors. As can be seen, as $\tilde{\Omega}/\gamma$ vanishes, the denominator approaches a finite value, but at the same time, at $\Omega \to 0$, the Drude permittivity $|\varepsilon| \to \infty$ and thus in the dc limit, the effect vanishes.

It is natural to expect [30] that a nontrivial Berry curvature and the bulk anomalous Hall effect should be accompanied with the orbital magnetization of the sample. Indeed, by adopting the approach of [35–37] we obtain the following ground-state magnetization M_z:

$$M_z = -\frac{e}{mc}P_0\frac{2\gamma^2\Delta}{\left(\tilde{\Omega}_0^2 + \gamma^2\right)^2}, \tag{18}$$

where P_0 is the ground-state pressure of 2DEG [38]. The rigorous derivation is presented in SM [35], however, this result can be understood from the following simple argument. Because of the anomalous Hall effect, the gradient ∇U of the confining potential near the edges will generate the edge charge current. For example, for the boundary along the x-axis, assuming a sufficiently smooth edge potential, and applying the Hall relation locally, we get the edge current density

$$j_x = -\frac{1}{e}\sigma_{xy}\partial_y U = -\frac{e}{\hbar}\mathcal{F}_{xy}\, n\partial_y U = \frac{e}{\hbar}\mathcal{F}_{xy}\partial_y P_0, \tag{19}$$

where we used the force balance condition $n\nabla U = -\nabla P_0$. By integrating the above expression across the boundary (from the interior to the exterior of the sample) we get the net edge current $I_{\text{edge}} = \frac{e}{\hbar}P_0\mathcal{F}_{xy}$, which produces the magnetization of Eq. (18).

It is worth emphasizing, that the cavity-induced correction to the ground state quantified by the Berry curvature, and the related observables, such as the anomalous Hall conductivity are extensive quantities, which at fixed plasma frequency $\gamma \sim g_0\sqrt{N}$ have finite values in the thermodynamics limit $\{N, S \to \infty\}$. This result is in stark contrast to the previously obtained thermodynamic observables of a gas of two-level systems in a cavity [28], where it was shown that under the same condition all cavity-induced corrections vanish in the limit $N \to \infty$.

We have seen from the above macroscopic consideration that the ground-state magnetization is produced by localized currents at the edges of a finite sample. Below we illustrate the appearance of these currents by considering a scattering of polaritonic states at the edge of a semi-infinite 2DEG that occupies a half plane $y \geqslant 0$.

We consider a one-electron plane-wave polariton falling on the boundary at $y = 0$. For the incident wave $\Psi_{in}(\mathbf{r}) = \Phi_{\mathbf{k}}(\mathbf{q})e^{i\mathbf{kr}}$, where $\Phi_{\mathbf{k}}(\mathbf{q})$ is defined by Eq. (11), the photonic part is assumed to be in its ground state $n_{in} = l_{in} = 0$. We then can obtain the expression for the polariton wave function for the scattered polariton from the boundary conditions and calculate the current as $j_x = \frac{e}{m}\langle\Psi|(\hbar k_x - \rho m\Omega q_x)|\Psi\rangle$. To compute the total current we integrate the single-particle current over the filled states, neglecting electron correlations induced by the coupling to the vacuum field. The approximate expression for the current density reads

$$j_x(y) = \frac{e}{\hbar}\epsilon_F n\mathcal{F}_{xy}\frac{4J_3(2k_F y)}{k_F y^2}, \tag{20}$$

where $J_3(x)$ is the third-order Bessel function of the first kind. The edge current $I_{\text{edge}} = \int dy J(y) = \frac{e}{\hbar}\epsilon_F n\mathcal{F}_{xy}$ then agrees with the result obtained via the many-body calculation of magnetization using periodic boundary conditions if we recall that the nonintercating pressure $P_0 \sim \epsilon_F n$. The expression for current density in the limit of weak coupling $\gamma \ll \Omega_0$ or equivalently $\alpha\epsilon_F \ll \hbar\Omega_0$ can be rewritten as

$$j(y) = nev_F\alpha\left(\frac{\epsilon_F}{\hbar\Omega}\right)^2\frac{\Delta}{\Omega_0}F(k_F y), \qquad F(x) = \frac{J_3(2x)}{x^2}, \tag{21}$$

where α is the fine-structure constant. As we can see the current oscillates and decays in the bulk of the structure as $(k_F y)^{-5/2}$.

We showed that the coupling of electrons to the vacuum electromagnetic field of the gyrotropic cavity induces the macroscopic orbital magnetization of the electron gas, and correspondingly, leads to the emergence of the anomalous Hall conductivity and the existence of edge currents at the boundaries of the sample. In our analysis we employed the translation invariance of the electronic subsystem. Apparently, the inevitable disorder-breaking translation invariance may affect the magnetization. Moreover, local inhomogeneities in the electron gas would couple the COM and the relative electron motion, and therefore the electron motion will also be influenced by the vacuum electromagnetic field. Quantitative estimation of the effect of disorder is an interesting problem for the future. Finally, it is worth mentioning that in a possible experiment, the gyrotropy in the mirrors is most naturally induced by using magnetic materials or applying an external magnetic field, which may generate the usual diamagnetic currents in the electron gas. While the full screening of the electron system from the external and/or stray field is a challenging task, the effect induced by the vacuum fluctuations can be extracted by studying the dependence on the cavity photon frequency.

To conclude, the extensive nature of the Berry curvature, the induced magnetization, and emergent edge currents indicate that the cavity engineering can be used to alter the macroscopic properties of the ground state of the low-dimensional electron systems, and thus further smears the boundaries between the fields of nanophotonics, quantum optics, and condensed matter theory.

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